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This study compares the abilities of Maple 6, Mathematica 4.0 and MATLAB Release 12 in solving differential equations. The capabilities of these software programs in solving differential equations both analytically and graphically are compared. Evaluations of the interfaces surrounding the solving of these equations are also included. The primary audience of this study includes students and teachers of differential equations. The underlying motivation for this study is the increasing popularity for introducing differential equations to high school and freshman level college students as well as the increased emphasis on utilizing technology for instructional purposes.

A COMPARATIVE STUDY OF MAPLE, MATHEMATICA, AND MATLAB IN SOLVING DIFFERENTIAL EQUATIONS

A Thesis
Presented to
The Department of Mathematics and Computer Science
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CHAPTER 1
INTRODUCTION

The reform movement in mathematics education [6] has a goal of making mathematics more accessible to all students. Computer algebra systems (CASs) offer a resource that can be used to achieve that goal. Computer algebra helps us simulate data, understand mathematical concepts, and solve tedious, time-consuming equations. In particular, computer algebra systems have drastically improved approaches in solving and manipulating differential equations. However, not all computer algebra systems have the same capabilities or purpose. Maple [12], Mathematica [14], and MATLAB [4] are three well-known general-purpose computer algebra systems used by many colleges and universities. This study compares the behavior of Maple, Mathematica, and MATLAB and their capabilities for solving a variety of differential equations.

A Brief History

The concept of symbolic manipulation with computers has evolved with the computer's development. The first computers were complex counting machines. Until the late 1970's [2], mathematical manipulations on the computer were limited to specific numerical, graphical, and algebraic tasks. The revival and development of programming languages including LISP and C in combination with faster computers, better computer interfaces, and development of new algorithms have helped in the advancement of computer algebra systems.

The following timeline summarizes some of the development of Maple, Mathematica, and MATLAB:

1981 First microcomputer implementation of Maple [16] developed.

1984 The Mathworks [17] founded (makers of MATLAB)

1986 Development of Mathematica [18] begins

1988 First release of Mathematica (version 1.0)

1990 Maple V released with new user interface and support for windows

1996 Release of Mathematica 3.0
Maple V Release 4 launched

1997 Waterloo Maple develops a toolkit for the student edition of MATLAB 5

1999 Casio and Waterloo Maple develop first hand-held version of Maple

2000 Release of Mathematica 4.1
Release of MATLAB Version 6 Release 12
Maple 6 ships on all platforms simultaneously (PC, MAC, UNIX, Linux)

Motivation to Study Computer Algebra Systems

Technology has changed the face of education. According to the 2000 National Council of Teachers of Mathematics (NCTM) Standards [5, p. 24], "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." The integration of computer algebra in education is creating a revolution that is similar to the introduction of the scientific calculator. Time-consuming algebraic manipulations have been "de-emphasized" or eliminated and replaced by higher-level concepts. For example, graphing large polynomial functions by hand has been de-emphasized in many high school mathematics programs. Just as algebraic concepts have been introduced in elementary schools, calculus, statistics, and differential equations will become more significant to the high school curriculum.

There are many general-purpose computer algebra systems available including Maple [12], Mathematica [14], MATLAB [4], MuPAD [11], and Derive [10]. Current comparative reviews are nearly non-existent. The most recent comparisons found ([6], [7], [9], and [13]) show that Maple and Mathematica perform better than their competitors in solving differential equations. MATLAB has never been considered a serious contender with Maple and Mathematica in terms of symbolic computation.

Most literature serves as an introduction or advertisement for a new version of software. They include short magazine articles from Computer Graphics World, Scientific Computing World, Macworld, and PC Week that point out new features added that have specific purposes. For example, Macworld [3] emphasized the improved graphics manipulation tools in Mathematica 4.0.

Computer algebra technology changes quickly and improves to accommodate very specialized tasks. As a result, current versions may not accurately reflect reviews of their predecessors. Therefore it is imperative that each system be continuously evaluated as technology advances.

Review of Literature

Detailed comparisons of modern computer algebra systems are difficult to find due to the continuous changes in technology. The most prominent set of recent comparisons is a collection of articles edited by Michael Wester [13] in 1999. These comparisons have been found in three basic forms:

1. The first [9] compares the speed at which computer algebra systems solve equations and simplify expressions. Processing speeds can often be machine-dependent and are only measurable on tasks that the computer algebra system can successfully complete. This study will not make a comparison based on speed.
2. The second type of comparison [13] takes a variety of problems from logic, algebra, analysis, statistics, number theory, matrix theory, calculus, and programming, and compares the output. Many of these problems do not have known solutions, or contain singularities. The object of this method is twofold:
   i. Find problems that one system solves that another cannot.
   ii. Compare each system's reaction to problems that do not have known solutions.
3. The third type of comparison [6] focuses on symbolics, graphics, numerics, word processing, and several other miscellaneous categories. Some characteristics are rated on a scale from 1 (poor) to 5 (excellent). The others are simply answered with a yes or no.

The variety of methods used for evaluating computer algebra systems has yielded mixed results. In 1999, Michael Wester [13] compared seven CASs including Mathematica 3.0 and Maple V Release 5.1. Included in a 542-problem test suite were twenty-three ordinary differential equations and six partial differential equations. The study revealed that Maple V successfully completed fifteen of the 29 differential equations problems while Mathematica 3.0 successfully completed thirteen. It also showed that Maple V produced partial and complete solutions for all six partial differential equations, while Mathematica 3.0 could do the same for only three [13, p. 58]. Although not explicitly stated by Wester, the evidence in this study indicates that Maple V could have a slight advantage over Mathematica 3.0 in the solving and manipulation of differential equations.

Frank Postel and Paul Zimmermann [7] also compared Maple V and Mathematica 3.0 and their abilities to solve ordinary differential equations:

> "Maple is rather powerful for linear equations and general methods. The ODE solver of Mathematica is weak for linear equations with polynomial coefficients, but does recognize many special kinds of equations. Since Mathematica 3.0, the user does not need to explicitly load the package Calculus 'Dsolve, which has been improved since version 2.2.3." [7, p. 208]

When solving ordinary differential equations, Postel and Zimmermann [7, p. 205] classified the possible outcomes. The most desired outcome is to have the CAS give a correct solution. However, incomplete solutions may be expected. There may also be problems that the CAS is unable to solve. In these cases, the program may simply echo the input, cause an internal error, give an "out of memory error", or give an incorrect answer. Internal error messages often appear out of context and are difficult to understand. They often occur when the CAS is not equipped to deal with a particular procedure or situation within a solving process. Certain problems [7, p. 206] may also cause "out of time or memory" errors. There are two possible interpretations of this error:

1) no solution or
2) a solution exists but more calculation is required.

The final possible outcome when solving differential equations is an incorrect solution. Critics discovered that Maple V Release 3 gave an incorrect solution to the equation $(x^2-1)y'^2-2xyy'+y^2-1=0$. However, the problem was fixed in Maple V Release 4 [8, p. 208].

Despite their in-depth presentation of possible results when solving differential equations, Frank Postel and Paul Zimmermann left two questions unanswered:

1) If two systems are compared and both encounter a problem, which type of problem should be considered easier to accept?
2) How could this information be used to obtain a quantitative comparison of computer algebra systems?

Another study of computer algebra systems focused on mathematics education. Bill Pletsch [6] discussed the varying degrees of implementation of technology in the classroom. He also evaluated seven computer algebra systems, including Mathematica 3.0 and Maple V. Pletsch's evaluation consisted of rating each system on a scale of 1 (poor) to 5 (excellent) in seven areas including symbolics, graphics, numerics, and word processing. His results indicate that Maple V and Mathematica 3.0 have similar strengths and weaknesses. For example, Pletsch gave both systems a rating of 3 for overall ease of use of symbolics, graphics, and word processing. Mathematica received slightly higher ratings for numerics than Maple. However Pletsch [6, p. 322] determined that Mathematica 3.0 was also more difficult to use in this area.

Pletsch's approach to comparing CASs is different in contrast to the technical problem sets presented by Wester, Postel, and Zimmermann. His primary focus was to evaluate the system interface and predict topics that educators may have trouble implementing in a classroom. Wester, Postel, and Zimmermann did not present an evaluation of the interface required to complete their problem sets. A combination of these two approaches is ideal for giving an audience a full evaluation of computer algebra systems.

Motivation for this Study

The combined total of each of these points prompts us to continue to study and compare computer algebra systems. Technology has allowed higher-level mathematics concepts to be presented to younger students. In particular, differential equations courses are now being offered in many high schools across the United States. Thus, a new population of students and teachers is being exposed to computer algebra. This new audience will have a strong interest in technology useful for manipulation of differential equations that is easy to learn, yet powerful enough to handle a wide variety of problems.

Most studies previously considered current are now outdated as a result of the introduction of newer versions of software. We must also consider that the focus of these comparisons has changed. Computer algebra is no longer limited to advanced college students, professors, and graduates in industry. This warrants comparisons that focus on the needs of students and teachers of differential equations in high school and college.

The three types of studies mentioned above will not stand alone as proper evaluations of computer algebra in education. A comparison of processing speeds will not provide significant information to an audience of novice users of computer algebra.

Michael Wester's comparison of seven CASs was very good for applied mathematicians to consider, and may be a good starting point for educators. The results provided on manipulation of differential equations is informative, yet needs further investigation on the most recent versions of software. Finally, Bill Pletsch's study gives significant information on the level of implementation of computer algebra in mathematics education. His focus is very broad and does not thoroughly address differential equations. Pletsch compares many environmental factors of computer algebra, but places little emphasis on computational abilities. Many problems presented to CASs in a classroom will be problems difficult or not reasonable to complete by hand. Thus a CAS capable of working with the "hard" problems is desirable.

This study will address the following goals:

1) Determine which computer algebra system provides the best tools for solving ordinary and partial differential equations. This includes focusing on which CAS works best for what type of problems.
2) Determine which computer algebra system provides the best interface surrounding the solving of differential equations.

CHAPTER 2
METHODS OF COMPARISON

Definitions

A **differential equation** [1] is an equation containing derivatives or differentials of an unknown function. If a differential equation contains partial derivatives of a function of more than one variable, then it is called a **partial differential equation** (PDE). Otherwise, the equation is called an **ordinary differential equation** (ODE).

A **computer algebra system** (CAS) [15] is a math engine that performs symbolic and numerical computations.

The **kernel** of a CAS is the set of fundamental commands required to convert the text and symbols typed by the user into machine instructions and algorithms for basic calculations and input and output operations.

Methodology

This study will compare three computer algebra systems used in numerous colleges and universities across the Unites States: Mathematica 4.0 [14], Maple 6 [12], and MATLAB Release 12 [4]. Maple and MATLAB will be run on a 350 MHz Pentium II with Windows 98. Mathematica will be run on a 450 MHz Pentium II with Windows 98. MATLAB Release 12 is a student package that includes MATLAB version 6 and the Symbolic Math Toolbox, which gives MATLAB access to the kernel from Maple V Release 5. The Partial Differential Equations Toolbox is also included in this package to aid in our evaluation of MATLAB's graphing capabilities. Two of the three systems reviewed are the most recent versions of the software packages. Mathematica 4.0 was used in this review instead of version 4.1. However, no significant changes were made [18] to the differential equation tools from version 4.0 to version 4.1.

Reviewing software can often be a tedious, subjective task. Determining whether one program is "superior" to another often depends on the task and the perspective of the evaluator. An educator may be seeking useful help files, clear error messages, and information on the methods used for solving, as well as exact numerical and graphical solutions. This study will focus on the perspective of educators and students who study analytic methods of solving differential equations.

The comparison of the three systems will occur in two parts. The first part will compare the results of solving forty-one differential equations and initial value problems. This will include the analytic solutions of linear and nonlinear ordinary differential equations, and partial differential equations. Consideration will also be given to different approaches to solving the same equation.

The test suite is compiled from a variety of sources ([1], [7], [8], [13], and [19]), some of which were used in the evaluation of previous versions of Maple and Mathematica (see Appendix A). Many of the problems selected produced errors and unexpected results. In this study, all the evaluations of Mathematica 3.0 (1996) and Maple V Release 5.1 (1998) have been summarized from the compilation of articles ([13] and [7]) edited by Michael Wester. One side effect of this study is that we may also determine whether improvements had been made in the new versions of Maple and Mathematica.

The first priority in selecting differential equations was to maintain variety. The following table indicates the type of equations included in the set as well as the number of problems in each category:

Type of Equation	Number
Ordinary Differential Equations	
Linear homogeneous	7
Linear non-homogeneous	10
Non-linear homogeneous	3
Non-linear non-homogeneous	7
Other (e.g. delay equation)	5
Systems of ordinary differential equations	5
Partial Differential Equations	
Parabolic (heat)	1
Hyperbolic (wave)	1
Elliptic (Laplace and Poisson)	2

The second priority in the selection was to find problems that might generate unsatisfactory results. Several of the problems chosen contain singularities. A computer algebra system that identifies these singularities and reacts appropriately is more desirable than one that does not. Equations containing more than one dependent variable, one integro-differential equation, and one time-delay differential equation are also included in this set (see equations 14, 22, and 5 respectively in Appendix A).

When solving differential equations using a CAS, we must first determine whether a solution is correct and complete. Determining whether a solution is correct will be easy for most of the given equations. Every solution will be verified using at least one of the three systems. Determining whether all solutions have been found is a more difficult issue. If equivalent solutions are given by all three systems, and other independent solutions are not known, then we may assume that all possible solutions have been found. Our priority will remain focused on the problems that yield different results.

If a correct solution is not given, we must then determine which types of responses are desirable. The following list gives priority to what we will consider most desirable to the CAS user:

1. Correct solution(s)
2. Partially correct solution(s)
3. Internal errors
4. Unable to solve (echoes the input)
5. Out of time or memory
6. Incorrect answer

We must be careful when working with CASs to not assume that all possible solutions are given. If users heed this warning, then finding one of many possible solutions set is still a partial success. Internal errors indicate that the CAS began the manipulation process. Most error messages give more information than the non-response characteristic of "unable to solve". As mentioned above, the "out of time or memory" error [7] leaves even more questions unanswered. Finally, an incorrect answer is very misleading. We rely on CASs to give correct output, and we assume its solutions to be correct. The effects of wrong answers can be catastrophic if used in scientific applications.

When comparing the output of Maple, Mathematica, and MATLAB, it is helpful to maintain a scorecard. Each result will be given a rating of 0 to 5 based on the following criteria:

Rating	Description
5	Correct solution. All known independent solutions are included. No complaints regarding the form in which the solution is presented.
4	Correct solution(s), but solution(s) could be presented in better form.
3	A solution is given, but other correct solutions are not given. Expected echo of input (e.g. general form of second order non-homogeneous ODE).
2	Internal errors (CAS could not solve, but seemed to make an attempt).
1	Out of time or memory (out of time indicates more than 20 minutes of processing with no result). Unable to solve (unexpectedly echoes the input).
0	Incorrect answer (verification fails).

Please note that all the evaluations of Mathematica 3.0 (1996) and Maple V Release 5.1 (1998) have been summarized from the compilation of articles edited by Michael Wester [13] in 1999. Although he used a much wider range of symbols to describe his results, each result met the requirements of one of the six categories listed above. Thus a rating from 0 to 5 could be obtained.

In addition to comparing the results of solving differential equations, we will also evaluate the software environment. We will use a five-point scale that will reflect the perspective of a novice CAS user pursuing studies in differential equations. This scale will evaluate the symbolic mathematics, graphics, online help, and user interface of each candidate (see Appendix C). This evaluation will be performed concurrently with the implementation of the test suite. The audience for this comparison will be teachers and students of differential equations who have had minimal experience with computer algebra systems.

CHAPTER 3
RESULTS

The purpose of this chapter is to give an overall picture of the results of this study. We shall illustrate the major differences between Mathematica 4.0, Maple 6, and MATLAB Release 12 shown by the problem test suite. And we shall describe similarities and differences between the environments and syntax surrounding the solving of these differential equations.

Part I: Summary of Test Suite

In the cases where all three systems gave similar results, discussion will be minimal. We will focus on the differences between the systems, and dedicate most of our time discussing the differential equations that reveal the largest differences. Most results are presented side-by-side within a box as shown:

differential equation	equation number (see Appendix A)
solutions determined by each CAS	

Twenty-four problems presented to each CAS were solved correctly. Eight of these problems yielded solutions that were in different forms. For example, all three CASs gave a correct solution set for the equation $2xy^2 + 2y + \frac{dy}{dx}(2x^2 y + 2x) = 0$. But MATLAB gave the simplest form: `y = [ -1/x]` and `[ C1/x]`. Maple 6 gave the following set of solutions:

$$y(x) = -\frac{1}{x},\ y(x) = \frac{-1 + \_C1}{x},\ y(x) = \frac{-1 - \_C1}{x}$$

Mathematica's solution is very similar. The form presented by MATLAB is obviously easier to read and verify.

Equation 12 presented similar issues. However, in this case, Maple and Mathematica gave solutions that were easier to read and dissect. MATLAB did not provide the notation for radicals and integrals:

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + e^{-x^2}y = 0$$ 12

Maple 6

$$y(x) = \_C1\ \cosh\left(\int \sqrt{-e^{(-x^2)}}\, dx\right) + \_C2\ \sinh\left(\int \sqrt{-e^{(-x^2)}}\, dx\right)$$

Mathematica 4.0

$$y[x] \to C[2]\ Cos\left[\sqrt{\frac{\pi}{2}}\ Erf\left[\frac{x}{\sqrt{2}}\right]\right] - C[1]\ Sin\left[\sqrt{\frac{\pi}{2}}\ Erf\left[\frac{x}{\sqrt{2}}\right]\right]$$

MATLAB

```
ans = C1*cos(1/2*exp(-t^2)^(1/2) * exp(1/2*t^2) * 2^(1/2) *
 pi^(1/2) * erf(1/2*t*2^(1/2))) + C2*sin(1/2*exp(-t^2)^(1/2)
 *exp(1/2*t^2)* 2^(1/2)* pi^(1/2)*erf(1/2*t*2^(1/2))
```

Maple 6 and MATLAB gave a better form of solution than Mathematica in problems 23 and 28. Problem 23 was solved using Laplace Transform. Later, we will see the results of solving this equation without specifying a particular method. Solutions appear as follows:

$$\frac{du}{dt} + 2u + 5\int_0^t u(\tau)d\tau = 10e^{(-4t)}$$ 23

Maple 6

$$u(t) = -\frac{40}{13}e^{(-4t)} + \left(\frac{40}{13} + u(0)\right)\cos(2t)\,e^{(-t)} - \frac{2}{13}I\left(\frac{5}{2}I - \frac{13}{4}I\,u(0)\right)e^{(-t)}\sin(2t)$$

Mathematica 4.0

$$u[t] \to -\frac{1}{52}e^{(-4-2i)t}(160\,e^{2it} - (1-8i)\,e^{3t}(10i + (2+3i)\,u[0]) - (8-i)\,e^{(3+4i)t}(10 + (3+2i)\,u[0]))$$

MATLAB

```
ans =  -40/13*exp(-4*t) + 40/13*exp(-t)*cos(2*t) + exp(-t)*u(0)
 *cos(2*t)-1/2*exp(-t)*u(0)*sin(2*t) + 5/13*exp(-t)*sin(2*t)
```

All three solutions are equivalent. MATLAB gave a real-valued function. Maple 6 included some imaginary values, but the solution easily simplifies to MATLAB's answer.

It is difficult to determine whether Mathematica's solution is real or complex without extensive simplification.

Solutions for equation 28 are lengthy. In this case, the first three terms of output from Maple and MATLAB are the particular solutions and the last three terms make up the homogeneous solution. Mathematica's output is difficult to separate into these homogeneous and particular forms. Also note that Mathematica uses Dsolve`t as its variable of integration rather than a single letter variable. Maple and MATLAB produce solutions that are easy to read and dissect.

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = g(x) \qquad 28$$

Maple 6 and MATLAB

$$y(x) = -\frac{1}{2}\int (\cos(x) + \sin(x))\, g(x)\, dx \sin(x) - \frac{1}{2}\int(-\sin(x) + \cos(x))\, g(x)\, dx \cos(x) + \frac{1}{2}\int g(x)\, e^{(-x)}\, dx\, e^x + \_C1 \sin(x) + \_C2 \cos(x) + \_C3\, e^x$$

Mathematica 4.0

$$y[x] \to \frac{1}{2}\left(e^x \int_{C[3]}^{x} e^{-DSolve`t}\, g[DSolve`t]\, dDSolve`t + Cos[x] \int_{C[2]}^{x} g[DSolve`t]\,(-Cos[DSolve`t] + Sin[DSolve`t])\, dDSolve`t - \left(\int_{C[1]}^{x} g[DSolve`t]\,(Cos[DSolve`t] + Sin[DSolve`t])\, dDSolve`t\right) Sin[x]\right)$$

All three CASs found explicit solutions for equation 29. However, three of the four solutions are very messy and are more readable in implicit form. Maple 6 gave the option of presenting an implicit form of solution. MATLAB allowed this option only when invoking the Maple V kernel.

$$\frac{d^2y}{dx^2} + y\left(\frac{dy}{dx}\right)^3 = 0$$ 29

Maple 6 and MATLAB with Maple V Kernel

$$y(x) = \_C1, \frac{1}{6}y(x)^3 + \_C1\ y(x) - x - \_C2 = 0$$

Mathematica 4.0

$$y[x] \to -\frac{C[1] + \left(-3x + \sqrt{-C[1]^3 + 9(x - C[2])^2} + 3C[2]\right)^{2/3}}{\left(-3x + \sqrt{-C[1]^3 + 9(x - C[2])^2} + 3C[2]\right)^{1/3}}$$

$$y[x] \to \left(\left(1 + i\sqrt{3}\right)C[1] + \left(1 - i\sqrt{3}\right)\left(-3x + \sqrt{-C[1]^3 + 9(x - C[2])^2} + 3C[2]\right)^{2/3}\right) \Big/ \left(2\left(-3x + \sqrt{-C[1]^3 + 9(x - C[2])^2} + 3C[2]\right)^{1/3}\right)$$

$$y[x] \to \left(\left(1 - i\sqrt{3}\right)C[1] + \left(1 + i\sqrt{3}\right)\left(-3x + \sqrt{-C[1]^3 + 9(x - C[2])^2} + 3C[2]\right)^{2/3}\right) \Big/ \left(2\left(-3x + \sqrt{-C[1]^3 + 9(x - C[2])^2} + 3C[2]\right)^{1/3}\right)$$

Finally, all three systems produced D'Alembert's solution to the wave equation given as problem 36. MATLAB required the use of the Maple V kernel to be successful. However, this solution left some needed simplification. Maple and Mathematica gave satisfactory solutions:

$$\frac{\partial^2}{\partial x^2}u(x,t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}u(x,t)$$ 36

Maple 6

$$u(x,t) = \_F1(ct + x) + \_F2(ct - x)$$

Mathematica 4.0

$$u[x, t] \to C[1]\left[t + \frac{x}{c}\right] + C[2]\left[t - \frac{x}{c}\right]$$

MATLAB with Maple V Kernel

u(x,t) = _F2((1/c^2)^(1/2)*x+t)+_F1(1/2*x-1/2*t/(1/c^2)^(1/2))

Maple, Mathematica, and MATLAB earned similar unsatisfactory ratings solving six problems from the test suite. This does not mean that the results were always identical. For example, when solving equation 5

$$\frac{dy}{dt} + a \cdot y(t-1) = 0$$

for *y(t)*, each system produced an error. Both Maple 6 and MATLAB had problems with the arguments for *y(t)* and *y(t-1)*:

```
"Error, (in ODEtools/info) found the indeterminate function
 y with different arguments {y(t-1)}"
```

Mathematica gave less information in its error message:

```
"The description of the equations appears to be ambiguous
 or invalid."
```

Solving equation 5 using Laplace transform achieved the same result from each system. Here, the equation was merely rewritten in an equivalent integral equation:

$$y(t) = y(0) - a \int_0^t y(\_U1 - 1)\, d\_U1$$

Use of Laplace transforms allowed us to avoid the variety of error messages. However, obtaining a rewritten form of the original equation can be easily misinterpreted as an explicit solution. We should not consider the solution by Laplace transforms an improvement over obtaining error messages. Converting this differential equation to an algebraic equation by the use of Laplace transform and back required no integration or differentiation of the undefined functions.

Each CAS also achieved the same rating of 3 solving equation 7. In this case, a general solution for this second order homogeneous equation was not expected.

$$\frac{dy}{dx} + P(x)y = Q(x)\frac{d^2y}{dx^2}$$

Again, each CAS gave a different message. Maple presented the solution as an object that could be used with well-defined functions P(x) and Q(x):

$$y(x) = \mathrm{DESol}\left(\{-Q(x)\left(\frac{\partial^2}{\partial x^2}\_Y(x)\right) + \left(\frac{\partial}{\partial x}\_Y(x)\right) + P(x)\,\_Y(x)\}, \{\_Y(x)\} \right)$$

MATLAB gave a similar result, however it was preceded by a commonly seen error message:

```
"Warning: Compact, analytic solution could not be found.
It is recommended that you apply PRETTY to the output.
Try mhelp dsolve, mhelp RootOf, mhelp DESol, or mhelp
allvalues for more information.

> In C:\MATLABR12\toolbox\symbolic\dsolve.m at line 299
  ans = DESol({-Q(t)*diff(Y(t),`$`(t,2))+diff(Y(t),t)+
  P(t)*Y(t)},{Y(t)})"
```

Mathematica gave the most detail in its error message, indicating why an explicit solution could not be found:

```
"Inverse Functions are being used. Values may be lost for
multivalued inverses."
```

The differences in error messages from equation 7 are not consistent with those shown by equation 5, where Mathematica gave the least amount of detail.

Equation 22 was to be solved by the conventional methods provided by each CAS (using 'dsolve' or 'DSolve'). Each system produced an error regarding the unknown integral. The responses are shown as follows:

$$\frac{du}{dt} + 2u + 5\int_0^t u(\tau)d\tau = 10e^{(-4t)} \qquad (22)$$

Maple 6

```
Error, (in ODEtools/info) found the indeterminate function u with
different arguments {u(tau)}
```

Mathematica 4.0

```
Dsolve::nvld : The description of the equations appears to be
ambiguous or invalid.
```

MATLAB with Maple V Kernel

```
dsolve('Dy+2*y+5*INT(y,k,0,t)=10*exp(-4*t)','t')
Warning: Explicit solution could not be found.
> In C:\MATLABR12\toolbox\symbolic\dsolve.m at line 326
     ans =[ empty sym ]
```

Maple 6 seemed to give the most detail in what caused the error. MATLAB gave no indication as to the reason for the error other than that it cannot solve the problem.

