AN EXPERIMENTAL STUDY DEALING WITH THE USE OF COLORED CHALK IN TEACHING PLANE GEOMETRY.

A THESIS

SUBMITTED TO THE DEPARTMENT OF EDUCATION AND THE GRADUATE COUNCIL OF THE KANSAS STATE TEACHERS COLLEGE OF EMPORIA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

BY

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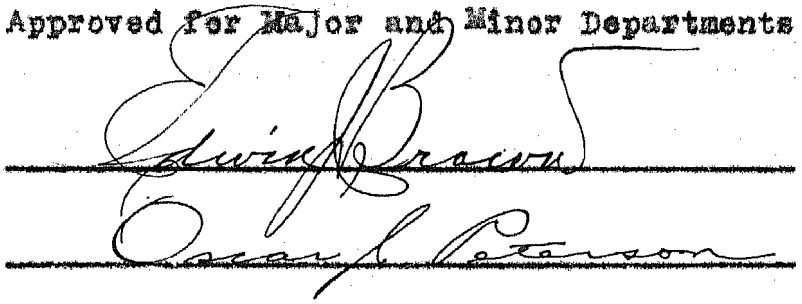
Approved for Major and Minor Departments



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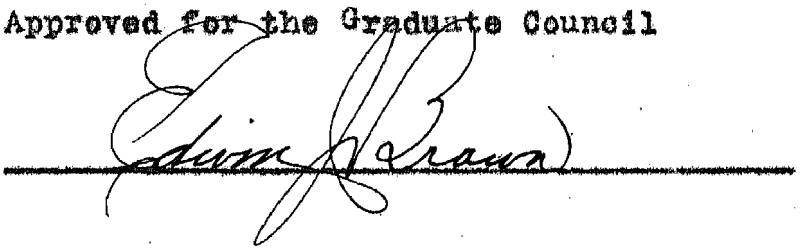

ACKNOWLEDGEMENT

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CHAPTER I.

INTRODUCTION

The value of using colored chalk in teaching plane geometry has been discussed in many of the books dealing with the subject of teaching mathematics. There seems to be nothing in the literature of an experimental nature. The opinions of some of the writers are favorable to its use.

David Eugene Smith[1] says:

> In the proof of the early propositions of plane geometry, and again at the beginning of solid geometry, there is a little advantage in using colored crayon to bring out more distinctly the equal parts of two figures, or the lines outside the plane or to differentiate one plane from another. This device, however, like that of models in solid geometry, can easily be abused and hence should be used sparingly and only until the purpose is accomplished. The student must learn to grasp the meaning of a figure drawn on white paper or white on a blackboard, and the sooner he is able to do this the better for him.

J. W. A. Young[2] states:

> The use of different colors to mark auxiliary parts, to distinguish parts given from those to be found and the like is very advantageous.

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[1] David Eugene Smith,- Teaching of Geometry, Ginn and Co., Boston, 1911, p.104.

[2] J. W. A. Young,- Teaching of Mathematics in the Elementary and Secondary Schools, Longmans, Green and Co., New York, 1924, p. 143.

Another opinion is that of Laura Blank;[3]

In the superposition proof the habit of drawing one figure with colored chalk whereas the other is drawn with white, then placing the colored one on the white, coloring the white lines, step by step as the various new positions are justified by the reasoning, is conducive to genuine understanding and tolerance for this type of demonstration, at best tedious. Colored chalk is invaluable in constructing auxiliary lines, added in the course of the proof, in contradistinction to those of the hypothesis, in such a proof as the one concerning angles the sides of which are perpendicular to each other, or the theorem concerning two triangles having two sides of one equal to two sides of the other but the included angle of one unequal, or the theorem concerning a series of parallel lines cutting off equal segments of one transversal, etc. The use of colored chalk to pick out from a complex figure a certain pair of parallel lines and a transversal will often remove a mass of confusion.

Still another opinion is th t of Arthur Schultze;[4]

Students who can reason logically sometimes forget the hypothesis or forget preceding parts of the proof, and hence are unable to continue. To make such forgetfulness almost impossible graphic methods may be employed. The hypothesis may be indicated by colors, equal colors representing equal lines or equal angles, arrows denoting parallel lines, a small colored square indicating a right angle. For the results obtained in the proof we use white chalk. In complex constructions various colors may be used to distinguish among different lines. It is impossible to mention all such cases and the resourceful teacher will have no difficulty in enlarging or modifying the above directions.

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[3] Laura Blank,- "Technique and Devices Conducive to Better Teaching of Geometry", MATHEMATICS TEACHER, Vol. XXI, March 1928, pp. 171-181.

[4] Arthur Schultze,- The Teaching of Mathematics in Secondary Schools, Macmillan Co., New York, 1912, pp 110-111.

Breslich[5] lists colored chalk among the materials needed to teach geometry effectively. Other writers such as Summers[6], and Smith and Reeves[7] make no mention of the use of colored chalk evidently thinking it in of no particular value in teaching geometry.

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5 Ernest R. Breslich,- The Teac ing of Secondary Mathematics, University of Chicago Press, Chicago, April 1930 p. 194.

6 S. Clayton Summers,- Supervised Study in Mathematics and Science, Macmillan Co., New York, 1922, 241 pages.

7 David Eugene Smith and William David Reeves, The Teaching of Junior High School Mathematics, Ginn and Co., Boston, 1927, 441 pages.

CHAPTER II.

METHOD OF PROCEDURE.

The data upon which this study was based were gathered in two ways. A letter of inquiry regarding the use of colored chalk in teaching plane geometry and asking the opinion of its value was sent to a number of high school teachers in the state of Kansas, and another letter of the same type was sent to mathematicians who have written textbooks on geometry. This was used merely to get the opinions of authorities on the subject.

The second method used was a comparison of an experimental class in which colored chalk was used and a control class in which only white chalk was used. Two classes, one meeting at eight-ten in the morning and the other at one-twenty-five in the afternoon were used. These classes were the regular classes in the Emporia, Kansas, City High School. The class period was fifty-five minutes long, thirty five for recitation and twenty for supervised study. The afternoon class was used as the experimental class.

The enrollment of the classes was twenty-six and twenty-seven respectively. From these, twenty-two pairs were obtained. The students were paired on four points: (1) intelligence quotient, (2) eighth grade record, (3) ninth grade record, and (4) the algebra mark. Of the twenty-two pairs,

fifteen were also paired in regard to sex. The intelligence quotient was obtained from scores made on the Terman Group Intelligence Test. The test was given by the writer to all the students on the same day, in the regualr class period after the class had been organized. The Terman Group Test may not be an exact test but it is as good a pairing agency as the algebra score, eighth grade record and ninth grade record. The tests were scored by students in the Measurement classes at the Kansas State Teachers College of Emporia under the supervision of Dr. H. E. Schrammel. The eighth grade and the ninth grade records were secured from the official record in the office of the Principal of the Emporia Junior High School, or in the case of children who were graduated from the Rural Schools from the office of the County Superintendent of Schools. The algebra record also was secured from the office of the Principal of the Junior High School. Of the forty-four pupils used in the experiment all except four had had the same algebra teacher. Eight pupils in the experimental division had failed in algebra either one semester or two, one in the control class had failed in algebra and one had failed in geometry. These failures may be a significant fact in pairing the pupils although there is no evidence to show that algebra

is a basis for geometry. However it may at least, be a factor in the results secured.

While this experiment may not be controlled sufficiently well to give positively reliable results, no better control could be worked out under normal working conditions in the ordinary school system. The complete control was:

1. The pairing of the students as described.
2. Same textbook was used.
3. Same teacher taught both classes.
4. Same theorems and exercises were used.
5. Same theorems were presented each day.
6. Same assignment was made each day.
7. Same home work was required.

In every way the teaching methods were the same except for the use of colored chalk in the experimental division.

Any new material, any difficult part, any important phase of the subject was presented to the experimental class by the use of colored chalk. Thepupils were not asked or expected to use colored chalk, although in one testone pupil used colored crayons on his test paper and one pupil asked after the first presentation with the use of colored chalk how he could represent that on paper. The control class presentation were at all times made entirely with white chalk.

Colored chalk was used to present the following:

1. In teaching the construction problems the arcs drawn using the same center were made with the colored chalk.

2. When the construction was complete the part that was required was then colored red.

3. In teaching the superposition theorems one figure was drawn with white chalk and the other with colored chalk, then the colored figure was placed upon the white figure coloring each part as soon as it was justified by the proof.

4. In other congruency theorems the equal parts were colored the same.

5. In over lapping figures the equal parts were also colored the same color, as when proving the bisectors of the base angles of an isosceles triangle were equal and also the medians drawn from the base angles to the opposite sides.

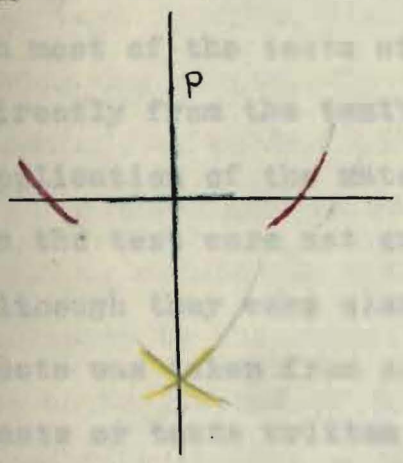
6. Parallel lines were always colored orange with the equal angles marked in colors also.

7. In proving the sum of the angles of a triangle equal to two right angles the parallel lines necessary for the proof were colored.

The colors used were red and yellow only when two colors were needed, and orange and light green added when more colors were needed.

Fig. 1. ILLUSTRATIONS OF USE OF COLORED CHALK

1&2



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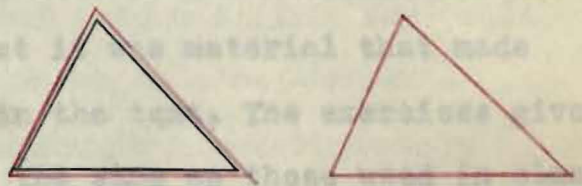

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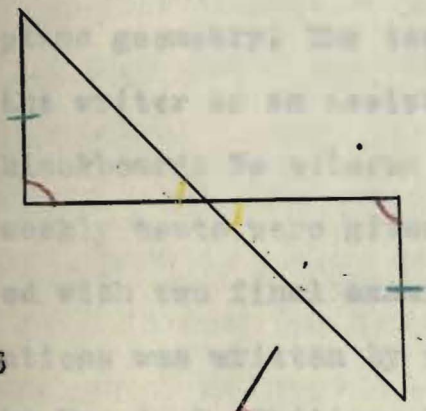

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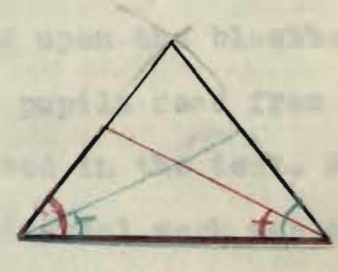

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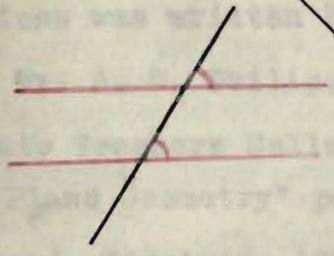

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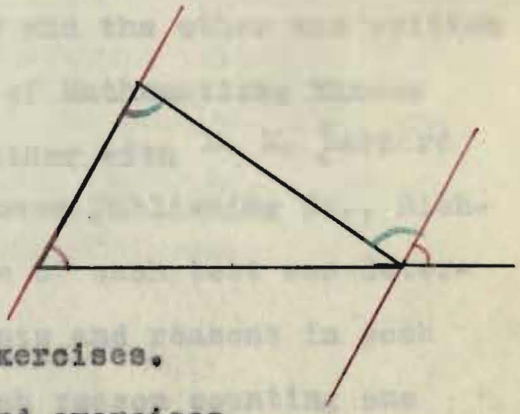


1&2. Construction problems.
3 Superposition theorems.
4 Congruent theorems and exercises.
5 Overlapping figures.
6 Parallel line theorems and exercises.
7 The sum of interior angles of a triangle equal 180.

A test was given each week, the same test being given to both divisions. The tests were constructed by the writer. In most of the tests at least half of the material was not directly from the textbook but it was material that made application of the material in the text. The exercises given in the test were not exactly the same as those used in class although they were closely related. The material for the tests was taken from supplementary text books, standard tests or tests written by the writer for former classes in plane geometry. The test was placed upon the blackboard by the writer or an assistant and the pupils read from the blackboard. No colored chalk was used in the test. Eight weekly tests were given. The experimental work was concluded with two final examinations. One of these final examinations was written by the writer and the other was written by Mr. A. W. Philips, Professor of Mathematics, Kansas State Teachers College and Co-author with D. M. Bernard "Plane Geometry" published Johnson Publishing Co., Richmond, Virginia, 1927. The value of each test was determined by the number of statements and reasons in each problem. Each statement and each reason counting one point. The value of each test is listed on the copy of the test in the appendix of this study.

The experiment was continued from January twentieth to March twenty-first, a period of nine weeks at the

beginning of the second semester of the school year of 1929-1930. The class completed the work on construction, congruent figures, parallel lines, and triangles and angles during the period. The text book used was Plane and Solid Geometry, by Walter Burton Ford and Charles Ammerman.

On the last day of the experiment, questions were asked to obtain the students' reaction to the experiment. In answer to the question,"Why do you think I have taught the class instead of the regular teacher?" Thirty of the forty-four answers were,"Because you were a practice teacher." This answer was likely due to the fact that the Emporia High School is used for teacher training purposes by the Kansas State Teachers College of Emporia. None of the students knew it was an experiment,although three knew that it was in some way connected with work of a graduate nature. Not one of the pupils knew that different practices were being employed in the two classes. The pupils in the experimental class were of the opinion that the colored chalk was helpful, especially in congruent figures.

CHAPTER III.

PRESENTATION OF DATA.

The purpose of this chapter is to present the material obtained.

Of the nine replies to the letter sent to the high school mathematics teachers : four instructors say they use colored chalk in teaching plane geometry; two say they use colored chalk to some extent, one uses it rarely, and two do not use colored chalk at all. Those who say they use it and those who use it rarely say it has a value. The ones who do not use colored chalk in plane geometry say it is a valuable aid in solid geometry. Some of the places in which teachers say it is valuable are: overlapping figures, measuring inscribed angles, the Pythagorean theorem, in similar figures, angles opposite unequal sides and unequal sides opposite unequal angles, congruent fugures, in proving bisectors of equal angles of isosceles triangles are equal, in proving medians through the equal angles are equal.

The following quotations are excerpts from letters sent in reply to the request for opinions as to the value of using colored chalk in teaching plane geometry. All of the writers are authors of geometry text-books.

I do not believe that its(colored chalk) use is essential; there are times, however, when colored chalk and colored pencils are desirable aids. They may be used at the beginning of the congruency theorems to visualize the equal pairs of parts. They may be used to advantage in calling attention to an identical line or angle. They are sometimes useful in indicating parts of overlapping figures. They may be used to mark four lines of a proposition upon a figure in order to bring out the pairs of triangles to prove similar. They enliven the drawing of figures because children like to play with colors.

In my own teaching, I reserve the use of colored chalk for my own work at the board. I do not require pupils to use colored chalk or pencils for their class work or for their home work. The use of colored chlak or pencils has considerable value, if it is not made a general practice.

W. W. Strader.
Wm. L. Dickinson High School, Jersey City, New Jersey.

I am not very enthusiastic about the use of colored chalk in teaching plane geometry. As far as the plane is concerned it is some use for a few figure. Usually however, the solid line, the dotted line, and the broken sketch line give sufficient variety.

Geo. A. Harper.
Tucson High School, Tucson, Arizona.

The more I think over it the more it seems as if you arelikely to be disappointed in the results. I just can not see how a class using the colors will show enough progress quantitatively measurable.

Theodore Lindquist.
Michigan State Normal College, Ypsilanti, Michigan.

My general attitude toward the use of colors in teaching geometry is th t like models they have their place in the beginning but should be regarded only as crutches which surely must be discarded as soon as pupils are able to limp along without the use of color.

E. R. Breslich.
School of Education, University of Chicago.

In reply to your letter asking about the use of colored chalk, I will saythat in my opinion it is a very valuable aid in teaching Plane Geometry. I would certainly advocate its use. There is a danger in the use of colored chalk that should be pointed out. It is like a crutch. There comes a time when a crutch should be put into the discard.

J. O.Hassler.
Department of Mathematics, University of Oklahoma.

I fear I cannot be of much assistance to you as I have never used it(colored chalk). I believe one reason I have never used it is because I never have had any on hand at the proper time, hence when it was necessary to distinguish one line from another in complicated figures I made the lines show up differently by making some of them wavy lines, some dotted, and some with little circles on them. Of course if I had had colored chalk at the time I would h have used it to good adavntage.

D. Meade Bernard.
Camp Director, Camp Carolina, Brevard, North Carolina.

Many pupils are lacking in visual-mindedness. Now, colored chalk is a kind of crutch for these pupils. I should not wish to continue the use of it through geometry.

John R. Clark.
Principal Lincln School, Teachers College, Columbia University.

I have a very strong conviction that a moderate use of colored chalk in more or less complicated figures helps out. I mean by moderate use, the use of one or possibly two colors in a figure. I think that it detracts often times from a drawing when the drawing is too ornate.

G. R. Mirick,
Lincln School, Teachers College, Columbia University.

I use colored chalk but not in any systematic way, but I keep it in my class room and when there is need for giving particular emphasis to some part of a figure I use it freely. The use of colored chalk frequently relieves eye fatigue and makes it possible to present a proof with greater brevity and clearness.

I would advocate its use in an informal way suggested above where it will add to the clearness of the figure and

to the ease and brevity of reference to it. Not, if used always according to some formal and prearranged system which may or may not be a real aid to the eye in following details of the argument.

A. W. Philips,
Department of Mathematics, Kansas State Teachers College of Emporia.

Prof. Raleigh Schorling, School of Education, University of Michigan and Prof. Geo. W. Mullins, Department of Mathematics, Barnard College, Teachers College, Columbia University did not express an opinion of the use of colore d chalk. The letters received from them will be found in Appendix B.

HOW PUPILS WERE PAIRED.

In Table 1 the manner in which the pupils were paired is shown. The intelligence quotient, eighth grade record, ninth grade record and algebra grade were secured as explained in Chapter II of this study.

The grades received from the office of the Principal of the Junior High School were weighted in this manner. An E grade was weighted as 5, a G grade as 4, M as 3, P as 2, and F as 1. In the chart a grade of 4.1 means a grade of G, while a grade of 2.8 means an unweighted grade of P +. The pairing is read across the page thus: L.A. with an intelligence quotient of 90 is paired with C. A. directly across the page who has an intelligence quotient of 90.

TABLE 1.

PAIRING OF THE PUPILS IN THE TWO CLASSES

CONTROL CLASS						EXPERIMENTAL CLASS				
Pair # number	Name of pupil	I.Q.	8th grade record	9th grade record	algebra grade	Name of Pupil	I.Q.	8th grade record	9th grade record	algebra grade
1.	L.A.	90	3.1	3.4	3.0	C.A.	90	3.1	3.0	3.0
2.	H.B.	116	2.8	2.7	2.0	O.A.	111	2.9	2.3	2.0
3.	J.C.	85	2.6	3.0	3.0	C.E.	89	2.8	2.9	3.0
4.	D.C.	97	2.5	2.7	3.0	L.B.	93	2.1	2.8	3.0
5.	C.D.	108	3.3	3.3	4.0	A.C.	102	3.5	3.5	4.0
6.	T.E.	81	3.1	3.0	3.0	R.S.	89	2.4	3.3	4.0
7.	J.F.	90	2.7	2.5	2.0	F.K.	102	3.8	2.2	2.0
8.	M.G.	84	3.0	2.6	2.0	G.F.	104	2.3	2.5	2.0
9.	R.H.	110	3.2	3.0	3.0	M.B.	107	2.3	2.8	3.0
10.	D.H.	91	3.2	3.0	3.0	M.A.	99	4.5	2.8	3.0
11.	L.J.	97	2.8	3.5	4.0	J.K.	101	3.5	3.0	3.0
12.	A.J.	119	4.1	3.2	3.0	W.H.	118	4.0	4.0	4.0
13.	V.J.	119	3.7	3.8	4.0	D.S.	119	3.8	3.8	4.0
14.	R.J.	104	3.7	3.8	4.0	S.N.	95	3.7	4.0	5.0
15.	E.K.	102	3.0	2.5	3.0	E.S.	102	3.7	2.8	3.0
16.	V.L.	106	3.1	2.7	3.0	H.R.	103	2.8	3.0	3.0
17.	P.L.	127	3.0	4.0	3.0	L.R.	124	3.8	4.6	5.0
18.	L.L.	93	3.0	3.0	4.0	D.E.	95	3.0	3.4	4.0
19.	F.M.	108	3.3	3.4	4.0	V.L.	107	3.8	3.8	4.0
20.	S.P.	103	2.2	2.5	3.0	G.G.	104	2.5	2.2	3.0
21.	K.S.	124	3.3	3.2	3.0	M.G.	106	3.9	3.9	3.0
22.	W.S.	98	3.4	3.0	3.0	B.F.	100	4.2	2.6	3.0
	Median	101.7	3.2	3.1	3.1		102.1	3.5	3.0	3.2
	Mean	102.7	3.2	3.0	3.1		102.3	3.3	3.2	3.3
	S.D.	12.5	.49	.4	.6		9.5	.7	.7	.8
	Range	82-127					84-124			

Read across page.

The comparison of their eighth grade record, ninth grade record, and algebra grade are read in the same manner.

The median intelligence quotient as derived from the Terman Group Test was 101.7 for the control class and for the experimental class 102.1. The eighth grade scholastic records put the median at 3.2 for the control class and 3.5 for the experimental class; in the ninth grade record the control class had a median of 3.1 and the experimental class a median of 3.0; and in the algebra grade the control class had a median of 3.1 and the experimental class a median of 3.2. It is evident that the differencesbetween the two classes were very slight. Since all had come from the same county and nearly all from the same school system the marks are likely quite well standardized.

RESULTS OF TEST I.

In Table 2 on the following page the score made by each pupil in Test I is shown. The pupils are arranged so that pair numbers are read across the page. The pupils initials were used rather than the entire name."L.A." is paired with "C.A." and so on down the page.

The score was obtained by evaluating each statement and each reason for a statement as one point as was explained in ChapterII. The point value of the first test was fifty-two. Since the reliability of the tests is not known this factor must be taken into account in evaluating results.

TABLE 2.

SCORES MADE BY PUPILS IN TEST I.

CONTROL CLASS			EXPERIMENTAL CLASS	
Pair# number	Name of pupil	Score	Name of pupil	Score
1.	L.A.	33	C.A.	10
2.	H.B.	7	O.A.	13
3.	J.C.	32	C.E.	23
4.	D.C.	7	L.B.	13
5.	C.D.	23	A.C.	25
6.	T.E.	19	R.S.	17
7.	J.F.	25	F.K.	20
8.	M.G.	10	G.F.	16
9.	R.H.	15	M.B.	27
10.	D.H.	11	M.AM	10
11.	L.J.	30	J.K.	16
12.	A.J.	23	W.H.	23
13.	V.J.	30	D.S.	35
14.	R.J.	25	S.N.	33
15.	E.K.	35	E.S.	15
16.	V.D.	13	H.R.	26
17.	P.L.	20	L.R.	37
18.	L.L.	10	D.E.	26
19.	F.M.	21	V.L.	31
20.	S.P.	23	G.G.	21
21.	K.S.	28	M.G.	16
22.	W.S.	28	B.F.	6
	Medain	23.1		21.0
	Mean	21.6		20.7
	S.D.	8.7		8.4
	Range	7-35		6-37

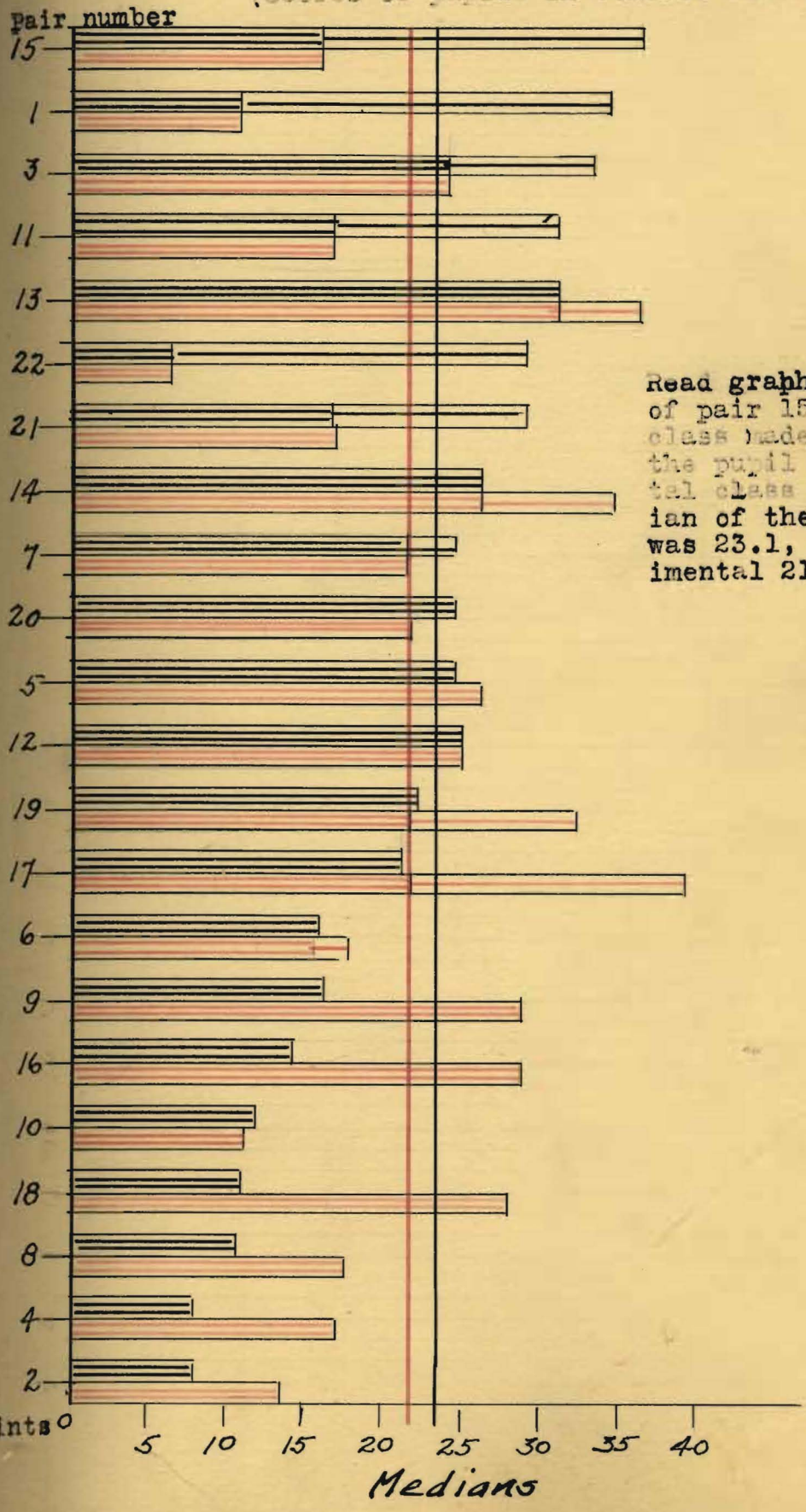
Read across page.
Score is expressed in points. Total number of points in test52.
Test questions may be found in Appendix A.

Legend

GRAPH I

Control
Experimental

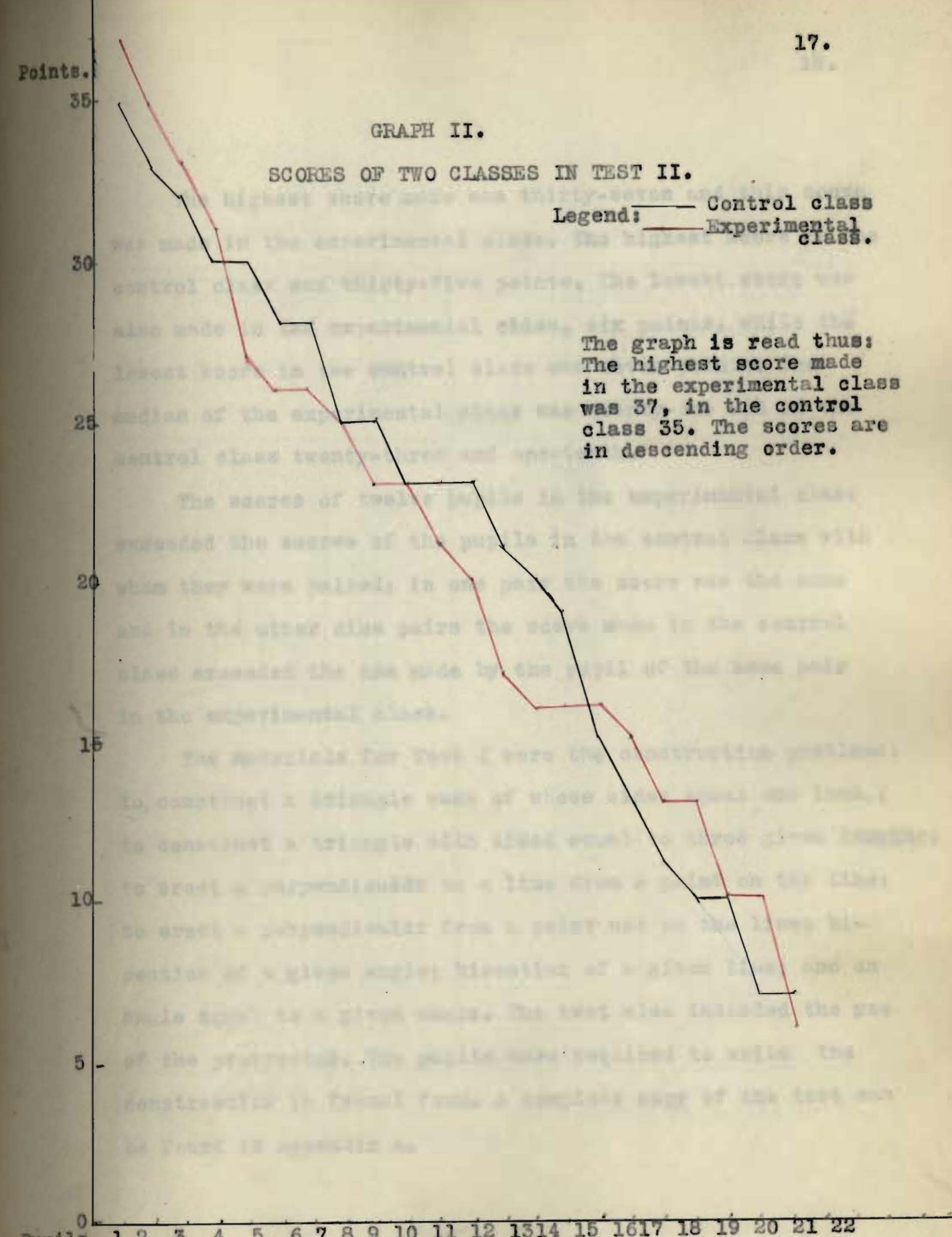
RANKING OF PUPILS IN TEST I
(Scores of pupils in control class in descending order.)



Read graph thus: the member of pair 15 in the control class made a score of 35, the pupil in the experimental class made 15. The median of the control class was 23.1, and of the experimental 21.0.

GRAPH II.

SCORES OF TWO CLASSES IN TEST II.



The highest score made was thirty-seven and this score was made in the experimental class. The highest score in the control class was thirty-five points. The lowest score was also made in the experimental class, six points, while the lowest score in the control class was seven points. The median of the experimental class was twenty-one and in the control class twenty-three and one-tenth.

The scores of twelve pupils in the experimental class exceeded the scores of the pupils in the control class with whom they were paired; in one pair the score was the same and in the other nine pairs the score made in the control class exceeded the one made by the pupil of the same pair in the experimental class.

The materials for Test I were the construction problems: to construct a triangle each of whose sides equal one inch,; to construct a triangle with sides equal to three given lengths; to erect a perpendicular to a line from a point on the line; to erect a perpendicular from a point not on the line; bisection of a given angle; bisection of a given line; and an angle equal to a given angle. The test also included the use of the protractor. The pupils were required to write the construction in fromal form. A complete copy of the test can be found in Appendix A.

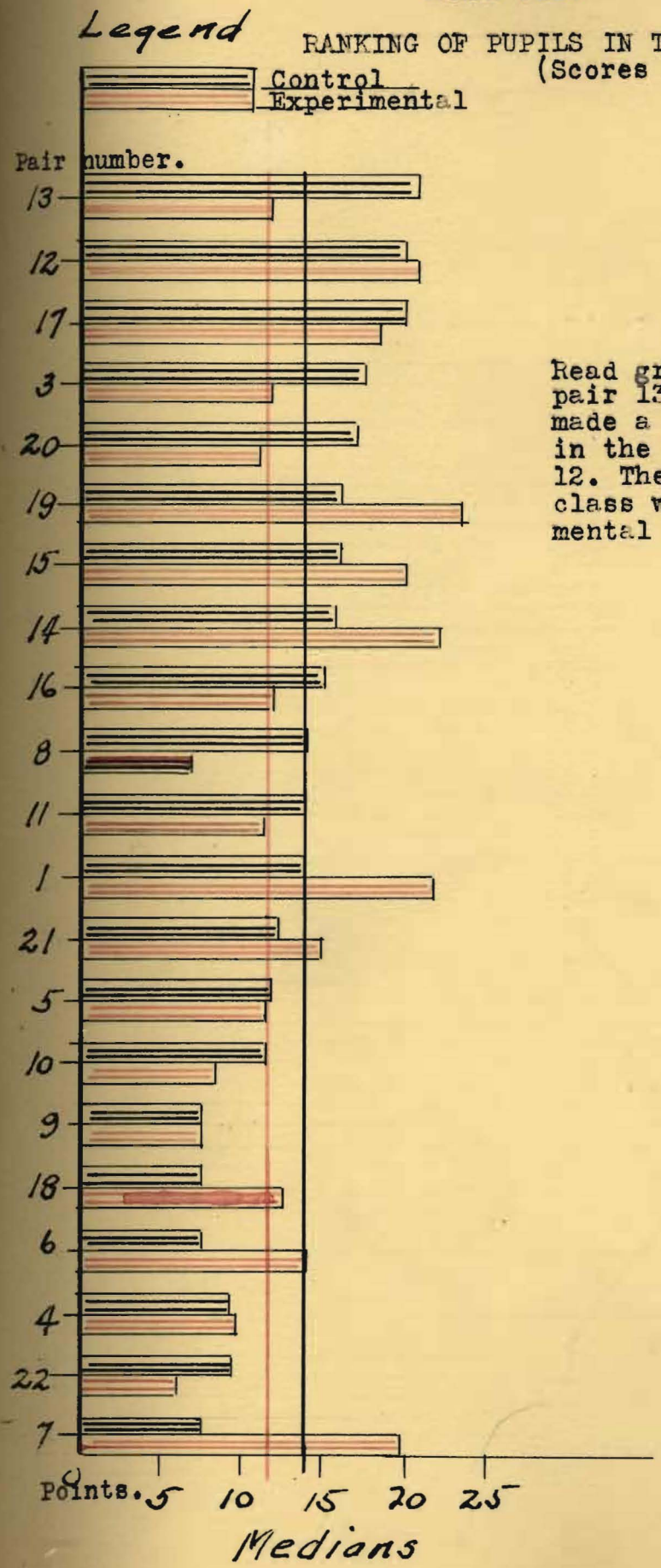
TABLE 3.

SCORES MADE BY PUPILS IN TEST II.

CONTROL CLASS			EXPERIMENTAL CLASS	
Pair# number	Name of pupil	Score	Name of pupil	Score
1.	L.A.	14	C.A.	11
2.	H.B.	9½	O.A.	12
3.	J.C.	17½	C.E.	11½
4.	D.C.	10	L.B.	16
5.	C.D.	12	A.C.	15
6.	T.E.	10	K.S.	15
7.	J.F.	7½	F.K.	19
8.	M.G.	15	G.F.	12
9.	R.H.	11	M.B.	8
10.	D.H.	11½	MA.	11
11.	L.J.	14	J.K.	7.
12.	A.J.	20	W.H.	21
13.	V.J.	21	D.S.	12
14.	R.J.	15½	S.N.	22
15.	E.K.	16	E.S.	20
16.	V.L.	15½	H.R.	12½
17.	P.L.	20	L.R.	18½
18.	L.L.	10	D.E.	10
19.	F.M.	16	V.L.	23
20.	S.P.	17	G.G.	11
21.	K.S.	13	M.G.	21½
22.	W.S.	9	B.F.	6
	Median	13.8		12.0
	Mean	13.8		14.3
	S.D.	3.8		4.8
	Range	7½-21		6-23

Read across the page.
Score is expressed in points. Total number of points is 26.
Tests may be found in Appendix A.

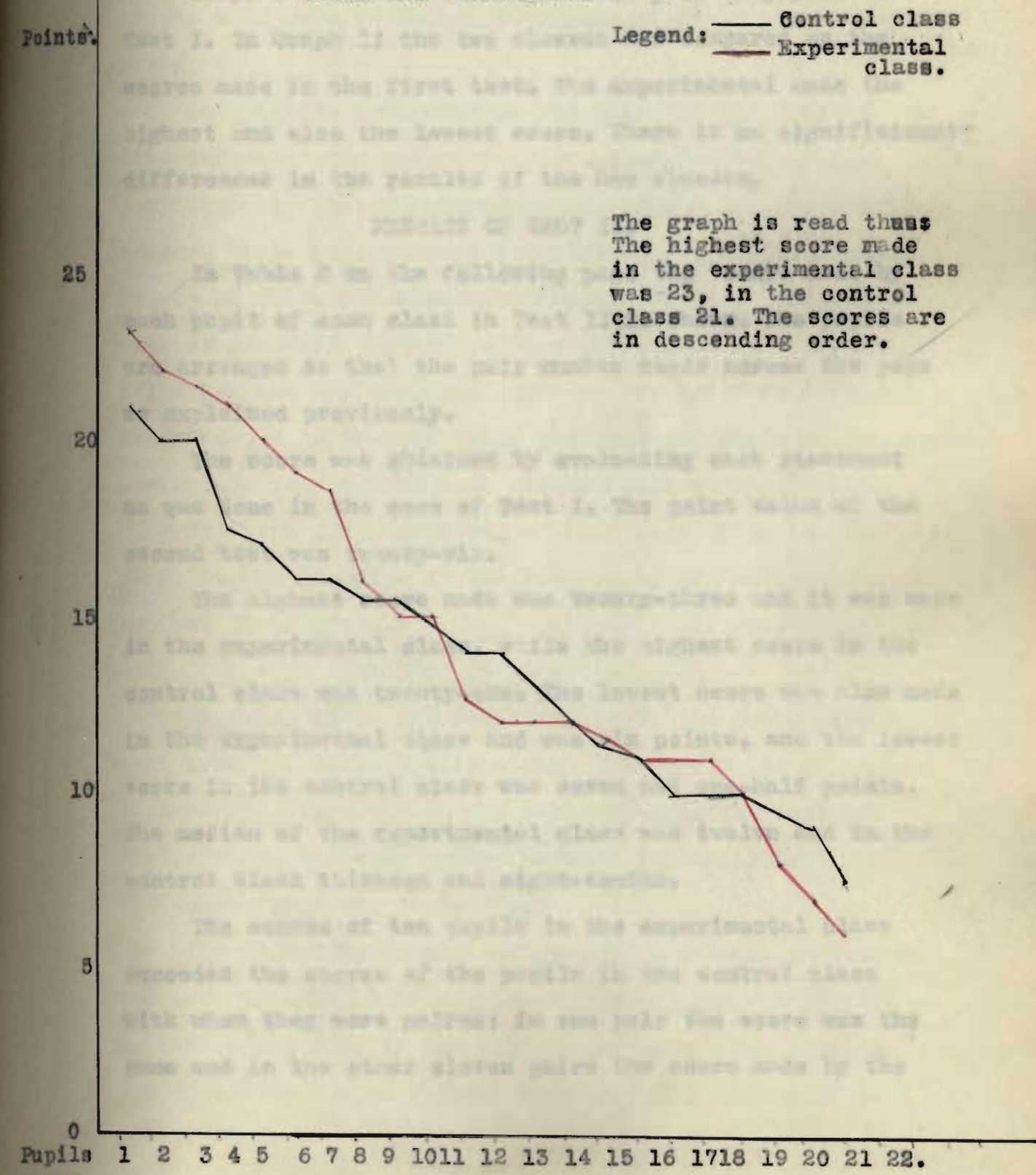
GRAPH III.



Read graph thus: The member of pair 13 in the control class made a score of 21, the pupil in the experimental class made 12. The median of the control class was 13.8 of the experimental class 12.0.

GRAPH IV.

SCORES OF TWO CLASSES IN TEST II.



Graph I shows the rankingof each pair of pupils in Test I. In Graph II the two classes are compared on the scores made in the first test. The experimental made the highest and also the lowest score. There is no signifficiannt differences in the results of the new classes.

RESULTS OF TEST II.

In Table 2 on the following page the score made by each pupil of each class in Test II is shown. The pupils are arranged so that the pair number reads across the page as explained previously.

The score was obtained by evaluating each statement as was done in the case of Test I. The point value of the second test was twenty-six.

The highest score made was twenty-three and it was made in the experimental class, while the highest score in the control class was twenty-one. The lowest score was also made in the experimetnal class and was six points, and the lowest score in the control class was seven and one-half points. The median of the experimental class was twelve and in the control class thirteen and eight-tenths.

The scores of ten pupils in the experimental class exceeded the scores of the pupils in the control class with whom they were paired; in one pair the score was the same and in the other eleven pairs the score made by the

pupil in the control class exceeded the one made by the pupil of the same pair in the experimental class in the second test given.

Definitions were used as the material for Test II. The test included: the definitions of supplementary angles, complementary angles, acute angles, obtuse angles, vertical angles, right angles, isosceles triangles, right triangles and in addition to these definitions the first theorem on congruency. This theorem is "Two triangles are congruent if two sides and the included angle of one triangle is equal respectively to two sides and the included angle of the other." A complete copy of the test can be found in Appendix A.

In Graph III the ranking of each pair of pupils in Test II is shown. In Graph IV the comparison of the two classes in Test II is given.

RESULTS OF TEST III.

In Table 3; on the following page, the score made by each pupil of each pair is shown. The pupils are arranged in the same manner as in the other charts. The test was evaluated in the same manner as were the other tests. The point value was forty-four.