The fourth example of problems receiving the same score required that the system solve for both *y(x)* and *k*, shown here as equation 9:

$$\frac{d^2y}{dx^2} + k^2 y = 0, \;\; y(0) = y'(1) = 0$$

The expected non-trivial solution is $y(x) = C\sin([\frac{\pi}{2} + n\pi]x), n \in \mathrm{Z}$. All three systems gave only the trivial solution $y(x) = 0$. However, Mathematica was kind enough to include an error message:

```
Solve::svars :
 Equations may not give solutions for all "solve" variables.
```

Missing non-trivial solutions like this creates a dangerous precedent for solving boundary value problems.

The heat equation presented as problem 35 produced similar ratings for each system. However, only Maple and MATLAB gave possible solutions. Mathematica gave an expected echo of input, knowing that there is not enough information to represent a complete solution set. MATLAB could only find its solution with the help of the Maple V kernel:

$$\frac{\partial^2}{\partial x^2} u(x,t) = \frac{1}{k}\frac{\partial}{\partial t} u(x,t) \qquad 35$$

Maple 6

$$\mathrm{u}(x,t) = \_C3\ \_C1\, \mathrm{e}^{(\sqrt{\_c_1}\, x + \_c_1 k t)} + \_C3\ \_C2\, \mathrm{e}^{(-\sqrt{\_c_1}\, x + \_c_1 k t)}$$

Mathematica 4.0

```
DSolve[u^(2,0)[x, t] == u^(0,1)[x, t]/k , u[x, t], {x, t}]
```

MATLAB with Maple V Kernel

```
u(x,t) = _C3*exp(_c[1]*k*t)*_C1*sinh(_c[1]^(1/2)*x)+
         _C3*exp(_c[1]*k*t)*_C2*cosh(_c[1]^(1/2)*x)
```

Determining which of these solutions is best depends on the perspective of the user. Giving a usable solution that does not include all possible solutions can be misleading. But it does give the user more specific information to work with. This situation could be improved if the solution came with a warning indicating that the solution might not be unique or requires initial or boundary conditions to be complete.

Mathematica also gave an expected echo of input for problems 37 (Laplace equation) and all variations of 38 (Poisson equaion). In these cases Maple 6 was much more liberal about solving. We must also reiterate that MATLAB required access to the Maple V kernel to solve equations with more than one independent variable. The following sequence of results illustrates how Maple attempted to give explicit solutions for equation 37 and the variations of problem 38.

$$\nabla^2 u(x,y,z)=0 \qquad 37$$

Maple 6

$$u(x,y,z) = \_C3\ \_C5 \cos(\%1)\ \_C1\ e^{(\sqrt{\_c_2}\,y+\sqrt{\_c_1}\,x)}$$
$$+\_C3\ \_C5\cos(\%1)\ \_C2\ e^{(\sqrt{\_c_2}\,y-\sqrt{\_c_1}\,x)} + \_C3\ \_C6\sin(\%1)\ \_C1\ e^{(\sqrt{\_c_2}\,y+\sqrt{\_c_1}\,x)}$$
$$+\_C3\ \_C6\sin(\%1)\ \_C2\ e^{(\sqrt{\_c_2}\,y-\sqrt{\_c_1}\,x)} + \_C4\ \_C5\cos(\%1)\ \_C1\ e^{(-\sqrt{\_c_2}\,y+\sqrt{\_c_1}\,x)}$$
$$+\_C4\ \_C5\cos(\%1)\ \_C2\ e^{(-\sqrt{\_c_2}\,y-\sqrt{\_c_1}\,x)} + \_C4\ \_C6\sin(\%1)\ \_C1\ e^{(-\sqrt{\_c_2}\,y+\sqrt{\_c_1}\,x)}$$
$$+\_C4\ \_C6\sin(\%1)\ \_C2\ e^{(-\sqrt{\_c_2}\,y-\sqrt{\_c_1}\,x)}$$
$$\%1 := \sqrt{\_c_1+\_c_2}\,z$$

Mathematica 4.0

$$\text{DSolve}[u^{(0,0,2)}[x,\ y,\ z] + u^{(0,2,0)}[x,\ y,\ z] + u^{(2,0,0)}[x,\ y,\ z] == 0,\ u[x,\ y,\ z],\ \{x,\ y,\ z\}]$$

MATLAB with Maple V Kernel

```
u(x,y,z) =
   _C1*sinh(_c[1]^(1/2)*x)*_C5*sin((_c[1]+_c[2])^(1/2)*z)*_C3*sinh(_c[2]^(1/2)
   *y)+_C1*sinh(_c[1]^(1/2)*x)*_C5*sin((_c[1]+_c[2])^(1/2)*z)*_C4*cosh(_c[2]^(
   1/2)*y)+_C1*sinh(_c[1]^(1/2)*x)*_C6*cos((_c[1]+_c[2])^(1/2)*z)*_C3*sinh(_c[
   2]^(1/2)*y)+_C1*sinh(_c[1]^(1/2)*x)*_C6*cos((_c[1]+_c[2])^(1/2)*z)*_C4*cosh
   (_c[2]^(1/2)*y)+_C2*cosh(_c[1]^(1/2)*x)*_C5*sin((_c[1]+_c[2])^(1/2)*z)*_C3*
   sinh(_c[2]^(1/2)*y)+_C2*cosh(_c[1]^(1/2)*x)*_C5*sin((_c[1]+_c[2])^(1/2)*z)*
   _C4*cosh(_c[2]^(1/2)*y)+_C2*cosh(_c[1]^(1/2)*x)*_C6*cos((_c[1]+_c[2])^(1/2)
   *z)*_C3*sinh(_c[2]^(1/2)*y)+_C2*cosh(_c[1]^(1/2)*x)*_C6*cos((_c[1]+_c[2])^(
   1/2)*z)*_C4*cosh(_c[2]^(1/2)*y)
```

Maple exceeded the 20-minute time limit trying to solve 38a while MATLAB and Mathematica both echoed the input: $\nabla^2 u(x,y,z) = -f(x,y,z)$. The results of this problem are not shown because none of the three systems gave definitive results. Again, this illustrates how Maple 6 continued to attempt to find explicit solutions for this type of partial differential equation.

The results for equation 38b are very similar to that of equation 37. Mathematica's solution is not shown because it is another echo of input.

$$\nabla^2 u(x,y,z) = -1 \qquad \text{38b}$$

Maple 6

$$sol38b := \mathrm{u}(x,y,z) = \%2\ \%1\ \_C1\ \_C3\ \_C5\ \sin(\%3) + \%2\ \%1\ \_C1\ \_C3\ \_C6\ \cos(\%3) + \frac{\%1\ \_C1\ \_C4\ \_C5\ \sin(\%3)}{\%2} + \frac{\%1\ \_C1\ \_C4\ \_C6\ \cos(\%3)}{\%2} + \frac{\%2\ \_C2\ \_C3\ \_C5\ \sin(\%3)}{\%1} + \frac{\%2\ \_C2\ \_C3\ \_C6\ \cos(\%3)}{\%1} + \frac{\_C2\ \_C4\ \_C5\ \sin(\%3)}{\%2\ \%1} + \frac{\_C2\ \_C4\ \_C6\ \cos(\%3)}{\%2\ \%1} - \frac{1}{2}y^2 - \frac{\_C3\ y}{\_C2} - \frac{\_C4}{\_C2}$$

$$\%1 := e^{(\sqrt{\_c_1}\,x)} \quad \%2 := e^{(\sqrt{\_c_2}\,y)}$$

$$\%3 := \sqrt{\_c_1 + \_c_2}\, z$$

MATLAB with Maple V Kernel

```
u(x,y,z) = -1/2*x^2*_c[2]-1/2*x^2*_c[3]-1/2*x^2+
_C1*x+_C2+1/2*_c[2]*y^2+_C3*y+_C4+1/2*_c[3]*z^2+_C5*z+_C6
```

Maple produced an unexpected error message when solving equation 38c. MATLAB (with the Maple V Kernel) and Mathematica both echoed the input as shown by Mathematica's solution.

$$\nabla^2 u(x,y,z) = -\frac{1}{x^2+y^2+z^2} \qquad \text{38c}$$

Maple 6

```
Error, (in pdsolve/sep/casesplit/do) invalid subscript selector
```

Mathematica 4.0

$$\mathrm{DSolve}\Big[\frac{1}{x^2+y^2+z^2} + u^{(0,0,2)}[x,y,z] + u^{(0,2,0)}[x,y,z] + u^{(2,0,0)}[x,y,z] == 0,\ u[x,y,z],\ \{x,y,z\}\Big]$$

Finally, Maple 6 was the only CAS to give a solution for equation 38d. Mathematica and MATLAB (with the Maple V Kernel) both echoed the input.

$$\nabla^2 u(x,y) = -\frac{1}{x^2+y^2} \tag{38d}$$

Maple 6

$$u(x,y) = \_F1(y - Ix) + \_F2(y + Ix) + \frac{1}{8}(-\ln((x^2+y^2)^2) + \ln(x+Iy))\ln(x+Iy)$$

The following differential equations each stimulated partially correct solutions in some of the CASs and complete solutions in the others.

All three systems produced a correct solution for equation 24. However, Mathematica 4.0 gave a complex solution to this problem while Maple and MATLAB gave only the real-valued solution (see Appendix D problem 24). In this case, Maple and MATLAB gave the same form of solution:

$$(1+x+x^2)\frac{d^3y}{dx^3} + (3+6x)\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 6x \tag{24}$$

Maple 6 and MATLAB

$$y(x) = \frac{\_C3}{1+x+x^2} + \frac{\_C2\ x}{1+x+x^2} + \frac{\_C1\ x^2}{1+x+x^2} + \frac{\frac{1}{4}x^4}{1+x+x^2}$$

Mathematica 4.0

$$y[x] \to \frac{1}{12\,(1+x+x^2)}\left(i\sqrt{3}\,(6\,C[1] - (1+2\,x)\,C[2]) + 3\,(x^4 - 2\,C[1] - 4\,x\,C[1] + C[2] + 4\,(1+x+x^2)\,C[3])\right)$$

Neither Maple nor MATLAB gave any warning of other solutions. Thus, Mathematica received a higher rating.

The differences illustrated by problem 31 are not consistent with the results of problem 24. Mathematica and MATLAB gave correct solutions for equation 31, but not a complete solution set. Maple was able to give a more complete solution, but only implicitly. Maple 6 ran out of time trying to find explicit solutions, and MATLAB failed to give the solutions *y(x)*=1 and *y(x)*=-1.

$$x\left(\frac{dy}{dx}\right)^2 - y^2 + 1 = 0 \qquad 31$$

Maple 6

$$y(x)^2 - 1 = 0,$$

$$-2\frac{\sqrt{x\,(y(x)^2-1)}}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}} + \frac{\sqrt{(y(x)-1)\,(y(x)+1)}\,\ln(y(x)+\sqrt{y(x)^2-1})}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}} + \_C1 = 0,$$

$$2\frac{\sqrt{x\,(y(x)^2-1)}}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}} + \frac{\sqrt{(y(x)-1)\,(y(x)+1)}\,\ln(y(x)+\sqrt{y(x)^2-1})}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}} + \_C1 = 0$$

Mathematica 4.0

```
y[x] → Cosh[2 √x + C[1]]
```

MATLAB with Maple V Kernel

```
_C1-2/(y(x)-1)^(1/2)*(x*(y(x)^2-1))^(1/2)/(y(x)+1)^(1/2)+1/(y(x)-1)^
(1/2)*log(y(x)+(y(x)^2-1)^(1/2))/(y(x)+1)^(1/2)*(y(x)^2-1)^(1/2)= 0,

_C1+2/(y(x)-1)^(1/2)*(x*(y(x)^2-1))^(1/2)/(y(x)+1)^(1/2)+1/(y(x)-1)^
(1/2)*log(y(x)+(y(x)^2-1)^(1/2))/(y(x)+1)^(1/2)*(y(x)^2-1)^(1/2)= 0
```

The differences in solutions for equation 19 are even more substantial than that of the previous cases. Maple 6 could not give an explicit solution within the 20-minute time limit, but a correct implicit solution was found. Mathematica ran for more than 30 minutes and still did not produce a solution. MATLAB's dsolve function was unable to find a solution. However, MATLAB was able to find the same solution as Maple 6 by accessing the Maple V kernel.

$$\frac{dy}{dx} = -\frac{1+2x\sin y}{1+x^2\cos y} \qquad 19$$

Maple 6 and MATLAB with Maple V Kernel

$$x + x^2\sin(y(x)) + y(x) + \_C1 = 0$$

Solving equation 10 also presented contrasting results. Maple 6 gave an explicit solution, while Mathematica and MATLAB surprisingly could not solve it, even with MATLAB's access to the Maple V Kernel. Maple 6's solution is not very attractive:

$$(56+59x)\frac{d^3y}{dx^3}+(13+19x)\frac{d^2y}{dx^2}+(-142-59x)\frac{dy}{dx}+(-199-9x)y=0 \qquad 10$$

Maple 6

$$\mathrm{y}(x)=e^{\left(\int \_\mathrm{b}(\_a)\,d\_a+\_C1\right)}\ \&\mathrm{where}\ \Bigg[\Bigg\{\Bigg(56\left(\frac{\partial^2}{\partial\_a^2}\_\mathrm{b}(\_a)\right)+59\,\_a\left(\frac{\partial^2}{\partial\_a^2}\_\mathrm{b}(\_a)\right)$$

$$+168\,\_\mathrm{b}(\_a)\left(\frac{\partial}{\partial\_a}\_\mathrm{b}(\_a)\right)+177\,\_\mathrm{b}(\_a)\,\_a\left(\frac{\partial}{\partial\_a}\_\mathrm{b}(\_a)\right)+56\,\_\mathrm{b}(\_a)^3$$

$$+59\,\_a\,\_\mathrm{b}(\_a)^3+13\left(\frac{\partial}{\partial\_a}\_\mathrm{b}(\_a)\right)+13\,\_\mathrm{b}(\_a)^2+19\left(\frac{\partial}{\partial\_a}\_\mathrm{b}(\_a)\right)\_a$$

$$+19\,\_a\,\_\mathrm{b}(\_a)^2-142\,\_\mathrm{b}(\_a)-59\,\_\mathrm{b}(\_a)\,\_a-199-9\,\_a\Bigg)\Big/(56+59\,\_a)=0\Bigg\},$$

$$\left\{\_\mathrm{b}(\_a)=\frac{\frac{\partial}{\partial x}\mathrm{y}(x)}{\mathrm{y}(x)},\ \_a=x\right\},\ \left\{x=\_a,\ \mathrm{y}(x)=e^{\left(\int \_\mathrm{b}(\_a)\,d\_a+\_C1\right)}\right\}\Bigg]$$

Each CAS also gave different results for Equation 11. Note that this initial value problem is invalid since there is a singularity at $x = 2$. The goal of presenting this problem was to determine the reaction of each CAS to this situation. Mathematica and MATLAB failed to find a solution as expected. Maple 6 surprisingly gave the series solution near the regular singular point ($x = 2$). This solution indicates the behavior of *y(x)* near the singularity.

The solution given by Maple 6 could also be produced with just one of the given initial conditions. Similarly, when no initial conditions were given, the system gave a series solution near $x = 0$. Maple provided no other means for finding a series solution near a regular singular point. Thus giving the invalid conditions was the only means available for obtaining this solution.

The solution found by Maple 6 could not be verified using the available CAS functions. However, it was verified by hand calculations. Mathematica's solving process indicated that a solution with the initial conditions could not be found. MATLAB ran out of time trying to find a series solution via the Maple V kernel. The equation and Maple's solution are shown:

$$(x-2)x^2\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}+e^{(x-1)}y=0,\ y(2)=0,\ y'(2)=1 \text{ series}$$ 11

Maple 6

$$y(x)=\_C1\left(1-\frac{1}{4}e(x-2)+\frac{1}{64}e^2(x-2)^2+\left(-\frac{1}{2304}e^2-\frac{1}{144}\right)e(x-2)^3+\left(\frac{1}{147456}e^3+\frac{1}{768}+\frac{5}{4608}e\right)e(x-2)^4+\left(-\frac{1}{1600}-\frac{23}{460800}e^2-\frac{17}{76800}e-\frac{1}{14745600}e^4\right)e(x-2)^5+O((x-2)^6)\right)+\_C2\left(\left(1-\frac{1}{4}e(x-2)+\frac{1}{64}e^2(x-2)^2+\left(-\frac{1}{2304}e^2-\frac{1}{144}\right)e(x-2)^3+\left(\frac{1}{147456}e^3+\frac{1}{768}+\frac{5}{4608}e\right)e(x-2)^4+\left(-\frac{1}{1600}-\frac{23}{460800}e^2-\frac{17}{76800}e-\frac{1}{14745600}e^4\right)e(x-2)^5+O((x-2)^6)\right)\ln(x-2)+\left(\frac{1}{2}e(x-2)-\frac{3}{64}e^2(x-2)^2+\left(-\frac{1}{48}e+\left(\frac{11}{6912}e^2+\frac{11}{432}\right)e\right)(x-2)^3+\left(\left(\frac{11}{2304}+\frac{1}{512}e\right)e+\left(-\frac{25}{884736}e^3-\frac{25}{4608}-\frac{125}{27648}e\right)e\right)(x-2)^4+\left(\left(-\frac{1}{384}-\frac{1}{15360}e^2-\frac{23}{46080}e\right)e+\left(\frac{137}{48000}+\frac{3151}{13824000}e^2+\frac{2329}{2304000}e+\frac{137}{442368000}e^4\right)e\right)(x-2)^5+O((x-2)^6)\right)\right)$$

Mathematica did not produce favorable results in problems 27 and 32, while both Maple and MATLAB gave complete solutions. The reverse occurs when solving the system of equations presented as problem 34. Mathematica was able to correctly find numerical solutions, while Maple and MATLAB both gave error messages:

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-.25)y=g(x)$$ 27

Maple 6 and MATLAB

$$y(x)=\frac{\int\frac{\cos(x)\,g(x)}{x^{(3/2)}}dx\,\sin(x)}{\sqrt{x}}-\frac{\cos(x)\int\frac{\sin(x)\,g(x)}{x^{(3/2)}}dx}{\sqrt{x}}+\frac{\_C1\sin(x)}{\sqrt{x}}+\frac{\_C2\cos(x)}{\sqrt{x}}$$

Mathematica 4.0

```
NIntegrate::nlim : DSolve`t = x is not a valid limit of integration.
NIntegrate::precw : The precision of the argument
             <<1>>
   function ( ------- ) is less than WorkingPrecision (25).
             DSolve`t
```

$$(x^2-1)\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} + y^2 - 1 = 0 \qquad 32$$

Maple 6 and MATLAB

$$x^2 - 1 + \mathrm{y}(x)^2 = 0,\ \mathrm{y}(x) = x\,\_C1 - \sqrt{\_C1^2+1},\ \mathrm{y}(x) = x\,\_C1 + \sqrt{\_C1^2+1}$$

Mathematica 4.0

```
DSolve[y[x]^2 + (-1 + x^2) y''[x] == 1 + 2 x y[x] y'[x], y[x], x]
```

Mathematica was the only CAS capable of finding graphical solutions for Equation 34. Maple 6 and MATLAB both produced an error message. The results of solving equation 34 also illustrate how graphical solutions are typically found for the purposes of creating a visual representation.

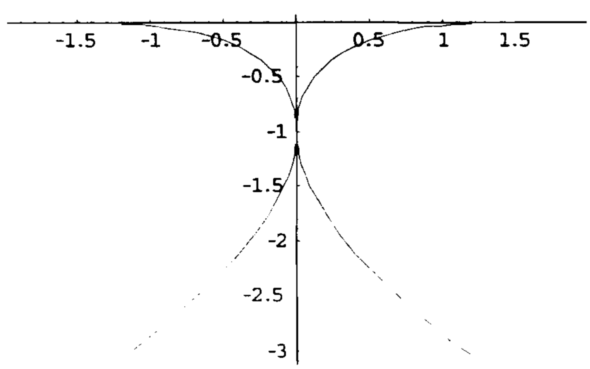
$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \frac{3}{2} \qquad (t - x(t))\frac{dy}{dt} + y(t)\frac{dx}{dt} = 0 \qquad 34$$

$x(0)=0$, $y(0)=-1$, find numeric solutions

Maple 6 and MATLAB with Maple V Kernel

```
Error, (in DEtools/convertsys) unable to convert to an explicit first-
order system
```

Mathematica 4.0

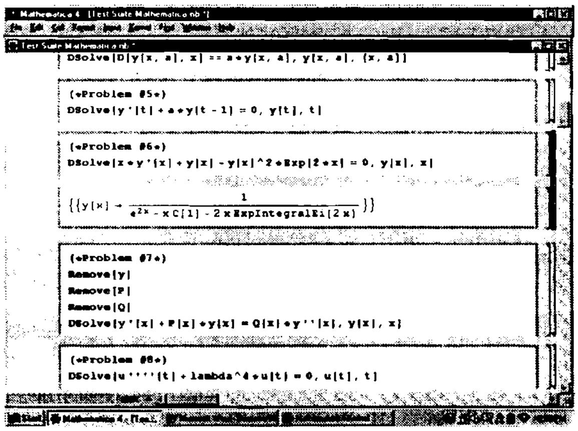


parametric plot of numeric solutions

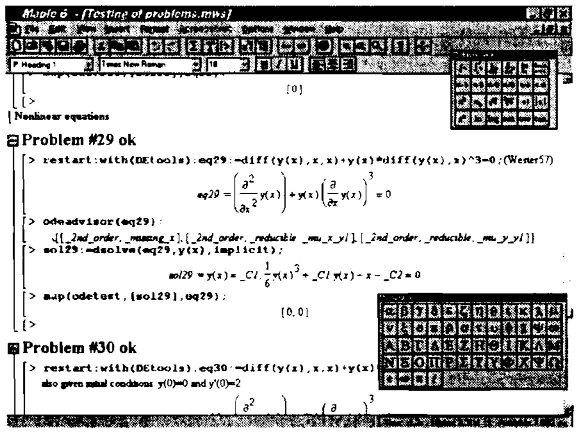
Part II: Summary of Syntax and Environment

The basic interface of each CAS is similar. Each includes a text-based editor with a series of menus to choose from at the top of the screen. Maple adds tool palettes for commonly used symbols as shown in the diagram below. Mathematica also includes tool palettes, but they disappear when the working window is maximized. Both Maple and Mathematica provide for commands to be grouped in collapsible sections. MATLAB's text editor allows for recall of previous commands and the environment allows for easy access to external files. However, it does not allow for notebook style editing.

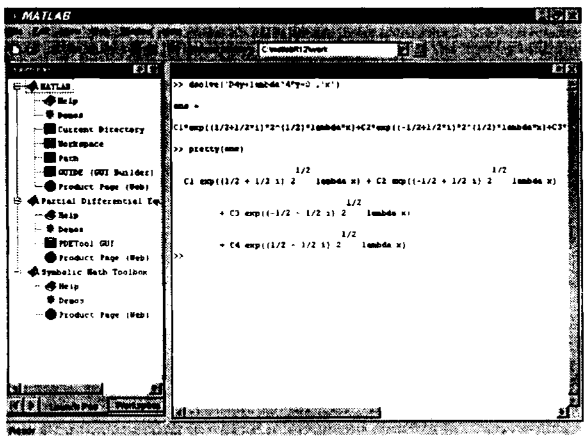
Mathematica 4.0



Maple 6

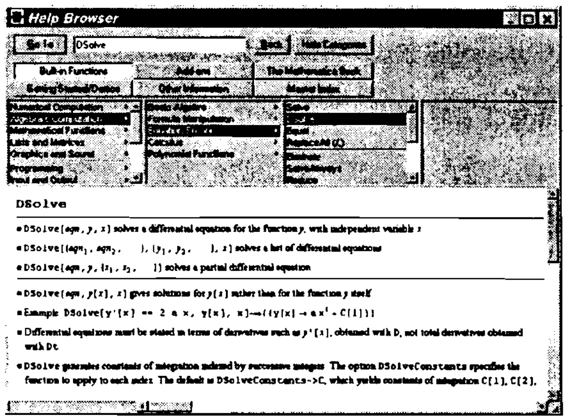


MATLAB Release 12

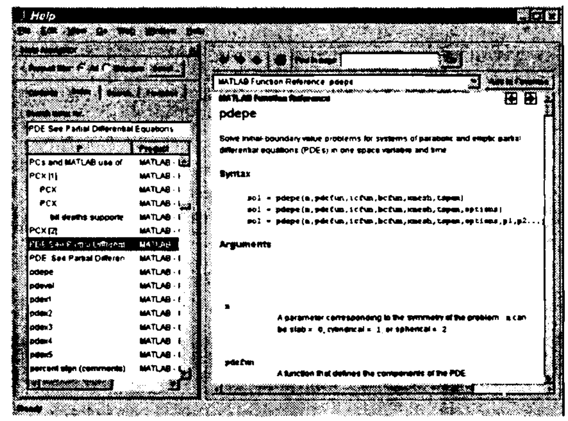


Each CAS comes with detailed help files. All three allow for searches by topic and text. The search engines in Maple 6 and Mathematica are more sophisticated than in MATLAB. Obtaining help for a particular command is as simple as highlighting the command and pressing F1. Mathematica includes an electronic user manual ("The Mathematica Book"). All three give adequate examples with each command. However, Maple 6 and Mathematica give explanations that a novice user can easily understand.

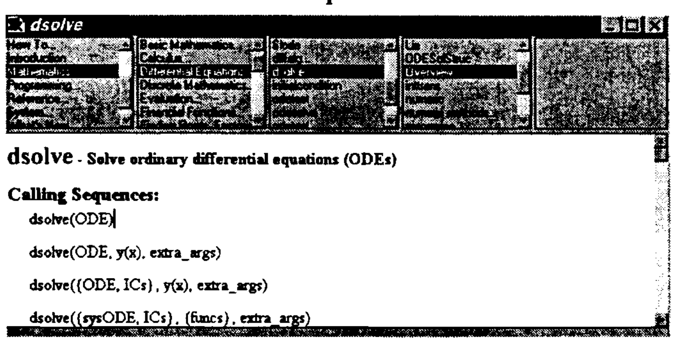
Mathematica 4.0



MATLAB Release 12



Maple 6



There are obvious differences in the problem and solution representations for each CAS. Entering ordinary differential equations in Mathematica is most closely related to actual math symbols used:

```
R'[r] + r * R''[r] - mu^2 * R[r]/r + lambda^2 * r * R[r] == 0
```

Maple 6 is not quite as easy to work with initially. However, the same function, "diff", is used for entering partial differential equations. Also note that the text "mu" converts to the character μ in the output of this statement.

```
eq13:=diff(r*diff(R(r),r),r)-(mu^2/r)*R(r)+lambda^2*r*R(r)=0
```

MATLAB's syntax is also not difficult to enter. However, determining when to use single quotes and when not to can be a frustrating task. For example, the dsolve function requires quotes around each of its parameters:

```
dsolve('(t^2+t+1)*D2y + (4*t+2)*Dy+2*y=3*t^2','t')
```

While the Laplace function only requires quotes around certain nested parameters:

```
Laplace(diff(diff(sym('y(x)')))+4*sym('y(x)'))
```

There are also obvious differences in solving by special procedures. For example, solving by Laplace Transforms is very easy to accomplish in Maple:

```
dsolve(diff(y(t),t)+a*y(t-1)=0,u(t),method=laplace);
```

Mathematica and MATLAB require several steps as shown here in MATLAB:

```
L=Laplace(diff(sym('y(t)'),t)+a*sym('y(t-1)'),t,s)
R=Laplace(0,t,s)
subs('s*laplace(y(t),t,s)-y(0)+a*laplace(y(t-1),t,s)',
  'laplace(y(t),t,s)','LAP')
solve(ans,'LAP')
Ilaplace(ans)
```

Another example involves the request for an implicit solution. Mathematica does not give the user an option to request a particular type of solution. Similarly, without access to the Maple V kernel, MATLAB also does not provide this option. Maple allows the user to request an implicit solution as shown here in Maple 6 and through MATLAB's access to the Maple kernel:

```
dsolve(diff(y(x),x)=-(1+2*x*sin(y(x)))/(1+x^2*cos(y(x)))
,y(x),implicit);
```

```
maple('dsolve','diff(y(x),x)=-
(1+2*x*sin(y(x)))/(1+x^2*cos(y(x)))','y(x)','implicit')
```

Similarly, verification of solutions is much easier to perform with Maple 6 than it is with Mathematica or MATLAB. Maple's verification process utilized the function

```
map(odetest,[sol2],eq2);
```

A return of zero from this function indicated that the solution was correct. Mathematica required a slightly more work, but the process was still fairly simple. In this case, each derivative had to be substituted individually. The "//." symbol indicates that a substitution is to be made and the responses returned are either "true" or "false":

```
FullSimplify[Equation //. Solution //. D[Solution,x]]
```

MATLAB's verification process depended upon whether the solution came from the Maple V kernel. As a result, most of the output from MATLAB was verified on either Maple 6 or Mathematica.

Finding analytic solutions to a differential equation is not always possible. And when they are found, they typically do not give a complete picture of how the solution behaves. Giving a visual representation of the solutions is also very important. Each of our three systems has wonderful graphing capabilities. The commands required to graph in Maple 6 and Mathematica require very little programming skill. However, once set up, Maple 6 and MATLAB have a wide variety of options available. For example, MATLAB allows the user to zoom in on two and three-dimensional graphs, and both Maple and MATLAB allow the user to rotate three-dimensional graphs with the mouse. Mathematica does not give the option to rotate graphs with the mouse, but the viewing perspective can be changed within the plot parameters.

CHAPTER 4
DISCUSSION

Part I: Implications of Test Suite Results

The data collected through the solving of forty-one differential equations indicates that overall there is little difference between Maple 6, Mathematica 4.0, and MATLAB Release 12 (see Appendix B). We can, however, see some differences in performance solving certain types of equations.

Maple 6 and MATLAB appear to have an advantage over Mathematica in solving linear and non-linear equations with singularities. Large differences appeared in three of the nine problems (see equations 10, 27, and 32 in Appendix A). Mathematica was unable to give a solution for any of these three equations.

Maple 6 and MATLAB also appear to have an advantage over Mathematica solving non-linear, non-homogeneous equations. Mathematica was unable to solve two of the seven equations (see equations 19 and 32 in Appendix A). Maple 6 and MATLAB were only successful by finding implicit solutions for these two problems. MATLAB required the use of the Maple V kernel. Otherwise, it would have given the same result as Mathematica.

MATLAB relies heavily on its access to the Maple V kernel to find analytic solutions to differential equations (see Appendix A). This is especially true for solving partial differential equations. MATLAB does not provide methods for representing and solving differential equations with more than one independent variable. Without access to the Maple V kernel, MATLAB's performance in this test suite would be very poor.

The technical aspects of this study are consistent with the results given by Wester [13], Postel, and Zimmermann [7] in 1999. Maple 6 and Mathematica 4.0 remain just as competitive with each other as Maple V and Mathematica 3.0. Improvements in these systems are evident in the results of the test suite of equations. For example, Mathematica showed improvement in its ability to solve problem 24 (see Appendix A). This process broke the kernel of Mathematica 3.0 [7, p. 206]. Mathematica 4.0 successfully solved this equation. Similarly, Maple 6 gave a correct solution for equation 10 (see Appendix A), while Maple V ran out of memory when solving [7, p. 207].

Part II: Implications of Syntax and Environment

Overall ratings of the environment surrounding the solving of differential equations shows that Maple 6 and Mathematica 4.0 are much easier to work with than MATLAB Release 12 (see Appendix C). Maple and Mathematica provided excellent online help and a presentation-ready notebook-style environment. Both systems provided lively interfaces desirable to modern computer users.

MATLAB received less than acceptable ratings in all areas considered except graphics. The PDE Toolbox included in this package provides an interactive environment for solving boundary value problems ranging from heat transfer and diffusion to structural mechanics and electrostatics. This is one of many graphical tools that are very easy to set up and manipulate

Contrasting MATLAB's results, Mathematica's weakest area was in graphics. Although determined reasonably easy to setup, Mathematica lacked zoom capabilities and lacked the capacity to rotate 3-d graphs with the mouse. However, its strengths in other areas help to compensate for this weakness and keep it a very favorable package for solving differential equations.

The ratings of the interfaces of each system are consistent with previous studies ([6] and [9]), with few exceptions. The interfaces of Mathematica and Maple remain competitive. The largest difference between previous studies and this one is that in this study, Maple's graphics achieved a rating that is higher than that of Mathematica. The appearance and ease of set-up of graphs in these two systems is similar. The difference in ratings occurs after the graph is produced. Here, Maple 6 allows the user to interactively manipulate the graph, including changes in axis positions and rotations of 3-dimensional graphs. The only way to perform these tasks in Mathematica 4.0 is to change a parameter in the corresponding graph command.

Recommendations

The following recommendations should be considered to improve the abilities of each CAS to find analytic solutions of differential equations:

1. Continue to develop algorithms for handling differential equations with singularities, especially those with multiple singularities.
2. Continue to develop tools for solving partial differential equations.

3. Improve functions required for verifying solutions to differential equations, especially verification of series solutions.
4. Mathematica needs to provide more flexibility in types of solutions desired. In particular, an option for the user to obtain implicit solutions is desirable.
5. MATLAB should either update its symbolic math toolbox to include access to the Maple 6 libraries or develop more extensive solving functions on its own.

Ideally, a combination of interface features from Mathematica and Maple would be most helpful to the user. Mathematica needs tools for interactively manipulating graphs, while Maple needs improvements in its input procedures by aligning its syntax with hand-written mathematics. The developers of MATLAB need to upgrade its interface to allow for notebook-style editing and graphical representation of mathematical symbols. The following recommendations would also be helpful for novice users to better utilize each computer algebra system:

1. Develop better error messages that suggest alternative tools that may be used for solving a differential equation.
2. Continue to improve the online help utilities by building more extensive examples of problems that work with the system as well as problems for which each system does not work.
3. Continue working to match the input and output of each system with hand-written mathematics.

In addition, computer algebra systems are excellent tools for presentation and exploration of differential equations. However, they are not always flawless in their results. For example, Maple, Mathematica, and MATLAB fail to give non-trivial solutions for some types of boundary value problems (see page 17), and Maple may give solutions for improperly formed initial value problems (see page 22). Thus, users of computer algebra systems must be vigilant about verifying and explaining their results, as well as seeking multiple methods for solving differential equations.

Conclusion

The evidence suggests that although the three computer algebra systems are technically competitive, the differences in environments give Maple 6 a small advantage over Mathematica and a large advantage over MATLAB. Maple 6 and Mathematica 4.0

are very good packages to learn and utilize for presentations. MATLAB would need to update its user interface to remain competitive in the twenty-first century.

Even as this study has progressed, these three computer algebra systems are quickly being replaced by newer versions. Continuous evaluation and feedback will ensure that the developers of computer algebra systems meet the needs of students and teachers of differential equations.

Future comparisons must address the needs of a changing population as younger students use technology to explore mathematics. Developers will need to continue to focus on both the technical abilities of the software as well as the convenience and flexibility of the environment. Current research on computer algebra systems directed at students and teachers must be available for educators to make informed decisions about software selection.

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APPENDIX A

Performance from Test Suite

Rating	Description
5	Correct solution. All known independent solutions are included. No complaints regarding the form in which the solution is presented.
4	Correct solution(s), but solution(s) could be presented in better form.
3	A solution is given, but other correct solutions are not given. Expected echo of input (e.g. general form of second order non-homogeneous ODE).
2	Internal errors (CAS could not solve, but seemed to make an attempt).
1	Out of time or memory (out of time indicates more than 20 minutes of processing with no result). Unable to solve (unexpectedly echoes the input).
0	Incorrect answer (verification fails).

Mp V	**Maple V Release 4**
Mm 3.0	**Mathematica 3.0**
Mp 6	**Maple 6**
Mm 4.0	**Mathmeatica 4.0**
MTB R12	**MATLAB Release 12**

Problem	Source	Mp V*	Mm 3.0*	Mp 6	Mm 4.0	MTB
1. $\frac{dy}{dx} = xe^{(y+\sin x)}$	[18]	n/a	n/a	5	5	5
2. $2xy^2 + 2y + \frac{dy}{dx}(2x^2y + 2x) = 0$	[18]	n/a	n/a	4	4	5
3. $y^2 - x + 2y\frac{dy}{dx} = 0$	[18]	n/a	n/a	5	5	5
4. $\frac{d}{dx}y(x,a) = a \cdot y(x,a)$	[12, p.57]	5	3	5	5	5^M
5a. $\frac{dy}{dt} + a \cdot y(t-1) = 0$ (dsolve)	[12, p.57]	2	2	2	2	2
5b. $\frac{dy}{dt} + a \cdot y(t-1) = 0$ (Laplace)	[12, p.57]	n/a	n/a	2	2	2
6. $x\frac{dy}{dx} + y - y^2e^{2x} = 0$	[2, p.86]	n/a	n/a	5	5	5
7. $\frac{dy}{dx} + P(x)y = Q(x)\frac{d^2y}{dx^2}$	[2, p.30]	n/a	n/a	3	3	3
8. $\frac{d^4u}{dt^4} + \lambda^4u = 0$	[9, p.11]	n/a	n/a	5	5	5

Problem	Source	Mp V*	Mm 3.0*	Mp 6	Mm 4.0	MTB
9. $\frac{d^2y}{dx^2}+k^2y=0,\ y(0)=y'(1)=0$ solve for y and k.	[12, p.57]	3	3	3	3	3
10. $(56+59x)\frac{d^3y}{dx^3}+(13+19x)\frac{d^2y}{dx^2}+(-142-59x)\frac{dy}{dx}+(-199-9x)y=0$	[8, p.205]	1	1	4	1	1
11. $(x-2)x^2\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}+e^{(x-1)}y=0$ $y(2)=0, y'(2)=1$, series	[8, p.206]	1	n/a	3	3	1^M
12. $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+e^{-x^2}y=0$	[2, p.142]	n/a	n/a	5	5	4
13. $\frac{dR}{dr}+r\frac{d^2R}{dr^2}-\frac{\mu^2R}{r}+\lambda^2rR=0$	[9, p.308]	n/a	n/a	5	5	5
14. $\frac{du}{dt}+K(t)\cdot u(t)=f(t)$	[9, p.1]	n/a	n/a	5	5	5
15. $3\frac{du}{dt}-2u(t)=\cos(t)$	[18]	n/a	n/a	5	5	5
16. $\frac{dy}{dx}=-y(x)-1$	[2, p.96]	n/a	n/a	5	5	5
17. $\frac{dy}{dx}=\frac{y}{x}+\frac{x}{y}$	[12, p.57]	3	5	5	5	5
18. $x^2\frac{dy}{dx}+3xy=\frac{\sin x}{x}$	[12, p.57]	5	5	5	5	5
19. $\frac{dy}{dx}=-\frac{1+2x\sin y}{1+x^2\cos y}$	[12, p.57]	5	4	5	1	5^M
20. $(x^2+x+1)\frac{d^2y}{dx^2}+(4x+2)\frac{dy}{dx}+2y=3x^2$	[8, p.198]	5	n/a	5	5	5
21. $\frac{d^2y}{dx^2}+4y=\sin(2x),\ y(0)=y'(0)=0$	[12, p.57]	5	5	5	5	5
22. $\frac{du}{dt}+2u+5\int_0^t u(\tau)d\tau=10e^{(-4t)}$ dsolve	[12, p.58]	2	1	2	2	2^M
23. $\frac{du}{dt}+2u+5\int_0^t u(\tau)d\tau=10e^{(-4t)}$ Laplace	[12, p.58]	5	4	5	4	5
24. $(1+x+x^2)\frac{d^3y}{dx^3}+(3+6x)\frac{d^2y}{dx^2}+6\frac{dy}{dx}=6x$	[8, p.206]	n/a	2	3	5	3
25a. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=\delta(x)$ Laplace	[8, p.208]	0	n/a	0	0	0^M

Problem	Source	Mp V*	Mm 3.0*	Mp 6	Mm 4.0	MTB
25b. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=\delta(x)$ dsolve	[8, p.208]	n/a	n/a	5	5	5^M
26. $(1-x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=g(x)$	[2, p.168]	n/a	n/a	5	5	5
27. $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-.25)y=g(x)$	[2, p.168]	n/a	n/a	5	2	5
28. $\frac{d^3y}{dx^3}-\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=g(x)$	[2, p.210]	n/a	n/a	5	4	5
29. $\frac{d^2y}{dx^2}+y\left(\frac{dy}{dx}\right)^3=0$	[12, p.57]	5	4	5	4	5
30. $\frac{d^2y}{dx^2}+y\left(\frac{dy}{dx}\right)^3=0, y(0)=0, y'(0)=2$	[12, p.57]	4	4	4	4	4
31. $x\left(\frac{dy}{dx}\right)^2-y^2+1=0$	[8, p.206]	n/a	3	4	3	3^M
32. $(x^2-1)\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx}+y^2-1=0$	[8, p.208]	5	n/a	5	1	5
33. $\frac{dx}{dt}=x(t)-y(t)$ $\frac{dy}{dt}=x(t)+y(t)$	[12, p.58]	4	5	5	5	5
34. $\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=\frac{3}{2}, x(0)=0, y(0)=-1$ $(t-x(t))\frac{dy}{dt}+y(t)\frac{dx}{dt}=0$, numeric	[8, p.205]	2	n/a	2	5	2^M
35. $\frac{\partial^2}{\partial x^2}u(x,t)=\frac{1}{k}\frac{\partial}{\partial t}u(x,t)$	[9, p.143] [12, p.58]	4	3^b	4	3^b	4^M
36. $\frac{\partial^2}{\partial x^2}u(x,t)=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}u(x,t)$	[9, p.221]	n/a	n/a	5	5	4^M
37. $\nabla^2u(x,y,z)=0$	[9, p.259]	n/a	n/a	4	3^b	4^M
38a. $\nabla^2u(x,y,z)=-f(x,y,z)$	[9, p.290]	n/a	n/a	1	3^b	$3^{b,M}$
38b. $\nabla^2u(x,y,z)=-1$	[9, p.290]	n/a	n/a	4	3^b	3^M
38c. $\nabla^2u(x,y,z)=-\frac{1}{x^2+y^2+z^2}$	[9, p.290]	n/a	n/a	2	3^b	$3^{b,M}$
38d. $\nabla^2u(x,y)=-\frac{1}{x^2+y^2}$	[9, p.290]	n/a	n/a	5	3^b	$3^{b,M}$

Problem	Source	Mp V*	Mm 3.0*	Mp 6	Mm 4.0	MTB
39. $\begin{cases} x'(t) = x(t) + y(t) + 2z(t), \\ y'(t) = x(t) + 2y(t) + z(t), \\ z'(t) = 2x(t) + y(t) + z(t) \end{cases}$	[2, p 357]	n/a	n/a	5	4	4
40. $\begin{cases} x'(t) = 2x(t) - 5y(t) + \csc(t), \\ y'(t) = x(t) - 2y(t) + \sec(t) \end{cases}$	[2, p.385]	n/a	n/a	4	5	5
41. $\begin{cases} x'(t) = 2x(t), \\ y'(t) = -2x(t) + y(t) - 2z(t), \\ z'(t) = x(t) + 3z(t) \end{cases}$	[12, p.58][W]	5	4	5	4	4

* Results summarized from Computer Algebra Systems, A Practical Guide. Edited by Michael J. Wester. John Wiley & Sons 1999.

M Access to Maple V kernel required

b Expected echo of input due to lack of initial or boundary conditions

W System of equations found at http://math.unm.edu/~wester/ cas_review.html.

APPENDIX B

Totals by Classification of Equation

Classification of equation	Mp 6	Mm 4.0	MTB R12
Linear Homogeneous # 7-13	28	25	22
Linear Non-Homogeneous # 14-16,18,20,21,24,26-28	48	46	48
Linear Equations with Singularities # 10,11,18,26,27	21	16	17
Non-Linear Homogeneous # 6,29,30	14	13	14
Non-Linear Non-Homogeneous # 1-3,17,19,31,32	33	24	33
Non-Linear Equations with Singularities # 2,17,31,32	18	13	18
Other Ordinary Differential Equations # 4,5,22,23,25	21	20	21
System of Ordinary Differential Equations # 33,34,39,40,41	21	23	20
Partial Differential Equations # 35-38	25	23	24
TOTAL SCORE	**190**	**174**	**182**

Notes on Partial Differential Equations

MATLAB cannot solve partial differential equations analytically without accessing the Maple V kernel. This requires the use of the Symbolic Math Toolbox [4], which must accompany MATLAB 6. Fortunately, the Symbolic Math Toolbox is standard in the MATLAB Release 12 package. The scores given to MATLAB in solving PDEs are completely dependent on the performance of the Maple V kernel.

Notes on Total Scores

The total scores shown in the table are the sum of the scores for each column corresponding to Maple 6, Mathematica 4.0, and MATLAB in Appendix A. This total will not reflect the total of the columns in Appendix B since the equations with singularities are also in other classifications. For example, equations 10 and 11 are both "linear homogeneous" and "linear equations with singularities." Similarly, equations 2, 17, 31, and 32 are "non-linear equations with singularities" as well as "non-linear non-homogeneous" equations. Thus the score corresponding to each equation is counted only once.

APPENDIX C

Ratings of Interface

Ratings of interface surrounding solving of differential equations.

0 => not available
1 => poor
2 => below expectations
3 => acceptable
4 => good
5 => excellent

Criteria	**Mp 6**	**Mm 4.0**	**MTB R12**
Help files:			
overall ease of use	5	5	3
searching	5	5	2
explanations	4	4	2
examples given	4	4	4
TOTAL	**18**	**18**	**11**
Symbolics: (use of actual math symbols)			
input	3	4	2
output	4	3	3
TOTAL	**7**	**7**	**5**
Graphics			
overall appearance	5	5	5
ease of setup	3	3	1 and 4[P]
detail/zoom adjustments	5	2	5
3-d rotations –perspective options	5	2	5
TOTAL	**18**	**12**	**19**
Environment			
editing of mathematics	5	5	3
exportability of work (html, text, LaTex, etc)	5	5	3
word processing (text between commands)	5	5	2
tool pallettes	4	3	0
TOTAL	**19**	**18**	**8**
Overall ease of use for solving differential equations:	3	3	2
Descriptive error messages	3	3	1[T]
TOTAL SUM OF SCORES	**68**	**61**	**46**

[P] Using PDE toolbox and other existing graphing tools

[T] Without consideration of access to Maple kernel

APPENDIX D

Maple 6 Syntax of Test Suite

(See Appendix A for references)

Problem #1 ok

```
> restart:with(DEtools):eq1:=diff(y(x),x)=x*exp(y(x) +
sin(x)); 
```
separable

$$eq1 := \frac{\partial}{\partial x} \mathrm{y}(x) = x\, \mathrm{e}^{(\mathrm{y}(x) + \sin(x))}$$

```
> odeadvisor(eq1);
```

$$[\_separable]$$

```
> sol:=dsolve(eq1,y(x));
```

$$sol := \mathrm{y}(x) = \ln\left(-\frac{1}{\int x\, \mathrm{e}^{\sin(x)}\, dx + \_C1}\right)$$

```
> map(odetest,[sol],eq1); 
```
verify solution

$$[0]$$

Problem #2 ok

```
> restart:with(DEtools):eq2:=2*x*y(x)^2 + 2*y(x) +
diff(y(x),x)*(2*x^2*y(x) + 2*x)=0;
```

$$eq2 := 2\, x\, \mathrm{y}(x)^2 + 2\, \mathrm{y}(x) + \left(\frac{\partial}{\partial x} \mathrm{y}(x)\right) (2\, x^2\, \mathrm{y}(x) + 2\, x) = 0$$

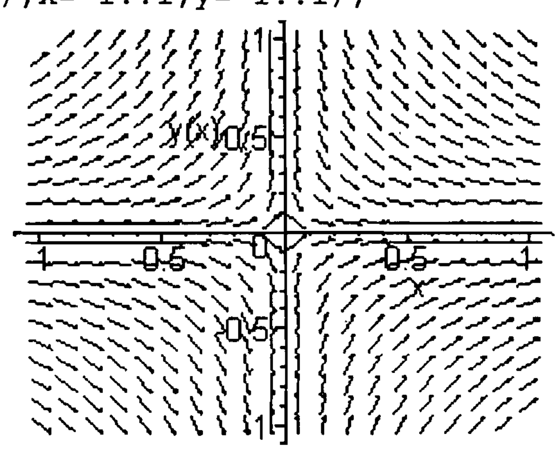
```
> odeadvisor(eq2);
```

$$[\_separable]$$

```
> sol2:=dsolve(eq2,y(x));
```

$$sol2 := \mathrm{y}(x) = -\frac{1}{x},\ \mathrm{y}(x) = \frac{-1 + \_C1}{x},\ \mathrm{y}(x) = \frac{-1 - \_C1}{x}$$

```
> DEplot(eq2,y(x),x=-1..1,y=-1..1);
```




```
> map(odetest,[sol2],eq2); 
```
verify solutions

$$[0, 0, 0]$$

Problem #3 ok

```
> restart:with(DEtools):eq3:=(y(x)^2 - x) +
2*y(x)*diff(y(x),x) = 0;
```

$$eq3 := y(x)^2 - x + 2\, y(x) \left(\frac{\partial}{\partial x} y(x)\right) = 0$$

```
> odeadvisor(eq3);
```

$$[\_rational, \_Bernoulli]$$

```
> sol3:=dsolve(eq3,y(x));
```

$$sol3 := y(x) = \frac{\sqrt{e^x (e^x x - e^x + \_C1)}}{e^x},\ y(x) = -\frac{\sqrt{e^x (e^x x - e^x + \_C1)}}{e^x}$$

```
> map(odetest,[sol3],eq3);
```

$$[0, 0]$$

Problem #4 (y(x,a) found via dsolve and pdsolve)

```
> restart:with(DEtools):eq4:=diff(y(x,a),x)=a*y(x,a);
```

$$eq4 := \frac{\partial}{\partial x} y(x, a) = a\, y(x, a)$$

```
> odeadvisor(eq4);
Error, (in ODEtools/info) Required a specification of the indeterminate
function
```

```
> sol4:=dsolve(eq4,y(x,a));
```

$$sol4 := y(x, a) = \_F1(a)\, e^{(a\, x)}$$

```
> map(odetest,[sol4],eq4);
Error, (in ODEtools/info) Required a specification of the indeterminate
function
```

```
> restart:with(PDEtools):eq4:=diff(y(x,a),x)=a*y(x,a):
> sol4:=pdsolve(eq4,build); pdetest(sol4,eq4);
```

solve as a pde. No difference appears in solution. However, testing the solution requires no identification of the unknown function _F1(a).

$$sol4 := y(x, a) = \_F1(a)\, e^{(a\, x)}$$

$$0$$

Problem #5 (time-delay, recevied a partial result with Laplace)

```
> restart:with(DEtools):eq5:=diff(y(t),t)+a*y(t-1)=0;
```

$$eq5 := \left(\frac{\partial}{\partial t} y(t)\right) + a\, y(t - 1) = 0$$

```
> odeadvisor(eq5);
Error, (in ODEtools/info) found the indeterminate function y with
different arguments {y(t-1)}
```

```
> dsolve(eq5,y(t));
Error, (in ODEtools/info) found the indeterminate function y with
different arguments {y(t-1)}

> sol5:=dsolve(eq5,y(t),method=laplace);
```

$$sol5 := y(t) = y(0) - a \int_0^t y(\_U1 - 1)\, d\_U1$$

```
> map(odetest,[sol5],eq5);
Error, (in ODEtools/info) found the indeterminate function y with
different arguments {y(t-1)}
```