The higest score was forty-four, a perfect score, and was made by a pupil in the experimental class, while the highest score made in the control class was forty-three. The lowest score, ten points, was made by a member of each

class. The median in the control class was twenty-eight and five-tenths, and the median in the experimental class was twenty-six and five-tenths.

The scores of nine pupils in the experimental class exceeded the scores of the pupils in the control class with whom they were paired; in the other thirteen pairs the score made by the pupil in the control class exceeded that made by the pupil in the experimental class.

The material for Test III was the congruent theorem, "Two triangles are congruent if two angles and the included side of one triangle is equal respectively to two angles and the included side of the other," and exercises and corollaries which apply to it.

In Graph V the ranking of each pair of pupils in Test III is shown. The graph VI shows the comparison of the two classes in Test III. The experimental class had one perfect paper or a score of forty-four. The lowest score for each group was the same.

RESULTS OF TEST IV.

In Table V on the next page the score made by each pupil of each pair in Test IV is shown. The pupils are arranged in the same manner as before. The pairs read across the page as before. The test was evaluated in the same way, one point for each statement and one point for each reason. The point value of the test was forty-four.

TABLE 4.

SCORES MADE BY PUPILS IN TEST III.

CONTROL CLASS			EXPERIMENTAL CLASS	
Pair# number	Name of pupil	Score	Name of pupil	Score
1.	L.A.	28	C.A.	27
2.	H.B.	14	O.A.	25
3.	J.C.	31	C.E.	27
4.	D.C.	11	L.B.	22
5.	C.D.	37	A.C.	44
6.	T.E.	23	R.S.	26
7.	J.F.	25	F.K.	20
8.	M.G.	31	G.F.	16
9.	R.H.	10	M.B.	22
10.	D.H.	32	M.A.	16
11.	L.J.	28	J.K.	10
12.	A.J.	37	W.H.	36
13.	V.J.	38	D.S.	34
14.	R.J.	43	S.N.	35
15.	E.K.	32	E.S.	20
16.	V.L.	27	H.R.	23
17.	P.L.	39	L.T.	42
18.	L.L.	36	D.E.	33
19.	F.M.	24	V.L.	39
20.	S.P.	21	G.G.	22
21.	K.S.	11	M.G.	35
22.	W.S.	24	B.F.	18
	Median	28.5		26.5
	Mean	27.5		27.3
	S.D.	9.6		8.5
	Range	10-43		10-44

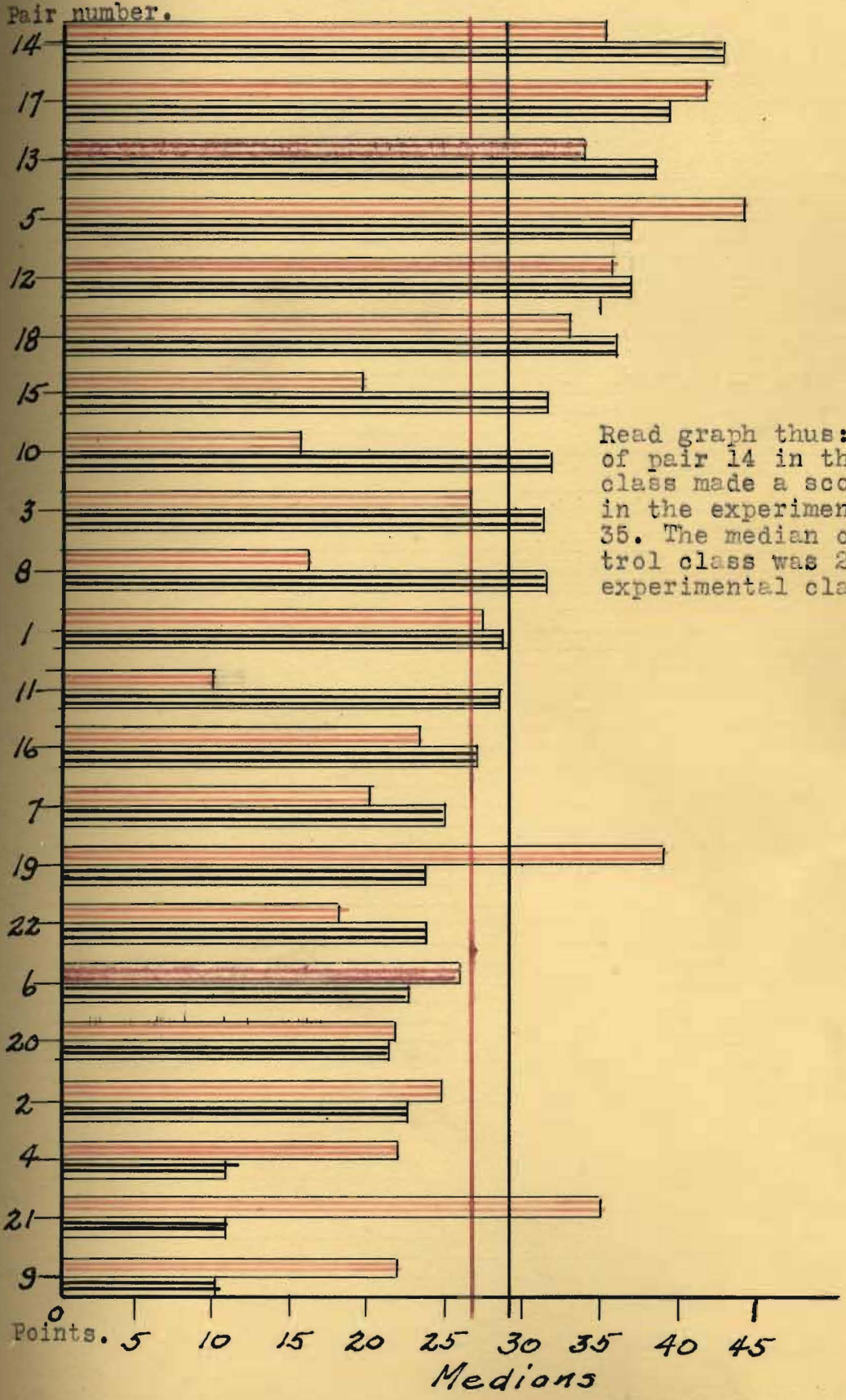
Read across page.
Scores are expressed in points. Total number of points 44.
Test questions may be found in Appendix A.

GRAPH V.

Legend

Experimental
Control

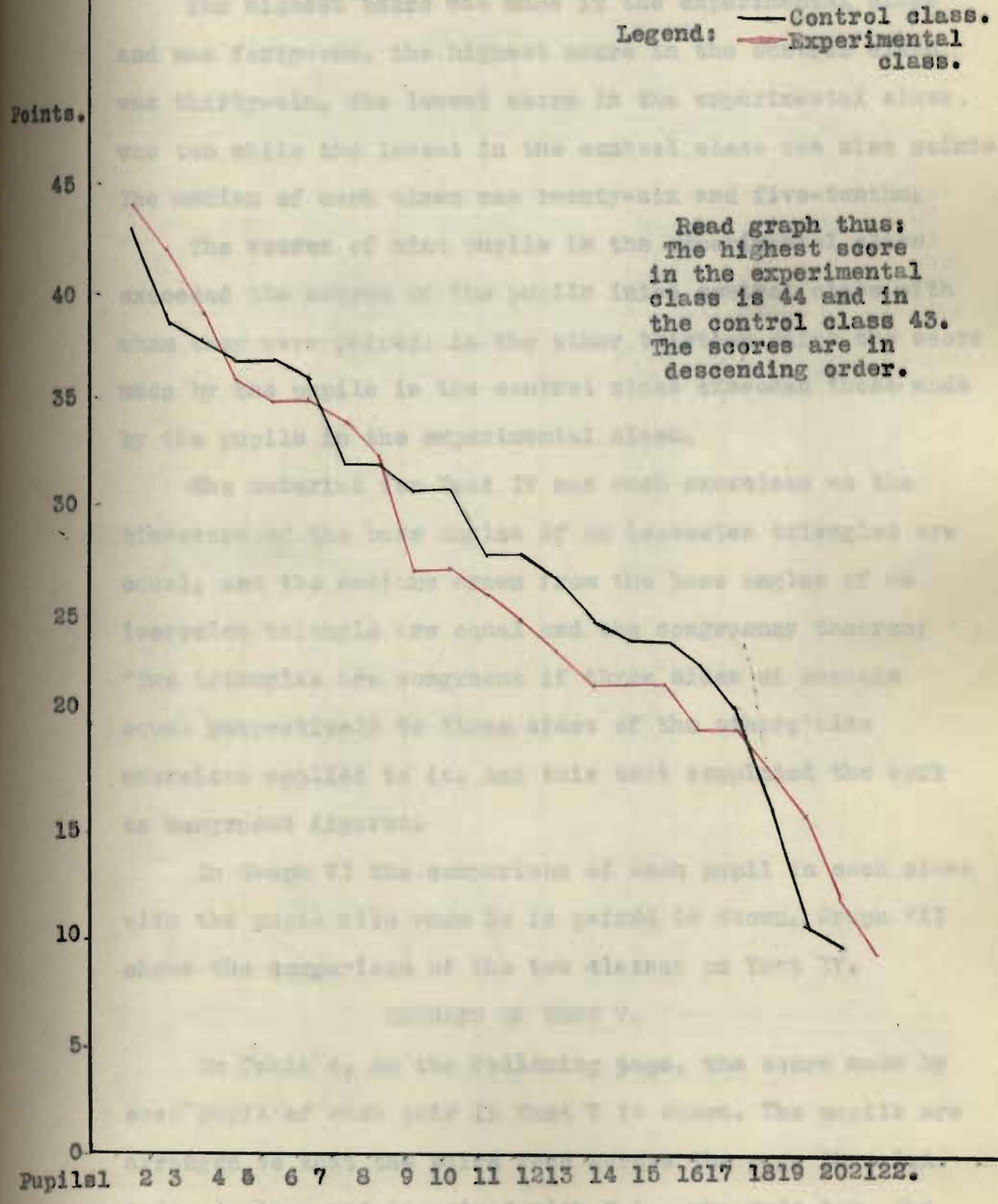
RANKING OF PUPILS IN TEST III.
(Scores of pupils in the control class in descending order.)



Read graph thus: The member of pair 14 in the control class made a score of 43, in the experimental class 35. The median of the control class was 28.5 of the experimental class 26.5.

GRAPH VI.

SCORES OF TWO CLASSES IN TESTIII.



The highest score was made in the experimental class and was forty-one, the highest score in the control class was thirty-six. The lowest score in the experimental class was ten while the lowest in the control class was nine points. The median of each class was twenty-six and five-tenths.

The scores of nine pupils in the experimental class exceeded the scores of the pupils inthe control class with whom they were paired; in the other thirteen pairs the score made by the pupils in the control class exceeded those made by the pupils in the experimental class.

The material for Test IV was such exercises as the bisectors of the base angles of an isosceles triangles are equal, and the medians drawn from the base angles of an isosceles triangle are equal and the congruency theorem; "Two triangles are congruent if three sides of one are equal respectively to three sides of the other;"also exercises applied to it. And this test completed the work on congruent figures.

In Graph VI the comparison of each pupil in each class with the pupil with whom he is paired is shown. Graph VII shows the comparison of the two classes on Test IV.

RESULTS OF TEST V.

In Table 6, on the following page, the score made by each pupil of each pair in Test V is shown. The pupils are arranged so that the pairs read across the page thus:L.A. made nineteen and is paired with C.A., who made ten. The

TABLE 5.

SCORES MADE BY PUPILS IN TEST IV.

CONTROL CLASS			EXPERIMENTAL CLASS.	
Pair # number	Name of pupil	Score	Name of pupil	Score
1.	L.A.	22	C.A.	24
2.	H.B.	26	C.A.	21
3.	J.C.	31	C.E.	20
4.	L.C.	9	L.B.	23
5.	C.D.	35	A.C.	36
6.	T.E.	32	R.S.	25
7.	J.F.	19	F.K.	29
8.	M.G.	26	G.F.	19
9.	R.H.	20	M.B.	28
10.	D.H.	20	M.A.	18
11.	L.J.	22	J.K.	17
12.	A.J.	43	W.H.	41
13.	V.J.	35	D.S.	34
14.	R.J.	35	S.N.	37
15.	E.K.	36	E.S.	28
16.	V.L.	24	H.R.	13
17.	P.L.	27	L.R.	37
18.	L.L.	26	D.E.	28
19.	E.M'	30	V.L.	36
20.	S.P.	27	G.G.	24
21.	K.S.	36	M.G.	30
22.	W.S.	20	B.E.	15
	Median	26.5		26.5
	Mean	27.2		26.6
	S.D.	7.5		7.8
	Range	9-36		13-41

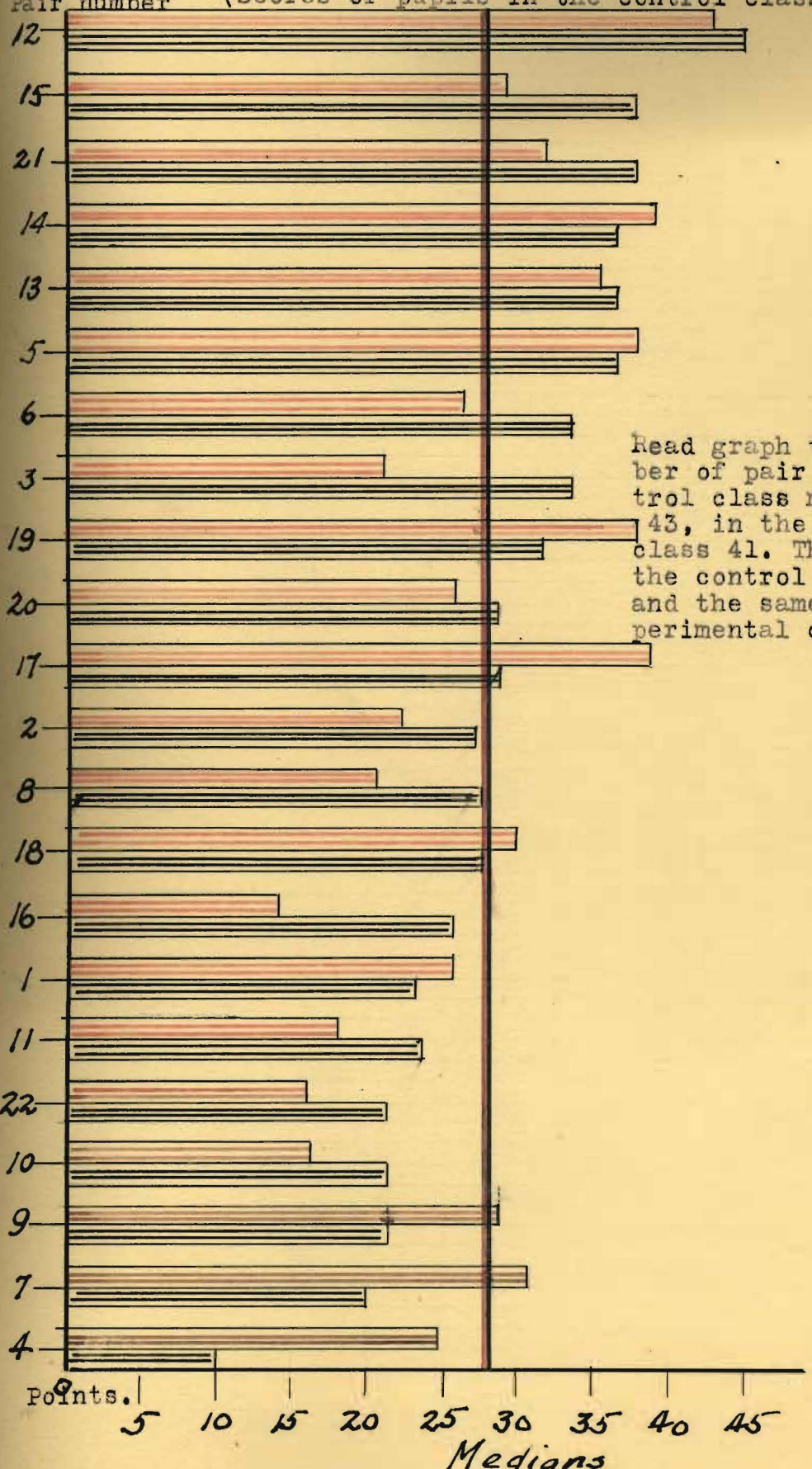
Read across the page.
Scores are expressed in points. Total number of points 44.
Tests may be found in Appendix A.

Legend

GRAPH VII.

Experimental
Control

RANKING OF PUPILS IN TEST IV.
(Scores of pupils in the control class in descending order.)

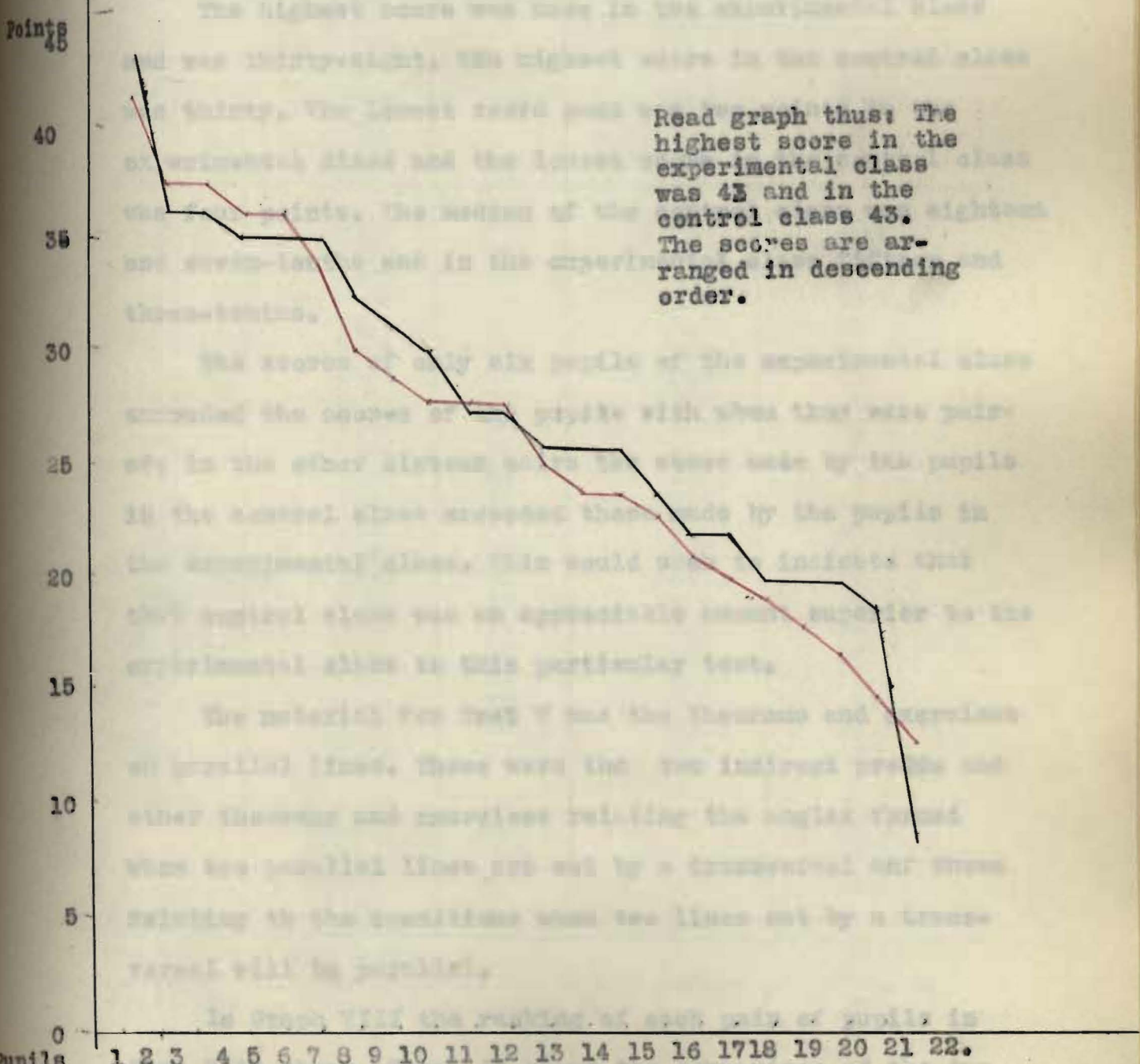


Read graph thus: The member of pair 12 in the control class made a score of 43, in the experimental class 41. The median of the control class was 26.5 and the same for the experimental class.

GRAPH VIII.

SCORES OF TWO CLASSES IN TEST IV.

Legend: ——Control class.
——Experimental class.



Read graph thus: The highest score in the experimental class was 43 and in the control class 43. The scores are arranged in descending order.

test was evaluated by allowing one point for each statement and one point for each reason for each statement. The point value was forty.

The highest score was made in the experimental class and was thirty-eight, the highest score in the control class was thirty. The lowest score made was two points in the ex erimental class and the lowest score in the control class was four points. The median of the control class was eighteen and seven-tenths and in the experimental class fifteen and three-tenths.

The scores of only six pupils of the experimental class exceeded the scores of the pupils with whom they were paired; in the other sixteen pairs the score made by the pupils in the control class exceeded those made by the pupils in the experimental class. This would seem to indicate that that control class was an appreciable amount superior to the experimental class in this particular test.