Problem #6 ok

```
> restart:with(DEtools):eq6:=x*diff(y(x),x)+y(x)-
y(x)^2*exp(2*x)=0;
```

$$eq6 := x\left(\frac{\partial}{\partial x} y(x)\right) + y(x) - y(x)^2 e^{(2x)} = 0$$

```
> odeadvisor(eq6);
```

$$[\_Bernoulli]$$

```
> sol6:=dsolve(eq6,y(x));
```

$$sol6 := y(x) = \frac{1}{e^{(2x)} + 2x\,\mathrm{Ei}(1, -2x) + x\,\_C1}$$

```
> map(odetest,[sol6],eq6);
```

$$[0]$$

Problem #7 echo of input

```
> restart:with(DEtools):eq7:=diff(y(x),x)+P(x)*y(x)=Q(x)*
diff(y(x),x,x);
```

$$eq7 := \left(\frac{\partial}{\partial x} y(x)\right) + P(x)\, y(x) = Q(x)\left(\frac{\partial^2}{\partial x^2} y(x)\right)$$

```
> odeadvisor(eq7);
```

$$[[\_2nd\_order, \_with\_linear\_symmetries]]$$

`> sol7:=dsolve(eq7,y(x));` **Maple stores this solution as an object so that other manipulations can be made on this object of more information is given (e.g. if P(x) is given).**

$$sol7 := y(x) = \mathrm{DESol}\left(\left\{-Q(x)\left(\frac{\partial^2}{\partial x^2}\_Y(x)\right) + \left(\frac{\partial}{\partial x}\_Y(x)\right) + P(x)\,\_Y(x)\right\}, \{\_Y(x)\}\right)$$

```
> map(odetest,[sol7],eq7);
```

$$[0]$$

Problem #8 ok

```
> restart:with(DEtools):eq8:=diff(u(t),t$4)+lambda^4*u(t)=0;
```

$$eq8 := \left(\frac{\partial^4}{\partial t^4} u(t)\right) + \lambda^4 u(t) = 0$$

```
> odeadvisor(eq8);
```

$$[[\_high\_order, \_missing\_x]]$$

```
> sol8:=simplify(dsolve(eq8,u(t)));
```

$$sol8 := u(t) = \_C1\, e^{((1/2 - 1/2\, I)\ 2\, \lambda\, t)} + \_C2\, e^{((1/2 + 1/2\, I)\ 2\, \lambda\, t)} + \_C3\, e^{((-1/2 - 1/2\, I)\ 2\, \lambda\, t)} + \_C4\, e^{((-1/2 + 1/2\, I)\ 2\, \lambda\, t)}$$

```
> map(odetest,[sol8],eq8);
```

$$[0]$$

Problem #9 ok

```
> restart:with(DEtools):eq9:=diff(y(x),x,x)+k^2*y(x) = 0;
```

y(0)=y'(0)=0 (Wester57)

$$eq9 := \left(\frac{\partial^2}{\partial x^2} y(x)\right) + k^2\, y(x) = 0$$

```
> init1:=D(y)(1)=0:init2:=y(0)=0:odeadvisor(eq9);
```

$$[[\_2nd\_order, \_missing\_x]]$$

```
> sol9:=dsolve(eq9,y(x));map(odetest,[sol9],eq9);
```

General solution

$$sol9 := y(x) = \_C1\cos(k\, x) + \_C2\sin(k\, x)$$

$$[0]$$

```
> dsolve({eq9,init2},y(x));
```

$$y(x) = \_C2\sin(k\, x)$$

```
> solve(_C2*cos(k)=0,k);
```

solving for k using initial conditions (and avoiding k=0)

$$\frac{1}{2}\pi$$

```
>
assume(k<>0):dsolve({eq9,init1,init2},{y(x),k});assume(k<>0
):dsolve({eq9,init1,init2},y(x));
Error, (in ODEtools/info) found wrong extra arguments: {{y(x), k}}
```

$$y(x) = 0$$

```
>
```

Problem #10 ok

```
>
restart:with(DEtools):eq10:=(56+59*x)*diff(y(x),x$3)+(13+19
*x)*diff(y(x),x$2)+(-142-59*x)*diff(y(x),x)+(-199-
9*x)*y(x)=0;
```

program either runs out of memory or indicates no solution in previous versions

```
>
```

$$eq10 := (56 + 59\, x)\left(\frac{\partial^3}{\partial x^3} y(x)\right) + (13 + 19\, x)\left(\frac{\partial^2}{\partial x^2} y(x)\right) + (-142 - 59\, x)\left(\frac{\partial}{\partial x} y(x)\right) + (-199 - 9\, x)\, y(x) = 0$$

```
> odeadvisor(eq10);
```

$$[[\_3rd\_order, \_with\_linear\_symmetries]]$$

```
> sol10:=dsolve(eq10,y(x));
```
strange looking

$$sol10 := y(x) = e^{\left(\int \_b(\_a)\,d\_a + \_C1\right)} \text{ \&where } \left[\left[\left\{ \_b(\_a)^3 + \frac{(19\_a+13)\,\_b(\_a)^2}{56+59\_a} + \frac{\left(-142 + 168\left(\frac{\partial}{\partial \_a}\_b(\_a)\right) + 177\left(\frac{\partial}{\partial \_a}\_b(\_a)\right)\_a - 59\_a\right)\_b(\_a)}{56+59\_a} + \left(56\left(\frac{\partial^2}{\partial \_a^2}\_b(\_a)\right) + 59\_a\left(\frac{\partial^2}{\partial \_a^2}\_b(\_a)\right) - 9\_a + 13\left(\frac{\partial}{\partial \_a}\_b(\_a)\right) - 199 + 19\left(\frac{\partial}{\partial \_a}\_b(\_a)\right)\_a\right)/(56+59\_a) = 0 \right\}, \left\{ \_b(\_a) = \frac{\frac{\partial}{\partial x}y(x)}{y(x)}, \_a = x \right\}, \left\{ x = \_a, y(x) = e^{\left(\int \_b(\_a)\,d\_a + \_C1\right)} \right\}\right]\right]$$

```
> map(odetest,[sol10],eq10);
```

$$[0]$$

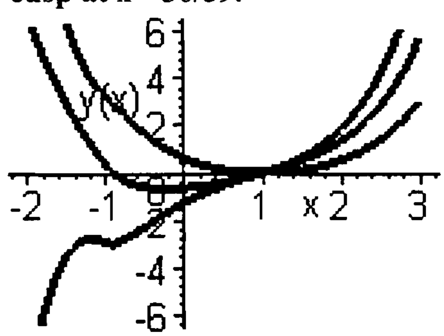
```
> 
inits:={eq10,y(1)=0,D(y)(1)=1,(D@@2)(y)(1)=1}:sol10i:=dsolv
e(inits,y(x),series);
```
initial value problem, series solution

$$sol10i := y(x) = x - 1 + \frac{1}{2}(x-1)^2 + \frac{169}{690}(x-1)^3 + \frac{4532}{39675}(x-1)^4 + \frac{151927}{12167000}(x-1)^5 + O((x-1)^6)$$

```
> 
inits:={eq10,y(1)=0,D(y)(1)=1,(D@@2)(y)(1)=1}:sol10j:=dsolv
e(inits,y(x));
```

$$sol10j :=$$

```
> DEplot(eq10,y(x),x=-
2..3,[[y(1)=0,D(y)(1)=1,(D@@2)(y)(1)=1],[y(1)=0,D(y)(1)=0,(
D@@2)(y)(1)=1],[y(1)=0,D(y)(1)=1,(D@@2)(y)(1)=0]],y=-
6..6,stepsize=.05,linecolor=[blue,black,red]);
```
plots of three initial value problems, notice the cusp at x=-56/59.



Problem #11 unexpected solution

```
> restart:with(DEtools):
> eq11:=(x-2)*x^2*diff(y(x),x,x)+x^2*diff(y(x),x)+exp(x-
1)*y(x)=0;
```
by series solution, 2 and 0 are regular singular points. (dsolve(eq,method=series) , we should not expect a solution.

$$eq11 := (x-2)\,x^2\left(\frac{\partial^2}{\partial x^2}\,y(x)\right) + x^2\left(\frac{\partial}{\partial x}\,y(x)\right) + e^{(x-1)}\,y(x) = 0$$

```
> odeadvisor(eq11);
```

$$[[\_2nd\_order, \_with\_linear\_symmetries]]$$

```
> inits:={eq11,y(2)=0,D(y)(2)=1}:expand(dsolve(inits,y(x),
series));
```

$$y(x) = \_C1\left(1 - \frac{1}{4}\,e\,(x-2) + \frac{1}{64}\,(e)^2\,(x-2)^2 + \left(-\frac{1}{2304}\,(e)^3 - \frac{1}{144}\,e\right)(x-2)^3 + \left(\frac{1}{147456}\,(e)^4 + \frac{1}{768}\,e + \frac{5}{4608}\,(e)^2\right)(x-2)^4 + \left(-\frac{1}{1600}\,e - \frac{1}{14745600}\,(e)^5 - \frac{23}{460800}\,(e)^3 - \frac{17}{76800}\,(e)^2\right)(x-2)^5 + O((x-2)^6)\right) + \_C2\,\ln(x-2)\left(1 - \frac{1}{4}\,e\,(x-2) + \frac{1}{64}\,(e)^2\,(x-2)^2 + \left(-\frac{1}{2304}\,(e)^3 - \frac{1}{144}\,e\right)(x-2)^3 + \left(\frac{1}{147456}\,(e)^4 + \frac{1}{768}\,e + \frac{5}{4608}\,(e)^2\right)(x-2)^4 + \left(-\frac{1}{1600}\,e - \frac{1}{14745600}\,(e)^5 - \frac{23}{460800}\,(e)^3 - \frac{17}{76800}\,(e)^2\right)(x-2)^5 + O((x-2)^6)\right) + \_C2\left(\frac{1}{2}\,e\,(x-2) - \frac{3}{64}\,(e)^2\,(x-2)^2 + \left(\frac{1}{216}\,e + \frac{11}{6912}\,(e)^3\right)(x-2)^3 + \left(-\frac{1}{1536}\,e - \frac{71}{27648}\,(e)^2 - \frac{25}{884736}\,(e)^4\right)(x-2)^4 + \left(\frac{1}{4000}\,e + \frac{131}{256000}\,(e)^2 + \frac{2251}{13824000}\,(e)^3 + \frac{137}{442368000}\,(e)^5\right)(x-2)^5 + O((x-2)^6)\right)$$

```
> subs(x=2,sol11);deriv11:=diff(sol11,x): subs(x=2,deriv11);
```
does not satisfy initial conditions, but solution indicates behavior near 2.

$$y(2) = \_C1 + \_C2\,\ln(0)$$

```
Error, division by zero
```

```
> map(odetest,[sol11],eq11);
```
this method of verification will not work for series solutions

Problem #12 ok

```
> restart:with(DEtools):eq12:=diff(y(x),x,x)+
x*diff(y(x),x)+exp(-x^2)*y(x)=0;
```

$$eq12 := \left(\frac{\partial^2}{\partial x^2}\mathrm{y}(x)\right) + x\left(\frac{\partial}{\partial x}\mathrm{y}(x)\right) + e^{(-x^2)}\,\mathrm{y}(x) = 0$$

```
> odeadvisor(eq12);
```

$$[[\_2nd\_order, \_with\_linear\_symmetries]]$$

```
> sol12:=simplify(dsolve(eq12,y(x)));
```

$$sol12 := \mathrm{y}(x) = \_C1\,\sinh\left(\int\sqrt{-e^{(-x^2)}}\,dx\right) + \_C2\,\cosh\left(\int\sqrt{-e^{(-x^2)}}\,dx\right)$$

```
> map(odetest,[sol12],eq12);
```

solution verified

$$[0]$$

```
> map(odetest,[y(x)=cos(int(sqrt(exp(-
x^2)),x))+sin(int(sqrt(exp(-x^2)),x))],eq12);
```

Boyce 5th edition solution is also verified

$$[0]$$

```
> evalb(rhs(sol12)=cos(int(sqrt(exp(-
x^2)),x))+sin(int(sqrt(exp(-x^2)),x)));
```

the two solutions are apparently not the same.

false

```
> convert(rhs(sol12),trig);
```

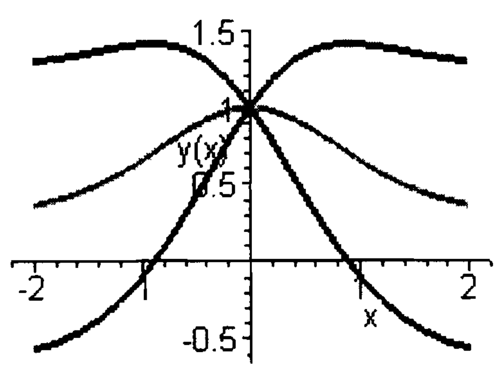
$$\_C1\,\sinh\left(\int\sqrt{-\cosh(x^2)+\sinh(x^2)}\,dx\right) + \_C2\,\cosh\left(\int\sqrt{-\cosh(x^2)+\sinh(x^2)}\,dx\right)$$

```
> s:=convert(rhs(sol12)-cos(int(sqrt(exp(-
x^2)),x))+sin(int(sqrt(exp(-x^2)),x)),trig):
map(odetest,[y(x)=s],eq12);
```

the difference between these solutions is also a solution

$$[0]$$

```
> with(DEtools):
DEplot([eq12],y(x),x=-
2..2,[[y(0)=1,D(y)(0)=1],[y(0)=1,D(y)(0)=0],[y(0)=1,D(y)(0)
=-1]],linecolor=[blue,red,green]);
```



Problem #13 ok, workable solution

```
> restart:with(DEtools):eq13:=diff(r*diff(R(r),r),r)-
(mu^2/r)*R(r)+lambda^2*r*R(r)=0;
```
Bessel

$$eq13 := \left(\frac{\partial}{\partial r}\mathrm{R}(r)\right) + r\left(\frac{\partial^2}{\partial r^2}\mathrm{R}(r)\right) - \frac{\mu^2\,\mathrm{R}(r)}{r} + \lambda^2\, r\,\mathrm{R}(r) = 0$$

```
> odeadvisor(eq13);
```

[[_2nd_order, _with_linear_symmetries]]

```
> sol13:=dsolve(eq13,R(r));
```
Same solution as given in Powers 314

$$sol13 := \mathrm{R}(r) = \_C1\ \mathrm{BesselJ}(\mu, \mathrm{csgn}(\lambda)\,\lambda\, r) + \_C2\ \mathrm{BesselY}(\mu, \mathrm{csgn}(\lambda)\,\lambda\, r)$$

```
> map(odetest,[sol13],eq13);
```

[0]

Problem #14 ok

```
> restart:with(DEtools):eq14:=diff(u(t),t)+K(t)*u(t)=f(t);
```

$$eq14 := \left(\frac{\partial}{\partial t}\mathrm{u}(t)\right) + \mathrm{K}(t)\,\mathrm{u}(t) = \mathrm{f}(t)$$

```
> odeadvisor(eq14);sol14:=dsolve(eq14,u(t));
map(odetest,[sol14],eq14);
```

[_linear]

$$sol14 := \mathrm{u}(t) = \left(\int \mathrm{f}(t)\, e^{\left(\int \mathrm{K}(t)\,dt\right)}\,dt + \_C1\right) e^{\left(\int -\mathrm{K}(t)\,dt\right)}$$

[0]

Problme #15 ok

```
> restart:with(DEtools):eq15:=3*diff(u(t),t)-2*u(t)=cos(t);
```

$$eq15 := 3\left(\frac{\partial}{\partial t}\mathrm{u}(t)\right) - 2\,\mathrm{u}(t) = \cos(t)$$

```
> odeadvisor(eq15);
```

[[_linear, class A]]

```
> sol15:=dsolve(eq15,u(t));map(odetest,[sol15],eq15);
```

$$sol15 := \mathrm{u}(t) = -\frac{2}{13}\cos(t) + \frac{3}{13}\sin(t) + e^{(2/3\,t)}\ \_C1$$

[0]

Problem #16 ok

```
> restart:with(DEtools):eq16:=diff(y(x),x)=-y(x)-1;
```

$$eq16 := \frac{\partial}{\partial x}\mathrm{y}(x) = -\mathrm{y}(x) - 1$$

```
> odeadvisor(eq16);sol16:=dsolve(eq16,y(x));
  map(odetest,[sol16],eq16);
```

[_quadrature]

$$sol16 := \mathrm{y}(x) = -1 + e^{(-x)}\ \_C1$$

[0]

Problem #17 ok

```
> restart:with(DEtools):eq17:=diff(y(x),x)  =y(x)/x+x/y(x);
```

$$eq17 := \frac{\partial}{\partial x}\mathrm{y}(x) = \frac{\mathrm{y}(x)}{x} + \frac{x}{\mathrm{y}(x)}$$

```
> odeadvisor(eq17);
```

[[_homogeneous, class A], [_1st_order, _with_linear_symmetries], _rational, _Bernoulli
]

```
> sol17:=dsolve(eq17,y(x));map(odetest,[sol17],eq17);
```

$$sol17 := \mathrm{y}(x) = \sqrt{2\ln(x) + \_C1}\, x,\ \mathrm{y}(x) = -\sqrt{2\ln(x) + \_C1}\, x$$

[0, 0]

```
> map(odetest,[y(x)=x*sqrt(2*log(x))],eq17);
```

Wester's solution checked.

[0]

Problem #18 ok

```
>
restart:with(DEtools):eq18:=x^2*diff(y(x),x)+3*x*y(x)=sin(x
)/x;
```

$$eq18 := x^2\left(\frac{\partial}{\partial x}\mathrm{y}(x)\right) + 3\,x\,\mathrm{y}(x) = \frac{\sin(x)}{x}$$

```
> odeadvisor(eq18);
```

[_linear]

```
> sol18:=dsolve(eq18,y(x));map(odetest,[sol18],eq18);
```

$$sol18 := \mathrm{y}(x) = \frac{-\cos(x) + \_C1}{x^3}$$

[0]

Problem #19 ok (includes plot of slope fields)

```
> restart:with(DEtools):eq19:=diff(y(x),x)=-
(1+2*x*sin(y(x)))/(1+x^2*cos(y(x)));
```

(Wester57)

$$eq19 := \frac{\partial}{\partial x}\mathrm{y}(x) = -\frac{1 + 2\,x\sin(\mathrm{y}(x))}{1 + x^2\cos(\mathrm{y}(x))}$$

```
> odeadvisor(eq19);
```

[y=_G(x,y')]

```
> sol19:=dsolve(eq19,y(x));
```

implicit solution without initial conditions

$$sol19 := x + \sin(\mathrm{y}(x))\,x^2 + \mathrm{y}(x) + \_C1 = 0$$

```
> solve(sol19,y(x));
```

attempt to also give explicit solution

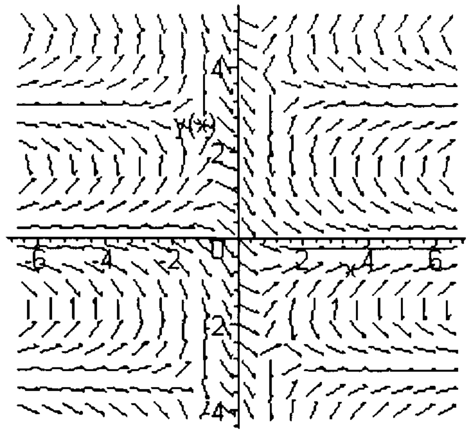
$$\mathrm{RootOf}(\_Z + x + \sin(\_Z)\,x^2 + \_C1)$$

```
> init1:=y(0)=0:
> sol19b:=dsolve({eq19,init1},y(x));
```

Explicit solutions with initial conditions

$$sol19b := \mathrm{y}(x) = \mathrm{RootOf}(\_Z + x + \sin(\_Z)\,x^2)$$

```
> with(DEtools):
DEplot(eq19,y(x),x=-2*Pi..2*Pi,y=-4..5, stepsize=.05,
color=blue);
```



Problem #20 ok

```
> restart:with(DEtools):eq20:=(x^2+x+1)*diff(y(x),x,x)+
(4*x+2)*diff(y(x),x)+2*y(x)=3*x^2;
```

$$eq20 := (x^2+x+1)\left(\frac{\partial^2}{\partial x^2}\mathrm{y}(x)\right)+(4x+2)\left(\frac{\partial}{\partial x}\mathrm{y}(x)\right)+2\,\mathrm{y}(x)=3\,x^2$$

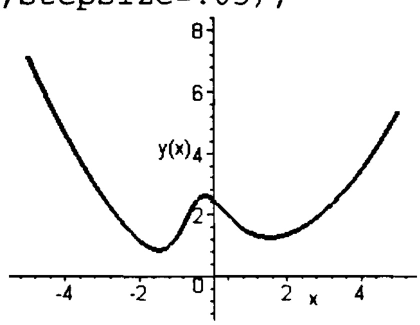
```
> odeadvisor(eq20);
```

[[_2nd_order, _exact, _linear, _nonhomogeneous]]

```
> sol20:=dsolve(eq20,y(x));
```

$$sol20 := \mathrm{y}(x)=\frac{\_C2}{x^2+x+1}+\frac{\_C1\,x}{x^2+x+1}+\frac{\frac{1}{4}x^4}{x^2+x+1}$$

```
> with(DEtools):
DEplot(eq20,y(x),x=-5..5,[[D(y)(0)=-1,y(0)=2.5]],y=-
1..8,linecolor=blue,stepsize=.05);
```



Problem #21ok and then some

```
> restart:with(DEtools):eq21:=diff(y(x),x,x)+4*y(x)=
sin(2*x);
```
by Laplace y(0)=y'(0)=0

$$eq21 := \left(\frac{\partial^2}{\partial x^2}\mathrm{y}(x)\right)+4\,\mathrm{y}(x)=\sin(2\,x)$$

```
> odeadvisor(eq21);
```

$$[[\_2nd\_order, \_reducible, \_mu\_x\_y1]]$$

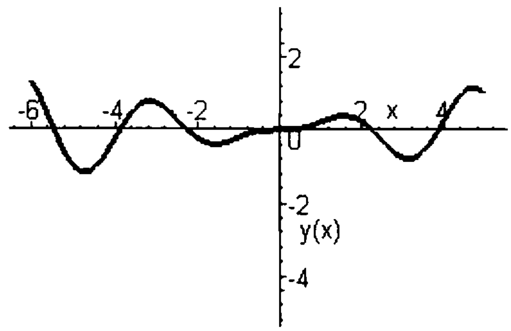
```
> sol21=dsolve(eq21,y(x),method=laplace);
```

$$sol21 = \left(y(x) = \frac{1}{8}\sin(2x) - \frac{1}{4}x\cos(2x) + y(0)\cos(2x) + \frac{1}{2}D(y)(0)\sin(2x) \right)$$

```
> sol21b=dsolve(eq21,y(x));
```
note: _C1 = 1/8 + (1/2)y'(0), and _C2 = y(0)

$$sol21b = \left(y(x) = -\frac{1}{4}x\cos(2x) + \_C1\cos(2x) + \_C2\sin(2x) \right)$$

```
> with(DEtools):
DEplot(eq21,y(x),x=-6..5,[[D(y)(0)=0,y(0)=0]],y=-
5..3,linecolor=magenta,stepsize=.05);
```



Problem #22 (error, integro-differential equation)

```
> restart:with(DEtools):
> eq22:=diff(u(t),t)+2*u(t)+5*int(u(tau),tau=0..t)=10*exp(-
4*t);
```

$$eq22 := \left(\frac{\partial}{\partial t}u(t)\right) + 2\,u(t) + 5\int_0^t u(\tau)\,d\tau = 10\,e^{(-4t)}$$

```
> odeadvisor(eq22);
Error, (in ODEtools/info) found the indeterminate function u with
different arguments {u(tau)}

> sol22:=dsolve(eq22,u(t));
Error, (in ODEtools/info) found the indeterminate function u with
different arguments {u(tau)}
```

Problem #23 ok (same problem, but by Laplace)

```
>
restart:with(DEtools):eq23:=diff(u(t),t)+2*u(t)+5*int(u(tau
),tau=0..t)=10*exp(-4*t);
```
by Laplace

$$eq23 := \left(\frac{\partial}{\partial t}u(t)\right) + 2\,u(t) + 5\int_0^t u(\tau)\,d\tau = 10\,e^{(-4t)}$$

```
> odeadvisor(eq23);
Error, (in ODEtools/info) found the indeterminate function u with
different arguments {u(tau)}
```

```
> sol23:=dsolve(eq23,u(t),method=laplace);
```
sol23 :=

$$u(t)=-\frac{40}{13}e^{(-4t)}+\left(\frac{40}{13}+u(0)\right)\cos(2t)e^{(-t)}-\frac{2}{13}I\left(\frac{5}{2}I-\frac{13}{4}I\,u(0)\right)e^{(-t)}\sin(2t)$$

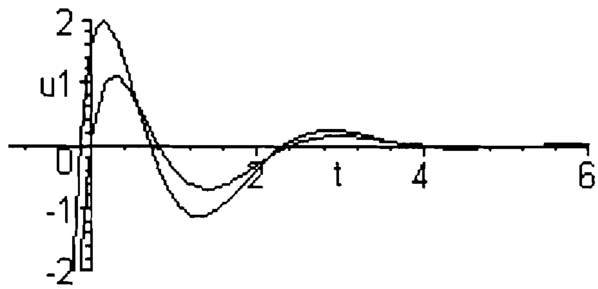
```
> sol23b:=dsolve({eq23,u(0)=1.5},u(t),method=laplace);
```

$$sol23b := u(t)=-\frac{40}{13}e^{(-4t)}+\frac{119}{26}e^{(-t)}\cos(2t)-\frac{19}{52}e^{(-t)}\sin(2t)$$

```
> sol23c:=dsolve({eq23,u(0)=0},u(t),method=laplace);
```

$$sol23c := u(t)=-\frac{40}{13}e^{(-4t)}+\frac{40}{13}e^{(-t)}\cos(2t)+\frac{5}{13}e^{(-t)}\sin(2t)$$

```
> plot([rhs(sol23b),rhs(sol23c)],t=-1..6,u=-
2..2,color=[blue,red]);
```



Problem #24 ok

```
>
restart:with(DEtools):eq24:=(1+x+x^2)*diff(y(x),x$3)+(3+6*x
)*diff(y(x),x$2)+6*diff(y(x),x)=6*x;
```

$$eq24 := (1+x+x^2)\left(\frac{\partial^3}{\partial x^3}y(x)\right)+(3+6x)\left(\frac{\partial^2}{\partial x^2}y(x)\right)+6\left(\frac{\partial}{\partial x}y(x)\right)=6x$$

```
> odeadvisor(eq24);
```

[[_3rd_order, _missing_y]]

```
> sol24:=dsolve(eq24,y(x));
```
looks like number 20

$$sol24 := y(x)=\frac{\_C3}{1+x+x^2}+\frac{\_C2\,x}{1+x+x^2}+\frac{\_C1\,x^2}{1+x+x^2}+\frac{\frac{1}{4}x^4}{1+x+x^2}$$

Problem #25 Laplace gives wrong solution

```
>
restart:with(DEtools):eq25:=diff(y(x),x,x)+2*diff(y(x),x)+y
(x)=Dirac(x);
```
by Laplace

$$eq25 := \left(\frac{\partial^2}{\partial x^2}y(x)\right)+2\left(\frac{\partial}{\partial x}y(x)\right)+y(x)=\mathrm{Dirac}(x)$$

```
> odeadvisor(eq25);
```

[[_2nd_order, _reducible, _mu_x_y1]]

```
> sol25:=expand(dsolve(eq25,y(x),method=laplace));
```

$$sol25 := y(x) = \frac{x\, D(y)(0)}{e^x} + \frac{x\, y(0)}{e^x} + \frac{x}{e^x} + \frac{y(0)}{e^x}$$

```
> map(odetest,[sol25],eq25);
```
Incorrect solution via Laplace transform

$$[-\mathrm{Dirac}(x)]$$

```
> sol25b:=expand(dsolve(eq25,y(x)));
```

$$sol25b := y(x) = \frac{\mathrm{Heaviside}(x)\, x}{e^x} + \frac{\_C1}{e^x} + \frac{\_C2\, x}{e^x}$$

```
> map(odetest,[sol25b],eq25);
```
Correct solution

$$[0]$$

Problem #26 ok

```
> restart:with(DEtools):eq26:=(1-
x)*diff(y(x),x,x)+x*diff(y(x),x)-y(x)=g(x);
```

$$eq26 := (1-x)\left(\frac{\partial^2}{\partial x^2} y(x)\right) + x\left(\frac{\partial}{\partial x} y(x)\right) - y(x) = g(x)$$

```
> odeadvisor(eq26);
```

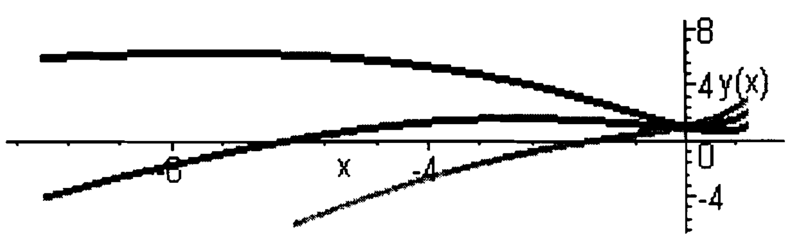
$$[[\_2nd\_order, \_with\_linear\_symmetries]]$$

```
> sol26:=dsolve(eq26,y(x));map(odetest,[sol26],eq26);
```

$$sol26 := y(x) = \int \frac{g(x)}{(-1+x)^2}\, dx\, x + \int -\frac{x\, g(x)\, e^{(-x)}}{(-1+x)^2}\, dx\, e^x + \_C1\, x + \_C2\, e^x$$

$$[0]$$

```
> with(DEtools):
DEplot(lhs(eq26)=x,y(x),x=-10...0.99,[[D(y)(0)=-
1,y(0)=1],[D(y)(0)=0,y(0)=1],[D(y)(0)=1,y(0)=1]],y=-
6..8,stepsize=.01,linecolor=[gold,green,red]);
>
```



Problem #27 ok

```
>
restart:with(DEtools):eq27:=x^2*diff(y(x),x,x)+x*diff(y(x),
x)+(x^2-.25)*y(x)=g(x);
```
Bessel

$$eq27 := x^2\left(\frac{\partial^2}{\partial x^2} y(x)\right) + x\left(\frac{\partial}{\partial x} y(x)\right) + (x^2 - .25)\, y(x) = g(x)$$

```
> odeadvisor(eq27);
```

$$[[\_2nd\_order, \_reducible, \_mu\_x\_y1]]$$

```
>
sol27:=expand(dsolve(eq27,y(x)));map(odetest,[sol27],eq27);
```

$$sol27 := y(x) = \frac{\int \frac{\cos(x)\, g(x)}{x^{(3/2)}}\, dx\, \sin(x)}{\sqrt{x}} - \frac{\int \frac{\sin(x)\, g(x)}{x^{(3/2)}}\, dx\, \cos(x)}{\sqrt{x}} + \frac{\_C1\cos(x)}{\sqrt{x}} + \frac{\_C2\sin(x)}{\sqrt{x}}$$

$$[0]$$

Problem #28 ok

```
> restart:with(DEtools):eq28:=diff(y(x),x$3)-
diff(y(x),x$2)+diff(y(x),x)-y(x)=g(x);
```

$$eq28 := \left(\frac{\partial^3}{\partial x^3} y(x)\right) - \left(\frac{\partial^2}{\partial x^2} y(x)\right) + \left(\frac{\partial}{\partial x} y(x)\right) - y(x) = g(x)$$

```
> odeadvisor(eq28);
```

$$[[\_3rd\_order, \_linear, \_nonhomogeneous]]$$

```
>
sol28:=simplify(dsolve(eq28,y(x)));map(odetest,[sol28],eq28
);
```

$$sol28 := y(x) = -\frac{1}{2}\int (\cos(x) + \sin(x))\, g(x)\, dx\, \sin(x)$$
$$-\frac{1}{2}\int (-\sin(x) + \cos(x))\, g(x)\, dx\, \cos(x) + \frac{1}{2}\int g(x)\, e^{(-x)}\, dx\, e^x + \_C1\sin(x)$$
$$+ \_C2\cos(x) + \_C3\, e^x$$

$$[0]$$

Problem #29 ok

```
> restart:with(DEtools):eq29:=diff(y(x),x,x)+y(x)*
diff(y(x),x)^3 = 0;
```

$$eq29 := \left(\frac{\partial^2}{\partial x^2} y(x)\right) + y(x)\left(\frac{\partial}{\partial x} y(x)\right)^3 = 0$$

```
> odeadvisor(eq29);
```

$$[[\_2nd\_order, \_missing\_x], [\_2nd\_order, \_reducible, \_mu\_x\_y1],$$
$$[\_2nd\_order, \_reducible, \_mu\_y\_y1]]$$

```
>
sol29:=dsolve(eq29,y(x),implicit);map(odetest,[sol29],eq29)
;
```

$$sol29 := y(x) = \_C1, \frac{1}{6}y(x)^3 + \_C1\, y(x) - x - \_C2 = 0$$

$$[0, 0]$$

Problem #30 ok

```
> restart:with(DEtools):eq30:=diff(y(x),x,x)+y(x)* diff(y(x)
,x)^3 = 0;
```
also given initial conditions: y(0)=0 and y'(0)=2.

$$eq30 := \left(\frac{\partial^2}{\partial x^2}\mathrm{y}(x)\right) + \mathrm{y}(x)\left(\frac{\partial}{\partial x}\mathrm{y}(x)\right)^3 = 0$$

```
> odeadvisor(eq30);
```

$$[[\_2nd\_order, \_missing\_x], [\_2nd\_order, \_reducible, \_mu\_x\_y1], [\_2nd\_order, \_reducible, \_mu\_y\_y1]]$$

```
> sol30b:=dsolve(eq30,y(x),implicit);
```

$$sol30b := \mathrm{y}(x) = \_C1, \frac{1}{6}\mathrm{y}(x)^3 + \_C1\,\mathrm{y}(x) - x - \_C2 = 0$$

```
> init1:=y(0)=0:init2:=D(y)(0)=2: sol30:=dsolve({eq30, init1,
init2},y(x));
```
only one real solution expected (due to initial value problem).

$$sol30 := \mathrm{y}(x) = \%1^{(1/3)} - \frac{1}{\%1^{(1/3)}},$$

$$\mathrm{y}(x) = -\frac{1}{2}\%1^{(1/3)} + \frac{\frac{1}{2}}{\%1^{(1/3)}} - \frac{1}{2}I\sqrt{3}\left(\%1^{(1/3)} + \frac{1}{\%1^{(1/3)}}\right),$$

$$\mathrm{y}(x) = -\frac{1}{2}\%1^{(1/3)} + \frac{\frac{1}{2}}{\%1^{(1/3)}} + \frac{1}{2}I\sqrt{3}\left(\%1^{(1/3)} + \frac{1}{\%1^{(1/3)}}\right)$$

$$\%1 := 3\,x + \sqrt{1 + 9\,x^2}$$

```
> expand(sol30);
```

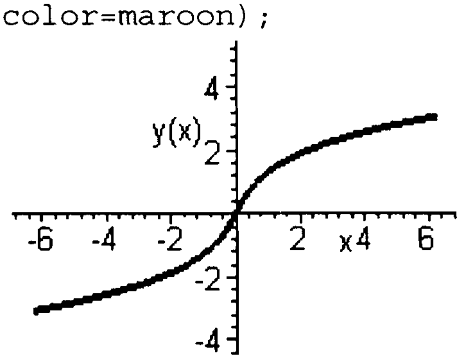
$$\mathrm{y}(x) = (3\,x + \sqrt{1 + 9\,x^2})^{(1/3)} - \frac{1}{(3\,x + \sqrt{1 + 9\,x^2})^{(1/3)}}$$

```
> s:=solve(1/6*y^3+1/2*y-x=0,y):
```
solving for y gives the three solutions shown above.

```
> map(odetest, [sol30],eq30);
```
indicates all three solutions satisfy the initial value problem, however the real valued function is a linear combination of the other two

$$[0, 0, 0]$$

```
> with(DEtools):
DEplot(eq30,y(x),x=-2*Pi..2*Pi,[[init1,init2]],y=-4..5,
stepsize=.05, linecolor=maroon);
```



Problem #31 ok, implicit required for time

```
> restart:with(DEtools):eq31:=x*diff(y(x),x)^2-y(x)^2+1=0;
```

$$eq31 := x\left(\frac{\partial}{\partial x}\mathrm{y}(x)\right)^2 - \mathrm{y}(x)^2 + 1 = 0$$

```
> odeadvisor(eq31);
```

$$[\_rational, [\_1st\_order, \_with\_symmetry\_[F(x),G(x)]]]$$

```
> sol31:=dsolve(eq31,y(x),implicit);
```

> **explicit solution was taking a long time**

$$sol31 := \mathrm{y}(x)^2 - 1 = 0,$$

$$-2\frac{\sqrt{x(\mathrm{y}(x)^2-1)}}{\sqrt{\mathrm{y}(x)-1}\sqrt{\mathrm{y}(x)+1}} + \frac{\sqrt{(\mathrm{y}(x)-1)(\mathrm{y}(x)+1)}\ln(\mathrm{y}(x)+\sqrt{\mathrm{y}(x)^2-1})}{\sqrt{\mathrm{y}(x)-1}\sqrt{\mathrm{y}(x)+1}} + \_C1 = 0,$$

$$2\frac{\sqrt{x(\mathrm{y}(x)^2-1)}}{\sqrt{\mathrm{y}(x)-1}\sqrt{\mathrm{y}(x)+1}} + \frac{\sqrt{(\mathrm{y}(x)-1)(\mathrm{y}(x)+1)}\ln(\mathrm{y}(x)+\sqrt{\mathrm{y}(x)^2-1})}{\sqrt{\mathrm{y}(x)-1}\sqrt{\mathrm{y}(x)+1}} + \_C1 = 0$$

`sol31:=dsolve(eq31,y(x));` **This takes a long time ... I have not seen a solution yet**

```
Warning, computation interrupted
```

```
>map(odetest,[sol31],eq31);
```

$$[0, 0, 0]$$

`>map(odetest,[y(x)=cosh(2*sqrt(x))],eq31);` **Another solution, not given here**

$$[0]$$

Problem #32 ok, implicit required for time

`> restart:with(DEtools):eq32:=(x^2-1)*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+y(x)^2-1=0;` Clairaut Equation

$$eq32 := (x^2-1)\left(\frac{\partial}{\partial x}\mathrm{y}(x)\right)^2 - 2\,x\,\mathrm{y}(x)\left(\frac{\partial}{\partial x}\mathrm{y}(x)\right) + \mathrm{y}(x)^2 - 1 = 0$$

```
>odeadvisor(eq32);
```

$$[[\_1st\_order, \_with\_linear\_symmetries], \_rational, \_Clairaut]$$

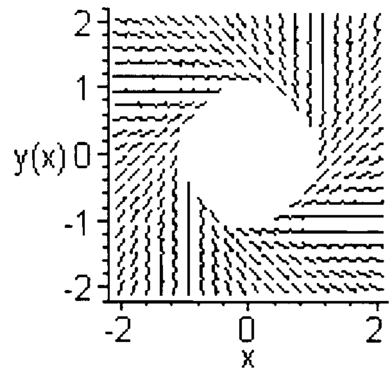
```
> sol32:=dsolve(eq32,y(x),implicit);
map(odetest,[sol32],eq32);
```

(the first solution is a unit circle, singularities at 1 and -1 are taken care of)

$$sol32 := x^2 - 1 + \mathrm{y}(x)^2 = 0,\ \mathrm{y}(x) = x\,\_C1 + \sqrt{\_C1^2+1},\ \mathrm{y}(x) = x\,\_C1 - \sqrt{\_C1^2+1}$$

$$[0, 0, 0]$$

```
> with(DEtools):DEplot(eq32,y(x),x=-2..2,y=-2..2,
stepsize=.05,color=blue);
```



Systems of ODE's

Problem #33 ok

```
> restart:with(DEtools):
> eq33a:=diff(x(t),t)=x(t)-y(t);
eq33b:=diff(y(t),t)=x(t)+y(t);
```

$$eq33a := \frac{\partial}{\partial t}\,\mathrm{x}(t) = \mathrm{x}(t) - \mathrm{y}(t)$$

$$eq33b := \frac{\partial}{\partial t}\,\mathrm{y}(t) = \mathrm{x}(t) + \mathrm{y}(t)$$

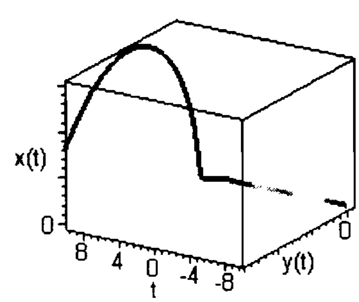
```
> sol33:=dsolve({eq33a,eq33b},{x(t),y(t)});
```

$$sol33 := \{\mathrm{y}(t) = \mathrm{e}^{t}\,(\_C1\,\sin(t) + \_C2\,\cos(t)),\ \mathrm{x}(t) = \mathrm{e}^{t}\,(\_C1\,\cos(t) - \_C2\,\sin(t))\}$$

```
> map(odetest, [sol33],{eq33a,eq33b});
```

$$[\{0\}]$$

```
> DEplot3d({eq33a,eq33b},{x(t),y(t)},
t=-10..10,stepsize=.1,[[x(0)=1,y(0)=1]],linecolor=t);
```



Problem #34 error

```
> restart:with(DEtools):
> eq34a:=diff(x(t),t)^2+diff(y(t),t)^2=3/2; eq34b:=(t-
x(t))*diff(y(t),t)+y(t)*diff(x(t),t)=0;
```

$$eq34a := \left(\frac{\partial}{\partial t}\mathrm{x}(t)\right)^2 + \left(\frac{\partial}{\partial t}\mathrm{y}(t)\right)^2 = \frac{3}{2}$$

$$eq34b := (t - \mathrm{x}(t))\left(\frac{\partial}{\partial t}\mathrm{y}(t)\right) + \mathrm{y}(t)\left(\frac{\partial}{\partial t}\mathrm{x}(t)\right) = 0$$

```
> sol34:=dsolve({eq34a,eq34b,x(0)=0,y(0)=-
1},{x(t),y(t)},numeric);
```

```
Error, (in DEtools/convertsys) unable to convert to an explicit first-
order system
```

Problem #35 ok

```
> restart:with(PDEtools):
```

```
> eq35:=diff(u(x,t),x,x)=(1/k)*diff(u(x,t),t);
```
Parabolic (heat) equation

0 < x < a, 0 < t, I may specify boundary conditions.

$$eq35 := \frac{\partial^2}{\partial x^2}\mathrm{u}(x,t) = \frac{\frac{\partial}{\partial t}\mathrm{u}(x,t)}{k}$$

```
> sol35:=combine(pdsolve(eq35,build));
```

$$sol35 := \mathrm{u}(x,t) = \_C3\ \_C1\ \mathrm{e}^{(\sqrt{\_c_1}\,x + \_c_1\,k\,t)} + \_C3\ \_C2\ \mathrm{e}^{(-\sqrt{\_c_1}\,x + \_c_1\,k\,t)}$$

```
> pdetest(sol35,eq35);
```
Solution Verified.

$$0$$

PDEplot requires that the pde be a first order pde. otherwise, the plot will not work.

Problem #36 ok

```
> restart:with(PDEtools):
```

```
> eq36:=diff(u(x,t),x,x)=(1/c^2)*diff(u(x,t),t,t);
```
Hyperbolic (wave) equation

0<x<a, 0<t, I may specify boundary conditions.

$$eq36 := \frac{\partial^2}{\partial x^2}\mathrm{u}(x,t) = \frac{\frac{\partial^2}{\partial t^2}\mathrm{u}(x,t)}{c^2}$$

```
> sol36:=pdsolve(eq36,build);
```

$$sol36 := \mathrm{u}(x,t) = \_F1(c\,t + x) + \_F2(c\,t - x)$$

```
> pdetest(sol36,eq36);
```
Solution Verified

$$0$$

Problem #37 ok

```
> restart:with(PDEtools):
```

```
> eq37:=diff(u(x,y,z),x,x)+diff(u(x,y,z),y,y)+
diff(u(x,y,z),z,z)=0;
```
Elliptic/Potential/Laplace

$$eq37 := \left(\frac{\partial^2}{\partial x^2}\,\mathrm{u}(x,y,z)\right)+\left(\frac{\partial^2}{\partial y^2}\,\mathrm{u}(x,y,z)\right)+\left(\frac{\partial^2}{\partial z^2}\,\mathrm{u}(x,y,z)\right)=0$$

```
> sol37:=combine(pdsolve(eq37,build));pdetest(sol37,eq37);
```

$$sol37 := \mathrm{u}(x,y,z) = \_C1\,\_C3\,\_C5\,\sin(\%1)\,e^{(\sqrt{\_c_1}\,x+\sqrt{\_c_2}\,y)}$$
$$+\_C1\,\_C3\,\_C6\,\cos(\%1)\,e^{(\sqrt{\_c_1}\,x+\sqrt{\_c_2}\,y)} + \_C1\,\_C4\,\_C5\,\sin(\%1)\,e^{(\sqrt{\_c_1}\,x-\sqrt{\_c_2}\,y)}$$
$$+\_C1\,\_C4\,\_C6\,\cos(\%1)\,e^{(\sqrt{\_c_1}\,x-\sqrt{\_c_2}\,y)} + \_C2\,\_C3\,\_C5\,\sin(\%1)\,e^{(-\sqrt{\_c_1}\,x+\sqrt{\_c_2}\,y)}$$
$$+\_C2\,\_C3\,\_C6\,\cos(\%1)\,e^{(-\sqrt{\_c_1}\,x+\sqrt{\_c_2}\,y)} + \_C2\,\_C4\,\_C5\,\sin(\%1)\,e^{(-\sqrt{\_c_1}\,x-\sqrt{\_c_2}\,y)}$$
$$+\_C2\,\_C4\,\_C6\,\cos(\%1)\,e^{(-\sqrt{\_c_1}\,x-\sqrt{\_c_2}\,y)}$$
$$\%1 := \sqrt{\_c_1+\_c_2}\,z$$

$$0$$

Problem #38 ok on most parts

```
> restart:with(PDEtools):
```

```
> eq38:=diff(u(x,y,z),x,x)+diff(u(x,y,z),y,y)+
diff(u(x,y,z),z,z) = -f(x,y,z);
```
Poisson, general form

$$eq38 := \left(\frac{\partial^2}{\partial x^2}\,\mathrm{u}(x,y,z)\right)+\left(\frac{\partial^2}{\partial y^2}\,\mathrm{u}(x,y,z)\right)+\left(\frac{\partial^2}{\partial z^2}\,\mathrm{u}(x,y,z)\right)=-\mathrm{f}(x,y,z)$$

>sol38b:=pdsolve(eq38,build); runs out of time.

```
> eq38b:=diff(u(x,y,z),x,x)+diff(u(x,y,z),y,y)+
diff(u(x,y,z),z,z)=-1;
```
Powers 290

$$eq38b := \left(\frac{\partial^2}{\partial x^2}\,\mathrm{u}(x,y,z)\right)+\left(\frac{\partial^2}{\partial y^2}\,\mathrm{u}(x,y,z)\right)+\left(\frac{\partial^2}{\partial z^2}\,\mathrm{u}(x,y,z)\right)=-1$$

```
> sol38b:=pdsolve(eq38b,build);
```

$$sol38b := \mathrm{u}(x,y,z) = \%2\ \%1\ \_C1\ \_C3\ \_C5 \sin(\%3) + \%2\ \%1\ \_C1\ \_C3\ \_C6 \cos(\%3) + \frac{\%1\ \_C1\ \_C4\ \_C5 \sin(\%3)}{\%2} + \frac{\%1\ \_C1\ \_C4\ \_C6 \cos(\%3)}{\%2} + \frac{\%2\ \_C2\ \_C3\ \_C5 \sin(\%3)}{\%1} + \frac{\%2\ \_C2\ \_C3\ \_C6 \cos(\%3)}{\%1} + \frac{\_C2\ \_C4\ \_C5 \sin(\%3)}{\%2\ \%1} + \frac{\_C2\ \_C4\ \_C6 \cos(\%3)}{\%2\ \%1} - \frac{1}{2} y^2 - \frac{\_C3\, y}{\_C2} - \frac{\_C4}{\_C2}$$

$$\%1 := e^{(\sqrt{\_c_1}\, x)}$$

$$\%2 := e^{(\sqrt{\_c_2}\, y)}$$

$$\%3 := \sqrt{\_c_1 + \_c_2}\, z$$

```
> dsubs(sol38b,eq38b);
```
solution verified

$$-1 = -1$$

```
> restart:with(PDEtools):
```

```
> eq38c:=diff(u(x,y,z),x,x)+diff(u(x,y,z),y,y)+
diff(u(x,y,z),z,z)=-1/(x^2+y^2+z^2);
```
a variation of Powers 290

$$eq38c := \left(\frac{\partial^2}{\partial x^2} \mathrm{u}(x,y,z)\right) + \left(\frac{\partial^2}{\partial y^2} \mathrm{u}(x,y,z)\right) + \left(\frac{\partial^2}{\partial z^2} \mathrm{u}(x,y,z)\right) = -\frac{1}{x^2+y^2+z^2}$$

```
> sol38c:=pdsolve(eq38c,build);
Error, (in pdsolve/sep/casesplit/do) invalid subscript selector
```

```
> restart:with(PDEtools):
```

```
> eq38d:=diff(u(x,y),x,x)+diff(u(x,y),y,y)=-1/(x^2+y^2);
sol38d:=pdsolve(eq38d,build);
```

$$eq38d := \left(\frac{\partial^2}{\partial x^2} \mathrm{u}(x,y)\right) + \left(\frac{\partial^2}{\partial y^2} \mathrm{u}(x,y)\right) = -\frac{1}{x^2+y^2}$$

$$sol38d := \mathrm{u}(x,y) = \_F1(y + I x) + \_F2(y - I x) - \frac{1}{8} \left(\ln((x^2+y^2)^2) - \ln(x - I y)\right) \ln(x - I y)$$

```
> test1:=factor(simplify(dsubs(sol38d,eq38d)));
```

$$test1 := -\frac{1}{(y + I x)(y - I x)} = -\frac{1}{x^2+y^2}$$

```
> pdetest(sol38d,eq38d);
```
zero indicates solution is correct.

$$0$$

Problem 39 (homogeneous linear system)

```
> restart:with(DEtools):
>
set39:={diff(x(t),t)=x(t)+y(t)+2*z(t),diff(y(t),t)=x(t)+2*y(t)+z(t),diff(z(t),t)=2*x(t)+y(t)+z(t)};
```

$$set39 := \{\frac{\partial}{\partial t}\mathrm{x}(t)=\mathrm{x}(t)+\mathrm{y}(t)+2\,\mathrm{z}(t),\ \frac{\partial}{\partial t}\mathrm{y}(t)=\mathrm{x}(t)+2\,\mathrm{y}(t)+\mathrm{z}(t),$$
$$\frac{\partial}{\partial t}\mathrm{z}(t)=2\,\mathrm{x}(t)+\mathrm{y}(t)+\mathrm{z}(t)\}$$

```
> sol39:=dsolve(set39,{x(t),y(t),z(t)});
map(odetest,[sol39],set39);
```

$$sol39 := \{\mathrm{x}(t)=\_C1\,\mathrm{e}^t+\_C2\,\mathrm{e}^{(-t)}+\_C3\,\mathrm{e}^{(4\,t)},\ \mathrm{y}(t)=-2\,\_C1\,\mathrm{e}^t+\_C3\,\mathrm{e}^{(4\,t)},$$
$$\mathrm{z}(t)=\_C1\,\mathrm{e}^t-\_C2\,\mathrm{e}^{(-t)}+\_C3\,\mathrm{e}^{(4\,t)}\}$$

[{0}]