The material for Test V was the theorems and exercises on parallel lines. These were the two indirect proofs and other theorems and exercises relating the angles formed when two parallel lines are cut by a transversal anf those relating to the conditions when two lines cut by a transversal will be parallel.

In Graph VIII the ranking of each pair of pupils in Test V is shown. Graph IX shows the comparison of the two classes in Test V.

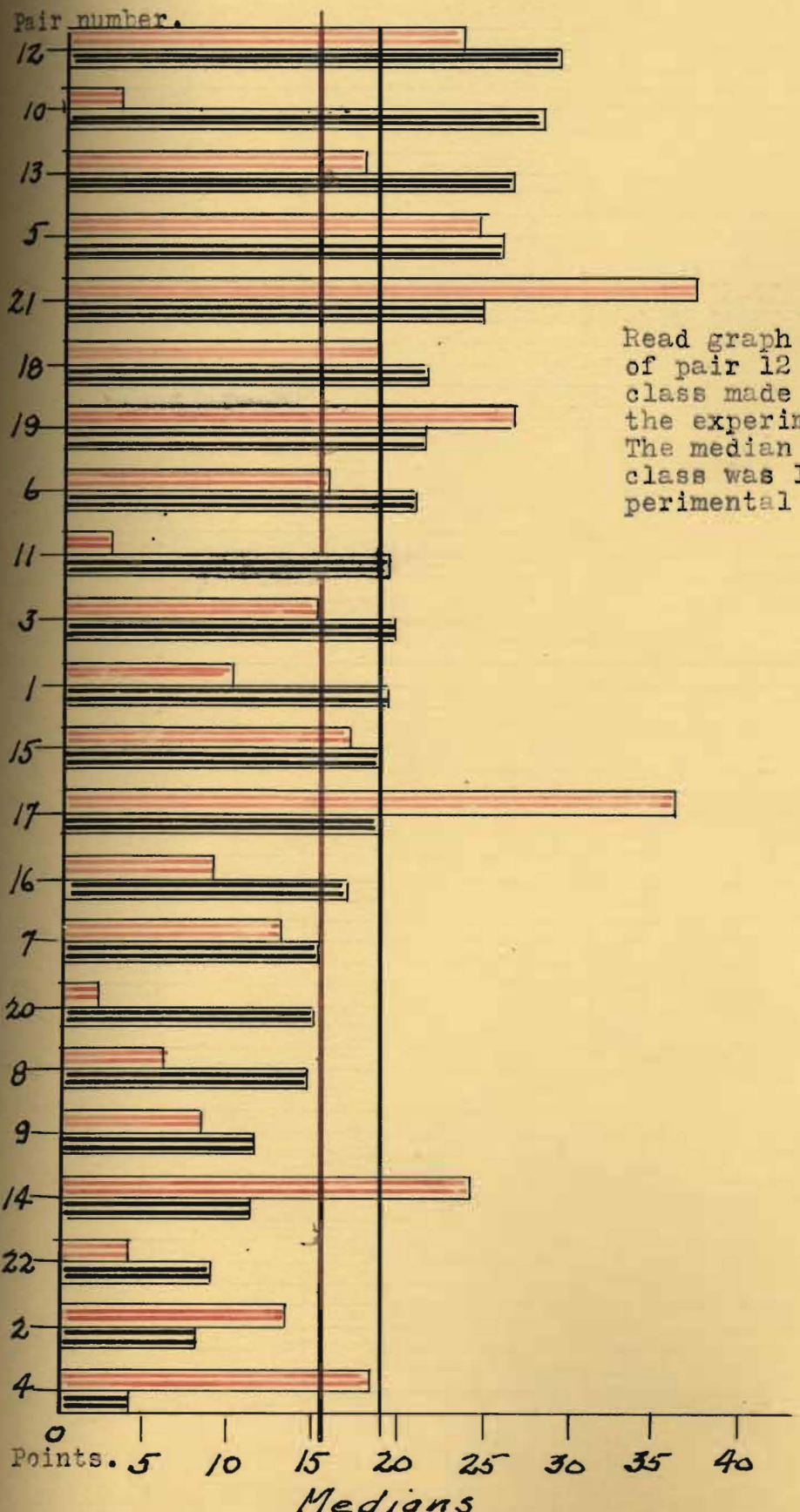
GRAPH IX.

Legend

Experimental

Control

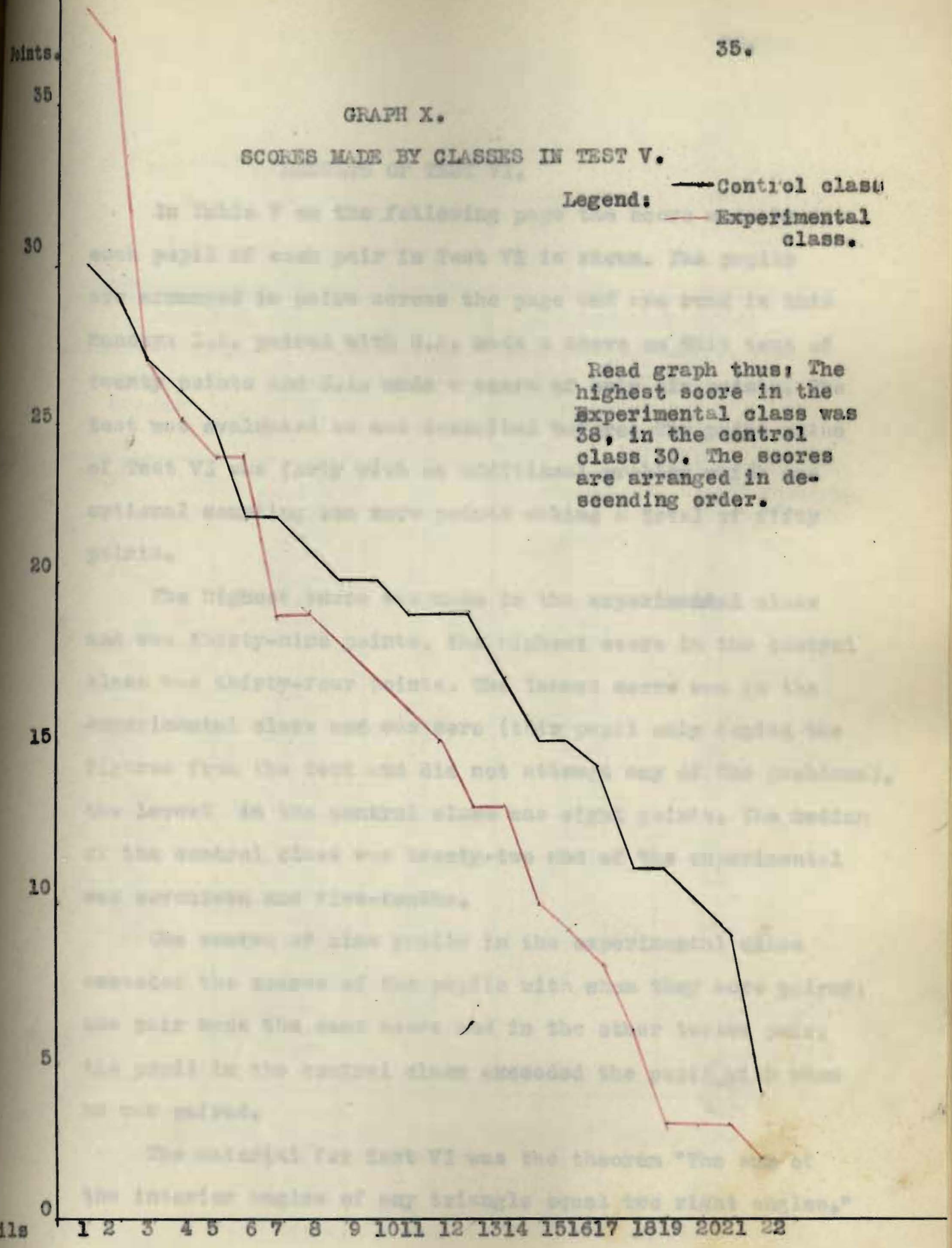
RANKING OF PUPILS IN TEST V.
(Scores of pupils in the control class in descending order.)



Read graph thus: The member of pair 12 in the control class made a score of 30, in the experimental class 24. The median of the control class was 18.7, of the experimental class 15.3.

GRAPH X.

SCORES MADE BY CLASSES IN TEST V.



RESULTS OF TEST VI.

In Table 7 on the following page the score made by each pupil of each pair in Test VI is shown. The pupils are arranged in pairs across the page and are read in this manner: L.A. paired with C.A. made a score on this test of twenty points and C.A. made a score of only six points. The test was evaluated as was described before. The point value of Test VI was forty with an additional problem which was optional counting ten more points making a total of fifty points.

The highest score was made in the experimental class and was thirty-nine points, the highest score in the control class was thirty-four points. The lowest score was in the experimental class and was zero (this pupil only copied the figures from the test and did not attempt any of the problems), the lowest in the control class was eight points. The median of the control class was twenty-two and of the experimental was seventeen and five-tenths.

The scores of nine pupils in the experimental class exceeded the scores of the pupils with whom they were paired; one pair made the same score and in the other twelve pair, the pupil in the control class exceeded the pupil with whom he was paired.

The material for Test VI was the theorem "The sum of the interior angles of any triangle equal two right angles,"

TABLE 7.

SCORES MADE BY PUPILS IN TEST VI.

CONTROL CLASS			EXPERIMENTAL CLASS	
Pair# number	Name of pupil	Score	Name of pupil	Score
1.	L.A.	20	C.A.	6.
2.	H.P.	19	O.A.	25
3.	J.C.	34	C.E.	16
4.	D.C.	17	L.B.	19
5.	C.D.	25	A.C.	35
6.	T.E.	21	R.S.	13
7.	J.F.	15	F.K.	27
8.	M.G.	14	G.F.	15
9.	R.H.	8	M.B.	11
10.	D.H.	15	M.A.	0
11.	L.J.	25	J.K.	7
12.	A.J.	28	W.H.	26
13.	V.J.	33	D.S.	18
14.	R.J.	27	S.N.	35
15.	E.K.	23	E.S.	23
16.	V.L.	18	H.R.	8
17.	P.L.	33	L.R.	39
18.	L.L.	32	D.E.	27
19.	F.M.	27	V.L.	18
20.	S.P.	15	G.G.	5
21.	K.S.	23	M.G.	29
22.	W.S.	8	B.F.	2
	Median	22.0		17.5
	Mean	23.6		18.6
	S.D.	7.5		10.5
	Range	8-34		0-39

Read across the page.
Score is given inpoints, Total number of points 50.
Tests may be found in Appendix A.

Legends

Experimental
Control

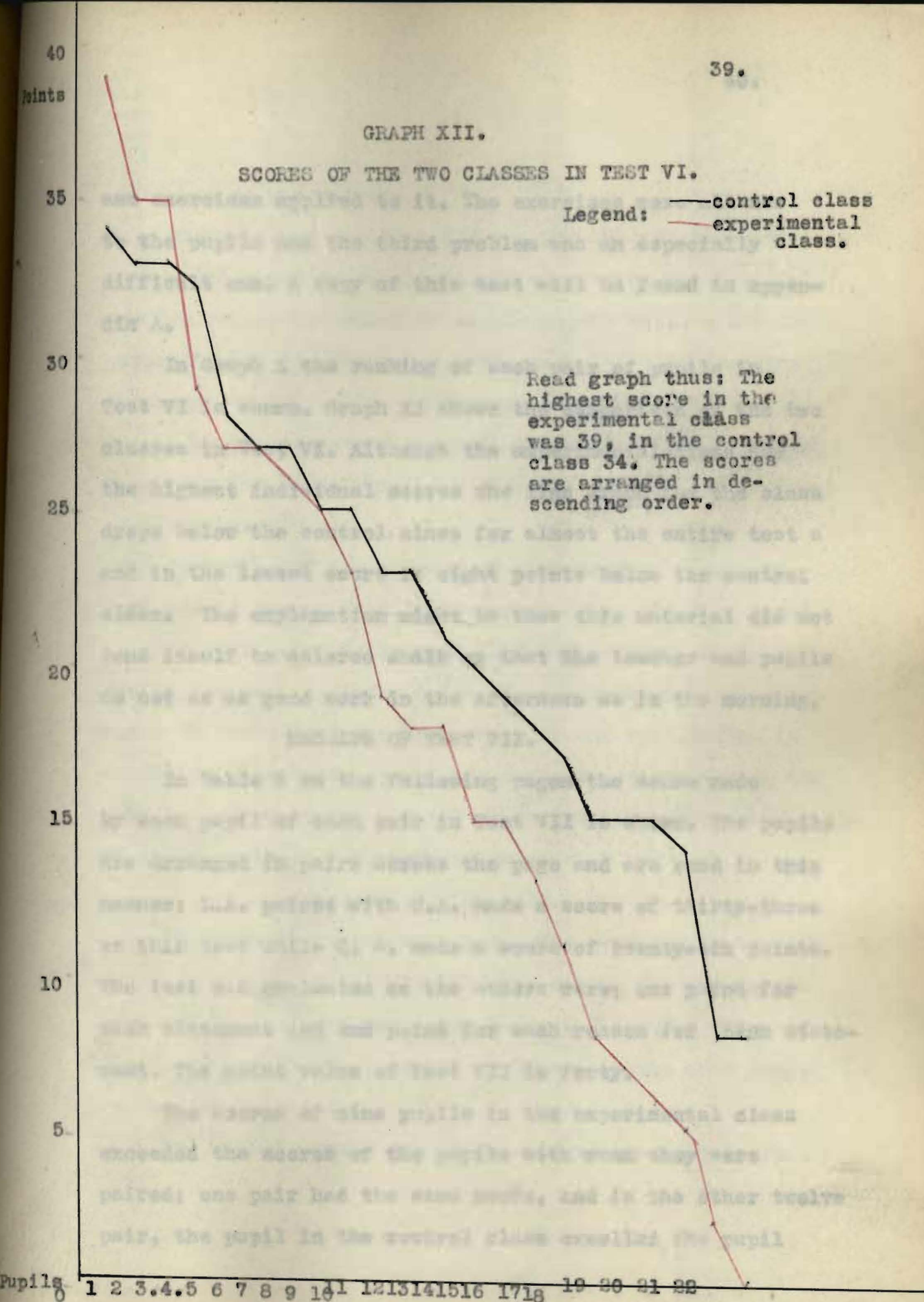
GRAPH XI.

RANKING OF PUPILS IN TEST VI.
(Scores of pupils in the control class in descending order.)

Read graph thus: The member of pair 3 in the control class made a score of 34, in the experimental class 15. The median of the control class was 22.0, of the experimental class 17.5.

GRAPH XII.

SCORES OF THE TWO CLASSES IN TEST VI.



and exercises applied to it. The exercises were all new to the pupils and the third problem was an especially difficult one. A copy of this test will be found in Appendix A.

In Graph X the ranking of each pair of pupils in Test VI is shown. Graph XI shows the comparison of the two classes in Test VI. Although the experimental class has the highest individual scores the line graph for the class drops below the control class for almost the entire test a and in the lowest score is eight points below the control class. The explanation might be that this material did not lend itself to colored chalk or that the teacher and pupils do not do as good work in the afternoon as in the morning.

RESULTS OF TEST VII.

In Table 8 on the following pagem the score made by each pupil of each pair in Test VII is shown. The pupils are arranged in pairs across the page and are read in this manner: L.A. paired with C.A. made a score of thirty-three on this test while C. A. made a score of twenty-six points. The test was evaluated as the others were; one point for each statement and one point for each reason for each statement. The point value of Test VII is forty.

The scores of nine pupils in the experimental class exceeded the scores of the pupils with whom they were paired; one pair had the same score, and in the other twelve pair, the pupil in the control class excelled the pupil

with whom he was paired.

The material for Test VII was the two theorems:"Two angles are equal or supplementary whose side are parallel," and"Two angles are equal or supplementary whose sides are perpendicular to each other." The material also included the theorems concerning the congruency of right triangles and exercises supplementary to them. The material of this test resembled the class work more closely than did the material of the other tests.

In Graph XII the ranking of each pair of pupils in Test VII is shown. It will be noticed that all the scores on this test were higher than in the previous test probably due to the nature of the material used in constructing the test. In Graph XIII the comparison of the two classes is shown. In this test, except for the one pupil who made the very low score in the control class the control class excelled the experimental clas noticeably.

RESULTS OF TEST VIII.

In Table IX on the next page, the scores made by each pupil of each pair in Test VIII are given. The pupils are arranged in pairs as explained before. The point value of the test was thirty. This was evaluated in the same manner as the other tests.

The highest score was again made in the experimental

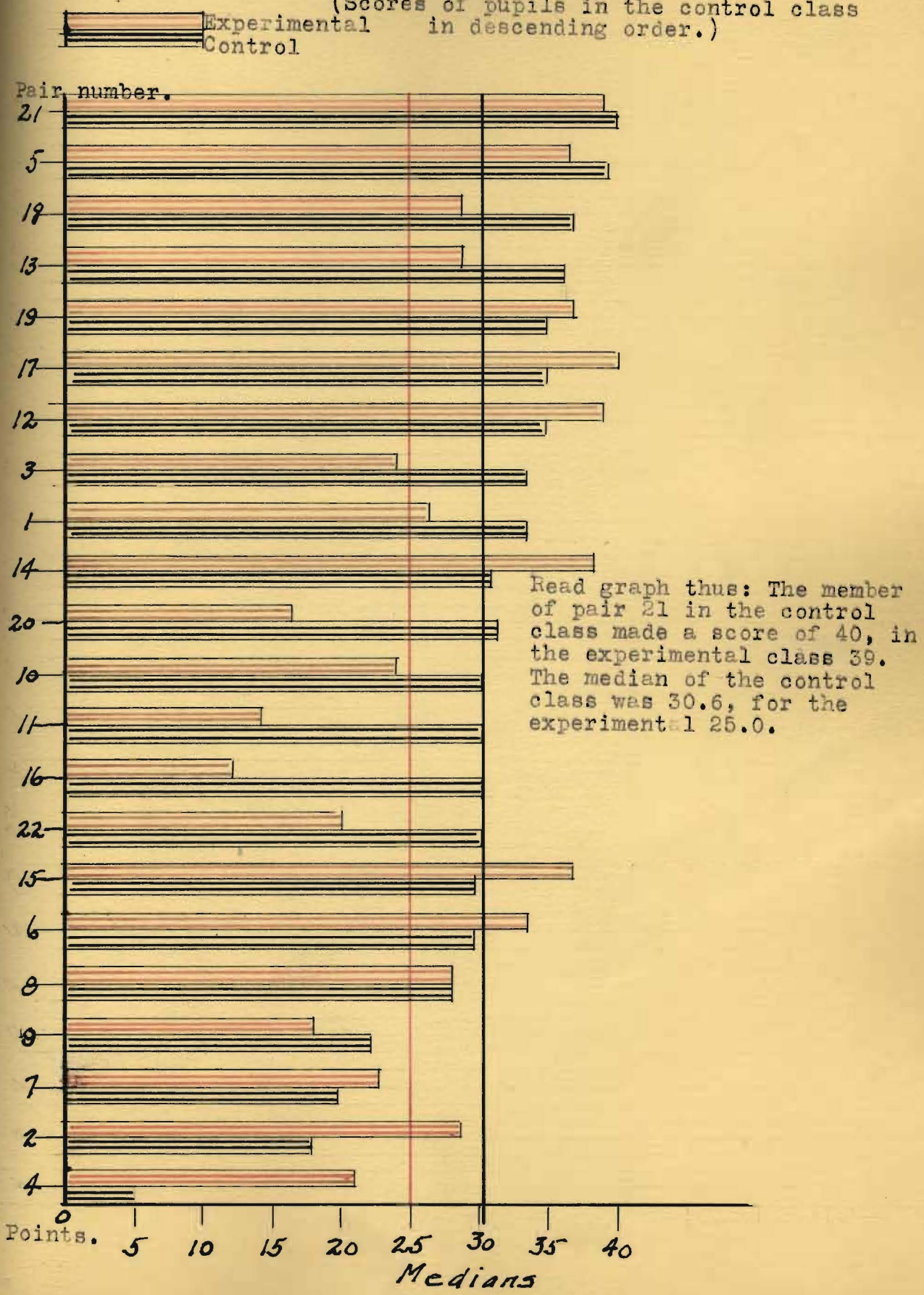
TABLE 8.

SCORES MADE BY PUPILS IN TEST VII.

CONTROL CLASS			EXPERIMENTAL CLASS	
Pair# number	Name of pupil	Score	Name of pupil	Score
1.	L.A.	33	C.A.	26
2.	H.B.	18	O.A.	29
3.	J.C.	33	C.E.	24
4.	D.C.	5	L.B.	21
5.	C.D.	39	A.C.	36
6.	T.E.	29	R.S.	33
7.	J.F.	20	F.K.	23
8.	M.G.	28	G.F.	28
9.	R.H.	22	M.B.	18
10.	D.H.	30	M.A.	24
11.	L.J.	30	J.K.	14
12.	A.J.	35	W.H.	39
13.	V.J.	36	D.S.	28
14.	R.J.	31	S.N.	38
15.	E.K.	29	E.S.	37
16.	V.L.	30	H.R.	12
17.	P.L.	35	L.R.	40
18.	L.L.	37	D.E.	28
19.	F.M.	35	V.L.	37
20.	S.P.	31	G.G.	16
21.	K.S.	40	M.C.	39
22.	W.S.	30	B.F.	20
	Medain	30.6		25.0
	Mean	32.4		27.2
	S.D.	10.5		8.5
	Range	5-40		12-40

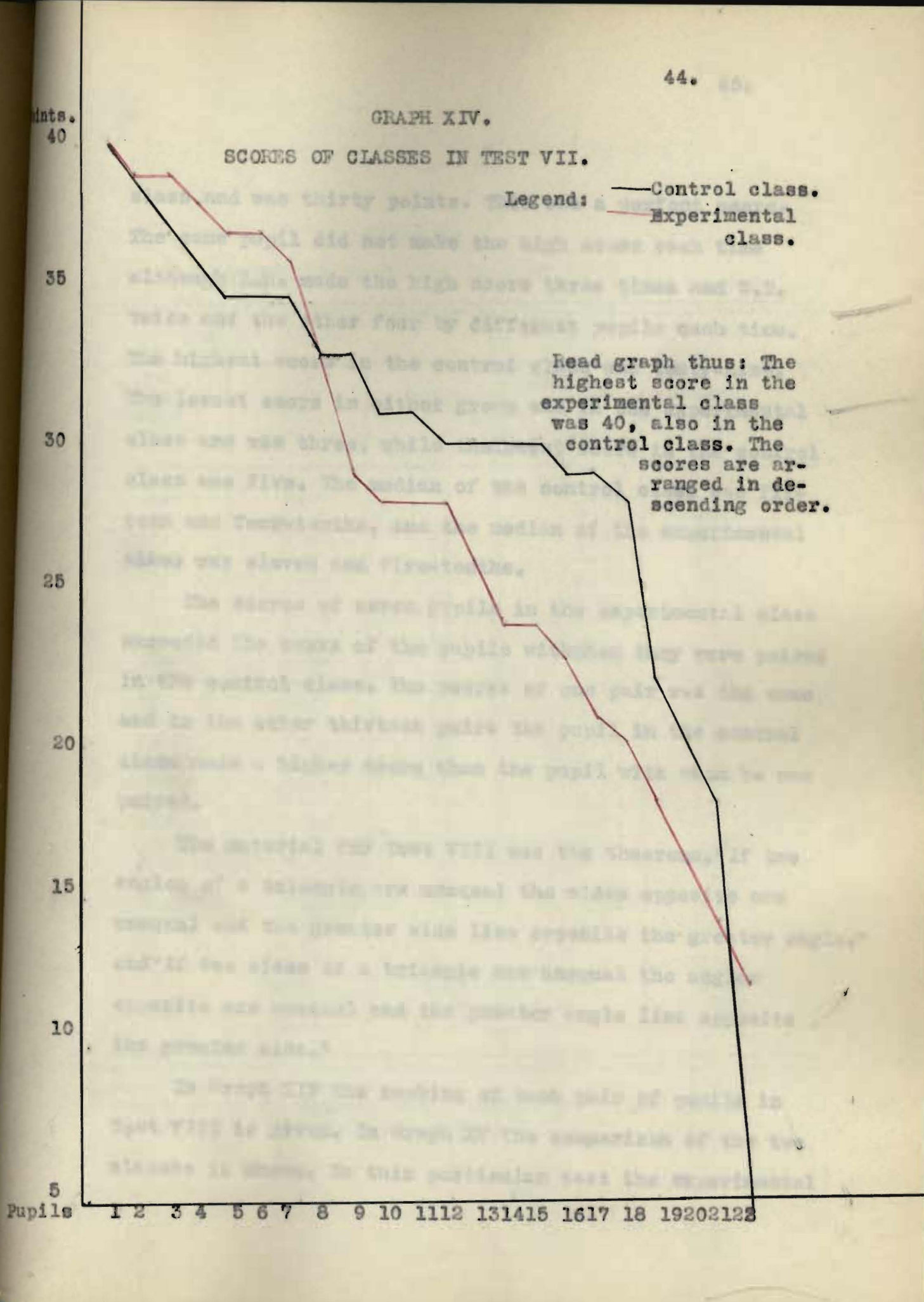
Read across the page.
Score is given in points. The total number of points is 40.
Tests may be found in Appendix A.

GRAPH XIII


Legends
RANKING OF PUPILS IN TEST VII.
(Scores of pupils in the control class in descending order.)
Experimental
Control
Pair number.
21
5
18
13
19
17
12
3
1
14
20
10
11
16
22
15
6
8
9
7
2
4
0
Points.
5
10
15
20
25
30
35
40
Medians
Read graph thus: The member of pair 21 in the control class made a score of 40, in the experimental class 39. The median of the control class was 30.6, for the experimental 25.0.

GRAPH XIV.

SCORES OF CLASSES IN TEST VII.



class and was thirty points. This was a perfect score. The same pupil did not make the high score each time although L.R. made the high score three times and W.H. twice and the other four by different pupils each time. The highest score in the control class was twnty-nine. The lowest score in either group was in the experimental class and was three, while thelowest score in the control class was five. The median of the control class was fifteen and four-tenths, and the median of the experimental class was eleven and five-tenths.

The scores of seven pupils in the experimental class exceeded the score of the pupils withwhom they were paired in the control class. The scores of one pair was the same and in the other thirteen pairs the pupil in the control class made a higher score than the pupil with whom he was paired.

The material for Test VIII was the theorems,"If two angles of a triangle are unequal the sides opposite are unequal and the greater side lies opposite the greater angle." and"If two sides of a triangle are unequal the angles opposite are unequaul and the greater angle lies opposite the greater side."

In Graph XIV the ranking of each pair of pupils in Test VIII is given. In Graph XV the comparison of the two classes is known. In this particular test the experimental

TABLE 9.

SCORES MADE BY PUPILS IN TEST VIII.

CONTROL CLASS			EXPERIMENTAL CLASS	
Pair# number#	Name of pupil	Score	Name of pupil	Score
1.	L.A.	20	C.A.	5
2.	H.B.	8	O.A.	9
3.	J.C.	10	C.E.	13
4.	D.C.	5	L.B.	14
5.	C.D.	15	A.C.	26
6.	T.E.	18	R.S.	12
7.	J.F.	18	F.K.	11
8.	M.G.	13	G.F.	7
9.	R.H.	8	M.B.	3
10.	D.H.	16	M.A.	3
11.	L.J.	19	J.K.	10
12.	A.J.	14	W.H.	30
13.	V.J.	12	D.S.	9
14.	R.J.	15	S.N.	30
15.	E.K.	20	E.S.	18
16.	V.L.	15	H.R.	6
17.	P.L.	29	L.R.	29
18.	L.L.	27	D.E.	25
19.	F.M.	20	V.L.	16
20.	S.P.	13	G.G.	3
21.	K.S.	13	M.G.	29.
22.	W.S.	6	B.F.	4
	Median	15.4		11.4
	Mean	15.5		14.0
	S.D.	6.0		9.5
	Range	5-29		3-30

TT

Read across the page.
Score given in points. Total number of points is 30.
Tests may be found in Appendix A.

Legend

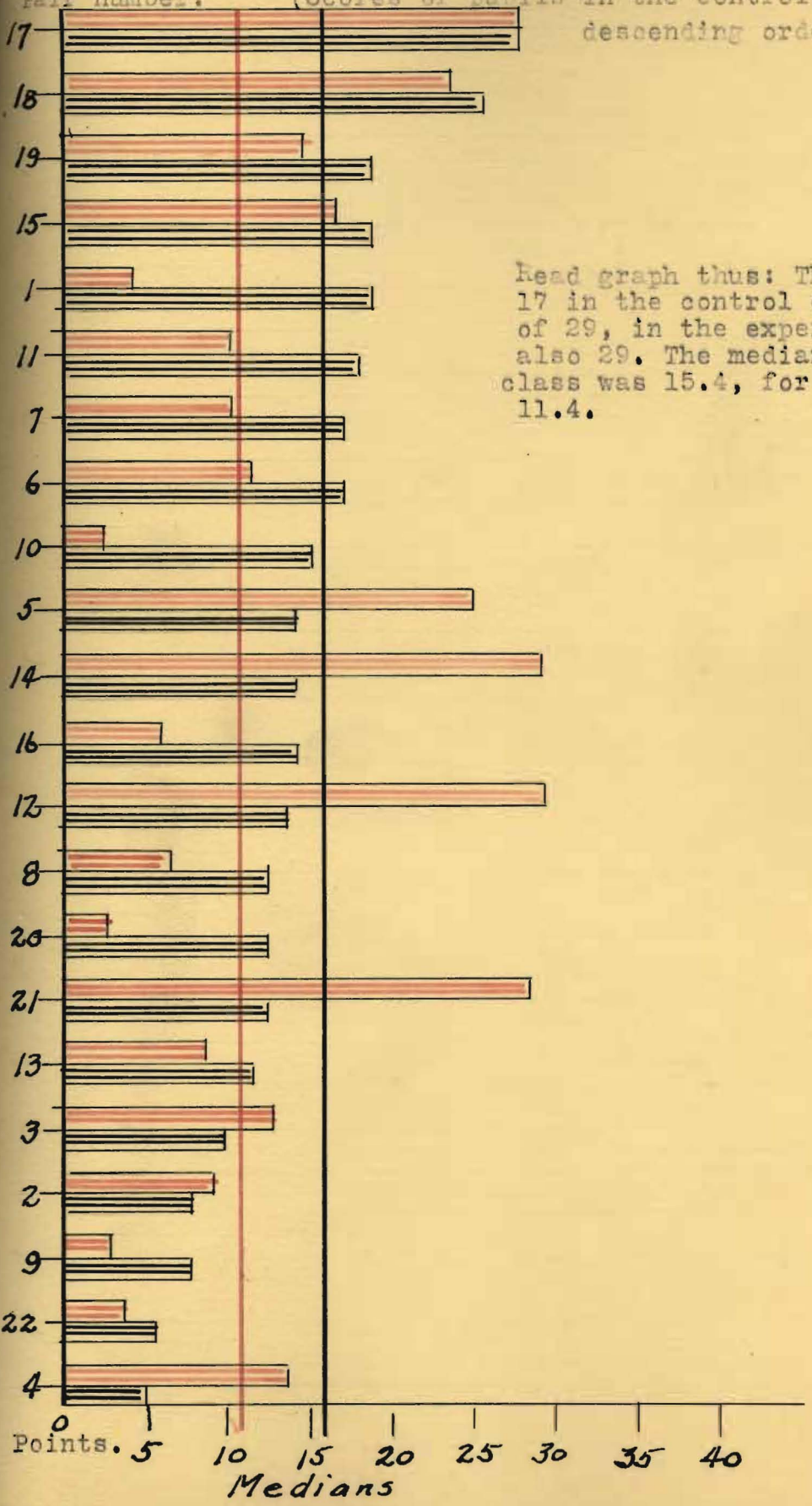
GRAPH XV.

Experimental

Control

RANKING OF PUPILS IN TEST VIII.

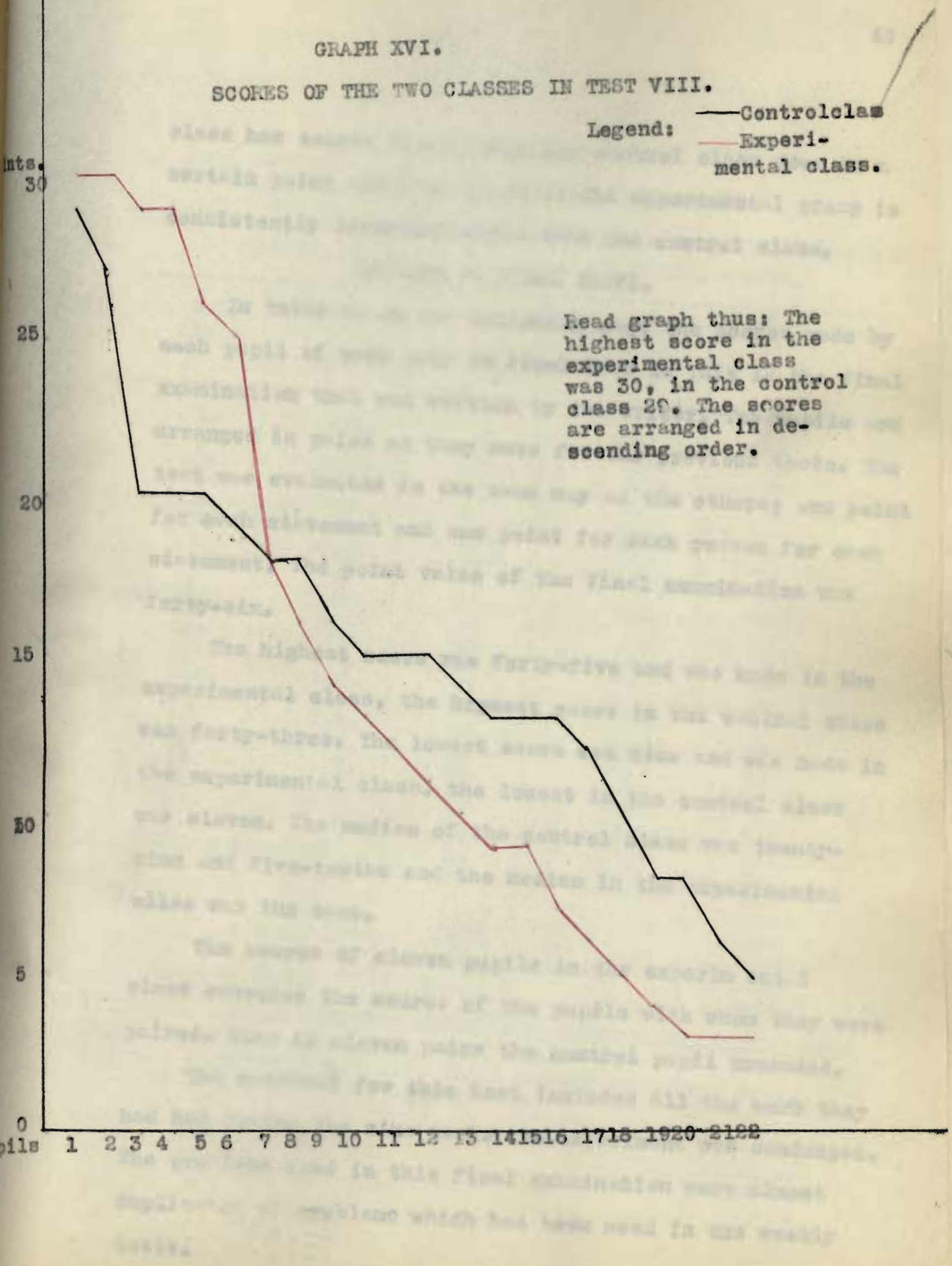
(Scores of pupils in the control class in descending order.)



Read graph thus: The member of pair 17 in the control class made a score of 29, in the experimental class also 29. The median of the control class was 15.4, for the experimental 11.4.

GRAPH XVI.

SCORES OF THE TWO CLASSES IN TEST VIII.



class has scores higher than the control class down to a certain point and from there on the experimental group is consistently lowerein scores than the control class.

RESULTS OF FINAL TEST I.

In Table X. on the following page the scores made by each pupil of each pair in Final Test I. This is the final examination that was written by the writer. The pupils are arranged in pairs as they were for the previous tests. The test was evaluated in the same way as the others; one point for each statement and one point for each reason for each statement. The point value of the final examination was forty-six.

The highest score was forty-five and was made in the experimental class, the highest score in the control class was forty-three. The lowest score was nine and was made in the experimental class, the lowest in the control class was eleven. The median of the control class was twenty-nine and five-tenths and the median in the experimental class was the same.

The scores of eleven pupils in the experim ental class exceeded the scores of the pupils with whom they were paired. Also in eleven pairs the control pupil exceeded.

The material for this test included all the work they had had during the nine weeks the experiment was conducted. The problems used in this final examination were almost duplicates of problems which had been used in the weekly tests.

TABLE 10.

SCORES MADE BY PUPILS IN FINAL TEST I.

CONTROL CLASS			EXPERIMENTAL CLASS	
Pair# number 6	Name of pupil	Score	Name of pupil	Score
1.	L.A.	26	C.A.	18
2.	H.B.	21	O.A.	30
3.	J.C.	32	C.E.	27
4.	D.C.	11	L.B.	18
5.	C.D.	37	A.C.	44
6.	T.E.	33	R.S.	35
7.	J.F.	20	F.K.	28
8.	M.G.	29	G.F.	21
9.	R.H.	12	M.B.	20
10.	D.H.	24	M.A.	17
11.	L.J.	28	J.K.	9
12.	A.J.	30	W.H.	44
13.	V.J.	39	D.S.	33
14.	R.J.	30	S.N.	44
15.	E.K.	43	E.S.	40
16.	V.L.	18	H.R.	16
17.	P.L.	40	L.R.	45
18.	L.L.	40	D.E.	30
19.	F.M.	41	V.L.	45
20.	S.P.	29	G.G.	15
21.	K.S.	38	M.C.	45
22.	W.S.	25	B.F.	12
	Median	29.5		29.5
	Mean	29.7		29.7
	S.D.	9.0		12.0
	Range	11-43		9-45

Read across the page.
Score is given in points. Total number of points 46.
Tests may be found in Appendix A.

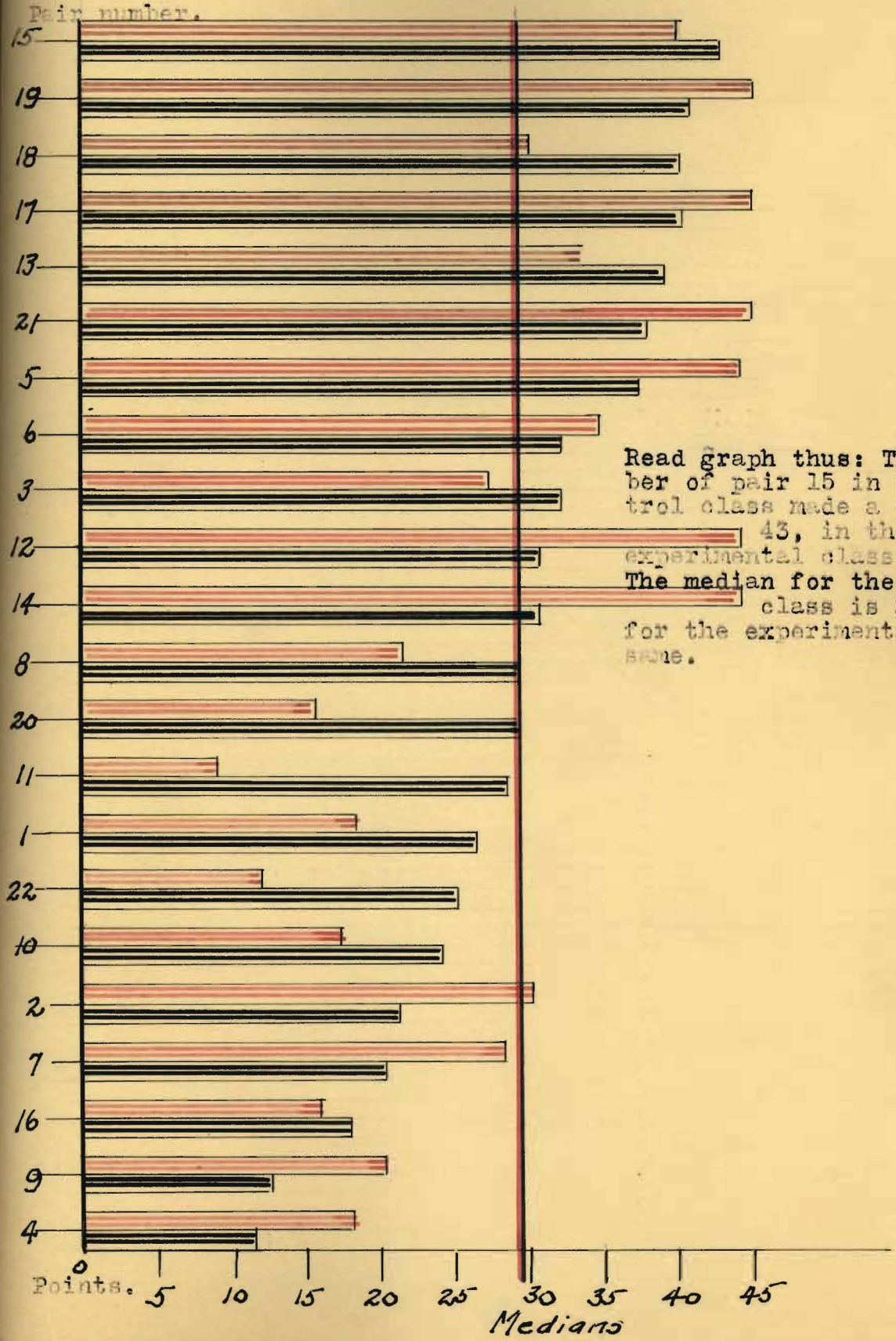
GRAPH XVII

Legend

RANKING OF PUPILS IN FINAL TEST I.