Problem 40 (nonhomogeneous linear system)

```
> restart:with(DEtools):set40:={diff(x(t),t)=2*x(t)-5*y(t)+csc(t),diff(y(t),t)=x(t)-2*y(t)+sec(t)};
```

$$set40 := \{\frac{\partial}{\partial t}\mathrm{y}(t)=\mathrm{x}(t)-2\,\mathrm{y}(t)+\sec(t),\ \frac{\partial}{\partial t}\mathrm{x}(t)=2\,\mathrm{x}(t)-5\,\mathrm{y}(t)+\csc(t)\}$$

```
> sol40:=combine(dsolve(set40,{x(t),y(t)}),trig);
map(odetest,[sol40],set40);
```

$$sol40 := \{\mathrm{y}(t)=-\sin(t)\ln(\cos(t))-2\sin(t)\,t+\sin(t)\ln(\sin(t))-\cos(t)\tan(t)$$
$$-2\cos(t)\ln(\cos(t))+\_C1\sin(t)+\_C2\cos(t),\ \mathrm{x}(t)=\frac{1}{2}(-5\ln(\cos(t))\cos(2\,t)$$
$$-5\ln(\cos(t))+1-\cos(2\,t)-2\,t\cos(2\,t)-2\,t+\ln(\sin(t))\cos(2\,t)+\ln(\sin(t))$$
$$+\tan(t)\sin(2\,t)-\tan(t)^2\cos(2\,t)-\tan(t)^2+\_C1\cos(2\,t)+\_C1-\_C2\sin(2\,t)$$
$$-4\,t\sin(2\,t)+2\ln(\sin(t))\sin(2\,t)-2\tan(t)\cos(2\,t)-2\tan(t)+2\,\_C1\sin(2\,t)$$
$$+2\,\_C2\cos(2\,t)+2\,\_C2-2\sec(t)\cos(t))/\cos(t)\}$$

[{0}]

Problem 41 (homogeneous linear system)

```
> restart:with(DEtools):set41:={diff(x(t),t)=2*x(t),
diff(y(t),t)=-2*x(t)+y(t)-2*z(t),diff(z(t),t)=x(t)+3*z(t)};
```

refer to [3]

$$set41 := \{\frac{\partial}{\partial t}\mathrm{y}(t)=-2\,\mathrm{x}(t)+\mathrm{y}(t)-2\,\mathrm{z}(t),\ \frac{\partial}{\partial t}\mathrm{z}(t)=\mathrm{x}(t)+3\,\mathrm{z}(t),\ \frac{\partial}{\partial t}\mathrm{x}(t)=2\,\mathrm{x}(t)\}$$

```
> sol41:=dsolve(set41,{x(t),y(t),z(t)});
map(odetest,[sol41],set41);
```

$$sol41 := \{\mathrm{z}(t)=-\_C3\,\mathrm{e}^{(2\,t)}+\mathrm{e}^{(3\,t)}\,\_C2,\ \mathrm{x}(t)=\_C3\,\mathrm{e}^{(2\,t)},\ \mathrm{y}(t)=-\mathrm{e}^{(3\,t)}\,\_C2+\mathrm{e}^t\,\_C1\}$$

[{0}]

APPENDIX E
Mathematica 4.0 Syntax of Test Suite

(See Appendix A for references)

```
In[12]:= (*Problem #1*)
         Remove[y]
         S = DSolve[y'[x] == x*Exp[y[x] + Sin[x]], y[x], x]
         FullSimplify[y'[x] == x*Exp[y[x] + Sin[x]] //. S //. D[S, x]]

       Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.
```

Out[13]= $\{\{y[x] \to -\mathrm{Log}[C[1] - \int e^{\mathrm{Sin}[x]}\, x\, dx]\}\}$

Out[14]= {{True}}

```
In[1]:= (*Problem #2*)
        Remove[y]
        DSolve[2*x*y[x]^2 + 2*y[x] + y'[x]*(2*x^2*y[x] + 2*x) = 0, y[x], x]
        FullSimplify[2*x*y[x]^2 + 2*y[x] + y'[x]*(2*x^2*y[x] + 2*x) = 0 /.
          DSolve[2*x*y[x]^2 + 2*y[x] + y'[x]*(2*x^2*y[x] + 2*x) = 0, y[x], x]]
```

Out[2]= $\left\{\left\{y[x] \to \frac{-1 - \sqrt{1 + C[1]}}{x}\right\}, \left\{y[x] \to \frac{-1 + \sqrt{1 + C[1]}}{x}\right\}\right\}$

Out[3]= $\left\{\frac{2\left(1 + C[1] + \sqrt{1 + C[1]}\,(1 - x^2\, y'[x])\right)}{x} == 0, \frac{1 + C[1] + \sqrt{1 + C[1]}\,(-1 + x^2\, y'[x])}{x} == 0\right\}$

```
(*Problem #3*)
DSolve[y[x]^2 - x + 2*y[x]*y'[x] == 0, y[x], x]
```

$\left\{\left\{y[x] \to -i\, e^{-x/2} \sqrt{e^x - e^x x + C[1]}\right\}, \left\{y[x] \to i\, e^{-x/2} \sqrt{e^x - e^x x + C[1]}\right\}\right\}$

```
In[15]:= (*Problem #4*)
         DSolve[D[y[x, a], x] == a*y[x, a], y[x, a], {x, a}]
```

Out[15]= $\{\{y[x, a] \to e^{a x}\, C[1][a]\}\}$

```
(*Problem #5*)
DSolve[y'[t] + a*y[t - 1] == 0, y[t], t]
L = LaplaceTransform[y'[t] + a*y[t - 1], t, s]
R = LaplaceTransform[0, t, s]
K = Solve[L == R, LaplaceTransform[y[t], t, s]]
InverseLaplaceTransform[K, s, t]
```

DSolve::nvld : The description of the equations appears to be ambiguous or invalid.

$$\left\{\left\{y[t] \to C[1] - a \int_0^t y[-1 + DSolve`t]\, dDSolve`t\right\}\right\}$$

a LaplaceTransform[y[-1 + t], t, s] + s LaplaceTransform[y[t], t, s] - y[0]

0

$$\left\{\left\{LaplaceTransform[y[t], t, s] \to -\frac{a\, LaplaceTransform[y[-1 + t], t, s] - y[0]}{s}\right\}\right\}$$

Unique::usym : {LaplaceTransform[y[t], t, s] -> -((a*<<1>> - y[0])/s)} is not a symbol or a valid symbol name.

$$\left\{\left\{y[t] \to -a \int_0^t y[-1 + s]\, ds + y[0]\right\}\right\}$$

```
(*Problem #6*)
DSolve[x*y'[x] + y[x] - y[x]^2*Exp[2*x] == 0, y[x], x]
```

$$\left\{\left\{y[x] \to \frac{1}{e^{2x} - x\, C[1] - 2x\, ExpIntegralEi[2x]}\right\}\right\}$$

```
(*Problem #7*)
Remove[y]
Remove[P]
Remove[Q]
DSolve[y'[x] + P[x]*y[x] == Q[x]*y''[x], y[x], x]
```

InverseFunction::ifun : Inverse functions are being used. Values may be lost for multivalued inverses.

InverseFunction::ifun : Inverse functions are being used. Values may be lost for multivalued inverses.

DSolve[P[x] y[x] + y'[x] == Q[x] y''[x], y[x], x]

```
(*Problem #8*)
DSolve[u''''[t] + lambda^4*u[t] == 0, u[t], t]
```

$$\left\{\left\{u[t] \to e^{-(-1)^{1/4}\, lambda\, t} C[1] + e^{(-1)^{1/4}\, lambda\, t} C[2] + e^{-(-1)^{3/4}\, lambda\, t} C[3] + e^{(-1)^{3/4}\, lambda\, t} C[4]\right\}\right\}$$

```
In[24]:= (*Problem #9*)
         Remove[y]
         inits = {y''[x] + k^2 * y[x] == 0, y'[1] == 0, y[0] == 0}
         DSolve[y''[x] + k^2 * y[x] == 0, y[x], x] (*general solution*)
         DSolve[inits, y[x], x]
         DSolve[inits, {y[x], k}, x]
```

```
Out[25]= {k^2 y[x] + y''[x] == 0, y'[1] == 0, y[0] == 0}

Out[26]= {{y[x] -> C[2] Cos[k x] + C[1] Sin[k x]}}

Out[27]= {{y[x] -> 0}}

         Solve::svars : Equations may not give solutions for all "solve" variables

Out[28]= DSolve[{k^2 y[x] + y''[x] == 0, y'[1] == 0, y[0] == 0}, {y[x], k}, x]
```

```
In[1]:= (*Problem #10*)
        DSolve[(56 + 59 * x) * y'''[x] + (13 + 19 * x) * y''[x] +
           (-142 - 59 * x) * y'[x] + (-199 - 9 * x) * y[x] == 0, y[x], x]
        DSolve[{(56 + 59 * x) * y'''[x] + (13 + 19 * x) * y''[x] + (-142 - 59 * x) * y'[x] +
            (-199 - 9 * x) * y[x] == 0, y[1] == 0, y'[1] == 1, y''[1] == 1}, y[x], x]
```

```
Out[1]= DSolve[(-199 - 9 x) y[x] + (-142 - 59 x) y'[x] +
           (13 + 19 x) y''[x] + (56 + 59 x) y^(3)[x] == 0, y[x], x]

Out[2]= DSolve[
          {(-199 - 9 x) y[x] + (-142 - 59 x) y'[x] + (13 + 19 x) y''[x] + (56 + 59 x) y^(3)[x] ==
            0, y[1] == 0, y'[1] == 1, y''[1] == 1}, y[x], x]
```

```
In[1]:= (*Problem #11  series , this does not find solution in nhood of x=2*)
        Remove[y]
        EQ11 = (x - 2) * x^2 * y''[x] + x^2 * y'[x] + Exp[x - 1] * y[x] == 0
         (*sol=DSolve[{eq,y[2]==0,y'[2]==1},y[x],x] produces many errors*)
         (*note that the fundamental exponents from the indicial
            equation for the series solution at x=2 are both zero*)
         y[x_] = Sum[a[n] (x - 2)^n, {n, 0, 4}] + O[x]^5
         (*Mathematica will not acceps O[x-2]*)
         EQ11;
         eqs = LogicalExpand[%]
         inits = {a[0] → 0, a[1] → 1};
         Solve[eqs /. inits, a[2]] (*indicates that we cannot find
           a recurrence relation with these initial conditions*)
```

Out[2]= $e^{-1+x}\, y[x] + x^2\, y'[x] + (-2+x)\, x^2\, y''[x] == 0$

Out[3]= $(a[0] - 2\,a[1] + 4\,a[2] - 8\,a[3] + 16\,a[4]) + (a[1] - 4\,a[2] + 12\,a[3] - 32\,a[4])\,x + (a[2] - 6\,a[3] + 24\,a[4])\,x^2 + (a[3] - 8\,a[4])\,x^3 + a[4]\,x^4 + O[x]^5$

Out[5]= $\frac{a[0] - 2\,a[1] + 4\,a[2] - 8\,a[3] + 16\,a[4]}{e} == 0\ \&\&$
$\frac{a[1] - 4\,a[2] + 12\,a[3] - 32\,a[4]}{e} + \frac{a[0] - 2\,a[1] + 4\,a[2] - 8\,a[3] + 16\,a[4]}{e} ==$
$0\ \&\&\ \frac{a[1] - 4\,a[2] + 12\,a[3] - 32\,a[4]}{6\,e} +$
$9\,(a[3] - 8\,a[4]) + \frac{a[3] - 8\,a[4]}{e} - 24\,a[4] + \frac{a[4]}{e} +$
$\frac{a[0] - 2\,a[1] + 4\,a[2] - 8\,a[3] + 16\,a[4]}{24\,e} + \frac{a[2] - 6\,a[3] + 24\,a[4]}{2\,e} == 0\ \&\&$
$a[1] - 4\,a[2] + 12\,a[3] + \frac{a[1] - 4\,a[2] + 12\,a[3] - 32\,a[4]}{e} -$
$32\,a[4] + \frac{a[0] - 2\,a[1] + 4\,a[2] - 8\,a[3] + 16\,a[4]}{2\,e} -$
$4\,(a[2] - 6\,a[3] + 24\,a[4]) + \frac{a[2] - 6\,a[3] + 24\,a[4]}{e} == 0\ \&\&$
$\frac{a[1] - 4\,a[2] + 12\,a[3] - 32\,a[4]}{2\,e} - 12\,(a[3] - 8\,a[4]) +$
$\frac{a[3] - 8\,a[4]}{e} + \frac{a[0] - 2\,a[1] + 4\,a[2] - 8\,a[3] + 16\,a[4]}{6\,e} +$
$4\,(a[2] - 6\,a[3] + 24\,a[4]) + \frac{a[2] - 6\,a[3] + 24\,a[4]}{e} == 0$

Out[7]= {}

```
In[136]:= (*Problem #12*)
          sol1 = DSolve[y''[x] + x*y'[x] + Exp[-x^2]*y[x] == 0, y[x], x]
          sol2 = y[x] -> Cosh[∫Sqrt[-Exp[-x^2]] dx] + Sinh[∫Sqrt[-Exp[-x^2]] dx]
          FullSimplify[
           y''[x] + x*y'[x] + Exp[-x^2]*y[x] == 0 //. sol1 //. D[sol1, x] //.
            D[sol1, {x, 2}]] (*solution 1 verified*)
          FullSimplify[y''[x] + x*y'[x] + Exp[-x^2]*y[x] == 0 //. sol2 //.
             D[sol2, x] //. D[sol2, {x, 2}]]
          (*solution 2 verified →this is Maple's solution*)
          sol1 === sol2 (*solutions are not equal*)
```

Out[136]= $\{\{y[x] \to C[2]\,\mathrm{Cos}[\sqrt{\frac{\pi}{2}}\ \mathrm{Erf}[\frac{x}{\sqrt{2}}]] - C[1]\,\mathrm{Sin}[\sqrt{\frac{\pi}{2}}\ \mathrm{Erf}[\frac{x}{\sqrt{2}}]]\}\}$

Out[137]= $y[x] \to \mathrm{Cosh}[e^{\frac{x^2}{2}}\sqrt{-e^{-x^2}}\ \sqrt{\frac{\pi}{2}}\ \mathrm{Erf}[\frac{x}{\sqrt{2}}]] + \mathrm{Sinh}[e^{\frac{x^2}{2}}\sqrt{-e^{-x^2}}\ \sqrt{\frac{\pi}{2}}\ \mathrm{Erf}[\frac{x}{\sqrt{2}}]]$

Out[138]= {{{True}}}

Out[139]= True

Out[140]= False

```
(*Problem #13*)
DSolve[R'[r] + r*R''[r] - mu^2*R[r]/r + lambda^2*r*R[r] == 0, R[r], r]
```

{{R[r] → BesselJ[-mu, lambda r] C[1] + BesselJ[mu, lambda r] C[2]}}

```
(*Problem #14*)
DSolve[u'[t] + k[t]*u[t] == p[t], u[t], t]
```

$\{\{u[t] \to e^{-\int_0^t k[\mathrm{DSolve`t}]\,d\mathrm{DSolve`t}}\, C[1] + e^{-\int_0^t k[\mathrm{DSolve`t}]\,d\mathrm{DSolve`t}} \int_0^t e^{\int_0^{\mathrm{DSolve`t}} k[\mathrm{DSolve`t}]\,d\mathrm{DSolve`t}}\, p[\mathrm{DSolve`t}]\, d\mathrm{DSolve`t}\}\}$

```
(*Problem #15*)
Remove[f]
Simplify[3*f'[t] - 2*f[t] == Cos[t]]
DSolve[3*f'[t] - 2*f[t] == Cos[t], f[t], t]
```

3 f'[t] == Cos[t] + 2 f[t]

$\{\{f[t] \to e^{2t/3}\, C[1] + \frac{1}{13}(-2\,\mathrm{Cos}[t] + 3\,\mathrm{Sin}[t])\}\}$

```
(*Problem #16*)
Remove[y]
EQ = FullSimplify[y'[x] == -y[x] - 1]
S = DSolve[y'[x] == -y[x] - 1, y[x], x]
S1 = D[%, x]
EQ //. S //. S1
```

$1 + y[x] + y'[x] == 0$

$\{\{y[x] \to -1 + e^{-x} C[1]\}\}$

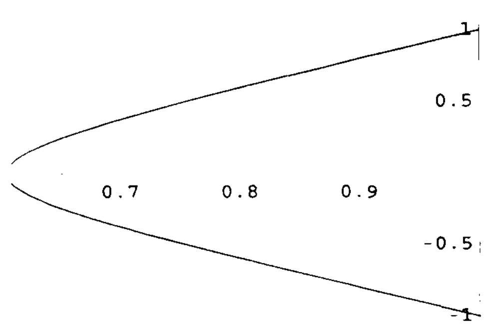
$\{\{y'[x] \to -e^{-x} C[1]\}\}$

{{True}}

```
In[8]:= (*Problem #17*)
Remove[y]
FullSimplify[y'[x] == y[x] / x + x / y[x]];
Sol := DSolve[y'[x] == y[x] / x + x / y[x], y[x], x]
(*general solution verified*)
FullSimplify[y'[x] == y[x] / x + x / y[x] /. Sol /. D[Sol, x]]

(*initial value problem and graph*)
DSolve[{y'[x] == y[x] / x + x / y[x], y[1] == 1}, y[x], x]
B :=
 y[x] /. NDSolve[{y'[x] == y[x] / x + x / y[x], y[1] == 1}, y[x], {x, .61, 1}]
A := y[x] /. NDSolve[{y'[x] == y[x] / x + x / y[x], y[1] == -1},
   y[x], {x, .61, 1}]
K = {A, B};
Plot[Evaluate[K], {x, .61, 1}]
```

Out[11]= $\left\{\left\{\text{True}, \frac{1 + C[1] + 2\,\text{Log}[x]}{\sqrt{C[1] + 2\,\text{Log}[x]}} == 0\right\}, \left\{\frac{1 + C[1] + 2\,\text{Log}[x]}{\sqrt{C[1] + 2\,\text{Log}[x]}} == 0, \text{True}\right\}\right\}$

Out[12]= $\left\{\left\{y[x] \to x\sqrt{1 + 2\,\text{Log}[x]}\right\}\right\}$



Out[16]= - Graphics -

```
(*Problem #18*)
Remove[y]
FullSimplify[x^2 * y'[x] + 3 * x * y[x] == Sin[x] / x]
DSolve[x^2 * y'[x] + 3 * x * y[x] == Sin[x] / x, y[x], x]

x^2 * y'[x] + 3 * x * y[x] == Sin[x] / x /.
 DSolve[x^2 * y'[x] + 3 * x * y[x] == Sin[x] / x, y[x], x]

(*NOTE that this has a singularity at zero!*)
```

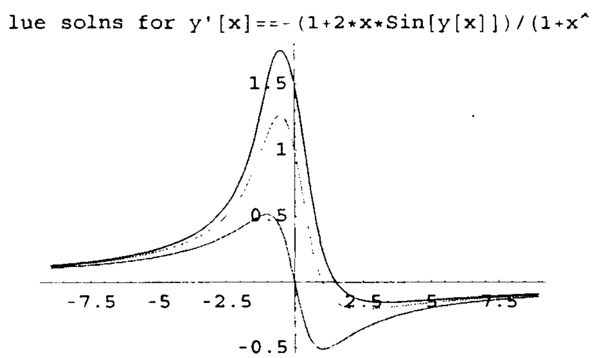
$$x\,(3\,y[x] + x\,y'[x]) == \frac{\text{Sin}[x]}{x}$$

$$\left\{\left\{y[x] \to \frac{C[1]}{x^3} - \frac{\text{Cos}[x]}{x^3}\right\}\right\}$$

$$\left\{3\,x\left(\frac{C[1]}{x^3} - \frac{\text{Cos}[x]}{x^3}\right) + x^2\,y'[x] == \frac{\text{Sin}[x]}{x}\right\}$$

```
(*Problem #19*)
Remove[y]
A := y[x] /.
  NDSolve[{y'[x] == -(1 + 2*x*Sin[y[x]]) / (1 + x^2*Cos[y[x]]), y[0] == 0},
   y[x], {x, -10, 10}]
B := y[x] /. NDSolve[{y'[x] == -(1 + 2*x*Sin[y[x]]) / (1 + x^2*Cos[y[x]]),
    y[0] == 1}, y[x], {x, -10, 10}]
B2 := y[x] /. NDSolve[{y'[x] == -(1 + 2*x*Sin[y[x]]) / (1 + x^2*Cos[y[x]]),
    y[1.5] == 0}, y[x], {x, -10, 10}]
K = {A, B, B2}
Plot[Evaluate[K], {x, -9, 9},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]},
 PlotLabel → "3 initial value solns for y'[
    x]==-(1+2*x*Sin[y[x]])/(1+x^2*Cos[y[x]]"]
(*DSolve[y'[x]==-(1+2*x*Sin[y[x]])/(1+x^2*Cos[y[x]]),y[x],x]
    y'[x]==-(1+2*x*Sin[y[x]])/(1+x^2*Cos[y[x]]) /. %
   this runs for a long time, do not try at home alone*)
```

```
{{InterpolatingFunction[{{-10., 10.}}, <>][x]},
 {InterpolatingFunction[{{-10., 10.}}, <>][x]},
 {InterpolatingFunction[{{-10., 10.}}, <>][x]}}
```



- Graphics -

```
(*Problem #20*)
DSolve[(x^2 + x + 1)*y''[x] + (4*x + 2)*y'[x] + 2*y[x] == 3*x^2, y[x], x]
(x^2 + x + 1)*y''[x] + (4*x + 2)*y'[x] + 2*y[x] == 3*x^2 /. %
```

$$\left\{\left\{y[x] \to \frac{x^4}{4\ (1 + x + x^2)} + \frac{C[1]}{1 + x + x^2} + \frac{x\, C[2]}{1 + x + x^2}\right\}\right\}$$

$$\left\{2\left(\frac{x^4}{4\ (1 + x + x^2)} + \frac{C[1]}{1 + x + x^2} + \frac{x\, C[2]}{1 + x + x^2}\right) + (2 + 4\, x)\ y'[x] + (1 + x + x^2)\ y''[x] == 3\, x^2\right\}$$

```
In[28]:= (*Problem #21 (Should be done by laplace)*)
         DSolve[{y''[x] + 4*y[x] == Sin[2*x], y[0] == 0, y'[0] == 0}, y[x], x]

         L = LaplaceTransform[y''[x] + 4*y[x], x, s];
         R = LaplaceTransform[Sin[2*x], x, s];
         L == R
         K = Solve[{L == R, y[0] == 0, y'[0] == 0}, LaplaceTransform[y[x], x, s]];
         InverseLaplaceTransform[K, s, x]
```

Out[28]= $\left\{\left\{y[x] \to \frac{1}{4}\left(-x\,\mathrm{Cos}[2\,x] + \frac{1}{2}\,\mathrm{Sin}[2\,x]\right)\right\}\right\}$

```
Unique::usym : {InterpolatingFunction[{{0.61, 1.}}, <<2>>, {<<2>>}][x]}
  is not a symbol or a valid symbol name.
```

Out[31]= $4\,\mathrm{LaplaceTransform}[y[x], x, s] + s^2\,\mathrm{LaplaceTransform}[y[x], x, s] - s\,y[0] - y'[0] == \frac{2}{4 + s^2}$

Out[33]= $\left\{\left\{y[x] \to \frac{1}{8}\,(-2\,x\,\mathrm{Cos}[2\,x] + \mathrm{Sin}[2\,x])\right\}\right\}$

```
(*Problem #22*)
Remove[u]
DSolve[
 u'[t] + 2*u[t] + 5*Integrate[u[tau], {tau, 0, t}] == 10*Exp[-4*t], u[t], t]

DSolve::nvld : The description of the equations appears to be ambiguous or invalid.
```

$\left\{\left\{u[t] \to e^{-2\,t}\,C[1] - 5\,e^{-2\,t}\int_0^t e^{-2\,\mathrm{DSolve`t}}\left(-2 + e^{4\,\mathrm{DSolve`t}}\int_0^{\mathrm{DSolve`t}} u[tau]\,dtau\right)d\mathrm{DSolve`t}\right\}\right\}$

In[34]:=
```
(*Problem #23*)
Remove[u]
u'[t] + 2 * u[t] + 5 * Integrate[u[tau], {tau, 0, t}] == 10 * Exp[-4 * t]
L := LaplaceTransform[
   u'[t] + 2 * u[t] + 5 * Integrate[u[tau], {tau, 0, t}], t, s];
R := LaplaceTransform[10 * Exp[-4 * t], t, s];
L == R;
A := Solve[L == R, LaplaceTransform[u[t], t, s]]
A
Ans := Expand[InverseLaplaceTransform[A, s, t]]
Ans
u'[t] + 2 * u[t] +
  5 * Integrate[u[tau], {tau, 0, t}] - 10 * Exp[-4 * t] //. {Ans}
(*this should equal zero*)
```

Out[35]= $5\int_0^t u[tau]\, dtau + 2\, u[t] + u'[t] == 10\, e^{-4t}$

Out[40]= $\left\{\left\{LaplaceTransform[u[t], t, s] \to -\frac{-\frac{10}{4+s} - u[0]}{2 + \frac{5}{s} + s}\right\}\right\}$

Out[42]= $\left\{\left\{u[t] \to -\frac{1}{52}\, e^{(-4-2i)t}\, (160\, e^{2it} - (1-8i)\, e^{3t}\, (10i + (2+3i)\, u[0]) - (8-i)\, e^{(3+4i)t}\, (10 + (3+2i)\, u[0]))\right\}\right\}$