Experimental
Control

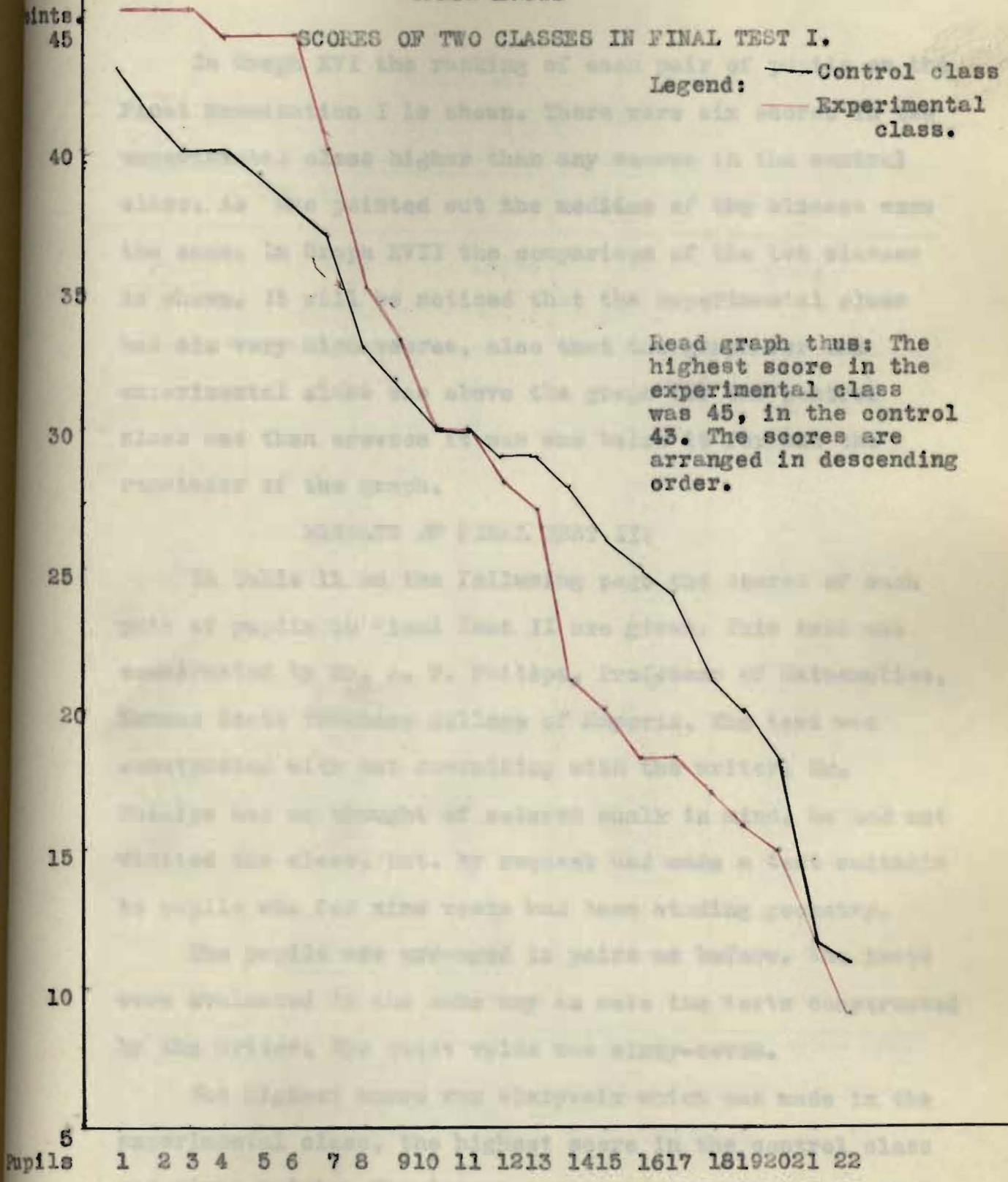
(Scores of pupils in the control class in descending order.)



Read graph thus: The member of pair 15 in the control class made a score of 43, in the experimental class 40. The median for the control class is 29.5, for the experimental the same.

GRAPH XVIII

SCORES OF TWO CLASSES IN FINAL TEST I.



In Graph XVI the ranking of each pair of pupils on the Final Examination I is shown. There were six scores in the experimental class higher than any scores in the control class. As was pointed out the medians of the classes were the same. In Graph XVII the comparison of the two classes is shown. It will be noticed that the experimental class had six very high scores, also that the graph for the experimental class was above the graph for the control class and then crosses it was was below it for all the remainder of the graph.

RESULTS OF FINAL TEST II.

In Table 11 on the followwing page the scores of each pair of pupils in Final Test II are given. This test was constructed by Mr. A. W. Philips, Professor of Mathematics, Kansas State Teachers College of Emporia. The test was constructed with out consulting with the writer. Mr. Philips had no thought of colored chalk in mind. He had not visited the class, but, by request had made a test suitable to pupils who for nine weeks had been studing geometry.

The pupils are arranged in pairs as before. The tests were evaluated in the same way as were the tests constructed by the writer. The point value was sixty-seven.

The highest score was sixty-six which was made in the experimental class, the highest score in the control class was sixty points. The lowest score made was in the control class and was sixteen while the lowest in the experimental

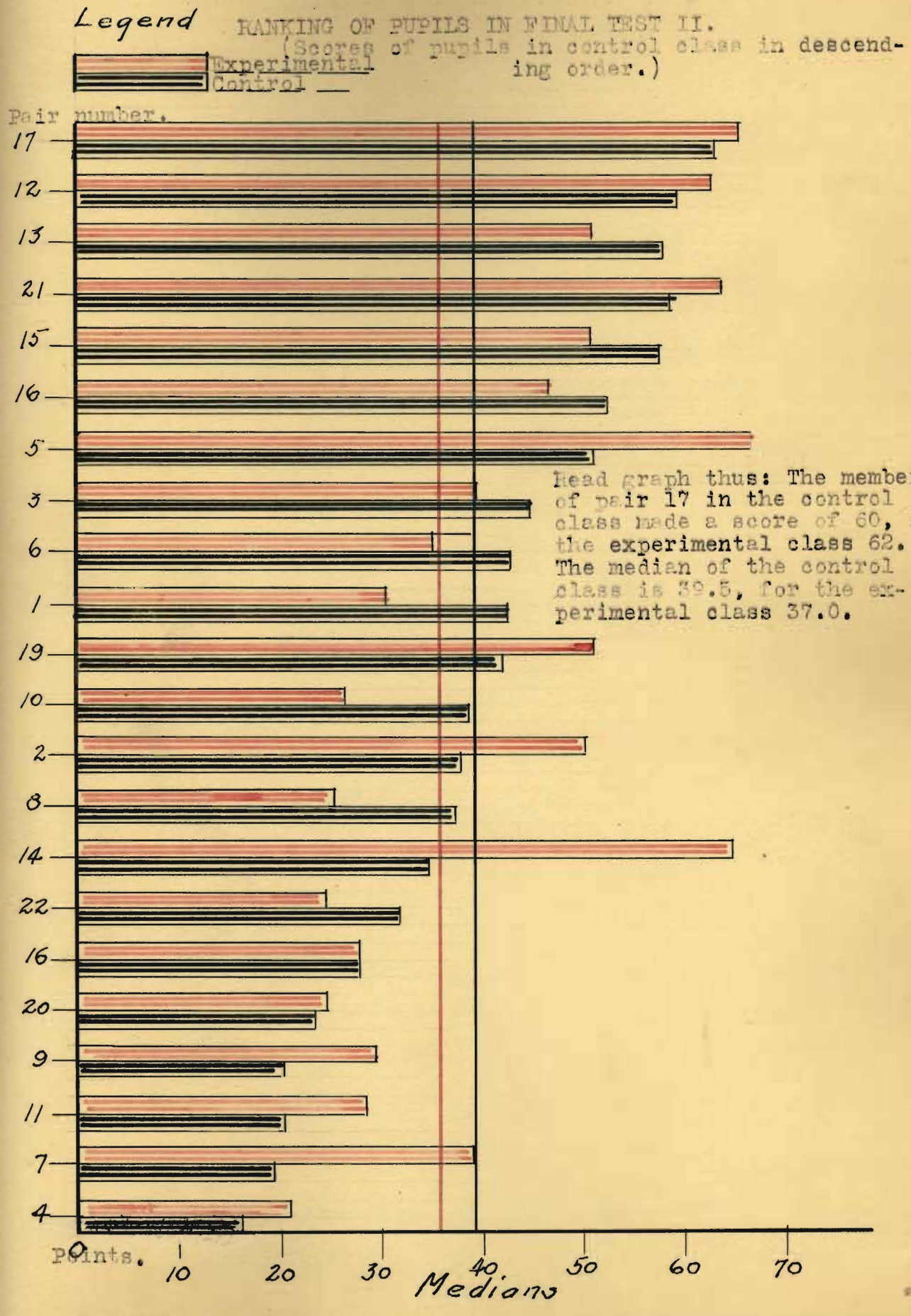
TABLE 11.

SCORES MADE BY PUPILS IN FINAL TEST II.

CONTROL CLASS			EXPERIMENTAL CLASS	
Pair# number	Name of pupil	Score	Name of pupil	Score
1.	L.A.	42	C.A.	30
2.	H.B.	38	O.A.	50
3.	J.C.	45	C.E.	39
4.	D.C.	16	L.E.	21
5.	C.D.	51	A.C.	66
6.	T.E.	43	R.S.	34
7.	J.F.	18	F.K.	39
8.	M.G.	37	G.F.	25
9.	R.H.	20	M.B.	29
10.	D.H.	39	M.A.	23
11.	L.JL	20	J.K.	28
12.	A.J.	69	W.H.	62
13.	V.J.	58	D.S.	51
14.	R.J.	35	S.N.	64
15.	E.K.	57	E.S.	51
16.	V.L.	27	H.R.	27
17.	P.L.	60	L.R.	62
18.	L.L.	52	D.E.	46
19.	F.M.	42	V.L.	51
20.	S.P.	23	G.G.	24
21.	K.S.	58	M.G.	64
22.	W.S.	31	F.F.	24.
	Median	39.5		37.0
	Mean	43.1		41.3
	S.D.	14.0		15.5
	Range	16-60		21-66

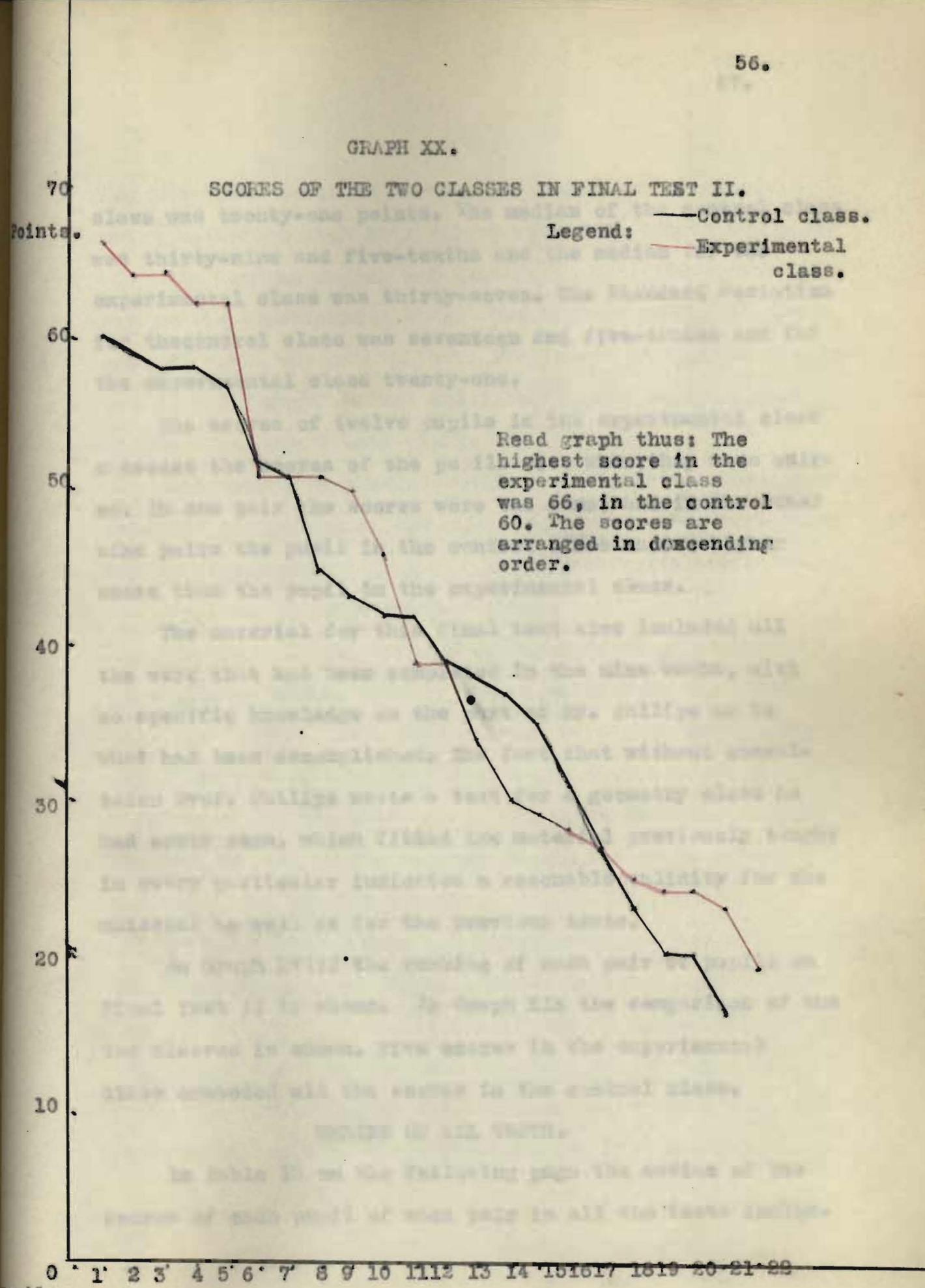
Read across the page.
Score is given in points. Total number of points is 67.
Test may be found in Appendix A.
This test was prepared by Prof. A. W. Phillips.

GRAPH XIX.


Legend
Experimental
Control
RANKING OF PUPILS IN FINAL TEST II.
(Scores of pupils in control class in descending order.)
Pair number.
17
12
13
21
15
16
5
3
6
1
19
10
2
8
14
22
16
20
9
11
7
4
Read graph thus: The member of pair 17 in the control class made a score of 60, in the experimental class 62. The median of the control class is 39.5, for the experimental class 37.0.
0
Points.
10
20
30
40
50
60
70
Medians

GRAPH XX.

SCORES OF THE TWO CLASSES IN FINAL TEST II.



class was twenty-one points. The median of the control class was thirty-nine and five-tenths and the median for the experimental class was thirty-seven. The Standard Deviation for thecontrol class was seventeen and five-tenths and for the experimental class twenty-one.

The scores of twelve pupils in the experimental class exceeded the scores of the pupils with whom they were paired. In one pair the scores were the same, and in the other nine pairs the pupil in the control class made a higher score than the pupil in the experimental class.

The material for this final test also included all the work that had been completed in the nine weeks, with no specific knowledge on the part of Mr. Philips as to what had been accomplished. The fact that without consultation Prof. Philips wrote a test for a geometry class he had never seen, which fitted the material previously taught in every particular indicates a reasonable validity for the material as well as for the previous tests.

In Graph XVIII the ranking of each pair of pupils on Final Test II is shown. In Graph XIX the comparison of the two classes is shown. Five scores in the experimental class exceeded all the scores in the control class.

MEDIAN OF ALL TESTS.

In Table 12 on the following page the median of the scores of each pupil of each pair in all the tests includ-

ing the final tests is shown. The pupils are arranged in pairs as before with L.A. paired with C.A. across the page.

The highest median was ninety-two and five-tenths and was in the experimental class; the highest median in the control class was eighty-four. The lowest median was twenty-two in the control class; the lowest score in the experimental class was twenty-eight.

The scores of eleven pupils in the experimental class exceeded the scores of the pupils with whom they were paired. In eleven pairs the pupil in the control class exceeded the pupil in the experimental class.

In Graph XX the ranking of the medians of each pair of pupils is shown. In pair number seventeen, the pupil from the control class made a score of eighty-four, while the pupil from the experimental class made a score of ninety-two and one-half. Graph XXI shows the comparison of the two classes in the medians of all the tests given including the two final tests.

Graph XXII presents a comparison of the two classes in all the tests given. The arithmetic mean was used in forming the graph. On Tests II,III,IV and on Final Test I, the median was the same for both classes. On all the other tests the control class did slightly better work than the experimental class. The medians varied from one and eight-

TABLE 12.

MEDIANS OF SCORES MADE BY PUPILS IN ALL TESTS.

CONTROL CLASS			EXPERIMENTAL CLASS	
Pair# number	Name of pupil	Score	Name of pupil	Score
1.	L.A.	58.0	C.A.	40.5
2.	H.B.	41.0	O.A.	52.5
3.	J.C.	68.5	C.E.	44.5
4.	D.C.	22.0	L.B.	47.0
5.	C.D.	70.5	A.C.	89.5
6.	T.E.	22.0	R.S.	54.0
7.	J.F.	43.0	F.K.	58.0
8.	M.G.	56.5	G.F.	37.5
9.	R.H.	27.5	M.B.	43.0
10.	D.H.	52.5	M.A.	35.0
11.	L.J.	58.0	J.K.	25.0
12.	A.J.	76.5	W.H.	87.5
13.	V.J.	81.5	D.S.	67.0
14.	R.J.	62.0	S.N.	86.5
15.	E.K.	69.5	E.S.	64.0
16.	V.L.	47.5	H.R.	32.5
17.	P.L.	87.0	L.R.	92.5
18.	L.L.	80.0	D.E.	66.0
19.	F.M.	64.0	V.L.	80.0
20.	S.P.	45.5	G.G.	37.0
21.	K.S.	60.5	M.G.	88.5
22.	W.S.	45.5	B.F.	29.0
	Median	56.7		52.5
	Mean	56.6		57.5
	S.D.	17.5		21.0
	Range	22.0-84.0		28.0-92.5.

Read across the page.

GRAPH XXI

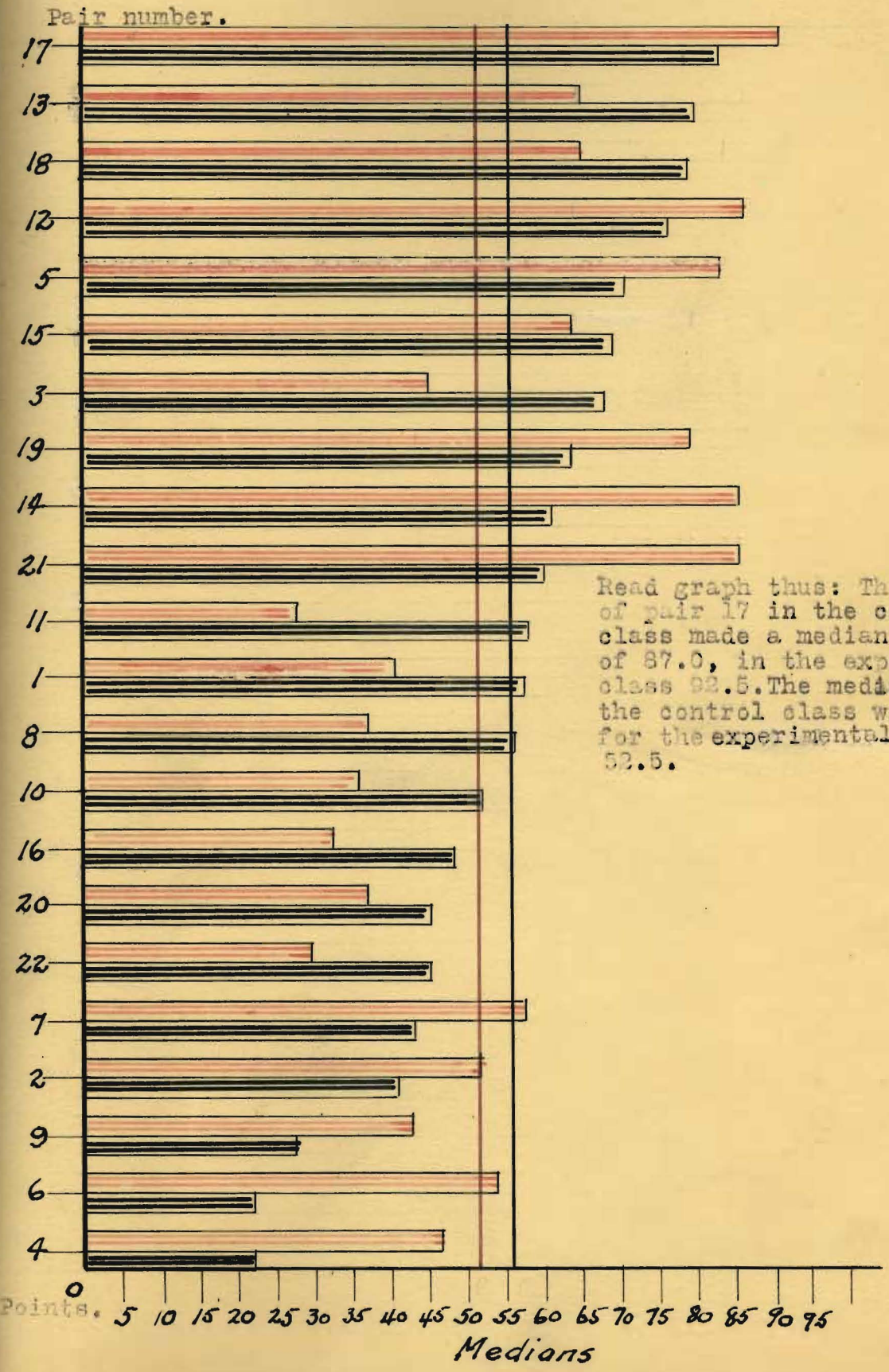
Legends

RANKING OF PUPILS IN MEDIANS OF ALL THE TESTS.

Experimental

Control

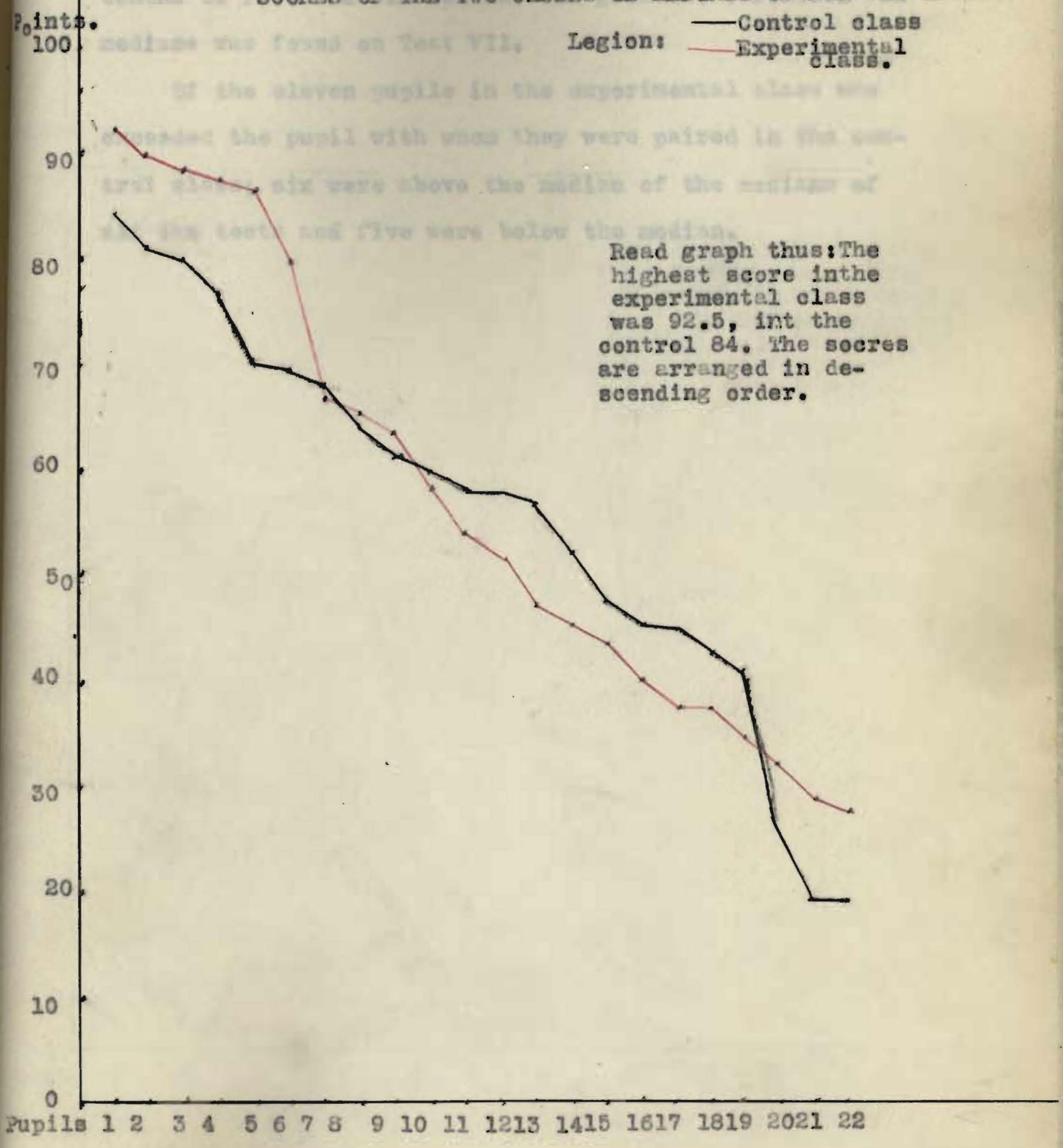
(Scores of pupils in control class in descending order.)



Read graph thus: The member of pair 17 in the control class made a median score of 87.0, in the experimental class 92.5. The median of the control class was 56.7, for the experimental class 52.5.

GRAPH XXII.

SCORES OF THE TWO CLSSES IN MEDIANS OF ALL THE TESTS.

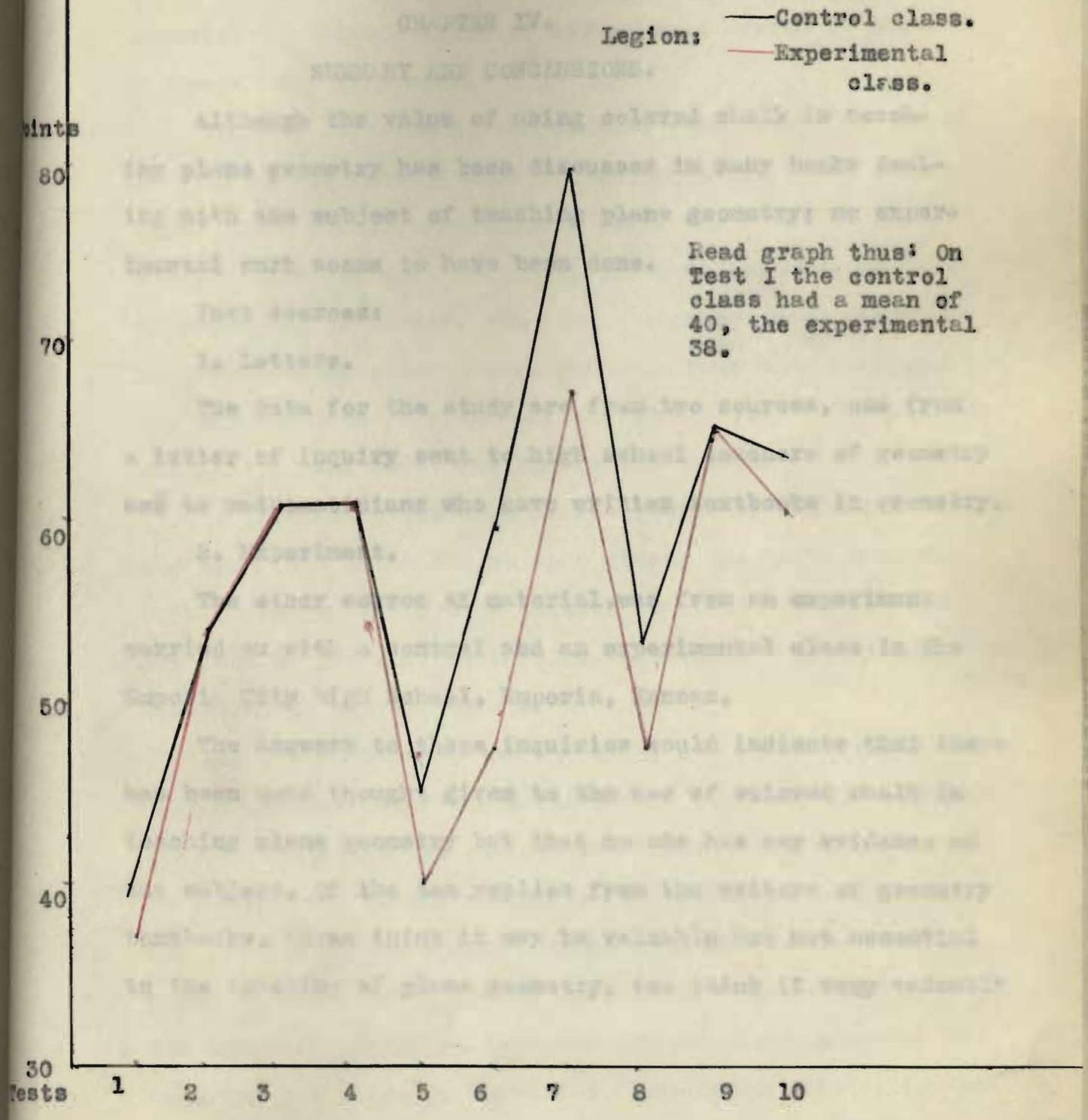


tenths to five and six-tenths. The greatest variation in medians was found on Test VII.

Of the eleven pupils in the experimental class who exceeded the pupil with whom they were paired in the control class; six were above the median of the medians of all the tests and five were below the median.

GRAPH XXIII.

MEAN OF THE CLAS ON ALL THE TESTS.



CHAPTER IV.

SUMMARY AND CONCLUSIONS.

Although the value of using colored chalk in teaching plane geometry has been discussed in many books dealing with the subject of teaching plane geometry; no experimental work seems to have been done.

Test sources:

1. Letters.

The data for the study are from two sources, one from a letter of inquiry sent to high school teachers of geometry and to mathematicians who have written textbooks in geometry.

2. Experiment.

The other source of material, was from an experiment carried on with a control and an experimental class in The Emporia City High School, Emporia, Kansas.

The answers to these inquiries would indicate that there has been some thought given to the use of colored chalk in teaching plane geometry but that no one has any evidence on the subject. Of the ten replies from the writers of geometry textbooks, three think it may be valuable but not essential to the teaching of plane geometry, two think it very valuable

one thinks it valuable but will not yield measurable differences, the others think it serves as a crutch to those who need help very much.

For the experiment, the classes were carefully paired on four bases: intelligence quotient, eighth grade record, ninth grade record, and algebra mark. The other items of control were the same textbook, the same teacher, the same theorems and exercises, the same assignment each day, the same theorems presented each day, the same home work required, and the same teaching methods except that colored chalk was used in one class and not in the other.

A test was given each week. A total of eight tests and two final examinations were given. The tests were all constructed by the writer except Final Test II which was prepared by Prof. A. W. Philips, Mathematics Department, Teachers College of Emporia.

In the first test the median of the control class exceeded the median of the experimental class by two and one-tenths points; in the second test the control class exceeded the experimental classone and eight-tenths points; in the third test the control class exceeded the experimental class by two points; in the fourth test the medians were the same; in the fifth test the control class exceeded the experimental class by three and four-tenths points; in the sixth test the control class exceeded by four and five-

tenths points; in the seventh test by five and six-tenths; in the eighth test by three and nine-tenths; in the first final the medians were the same; and in the second final the control class again exceeded the experimental by two and five-tenths points.

In all except two tests in which the medians were the same the control class exceeded the experimental class by a very small amount. This small amount is an appreciable amount and statistically the difference is not significant.

Of the eleven pupils in the experimental class who exceeded the pupil with whom they were paired in the control class; six were above the median of the medians of all the tests and five were below the median. This would seem to indicate that the colored chalk was not more helpful to the students above the median of the class than to those below the median and vice versa.

The purpose of this study has been an attempt under as carefully controled conditions as was possible using normal class room conditions to make an experimental study dealing with the use of colored chalk in teaching plane geometry. The results are not conclusive as the control situation leaves something to be desired and the time (nine weeks) is insufficient. However, the evidence, without exception,

points to no appreciable difference in the results secured from the two classes. What slight advantage there might have been went to the control class (white chalk).

When consideration is given to the fact that the colored chalk is more expensive; that it soils hands and clothing easily; and, that there is some evidence that it produces no better results than does the ordinary white chalk the conclusion seems justifiable that its usage has depended to a great extent upon traditional practice rather than real values.

TABLE OF APPENDIXES

APPENDIX A

A Complete List of Tests Used.

Test I.

1. Construct two lines perpendicular to each other and equal to 2 and 3 inches respectively.
2. Make an angle of thirty degrees on your paper by use of the protractor.
3. Then make an angle at the other end of the three inch line in (1) equal to this angle in (2).
4. Make the figure into a triangle.
5. Measure with a protractor the number of degrees in the third angle of the triangle.
6. Bisect this angle.
7. Make or erect a perendicular from the vertex of the right angle to the opposite side.
8. Find the center of the base line.

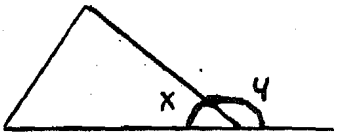
Write this in the best form possible.

Values assigned to each problem.

1.--10 points.
2. --2 points.
3.--10 points.
4.-- 1 point.
5.-- 2 points.
6.--10 points.
7.--10 points.
8.--10 points.
Total 52 points.

Test II.

A. 1.



If x 60, y ?

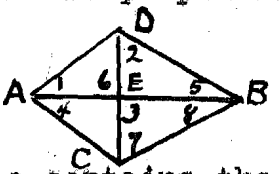
2.

Values assigned to each problem.

A. 1-12 1 point each.
B. 12 points -Total 26.

CA. and AB are perpendicular to each other then $\angle CAB = ?^{o} = -\angle$

3&4



What angle contains the number 1?

What angle contains the number8?

What number does $\angle DEA$ contain?

What number does $\angle BDE$ contain?

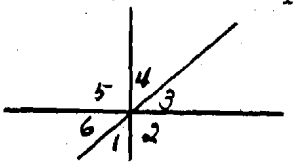
5. Is an angle of 72 acute or obtuse?

6. An angle of 180 is called a -- angle.

7. What is the complement of 60^{0}?

8. What is the supplement of 39^{0}?

9.



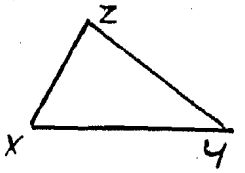
Name the angle equal to angle 3.

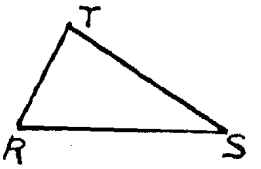
10. Name a right angle in the figure.

11. Name vertical angles in the figures.

12. An isosceles triangle has -- -- equal.

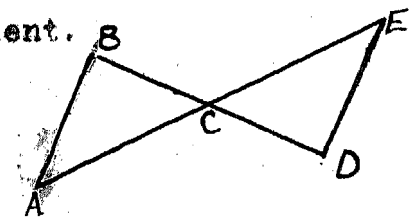
B. If 2 triangles have -- and-- of one equal respectively to -- and -- of the other the -- --. Prove the theorem. Write it out completely.





Test III.

1. If 2 triangles have 2 angles and the included side of one equal respectively to 2 angles and the included side of the other, the triangles are congruent.



2. Given: BC = CD

AC = EC

To prove: AB = DE

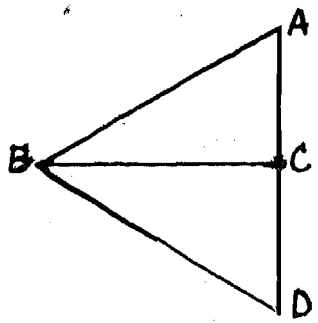
Proof: -------

3. Given: $\angle ABC$ $\angle CBD$

BC $\perp$ AD

To prove: AB = BD

Proof: --------



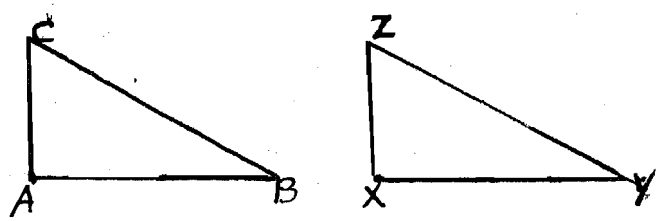
4. Given: Rt.$\triangle$ ABC and Rt.$\triangle$ XYZ.

with AC = XZ

AB = XY

To Prove: $\triangle ABC \cong \triangle XYZ$

Proof:--------



5. State three ways that any triangles may be congruent.

Values assigned to each problem.

1.-- 10 points
2.-- 10 points.
3. --10 points.
4. -- 8 points.
5 -- 6 points.
Total 44 points.

Test IV.

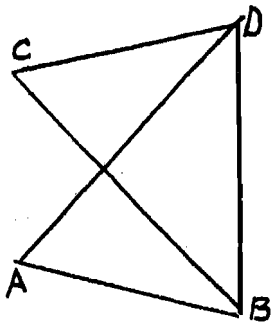
1. Given: ΔABD & ΔBCD

with $CD = AB$

$BC = AD$

To prove: $\angle A = \angle C$

Proof: ------



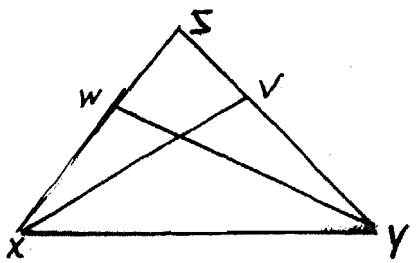
2. Given: Isos. ΔXYZ

with $XZ = YZ$

medians $XV = YW$

To prove: $XV = YW$

Proof: -----



3. Given: ΔABC & $A'B'C'$

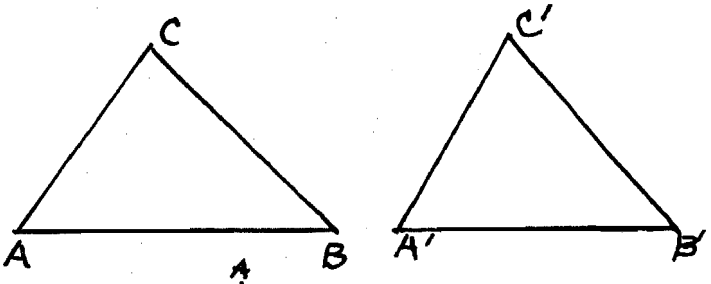
with $AB = A'B'$

$AC = A'C'$

$BC = B'C'$

To Prove: $\Delta ABC \cong \Delta A'B'C'$

Proof:------



4. Given: Isos Δ ABC

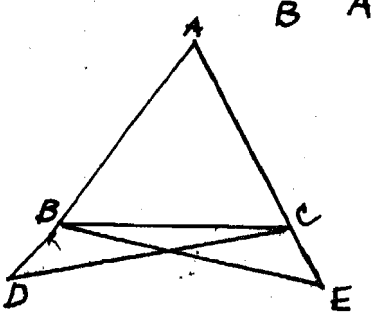
with $AB = AC$

and constructed so that

$BD = CE$

To Prove: $CD = BE$

Proof: ------



Values assigned each problem.
1.-- 10 points
2.-- 10 points.
3.-- 12 points.
4.-- 12 points
Total 44 points.

Test V.

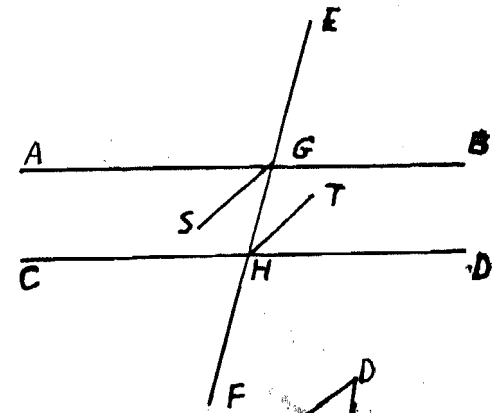
1. Given: AB // CD

cut by the transversal EF at G&H

with SG bisecting ∠AGH

HT bisecting ∠GHD

To prove: SG // HT.



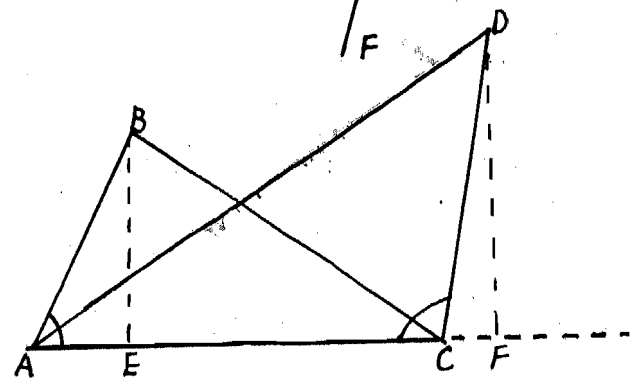
2. Given: △ ABC & ADC

with Alt. BE and DF

∠CAB supplementary ∠ACD

To Prove: (1) AB // CD

(2) EB // DF



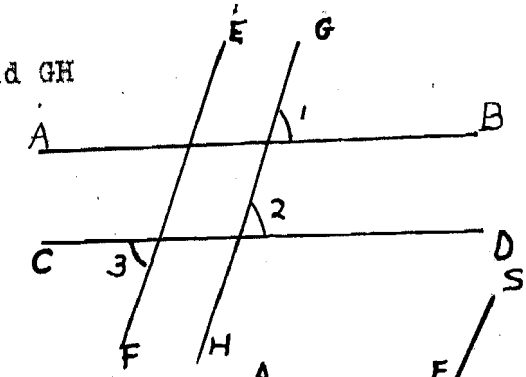
3. Given: AB & CD

cut by the transversal EF and GH

with ∠1 = ∠2, ∠1 = ∠3

To prove: (1) AB // CD

(2) EF // GH

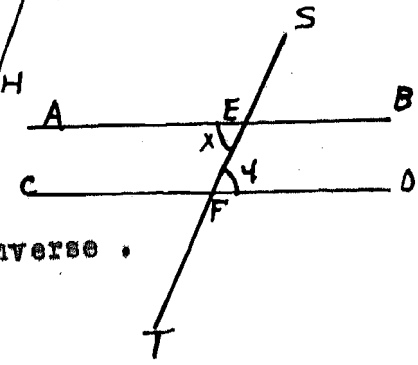


4. Given: AB // CD

cut by the transversal ST at E & F

To Prove: ∠x = ∠y

Use inderect proof. State the converse.