Out[43]= $\left\{\left\{-10\, e^{-4t} + 5\int_0^t u[tau]\, dtau - \frac{1}{26}\, e^{(-4-2i)t}\, (160\, e^{2it} - (1-8i)\, e^{3t}\, (10i + (2+3i)\, u[0]) - (8-i)\, e^{(3+4i)t}\, (10 + (3+2i)\, u[0])) + u'[t]\right\}\right\}$

```
(*Problem #24*)
Remove[y]
EQ = (1 + x + x^2) * y'''[x] + (3 + 6 * x) * y''[x] + 6 * y'[x] == 6 * x
Sol = FullSimplify[DSolve[EQ, y[x], x]]
(*verifying solution*)
f[x_] := 1/(12 (1 + x + x^2)) (ⅈ Sqrt[3] (6 C[1] - (1 + 2 x) C[2]) +
    3 (x^4 - 2 C[1] - 4 x C[1] + C[2] + 4 (1 + x + x^2) C[3]))
FullSimplify[EQ /. {y'[x] → f'[x], y''[x] → f''[x], y'''[x] → f'''[x]}]
(*check for equivalence with Maple's solution  (fails)*)
FullSimplify[f[x_] == (C[3] + C[2] * x + C[1] * x^2 + 1 / 4 * x^4) / (1 + x + x^2)]
g[x_] := (C[3] + C[2] * x + C[1] * x^2 + 1 / 4 * x^4) / (1 + x + x^2)
(*verify Maple's solution here (yes it works*)
FullSimplify[EQ /. {y'[x] → g'[x], y''[x] → g''[x], y'''[x] → g'''[x]}]
FullSimplify[f[x] - g[x]]
```

$$6\, y'[x] + (3 + 6\, x)\, y''[x] + (1 + x + x^2)\, y^{(3)}[x] == 6\, x$$

$$\left\{\left\{y[x] \to \frac{1}{12\,(1 + x + x^2)} \left(i \sqrt{3}\,(6\, C[1] - (1 + 2\, x)\, C[2]) + 3\,(x^4 - 2\, C[1] - 4\, x\, C[1] + C[2] + 4\,(1 + x + x^2)\, C[3])\right)\right\}\right\}$$

True

$$\frac{1}{12\,(1 + x\_ + x\_^2)} \left(i \sqrt{3}\,(6\, C[1] - C[2]\,(1 + 2\, x\_)) + 3\,(-2\, C[1] + C[2] - 4\, C[1]\, x\_ + x\_^4 + 4\, C[3]\,(1 + x\_ + x\_^2))\right) == \frac{\frac{x^4}{4} + x^2\, C[1] + x\, C[2] + C[3]}{1 + x + x^2}$$

True

$$\frac{1}{12\,(1 + x + x^2)} \left(i \sqrt{3}\,(6\, C[1] - (1 + 2\, x)\, C[2]) + 3\,(-2\,(1 + 2\, x\,(1 + x))\, C[1] + C[2] - 4\, x\, C[2] + 4\, x\,(1 + x)\, C[3])\right)$$

```
In[73]:= (*Problem #25*)
         EQ = y''[x] + 2*y'[x] + y[x] == DiracDelta[x]
         L = LaplaceTransform[y''[x] + 2*y'[x] + y[x], x, s]
         R = LaplaceTransform[DiracDelta[x], x, s]
         L == R
         K = Solve[L == R, LaplaceTransform[y[x], x, s]]
         FullSimplify[InverseLaplaceTransform[K, s, x]]
         DSolve[EQ, y[x], x]

Out[73]= y[x] + 2 y'[x] + y''[x] == DiracDelta[x]

Out[74]= LaplaceTransform[y[x], x, s] + s^2 LaplaceTransform[y[x], x, s] +
          2 (s LaplaceTransform[y[x], x, s] - y[0]) - s y[0] - y'[0]

       Unique::usym : {LaplaceTransform[Removed["y"] ..., s] -> -((-1 + <<3>>)/<<1>>)}
          is not a symbol or a valid symbol name.

Out[75]= 1

Out[76]= LaplaceTransform[y[x], x, s] + s^2 LaplaceTransform[y[x], x, s] +
           2 (s LaplaceTransform[y[x], x, s] - y[0]) - s y[0] - y'[0] == 1
```

Out[77]= $\{\{\text{LaplaceTransform}[y[x],\ x,\ s] \to -\frac{-1 - 2\,y[0] - s\,y[0] - y'[0]}{(1+s)^2}\}\}$

Out[78]= $\{\{y[x] \to e^{-x}\ (y[0] + x\ (1 + y[0] + y'[0]))\}\}$

Out[79]= $\{\{y[x] \to e^{-x}\ (C[1] + x\,C[2] + x\,\text{UnitStep}[x])\}\}$

```
(*Problem #26*)
Remove[y]
Remove[g]
EQ = (1 - x) * y''[x] + x * y'[x] - y[x] == g[x]
S = FullSimplify[DSolve[EQ, y[x], x]]

-y[x] + x y'[x] + (1 - x) y''[x] == g[x]
```

$$\{\{y[x] \to e^{x}\,C[1] - x\,C[2] + \int_0^x \frac{e^{-\text{DSolve`t}}\ (-e^{x}\,\text{DSolve`t} + e^{\text{DSolve`t}}\,x)\ g[\text{DSolve`t}]}{(-1 + \text{DSolve`t})^2}\, d\text{DSolve`t}\}\}$$

```
(*Problem #27*)
Remove[y]
Remove[g]
EQ = x^2 * y''[x] + x * y'[x] + (x^2 - .25) * y[x] == g[x]
S = FullSimplify[DSolve[EQ, y[x], x]]
```

$(-0.25 + x^2)\, y[x] + x\, y'[x] + x^2\, y''[x] == g[x]$

```
NIntegrate::nlim : DSolve`t = x is not a valid limit of integration.

NIntegrate::precw :
 The precision of the argument function (<<1>>/DSolve`t) is less than WorkingPrecision (25).

NIntegrate::nlim : DSolve`t = x is not a valid limit of integration.

NIntegrate::precw :
 The precision of the argument function (<<1>>/DSolve`t) is less than WorkingPrecision (25).

NIntegrate::nlim : DSolve`t = x is not a valid limit of integration.

General::stop :
 Further output of NIntegrate::nlim will be suppressed during this calculation.

NIntegrate::precw :
 The precision of the argument function (<<1>>/DSolve`t) is less than WorkingPrecision (25).

General::stop :
 Further output of NIntegrate::precw will be suppressed during this calculation.
```

$$\left\{\left\{y[x] \to \frac{e^{-1.\,i\,x}\,(C[1] - (0. + 0.5\,i)\,e^{2.\,i\,x}\,C[2])}{\sqrt{x}} + \text{NIntegrate}\left[\frac{\left(\frac{(0.+0.5\,i)\,e^{i\,\text{DSolve`t}-i\,x}}{\sqrt{\text{DSolve`t}}\,\sqrt{x}} - \frac{(0.+0.5\,i)\,e^{-i\,\text{DSolve`t}+i\,x}}{\sqrt{\text{DSolve`t}}\,\sqrt{x}}\right) g[\text{DSolve`t}]}{\text{DSolve`t}}, \{\text{DSolve`t}, 0., x\}, \text{WorkingPrecision} \to 25., \text{AccuracyGoal} \to \infty, \text{PrecisionGoal} \to 15.\right]\right\}\right\}$$

```
(*Problem #28*)
Remove[y]
Remove[g]
EQ = y'''[x] - y''[x] + y'[x] - y[x] = g[x]
S = FullSimplify[DSolve[EQ, y[x], x], g[x]]
```

$-y[x] + y'[x] - y''[x] + y^{(3)}[x] == g[x]$

$$\left\{\left\{y[x] \to \frac{1}{2}\left(e^x \int_{C[3]}^{x} e^{-\text{DSolve`t}}\, g[\text{DSolve`t}]\, d\text{DSolve`t} + \text{Cos}[x] \int_{C[2]}^{x} g[\text{DSolve`t}]\, (-\text{Cos}[\text{DSolve`t}] + \text{Sin}[\text{DSolve`t}])\, d\text{DSolve`t} - \left(\int_{C[1]}^{x} g[\text{DSolve`t}]\, (\text{Cos}[\text{DSolve`t}] + \text{Sin}[\text{DSolve`t}])\, d\text{DSolve`t}\right) \text{Sin}[x]\right)\right\}\right\}$$

```
(*Problem #29*)
Remove[y]
EQ = y''[x] + y[x] * (y'[x]) ^3 == 0
S = FullSimplify[DSolve[EQ, y[x], x]]
S1 = D[%, x]
S2 = D[%, x]
FullSimplify[EQ //. S //. S1 //. S2]
```

$y[x]\, y'[x]^3 + y''[x] == 0$

$$\Big\{\Big\{y[x] \to -\frac{C[1] + \left(-3\,x + \sqrt{-C[1]^3 + 9\,(x - C[2])^2} + 3\,C[2]\right)^{2/3}}{\left(-3\,x + \sqrt{-C[1]^3 + 9\,(x - C[2])^2} + 3\,C[2]\right)^{1/3}}\Big\},\ \Big\{y[x] \to$$

$$\left(\left(1 + i\sqrt{3}\right) C[1] + \left(1 - i\sqrt{3}\right)\left(-3\,x + \sqrt{-C[1]^3 + 9\,(x - C[2])^2} + 3\,C[2]\right)^{2/3}\right) \Big/$$

$$\left(2\left(-3\,x + \sqrt{-C[1]^3 + 9\,(x - C[2])^2} + 3\,C[2]\right)^{1/3}\right)\Big\},\ \Big\{y[x] \to$$

$$\left(\left(1 - i\sqrt{3}\right) C[1] + \left(1 + i\sqrt{3}\right)\left(-3\,x + \sqrt{-C[1]^3 + 9\,(x - C[2])^2} + 3\,C[2]\right)^{2/3}\right) \Big/$$

$$\left(2\left(-3\,x + \sqrt{-C[1]^3 + 9\,(x - C[2])^2} + 3\,C[2]\right)^{1/3}\right)\Big\}\Big\}$$

```
$Aborted
```

```
In[67]:= (*Problem #30*)
         Remove[y]
         EQ = y''[x] + y[x] * (y'[x])^3 == 0
         S = FullSimplify[DSolve[{EQ, y[0] == 0, y'[0] == 2}, y[x], x]]
         S1 = D[%, x]
         S2 = D[%, x]
         FullSimplify[EQ //. S //. S1 //. S2]
```

Out[68]= $y[x]\, y'[x]^3 + y''[x] == 0$

```
Unique::usym : {LaplaceTransform[Removed["y"] ..., s] -> -((-1 + <<3>>)/<<1>>)}
  is not a symbol or a valid symbol name.
```

Out[69]= $\left\{y[x] \to \frac{1}{\left(-3x+\sqrt{1+9x^2}\right)^{1/3}} - \left(-3x+\sqrt{1+9x^2}\right)^{1/3}\right\}$

Out[70]= $\left\{y'[x] \to -\frac{-3+\frac{9x}{\sqrt{1+9x^2}}}{3\left(-3x+\sqrt{1+9x^2}\right)^{4/3}} - \frac{-3+\frac{9x}{\sqrt{1+9x^2}}}{3\left(-3x+\sqrt{1+9x^2}\right)^{2/3}}\right\}$

Out[71]= $\left\{y''[x] \to \frac{4\left(-3+\frac{9x}{\sqrt{1+9x^2}}\right)^2}{9\left(-3x+\sqrt{1+9x^2}\right)^{7/3}} + \frac{2\left(-3+\frac{9x}{\sqrt{1+9x^2}}\right)^2}{9\left(-3x+\sqrt{1+9x^2}\right)^{5/3}} - \frac{-\frac{81x^2}{(1+9x^2)^{3/2}}+\frac{9}{\sqrt{1+9x^2}}}{3\left(-3x+\sqrt{1+9x^2}\right)^{4/3}} - \frac{-\frac{81x^2}{(1+9x^2)^{3/2}}+\frac{9}{\sqrt{1+9x^2}}}{3\left(-3x+\sqrt{1+9x^2}\right)^{2/3}}\right\}$

Out[72]= True

```
(*Problem #31*)
EQ = x * (y'[x])^2 - (y[x])^2 + 1 == 0
S = FullSimplify[DSolve[EQ, y[x], x]]
S1 = D[%, x]
FullSimplify[EQ //. S //. S1]
```

$1 - y[x]^2 + x\, y'[x]^2 == 0$

$\{\{y[x] \to \mathrm{Cosh}[2\sqrt{x} + C[1]]\}\}$

$\left\{\left\{y'[x] \to \frac{\mathrm{Sinh}[2\sqrt{x} + C[1]]}{\sqrt{x}}\right\}\right\}$

{{True}}

```
(*Problem #32*)
EQ = (x^2 - 1) * y''[x] - 2 * x * y[x] * y'[x] + y[x]^2 - 1 == 0
S = FullSimplify[DSolve[EQ, y[x], x]]
```

$-1 + y[x]^2 - 2\,x\,y[x]\,y'[x] + (-1 + x^2)\,y''[x] == 0$

$\text{DSolve}[y[x]^2 + (-1 + x^2)\,y''[x] == 1 + 2\,x\,y[x]\,y'[x],\ y[x],\ x]$

```
In[57]:= (*Problem #33*)
         EQ1 = x'[t] == x[t] - y[t]
         EQ2 = y'[t] == x[t] + y[t]
         S = FullSimplify[DSolve[{EQ1, EQ2}, {x[t], y[t]}, t]]
         S2 = D[S, t];
         FullSimplify[EQ1 //. S //. S2] (*soltutions verified*)
         FullSimplify[EQ2 //. S //. S2]
```

Out[57]= $x'[t] == x[t] - y[t]$

Out[58]= $y'[t] == x[t] + y[t]$

Out[59]= $\{\{x[t] \to e^t\,(C[1]\,\text{Cos}[t] - C[2]\,\text{Sin}[t]),$
$y[t] \to e^t\,(C[2]\,\text{Cos}[t] + C[1]\,\text{Sin}[t])\}\}$

Out[61]= {{True}}

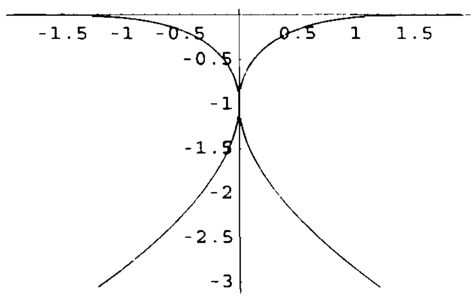
Out[62]= {{True}}

```
In[63]:= (*Problem #34*)
         EQ1 = x'[t]^2 + y'[t]^2 == 3 / 2
         EQ2 = (t - x[t]) * y'[t] + y[t] * x'[t] == 0
         S = NDSolve[{EQ1, EQ2, x[0] == 0, y[0] == -1}, {x[t], y[t]}, {t, -2, 2}]
         ParametricPlot[Evaluate[{x[t], y[t]} /. S], {t, -2, 2}]
```

Out[63]= $x'[t]^2 + y'[t]^2 == \frac{3}{2}$

Out[64]= y[t] x'[t] + (t - x[t]) y'[t] == 0

```
Out[65]= {{x[t] → InterpolatingFunction[{{-2., 2.}}, <>][t],
           y[t] → InterpolatingFunction[{{-2., 2.}}, <>][t]},
          {x[t] → InterpolatingFunction[{{-2., 2.}}, <>][t],
           y[t] → InterpolatingFunction[{{-2., 2.}}, <>][t]}}
```



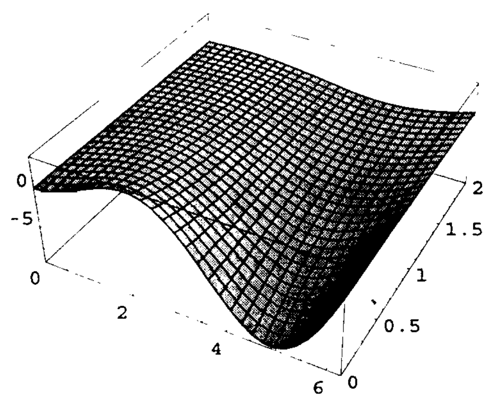
Out[66]= - Graphics -

```
(*Problem #35  PDE  Parabolic/Heat Equation*)
Remove[u]
EQ = ∂x,x u[x, t] == 1 / k * ∂t u[x, t]
S = FullSimplify[DSolve[EQ, u[x, t], {x, t}]]

(*Graphical Solution of Boundary Value Problem*)
solution = NDSolve[{∂t u[x, t] == ∂x,x u[x, t], u[x, 0] == 2 x * Sin[x],
   u[0, t] == 0, u[2 * Pi, t] == 0}, u, {x, 0, 2 * Pi}, {t, 0, 2}]

Plot3D[Evaluate[u[x, t] /. First[%]],
 {x, 0, 2 * Pi}, {t, 0, 2}, PlotPoints → 30]
```

$$u^{(2,0)}[x, t] == \frac{u^{(0,1)}[x, t]}{k}$$

$$\text{DSolve}\left[u^{(2,0)}[x, t] == \frac{u^{(0,1)}[x, t]}{k}, u[x, t], \{x, t\}\right]$$

{{u → InterpolatingFunction[{{0., 6.28319}, {0., 2.}}, <>]}}



- SurfaceGraphics -

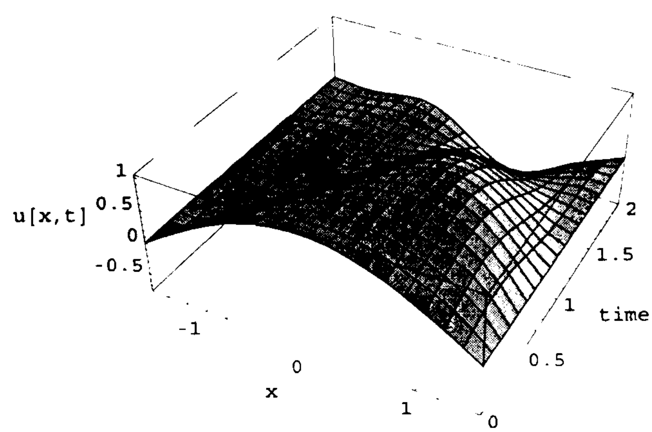
```
In[230]:= (*Problem #36  PDE  Hyperbolic/Wave Equation*)
          Remove[u]
          EQ = D[D[u[x, t], x], x] == 1 / c^2 * D[D[u[x, t], t], t]
          S = FullSimplify[DSolve[EQ, u[x, t], {x, t}]]
          S1 = D[D[S, t], t]; S2 = D[D[S, x], x];
          FullSimplify[EQ //. S1 //. S2] (*solution verified*)

          EQ2 = D[D[u[x, t], x], x] == D[D[u[x, t], t], t];
          boundary1 = u[-Pi / 2, t] == 0; boundary2 = u[Pi / 2, t] == 0;

          init2 = u[x, 0] == Cos[x];
          init4 = Derivative[0, 1][u][x, 0] == Sin[x];

          NDSolve[{EQ2, init2, init4, boundary1, boundary2},
           u, {x, -Pi / 2, Pi / 2}, {t, 0, 2}]

          Plot3D[Evaluate[u[x, t] /. First[%]], {x, -Pi / 2, Pi / 2},
           {t, 0, 2}, PlotPoints → 20, AxesLabel → {"x", "time", "u[x,t] "}]
```

Out[231]= $u^{(2,0)}[x, t] == \frac{u^{(0,2)}[x, t]}{c^2}$

Out[232]= $\{\{u[x, t] \to C[1]\left[t + \frac{x}{c}\right] + C[2]\left[t - \frac{x}{c}\right]\}\}$

Out[234]= {{True}}

Out[239]= {{u → InterpolatingFunction[{{-1.5708, 1.5708}, {0., 2.}}, <>]}}



Out[240]= - SurfaceGraphics -

```
(*Problem #37  PDE  Elliptic/Potential/Laplace Equation*)
Remove[u]

EQ = ∂x,x u[x, y, z] + ∂y,y u[x, y, z] + ∂z,z u[x, y, z] == 0
S = FullSimplify[DSolve[EQ, u[x, y, z], {x, y, z}]]
```

$u^{(0,0,2)}[x, y, z] + u^{(0,2,0)}[x, y, z] + u^{(2,0,0)}[x, y, z] == 0$

$\text{DSolve}[u^{(0,0,2)}[x, y, z] + u^{(0,2,0)}[x, y, z] + u^{(2,0,0)}[x, y, z] == 0,$
$u[x, y, z], \{x, y, z\}]$

```
(*Problem #38 a  PDE  Poisson-general form Equation*)
Remove[u]

EQ = ∂x,x u[x, y, z] + ∂y,y u[x, y, z] + ∂z,z u[x, y, z] == -f[x, y, z]
S = FullSimplify[DSolve[EQ, u[x, y, z], {x, y, z}]]
```

$u^{(0,0,2)}[x, y, z] + u^{(0,2,0)}[x, y, z] + u^{(2,0,0)}[x, y, z] == -f[x, y, z]$

$\text{DSolve}[f[x, y, z] + u^{(0,0,2)}[x, y, z] + u^{(0,2,0)}[x, y, z] + u^{(2,0,0)}[x, y, z] == 0,$
$u[x, y, z], \{x, y, z\}]$

```
(*Problem #38 b  PDE  Poisson Equation*)
Remove[u]

EQ = ∂x,x u[x, y, z] + ∂y,y u[x, y, z] + ∂z,z u[x, y, z] == -1
S = FullSimplify[DSolve[EQ, u[x, y, z], {x, y, z}]]
```

$u^{(0,0,2)}[x, y, z] + u^{(0,2,0)}[x, y, z] + u^{(2,0,0)}[x, y, z] == -1$

$\text{DSolve}[u^{(0,0,2)}[x, y, z] + u^{(0,2,0)}[x, y, z] + u^{(2,0,0)}[x, y, z] == -1,$
$u[x, y, z], \{x, y, z\}]$

```
(*Problem #38 c  PDE  Poisson Equation*)
Remove[u]

EQ = ∂x,x u[x, y, z] + ∂y,y u[x, y, z] + ∂z,z u[x, y, z] == -1 / (x^2 + y^2 + z^2)
S = FullSimplify[DSolve[EQ, u[x, y, z], {x, y, z}]]
```

$u^{(0,0,2)}[x, y, z] + u^{(0,2,0)}[x, y, z] + u^{(2,0,0)}[x, y, z] == -\frac{1}{x^2 + y^2 + z^2}$

$\text{DSolve}\left[\frac{1}{x^2 + y^2 + z^2} + u^{(0,0,2)}[x, y, z] + u^{(0,2,0)}[x, y, z] + u^{(2,0,0)}[x, y, z] == 0,\right.$
$\left.u[x, y, z], \{x, y, z\}\right]$

```
(*Problem #38 d  PDE  Poisson Equation  -  two variables*)
Remove[u]

EQ = ∂x,x u[x, y] + ∂y,y u[x, y] == -1 / (x^2 + y^2)
S = FullSimplify[DSolve[EQ, u, {x, y}]]
```

$u^{(0,2)}[x, y] + u^{(2,0)}[x, y] == -\frac{1}{x^2 + y^2}$

$DSolve\left[\frac{1}{x^2 + y^2} + u^{(0,2)}[x, y] + u^{(2,0)}[x, y] == 0, u, \{x, y\}\right]$

```
In[42]:= (*Problem #39 homogeneous linear system*)
         Remove[x] Remove[y] Remove[z];
         system = {x'[t] == x[t] + y[t] + 2 * z[t],
           y'[t] == x[t] + 2 * y[t] + z[t], z'[t] == x[t] + y[t] + 2 * z[t]}
         S = DSolve[system, {x[t], y[t], z[t]}, t]
         FullSimplify[system //. S //. D[S, t]]

Out[43]= {x'[t] == x[t] + y[t] + 2 z[t],
          y'[t] == x[t] + 2 y[t] + z[t], z'[t] == x[t] + y[t] + 2 z[t]}
```

Out[44]= $\{\{x[t] \to \frac{1}{12}(9\,C[1] + 3\,e^{4t}\,C[1] - 4\,e^{t}\,C[2] + 4\,e^{4t}\,C[2] - 9\,C[3] + 4\,e^{t}\,C[3] + 5\,e^{4t}\,C[3]),$

$y[t] \to \frac{1}{12}(-3\,C[1] + 3\,e^{4t}\,C[1] + 8\,e^{t}\,C[2] + 4\,e^{4t}\,C[2] + 3\,C[3] - 8\,e^{t}\,C[3] + 5\,e^{4t}\,C[3]),$

$z[t] \to \frac{1}{12}(-3\,C[1] + 3\,e^{4t}\,C[1] - 4\,e^{t}\,C[2] + 4\,e^{4t}\,C[2] + 3\,C[3] + 4\,e^{t}\,C[3] + 5\,e^{4t}\,C[3])\}\}$

```
Out[45]= {{{True, True, True}}}

In[38]:= (*Problem #40 nonhomogeneous linear system*)
         Remove[x] Remove[y];
         system = {x'[t] == 2 * x[t] - 5 * y[t] + Csc[t], y'[t] == x[t] - 2 * y[t] + Sec[t]}
         S = FullSimplify[DSolve[system, {x[t], y[t]}, t]]
         FullSimplify[system //. S //. D[S, t]]

Out[39]= {x'[t] == Csc[t] + 2 x[t] - 5 y[t], y'[t] == Sec[t] + x[t] - 2 y[t]}

Out[40]= {{x[t] → -5 (C[2] - 2 Log[Cos[t]]) Sin[t] +
             (-2 t + C[1] - 5 Log[Cos[t]] + Log[Sin[t]]) (Cos[t] + 2 Sin[t]),
           y[t] → Cos[t] (C[2] - 2 Log[Cos[t]]) +
             (-2 t + C[1] - 2 C[2] - Log[Cos[t]] + Log[Sin[t]]) Sin[t]}}

Out[41]= {{{True, True}}}
```

```
In[183]:= (*Problem #41 homogeneous linear system*)
          Remove[x] Remove[y] Remove[z];
          system =
           {x'[t] == 2*x[t], y'[t] == -2*x[t] + y[t] - 2*z[t], z'[t] == x[t] + 3*z[t]}
          S = DSolve[system, {x[t], y[t], z[t]}, t]
          FullSimplify[system //. S //. D[S, t]]

Out[184]= {x'[t] == 2 x[t], y'[t] == -2 x[t] + y[t] - 2 z[t], z'[t] == x[t] + 3 z[t]}

Out[185]= {{x[t] → e^(2 t) C[1], y[t] → -e^t (-C[1] + e^(2 t) C[1] - C[2] - C[3] + e^(2 t) C[3]),
            z[t] → e^(2 t) (-C[1] + e^t C[1] + e^t C[3])}}

Out[186]= {{{True, True, True}}}
```

APPENDIX F

MATLAB Syntax of Test Suite

(See Appendix A for references)

1.

```
>> dsolve('Dy=x*exp(y+sin(x))','x')

ans =

-log(-Int(x*exp(sin(x)),x)-C1)

>> pretty(ans)
                                      /
                                     |
                            -log(-   |   x exp(sin(x)) dx - C1)
                                     |
                                    /
```