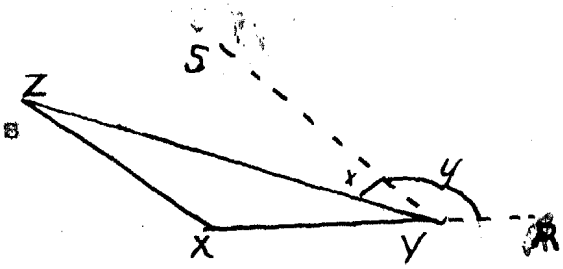


Value assigned to each problem.

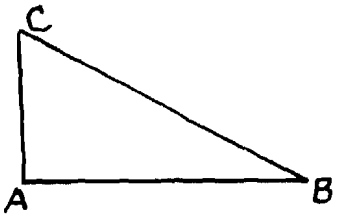
1.-- 8 points.
2.--10 points.
3.--10 points.
4.--12 points.
Total 40 points.

Test VI.

1. Prove the sum of the interior angles of any triangle equal 180 degrees. (Use the figure).



2. Given: Rt. Δ ABC with $\angle C = 2\ \angle B$

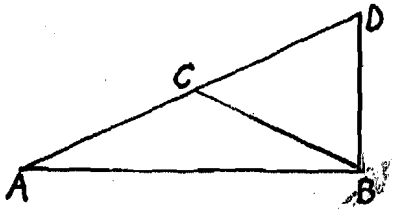


Find the number of degree in each angle of triangle.

3. Given: AC = BC

CD = BC

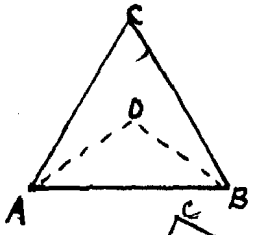
To Prove: $\angle ABD = 90^\circ$



4. Given: Equil. triangle Δ ABC with AD bisecting $\angle A$

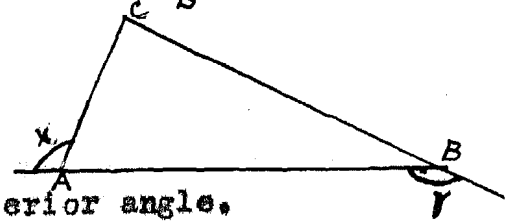
BD bisecting $\angle B$

To prove: $\angle D = 2\ \angle C$



5. Given: Δ APC with exterior $\angle x = 120^\circ$

exterior $\angle y = 150^\circ$



Find the number of degree in each interior angle.

Value assigned to each problem.

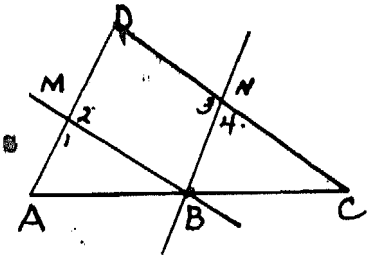
1.-- 8 points.
2.-- 6 points.
3.-- 12 points.
4.-- 8 points.
5.-- 6 points
Total 40 points.

Optional.

5. Given: ΔACD

with $\angle 1$, $\angle 2$, $\angle D$, $\angle 3$, $\angle 4$ each $\cong$ rt. angles

and B the midpoint of AC.



To Prove: $\Delta ABM \cong \Delta BCM$.

(value 10 points)

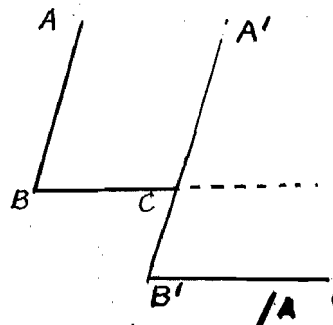
Test VII.

1. (A) Given: $\triangle ABC$ & $A'B'C'$

with $AB \parallel A'B'$

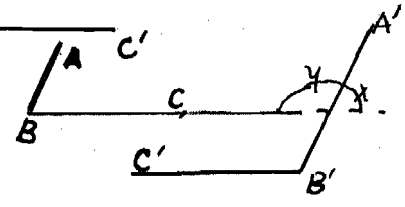
$BC \parallel B'C'$

To Prove: $\angle B = \angle B'$.



(B) Given: Same as (A)

To prove: $\angle B$ is supplementary to $\angle B'$.

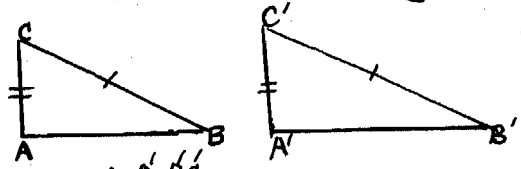


2. Given: Rt $\triangle ABC$ & $A'B'C'$

with $BC = B'C'$

$AC = A'C'$

To Prove: Rt $\triangle ABC \cong$ Rt $\triangle A'B'C'$.

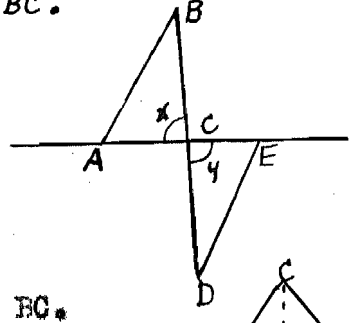


3. Given: $AC = CE$

$AB \nparallel DE$

$\angle x$ and $\angle y$ each = rt. $\angle$

To prove: $AB = DE$.

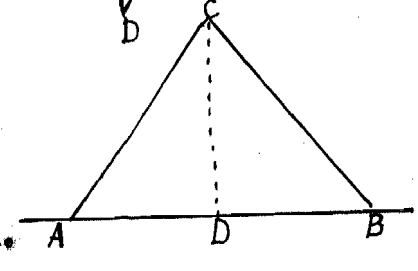


4. Given: 2 oblique lines AC And BC.

with $CD \perp AB$

$\angle A = \angle B$

To prove: $AC = BC$

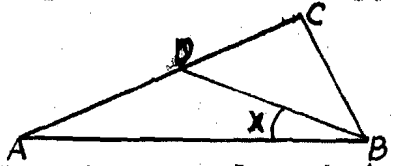


Values assigned to each problem.

1.-- 6 points.
2.-- 8 points.
3.-- 8 points.
4.-- 10 points.
5.-- 8 points.
Total 40 points.

Test VIII.

1. If 2 $\angle$s of a triangle are unequal the sides opposite them are unequal and the greater side is opposite the greater angle. Use this figure.

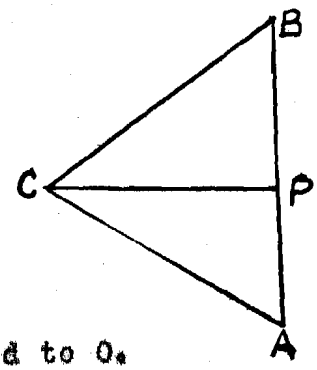


2. Prove that the hypotenuse of a rt.$\triangle$ is its longest side.

3. Given: $CP \perp AB$

$\angle A > \angle B$

To Prove: $BP > PA$.

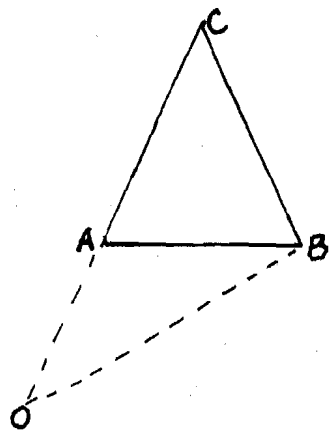


4. Given: Isos.$\triangle$ ABC

with $AC = BC$

and AC prolonged to O.

To Prove: $\angle CBO > \angle COB$.

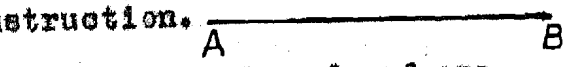


Value assigned to each problem

1.-- 8 points.
2.-- 6 points.
3.-- 8 points.
4.-- 8 points.
Total 30 points.

Final Test I.

1. Erect a perpendicular to AB from a point P and bisect the right angle formed. Show all construction lines, but you need not write out the construction.

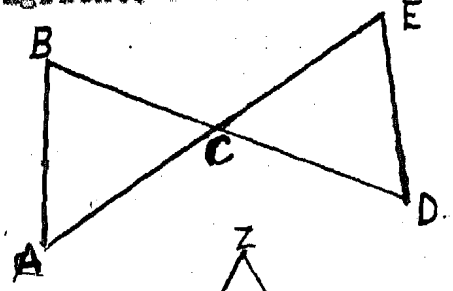


2. If 2 triangles have 2 sides and the included angle of one equal respectivly to 2 sides and the included angle of the other the triangles are congruent. Prove the theorem.

3. Given: BC = CD

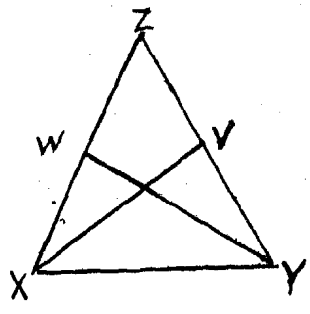
AC = CE

To prove: AB = DE



4. Given: Isos. $\triangle XYZ$ with XZ = YZ and medians XV & YW

o Prove: XV = YW.



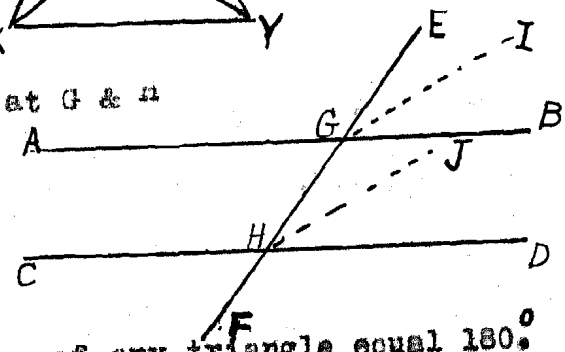
5. Given: AB // CD

cut by the transversal EF at G & H

with GI bisecting $\angle EGB$

HJ bisecting $\angle GHD$

To Prove: GI // HJ.



6. The sum of the interior angles of any triangle equal 180°. Prove the theorem using this figure.

Value assigned to each problem.

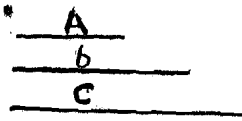
1.-- 6 points.
2.-- 8 points.
3.-- 8 points.
4.--10 points.
5.-- 6 points.
6.-- 8 points.
Total 46 points.

Final Test II-written by Prof. A. W. Phillips.

1. (a) With your compasses and ruler construct accurately an angle of 90.

(b) Using the above figure, construct and angle of 45.

2. Make a triangle having sides equal to three lines.



3. Illustrate with simple sketches the following:

(a) complementary angles.
(b) vertical angles.
(c) an exterior angle of a triangle.
(d) corresponding angles when 2 parallel lines are cut by a transversal.
(e) an obtuse angle.
(f) median of a triangle.

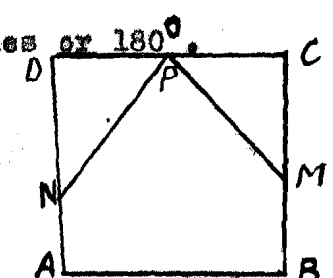
4. This is a sketch of a gate. The brace AB keeps the gate from sagging. What geometric principle explains this fact?

5. (a) Sate the general tests for congruency of triangles.

(b) State the general tests for congruency which apply only to right triangles.

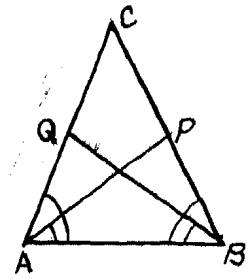
6. Prove in a formal way that the sum of the three angles of any triangle is equal to two right angles or 180°.

7. In the square ABCD the point P bisects CD and BM is made equal to AN, as shown in the figure. Prove PM = PN.



8. (a) State the converse of the following theorem: If 2 angles of a triangle are equal, the sides opposite are equal, and the triangle is isosceles.

(b) Why is it necessary to prove the converse of a geometric truth before this converse may be counted true? Illustrate your answer.



Optional problems for those who have time.

A. The triangle ABC has $\angle A = \angle B$. The lines AP and BQ are so drawn that $\angle BAP = \angle QBA$. Prove that $AP = BQ$

B. Prove that in a right triangle the longest side is the hypotenuse.

Value assigned to each problem.

1.--(a) 4 points.
(b) 3 points.
2.-- 4 points.
3.-- 6 points.
4.-- 3 points.
5.-- (a)3 points.
(b)4 points.
6.-- 8 points.
7.-- 10 points.
8.-- (a)3points.
(b)3 points.
9.-- 10 points.
10.-- 6 points.

Total 67 points.

APPENDIX B.

A copy of letter sent to the high school teachers of geometry.

My Dear Miss------:

I am working on the thesis for my Master's degree and would like to have your opinion of it. The subject is,"To determine the value of using colored chalk in teaching plane geometry."

Do you use colored chalk? Yes --- No----

Do you think the use of colored chalk valuable? Yes ---
No ---

With what particular class or grade of pupils do you think it is most valuable?

In what particular parts of geometry do you feel it is most valuable?

I thank you for your help.

Yours respectfully.

Acopy of letter sent to mathematicans who are writers of textbooks.

My Dear Mr. -------:

I am working on my Master's degree and the subject for my thesis is,"To determine the value of using colored chalk in teaching plane geometry."

Would you please tell me what is your opinion of the use of colored chalk? Do you think it is valuable? Do you advocate its use?

I thank you for your trouble and the help you have been to me.

Yours respectfully.

Appendix C.

Letters from Mathematicans Who Are Textbook Writers.

April 14, 1930

Miss Ruth R. Harris

Emporia, Kansas

Dear Miss Harris:

I have the pleasure to reply to your inquiry relating to the use of colored chalk in the teachingof plane geometry.

I do not believe that its use is essential; there are times, however, when colored chlak and colored pencils are desirable aids. They may be used at the beginning of the congruence theorems to visualize the equal pairs of parts. They may be used to advantage in calling attention toan identical line or angle. They are sometimes wseful in indicating parts in overlapping figures. They may be used to mark four lines of a proportion upon a figur in order to bring out the pair of triangles to prove similar. They enliven the dra ing of figures because children likr to play with colors. Etc.

The use of colored chalk or pencils should not be carried too far as an aid, otherwise, such aid becomes a crutch to visualization and prevents strong growth. Care should be taken in their use so that pupils do not waste time in drawing ornate and pretty designs when they should be gaining power in building proofs. In my own teaching, I reserve the use of colored chalk for my own work at the board. I do not require pupils to use colored chalk or pencils for their class work or for their home work. The use of colored chalk or pencils has a considerable value, if it is not made a general practice.

Very truly yours,

WM. W. Strader
Wm. L. Dickenson High School
Jersey City, New Jersey.

Miss Ruth R. Harris,
Kansas State Teachers College,
Emporia, Kansas.

My dear Miss Harris:

I am not very enthusiastic about the use of colored chalk in teaching plane geometry. I do find it of considerable value in the first month's work in solid geometry.

As far as the plane is concerned it i is of some use for a few figure. Usually however, the solid line, the dotted line, and a broken sketch line give us sufficient variety.

Very sincerly yours,

Geo. A. Harper.

Tucson High School,

Tucson, Arizona.

November 21, 1929

Miss Ruth R. Harris,
1107 Merchant Street,
Emporia, Kansas.

My dear Miss Harris:

The topic which you suggest as a thesis subject in my opinion will be rather a difficult problem which will require a treat deal of reading along the line of psychological investigations done in that particular field, that is if you expect to do a real piece of work,

My general attitude toward the use of colors in teaching geometry is that like models they have their place in thebeginning but should be regarded only as crutches which surely must be discarded as soon as pupils are able to limp along without the use of color.

I would threrfore concentrate my study on comparisons which may be made in the very early study of geometry. I should then later give tests to both groups to see if different studies exist.

If you have any further questions, I shall be glad to have you write me again.

Very sincerly yours,

E. R. Breslich,

School of Education,

University of Chicago.

April 11, 1930

Miss Ruth Harris,
1107 Merchant Street,
Emporia, Kansas.

My dear Miss Harris:

In reply to your letter asking about the use of colored chalk, I will say that in my opinion it is a very valuable aid in the teaching of Plane Geometry. I would certainly advocate its use. There is a danger in the use of colored chalk that shoul be pointed out. It is like a crutch. There comes atime when a crutch should be put into the discard. The so-called crutches in Mathematics about which you have read in eduacational literature are sometimes carried along too long by the student. In a similar manner colored crayon can be used to such an extent that the pu il will not develop spacial imagination as he should. He leans too much on his crutch.

Cordially yours,

J. O. Hassler,
Department of Mathematics,
University of Oklahoma.

April 10, 1930.

Miss Ruth R. Harris,
1207 Merchant Street,
Emporia, Kansas.

My dear Miss Harris:

Your nice letter of April 7 has been received. I wish I were in a position to give you some real help on the question, "The value of using colored chalk in teaching Plane Geometry" but I fear I cannot be of much assistance to you as I have never used it.

I expect one reason I have never used it is because I never have had any on hand at the proper time, hence when it was nemessary to distinguish one line from another in complicated figures I made the lines show up differently by making some of them wavy lines, some dotted, and some with little circles on them. Ofcourse if I had had colored chalk at the time I would have used it to good advantage.

I do not think there is any question about the fact that colored chalk might be helpful in some cases. It is not essential of course, but it certainly can be used to advantage.

I hope you will have fine luck with your thesis. I know I would hate to undertake to write a thesis on your subject although I do believe colored chalk can be used to advantage.

With best wishes, I am.

Sincerly yours,
D. Meade Fernard.
Camp Director, Camp Carlina,
Brevard, North Carolina.

April 10, 1930.

Miss Ruth R. Harris
1107 Merchant ST.
Emporia, Kansas

My dear Miss Harris:

I think that colored chalk has some value in teaching three-dimensional geometry. It facilitates perceptual analysis. Pupils need some guidance in this process during their early experiences in practicing it. It is a fact of common experience of teachers, that many pupils are lacking visual-mindedness. Now, colored chalk is a kind of crutch for those pupils. I shouldn't wish to continue the use of it all through geometry.

Would you please send me a copy of your report?

Very truly yours,

John R. Clark
Principal, High School Division
The Lincoln School,
Teachers College.

April 12, 1930

Miss Ruth R. Harris
1107 Merchant Street
Emporia, Kansas

Dear Miss Harris:

Your letter of April 7 has been received with a request for my opinion concerning the use of colored chalk in the teaching of geometry. I have a vey strong conviction that a moderate use of colored chalk in more or less complicated figures help out. I mean by moderate use, the use of one or possibly two colors in a figue. I think that it detracts oftentimes from a drawing when the drawing is too ornate. I find that the colored chalk in solid geometry is especially helpful in complicated work.

I trust that these few comments will be of some service. I would be glad to comment further if you have additional questions.

Sincerely yours,

Gordon R. Mirick,
Lincoln School,
Teachers College.

May 9, 1930.

Mrs. Ruth Harris,
Emporia, Kansas.

My dear Mrs. Harris:

In reply to your request for my opinion concerning the use of colored chalk in teaching plane geometry, I shall answer directly and briefly the question which you ask.

1. "Do you use colored chalk?"
Yes. Not in any systematic way, but I keep it in my class room and when there is need for giving particular emphasis to some part of a figure I use it freely. The use od colored chalk frequently relieves eye fatigue and makes it possible to present a proof with greater brevity and clearness. For example, I have a mind a fairly involoved orginal problem in which it is necessary to establish the congruency of a pair of overlapping triangles which are rather difficult to distinguish from other triangles in the figure. By drawing one of these in yellow and the other in green say, it is possible for members of the class to keep their attention on the figure and to follow the argument easily by merely referring to the yellow and green triangles.

2."Would you advocate its use?"
Yes, and No. Yes, if used in the inforaml way suggested above where it will add to the clearness of the figure and to the ease and brevity of refernce to it. No, if used always according to some formal and prearranged system which may or may not be a real aid to the eye in following details of the argument.

Very truly yours,

A. W. Philips.
Department of Mathematics,
Kansas State Teachers College
Emporia, Kansas.

November 20, 1929.

Dear Mrs. Harris:

As you know I used to say that I could not teach mathematics without colored chalk and we bought it for th e Department by the box of the good solid colors. Since your letter came I have "mulled" over it and your suggestion for a thesis subject quite a bit. The more I think over it the more it seems as if you are likely to be disapointed in the results. I just can not see how a class using the colors will show enough progress so as to make a difference quantitatively measurable.

With all good wishes from the Lindquists.

Theodore Lindquist,
Michigan State Normal College,
Ypsilanti, Michigan.

April 10, 1930

Miss Ruth R. Harris
1107 Merchant Street
Emporia, Kansas

Dear Miss Harris:

I am afraid I am not able to give you any information with reference to the use of colored chalk in teaching plane geometry. I do not teach the subject and have not taught it for years. Whatever I might have to say could not be backed up by experience.

Very sincerely yours,

Geo. W. Mullins,
Department of Mathematics,
Barnard College,
Teachers College.

April 11, 1930

Miss Ruth R. Harris
1107 Merchant Street
Emporia, Kansas

Dear Miss Harris:

So far as I know there is nothing in the literature on the use of colored chalk in teaching plane geometry. I would be very much interested in such a study but I should want it organized very carefully so as to make it an experiment in educational psychology. Unless this were done ther is a chance that it might be a very trivial subject for a master's dissertation.

I shall be greatly pleased to receive a copy of your study.

Sincerely yours,

Raleigh Schorling
School of Education
University of Michigan