2.

```
>> y=dsolve('2 * x * y ^ 2 + 2 * y + Dy * (2 * x ^ 2 * y + 2 * x) =
0','x')

y =

[ -1/x]
[ C1/x]
```

3.

```
>> y=dsolve('y^2-t+2*y*Dy=0')

y =

[  1/exp(t)*(exp(t)*(exp(t)*t-exp(t)+C1))^(1/2)]
[ -1/exp(t)*(exp(t)*(exp(t)*t-exp(t)+C1))^(1/2)]

>> pretty(y)

                    [                                   1/2 ]
                    [  (exp(t)  (exp(t)  t - exp(t)  + C1))     ]
                    [  ------------------------------------ ]
                    [                exp(t)                 ]
                    [                                       ]
                    [                                    1/2]
                    [   (exp(t)  (exp(t)  t  - exp(t)  + C1))   ]
                    [-  ------------------------------------]
                    [                exp(t)                 ]
```

4.

```
>> maple('dsolve','diff(y(x,a),x)=a*y(x,a)','y(x,a)')

ans =

y(x,a)  =  _F1(a)*exp(a*x)
```

5.

```
>> dsolve('Dy+a*y(t-1)=0','t')
??? Index exceeds matrix dimensions.

Error in ==> C:\MATLABR12\toolbox\symbolic\dsolve.m
On line 227  ==> if isequal(Eqn(1),'[') & isequal(Eqn(end),']')

>> L=Laplace(diff(sym('y(t)'),t)+a*sym('y(t-1)'),t,s)

L = s*laplace(y(t),t,s)-y(0)+a*laplace(y(t-1),t,s)

>> R=Laplace(0,t,s)

R = 0

>> subs('s*laplace(y(t),t,s)-y(0)+a*laplace(y(t-
1),t,s)','laplace(y(t),t,s)','LAP')

ans =

s*(LAP)-y(0)+a*laplace(y(t-1),t,s)

>> solve(ans,'LAP')

ans =

-(-y(0)+a*laplace(y(t-1),t,s))/s

>> Ilaplace(ans)

ans =

y(0)-a*int(y(_U1-1),_U1 = 0 .. t)
```

6. <<SOLUTION APPEARS TO BE CORRECT>>

```
>> y=dsolve('t*Dy+y-y^2*exp(2*t)=0')

y =

1/(exp(2*t)+2*t*Ei(1,-2*t)+t*C1)

>> pretty(y)
                                        1
                   ---------------------------------
                   exp(2 t) + 2 t Ei(1, -2 t) + t C1
```

7. <<CANNOT FIND ANALYTIC SOLUTION>>

```
>> dsolve('Dy+P(t)*y=Q(t)*D2y')
Warning: Compact, analytic solution could not be found.
         It is recommended that you apply PRETTY to the output.
         Try mhelp dsolve, mhelp RootOf, mhelp DESol, or mhelp
allvalues
         for more information.
> In C:\MATLABR12\toolbox\symbolic\dsolve.m at line 299
```

```
ans =

DESol({-Q(t)*diff(Y(t),`$`(t,2))+diff(Y(t),t)+P(t)*Y(t)},{Y(t)})

>> pretty(dsolve('Dy+P(t)*y=Q(t)*D2y'))
Warning: Compact, analytic solution could not be found.
         It is recommended that you apply PRETTY to the output.
         Try mhelp dsolve, mhelp RootOf, mhelp DESol, or mhelp
allvalues
         for more information.
> In C:\MATLABR12\toolbox\symbolic\dsolve.m at line 299

                         / 2         \
                         |d          |    /d         \
            DESol({-Q(t) |--- Y(t)| + |-- Y(t)| + P(t) Y(t)}, {Y(t)})
                         |  2        |    \dt        /
                         \dt         /
```

8. <<SOLUTION APPEARS TO BE CORRECT>>

```
>> dsolve('D4u+a^4*u=0')

ans =

C1*exp((1/2+1/2*i)*2^(1/2)*a*t)+C2*exp((-
1/2+1/2*i)*2^(1/2)*a*t)+C3*exp((-1/2-1/2*i)*2^(1/2)*a*t)+C4*exp((1/2-
1/2*i)*2^(1/2)*a*t)

>> pretty(ans)
                                1/2                                1/2
  C1 exp((1/2 + 1/2 i) 2     a t) + C2 exp((-1/2 + 1/2 i) 2     a t)
                                          1/2                              1/2
    + C3 exp((-1/2 - 1/2 i) 2     a t) + C4 exp((1/2 - 1/2 i) 2     a t)
```

9. <<SOLUTION APPEARS TO BE CORRECT, however only gives trivial solution>>

```
>> dsolve('D2y+k^2*y=0')

ans =

C1*sin(k*t)+C2*cos(k*t)

>> dsolve('D2y+k^2*y=0','Dy(1)=0','y(0)=0')

ans = 0
```

10. <<EXPLICIT SOLUTION NOT FOUND>>

```
>> dsolve('(56+59*t)*D3y+(13+19*t)*D2y+(-142-59*t)*Dy+(-199-9*t)*y=0')
Warning: Compact, analytic solution could not be found.
         It is recommended that you apply PRETTY to the output.
         Try mhelp dsolve, mhelp RootOf, mhelp DESol, or mhelp
allvalues
         for more information.
> In C:\MATLABR12\toolbox\symbolic\dsolve.m at line 299
```

```
ans =

DESol({(56+59*t)*diff(Y(t),`$`(t,3))+(13+19*t)*diff(Y(t),`$`(t,2))+(-
142-59*t)*diff(Y(t),t)+(-199-9*t)*Y(t)},{Y(t)})

>> pretty(dsolve('(56+59*t)*D3y+(13+19*t)*D2y+(-142-59*t)*Dy+(-199-
9*t)*y=0'))
Warning: Compact, analytic solution could not be found.
         It is recommended that you apply PRETTY to the output.
         Try mhelp dsolve, mhelp RootOf, mhelp DESol, or mhelp
allvalues
         for more information.
> In C:\MATLABR12\toolbox\symbolic\dsolve.m at line 299
                      / 3        \                 / 2        \
                      |d         |                 |d         |
  DESol({(56 + 59 t)  |--- Y(t)| + (13 + 19 t)  |--- Y(t)|
                      |  3       |                 |  2       |
                      \dt        /                 \dt        /

                          /d        \
         + (-142 - 59 t)  |-- Y(t)| + (-199 - 9 t) Y(t)}, {Y(t)})
                          \dt       /
>> maple('dsolve','(56+59*t)*diff(y(t),t,t,t)+(13+19*t)*diff(y(t),t,t)
+ (-142-59*t)*diff(y(t),t)+(-199-9*t)*y(t)=0','y(t)')

ans =
y(t) = DESol({(56+59*t)*diff(_Y(t),`$`(t,3))+ (13+19*t) *
diff(_Y(t),`$`(t,2)) + (-142 - 59*t) * diff(_Y(t),t) + (-199 -
9*t)*_Y(t)},{_Y(t)})

11. <<tried with Maple V, runs out of time without initial conditions>>

>> maple('dsolve','{(t-2)*t^2*diff(y(t),t,t)+t^2*diff(y(t),t)+exp(t-
1)*y(t)=0,y(2)=0, D(y)(2)=0}','y(t)','series')

ans =

y(t) = series(O((t-2)^6),t=-(-2),6)        <<partially correct>>

12.
>> dsolve('D2y+t*Dy+exp(-t^2)*y=0')

ans =

C1*cos(1/2*exp(-
t^2)^(1/2)*exp(1/2*t^2)*2^(1/2)*pi^(1/2)*erf(1/2*t*2^(1/2)))+C2*sin(1/2
*exp(-t^2)^(1/2)*exp(1/2*t^2)*2^(1/2)*pi^(1/2)*erf(1/2*t*2^(1/2)))

>> pretty(ans)

                     2 1/2            2   1/2    1/2                 1/2
  C1 cos(1/2 exp(-t )     exp(1/2 t ) 2     pi     erf(1/2 t 2    ))

                               2 1/2            2   1/2    1/2                 1/2
         + C2 sin(1/2 exp(-t )     exp(1/2 t ) 2     pi     erf(1/2 t 2
))
```

```
13.  <<SOLUTION APPEARS TO BE CORRECT>>
>> dsolve('Dy+t*D2y-m^2*y/t+a^2*t*y=0')

ans =

C1*besselj(m,csgn(a)*a*t)+C2*bessely(m,csgn(a)*a*t)

14.  <<SOLUTION APPEARS TO BE CORRECT>>

>> dsolve('Du+k(t)*u=f(t)'); pretty(ans)

            /             /             /                        /
           |             |             |                        |
     exp(- |  k(t) dt)   |  f(t) exp(  |  k(t) dt) dt + exp(-  |  k(t) dt)
C1
           |             |             |                        |
          /             /             /                        /
15. <<SOLUTION APPEARS TO BE CORRECT>>

>> dsolve('3*Du-2*u=cos(t)','t'); pretty(ans)

                  - 2/13 cos(t) + 3/13 sin(t) + exp(2/3 t) C1

16.  <<SOLUTION APPEARS TO BE CORRECT>>

>> dsolve('Dy=-y-1','t'); pretty(ans)

                                -1 + exp(-t) C1

17.  <<SOLUTION APPEARS TO BE CORRECT>>

>> dsolve('Dy=y/x + x/y','x'); pretty(ans)

                             [                 1/2   ]
                             [(2 log(x) + C1)      x ]
                             [                       ]
                             [                  1/2  ]
                             [-(2 log(x) + C1)     x]

18. <<SOLUTION APPEARS TO BE CORRECT>>

>> dsolve('t^2*Dy+3*t*y=sin(t)/t','t'); pretty(ans)

                                 -cos(t) + C1
                                 ------------
                                       3
                                      t
19. <<implicit solution found>>

>> maple('dsolve','diff(y(x),x)=-
(1+2*x*sin(y(x)))/(1+x^2*cos(y(x)))','y(x)','implicit')

ans =

_C1+x+x^2*sin(y(x))+y(x) = 0
                                        []
```

20. <<SOLUTION APPEARS TO BE CORRECT>>

```
dsolve('(t^2+t+1)*D2y + (4*t+2)*Dy+2*y=3*t^2','t')

ans =

1/4*t^4/(t^2+t+1)+C1/(t^2+t+1)+C2/(t^2+t+1)*t

>> pretty(ans)
                         4
                        t             C1            C2 t
              1/4 ---------- + ---------- + ----------
                    2              2              2
                   t  + t + 1     t  + t + 1     t  + t + 1
```

21.

```
>> L=Laplace(diff(diff(sym('y(x)')))+4*sym('y(x)'))

L =

s*(s*laplace(y(x),x,s)-y(0))-D(y)(0)+4*laplace(y(x),x,s)

>> R=Laplace(sin(2*x),x,s)

R = 2/(s^2+4)

>> subs(L,'laplace(y(x),x,s)',sym('LAP'))

ans =

s*(s*LAP-y(0))-D(y)(0)+4*LAP

>> solve('s*(s*LAP-y(0))-D(y)(0)+4*LAP=2/(s^2+4)','LAP')

ans =

(y(0)*s^3+4*s*y(0)+D(y)(0)*s^2+4*D(y)(0)+2)/(s^4+8*s^2+16)

>> ILaplace(ans)

ans =

-1/4*t*cos(2*t)+1/8*sin(2*t)+y(0)*cos(2*t)+1/2*D(y)(0)*sin(2*t)
```

22. <<ERROR, even with maple kernel>>

```
dsolve('Dy+2*y+5*INT(y,k,0,t)=10*exp(-4*t)','t')
Warning: Explicit solution could not be found.

>> maple('dsolve','diff(y(x),x)+2*y(x)+5*int(y(tau),tau=0..x)=10*exp(-
4*x)','y(x)')
??? Error using ==> maple
Error, (in ODEtools/info) Found the indeterminate function y with
different arguments, {y(tau)}
```

23.

```
>> L=Laplace(diff(sym('y(x)'))+2*sym('y(x)')+5*Int(sym('y(tau)'),0,x))
```

```
Warning: Explicit integral could not be found.
> In C:\MATLABR12\toolbox\symbolic\@sym\int.m at line 58

L =

s*laplace(y(x),x,s)-y(0)+2*laplace(y(x),x,s)+5*laplace(y(x),x,s)/s

>> R=Laplace(10*exp(-4*x))

R = 10/(s+4)

>> subs(L,'laplace(y(x),x,s)',sym('LAP'))

ans =

s*LAP-y(0)+2*LAP+5*LAP/s

>> solve('s*LAP-y(0)+2*LAP+5*LAP/s=10/(s+4)','LAP')

ans =

s*(s*y(0)+4*y(0)+10)/(s^3+6*s^2+13*s+20)

>> ILaplace(ans)

ans =

-40/13*exp(-4*t)+40/13*exp(-t)*cos(2*t)+exp(-t)*y(0)*cos(2*t)-1/2*exp(-
t)*y(0)*sin(2*t)+5/13*exp(-t)*sin(2*t)
```

24. <<SEEMS TO BE CORRECT>>

```
b=dsolve('(1+t+t^2)*D3y+(3+6*t)*D2y+6*Dy=6*t')

b =

1/4*t^4/(1+t+t^2)+C1/(1+t+t^2)+C2*t/(1+t+t^2)+C3*t^2/(1+t+t^2)

>> pretty(b)
                          4                                                2
                         t              C1            C2 t          C3 t
                 1/4 ---------- + ---------- + ---------- + ----------
                                2             2             2             2
                     1 + t + t     1 + t + t     1 + t + t     1 + t + t
```

25. << with laplace gives wrong answer >>
<<with laplace>>

```
>>
k=maple('dsolve','diff(y(x),x,x)+2*diff(y(x),x)+y(x)=Dirac(x)','y(x)','
method=laplace')

k =

y(x) = exp(-x)*x*D(y)(0)+exp(-x)*x*y(0)+exp(-x)*x+y(0)*exp(-x)
```

```
<<without laplace>>
>> k=maple('dsolve','diff(y(x),x,x)+2*diff(y(x),x)+
y(x)=Dirac(x)','y(x)')

k =

y(x) = Heaviside(x)*exp(-x)*x+_C1*exp(-x)+_C2*exp(-x)*x
```

26. <<appears to be correct>>

```
b=dsolve('(1-t)*D2y+t*Dy-y=g(t)','t')

b =
 (int(g(t)/(-1+t)^2,t)*t*exp(-t)-int(t*g(t)/(-1+t)^2*exp(-
t),t))*exp(t)+C1*t+C2*exp(t)

>> pretty(b)

  /  /                                  /                      \
  | |    g(t)                          |  t g(t)  exp(-t)      |
  | |  --------- dt t exp(-t)  -       |  -------------- dt|  exp(t)  + C1 t
  | |          2                        |            2         |
  \/   (-1 + t)                        /      (-1 + t)         /

          + C2 exp(t)
```

27. <<solution appears to be correct>>

```
b=dsolve('x^2*D2y+x*Dy+(x^2-.25)*y=g(x)','x')

b =

-(int(1/x^(3/2)*sin(x)*g(x),x)*cos(x)-
int(1/x^(3/2)*cos(x)*g(x),x)*sin(x))/x^(1/2)+C1/x^(1/2)*cos(x)+C2/x^(1/
2)*sin(x)

>> pretty(b)
        /                              /
       |  sin(x)  g(x)                |  cos(x)  g(x)
       |  ----------- dx cos(x)  -    |  ----------- dx sin(x)
       |      3/2                     |      3/2
      /      x                       /      x                  C1 cos(x)
  - ----------------------------------------------------- + ---------
                             1/2                                1/2
                            x                                  x
           C2 sin(x)
         + ---------
              1/2
             x
```

28. <<APPEARS TO BE CORRECT>>

```
b=dsolve('D3y-D2y+Dy-y=g(x)','x')
```

```
b =

1/2*(int(-g(x)*cos(x)-g(x)*sin(x),x)*sin(x)*exp(-x)+int(g(x)*sin(x)-
g(x)*cos(x),x)*cos(x)*exp(-x)+int(g(x)*exp(-
x),x))*exp(x)+C1*sin(x)+C2*cos(x)+C3*exp(x)

>> pretty(b)
       /  /
      | |
  1/2 | |  -g(x) cos(x) - g(x) sin(x) dx sin(x) exp(-x)
      | |
      \/

             /
            |
        +   |  g(x) sin(x) - g(x) cos(x) dx cos(x) exp(-x)
            |
           /

             /                  \
            |                   |
        +   |  g(x) exp(-x) dx| exp(x) + C1 sin(x) + C2 cos(x) + C3
exp(x)
            |                   |
           /                   /
```

29. <<CORRECT, implicit solution found via Maple V kernel>>

```
>>
maple('dsolve','diff(y(x),x,x)+y(x)*diff(y(x),x)^3=0','y(x)','implicit'
)
ans =
1/6*y(x)^3+_C1*y(x)-x-_C2 = 0, y(x) = _C2

 b=dsolve('D2y+y*(Dy)^3=0','x')

b =
[
(3*x+3*C2+(8*C1^3+9*x^2+18*x*C2+9*C2^2)^(1/2))^(1/3)-
2*C1/(3*x+3*C2+(8*C1^3+9*x^2+18*x*C2+9*C2^2)^(1/2))^(1/3)]
[ -
1/2*(3*x+3*C2+(8*C1^3+9*x^2+18*x*C2+9*C2^2)^(1/2))^(1/3)+C1/(3*x+3*C2+(
8*C1^3+9*x^2+18*x*C2+9*C2^2)^(1/2))^(1/3)+1/2*i*3^(1/2)*((3*x+3*C2+(8*C
1^3+9*x^2+18*x*C2+9*C2^2)^(1/2))^(1/3)+2*C1/(3*x+3*C2+(8*C1^3+9*x^2+18*
x*C2+9*C2^2)^(1/2))^(1/3))]
[ -
1/2*(3*x+3*C2+(8*C1^3+9*x^2+18*x*C2+9*C2^2)^(1/2))^(1/3)+C1/(3*x+3*C2+(
8*C1^3+9*x^2+18*x*C2+9*C2^2)^(1/2))^(1/3)-
1/2*i*3^(1/2)*((3*x+3*C2+(8*C1^3+9*x^2+18*x*C2+9*C2^2)^(1/2))^(1/3)+2*C
1/(3*x+3*C2+(8*C1^3+9*x^2+18*x*C2+9*C2^2)^(1/2))^(1/3))]
[
C2]
>> pretty(b)
             [                  1/3      C1                ]
             [               %1     - 2 -----              ]
             [                           1/3               ]
             [                         %1                  ]
             [                                             ]
```

```
      [           1/3      C1                  1/2  /   1/3        C1   \]
      [- 1/2 %1      + ----- + 1/2 i 3     |%1      + 2 -----|]
      [                1/3                      |                1/3|]
      [              %1                         \              %1    /]
      [                                                                 ]
      [           1/3      C1                  1/2  /   1/3        C1   \]
      [- 1/2 %1      + ----- - 1/2 i 3     |%1      + 2 -----|]
      [                1/3                      |                1/3|]
      [              %1                         \              %1    /]
      [                                                                 ]
      [                             C2                                  ]

                              3       2                      2 1/2
       %1 := 3 x + 3 C2 + (8 C1   + 9 x   + 18 x C2 + 9 C2 )
```

30. <<correct>>

```
dsolve('D2y+y*(Dy)^3=0','y(0)=0, Dy(0)=2','x')

ans =

[ -
1/2*(3*(3*x+(1+9*x^2)^(1/2))^(2/3)*x+(3*x+(1+9*x^2)^(1/2))^(2/3)*(1+9*x
^2)^(1/2)+3*i*3^(1/2)*(3*x+(1+9*x^2)^(1/2))^(2/3)*x+i*3^(1/2)*(3*x+(1+9
*x^2)^(1/2))^(2/3)*(1+9*x^2)^(1/2)+i*3^(1/2)*(18*x^2+6*(1+9*x^2)^(1/2)*
x+1)^(1/3)*(3*x+(1+9*x^2)^(1/2))^(1/3)-3*x-
(1+9*x^2)^(1/2))/(3*x+(1+9*x^2)^(1/2))^(4/3)]
[
(3*x+(1+9*x^2)^(1/2))^(1/3)-1/(3*x+(1+9*x^2)^(1/2))^(1/3)]
[ 1/2*(-3*(3*x+(1+9*x^2)^(1/2))^(2/3)*x-
(3*x+(1+9*x^2)^(1/2))^(2/3)*(1+9*x^2)^(1/2)+3*i*3^(1/2)*(3*x+(1+9*x^2)^
(1/2))^(2/3)*x+i*3^(1/2)*(3*x+(1+9*x^2)^(1/2))^(2/3)*(1+9*x^2)^(1/2)+i*
3^(1/2)*(18*x^2+6*(1+9*x^2)^(1/2)*x+1)^(1/3)*(3*x+(1+9*x^2)^(1/2))^(1/3
)+3*x+(1+9*x^2)^(1/2))/(3*x+(1+9*x^2)^(1/2))^(4/3)]

>> pretty(ans)
```

```
    [              2/3          2/3              2 1/2            1/2    2/3
    [- 1/2 (3 %1      x + %1      (1 + 9 x )      + 3 i 3     %1      x
    [

           1/2    2/3              2 1/2
     + i 3     %1      (1 + 9 x )

           1/2         2                  2 1/2            1/3      1/3
     + i 3     (18 x   + 6 (1 + 9 x )      x + 1)      %1       - 3 x

                   2 1/2      /    4/3]
     - (1 + 9 x )      )   /   %1     ]
                          /           ]

    [  1/3       1  ]
    [%1     - -----]
    [            1/3]
    [          %1   ]

    [              2/3          2/3              2 1/2            1/2    2/3
    [1/2 (-3 %1      x - %1      (1 + 9 x )      + 3 i 3     %1      x
```

```
      [

              1/2    2/3             2 1/2
      + i 3      %1      (1 + 9 x )

              1/2        2                  2 1/2           1/3    1/3
      + i 3      (18 x   + 6 (1 + 9 x )      x + 1)      %1     + 3 x

                        2 1/2     /    4/3]
      + (1 + 9 x )      )  /   %1     ]
                              /          ]

                            2 1/2
  %1 := 3 x + (1 + 9 x )
```

31. (takes a long time without maple)...

```
>> k=maple('dsolve','x*diff(y(x),x)^2-y(x)^2+1=0','y(x)')

k =

_C1-2/(y(x)-1)^(1/2)*(x*(y(x)^2-1))^(1/2)/(y(x)+1)^(1/2)+
1/(y(x)-1)^(1/2)*log(y(x)+(y(x)^2-1)^(1/2))/(y(x)+1)^(1/2)*
(y(x)^2-1)^(1/2) = 0,
 _C1+2/(y(x)-1)^(1/2)*(x*(y(x)^2-1))^(1/2)/
(y(x)+1)^(1/2)+1/(y(x)-1)^(1/2)*log(y(x)+(y(x)^2-1)^(1/2))/
(y(x)+1)^(1/2)*(y(x)^2-1)^(1/2) = 0
```

32.

```
>> dsolve('(x^2-1)*Dy^2-2*x*y*Dy+y^2-1=0','x')

ans =

[          x*C1+(C1^2+1)^(1/2)]
[   (x^2-1)*(-1/(x^2-1))^(1/2)]
[  (-x^2+1)*(-1/(x^2-1))^(1/2)]
[          x*C1-(C1^2+1)^(1/2)]
```

33.

```
>> S = dsolve('Df = f-g', 'Dg = f+g')

S =

    f: [1x1 sym]
    g: [1x1 sym]

>> f=S.f

f =

exp(t)*(cos(t)*C1-sin(t)*C2)

>> g=S.g

g = exp(t)*(sin(t)*C1+cos(t)*C2)
```

34.

```
>> maple('dsolve','{diff(x(t),t)^2+diff(y(t),t)^2=3/2,
      (t-x(t))*diff(y(t),t)+y(t)*diff(x(t),t)=0,x(0)=0,y(0)=-1}'
      ,'{x(t),y(t)}','numeric')
??? Error using ==> maple
Error, (in DEtools/convertsys) unable to convert to an explicit
      first-order system
```

35.

```
>> k=maple('pdsolve','diff(u(x,t),x,x)=1/k*diff(u(x,t),t)','u(x,t)',
'build')

k =
u(x,t) = _C3*exp(_c[1]*k*t)*_C1*sinh(_c[1]^(1/2)*x)+
      _C3*exp(_c[1]*k*t)*_C2*cosh(_c[1]^(1/2)*x)
```

36.

```
>> k=maple('pdsolve','diff(u(x,t),x,x)=1/c^2*diff(u(x,t),t,t)' ,
'u(x,t)')

k =

u(x,t) = _F2((1/c^2)^(1/2)*x+t)+_F1(1/2*x-1/2*t/(1/c^2)^(1/2))
```

37

```
>> k=maple('pdsolve','diff(u(x,y,z),x,x)+diff(u(x,y,z),y,y) +
diff(u(x,y,z),z,z)=0','u(x,y,z)','build')

k =
u(x,y,z) = _C1*sinh(_c[1]^(1/2)*x)*_C5*sin((_c[1]+_c[2])^(1/2)*z)*
          _C3*sinh(_c[2]^(1/2)*y)+_C1*sinh(_c[1]^(1/2)*x)*
          _C5*sin((_c[1]+_c[2])^(1/2)*z)*_C4*cosh(_c[2]^(1/2)*y)+
          _C1*sinh(_c[1]^(1/2)*x)*_C6*cos((_c[1]+_c[2])^(1/2)*z)*
          _C3*sinh(_c[2]^(1/2)*y)+_C1*sinh(_c[1]^(1/2)*x)*
          _C6*cos((_c[1]+_c[2])^(1/2)*z)*_C4*cosh(_c[2]^(1/2)*y)+
          _C2*cosh(_c[1]^(1/2)*x)*_C5*sin((_c[1]+_c[2])^(1/2)*z)*
          _C3*sinh(_c[2]^(1/2)*y)+_C2*cosh(_c[1]^(1/2)*x)*
          _C5*sin((_c[1]+_c[2])^(1/2)*z)*_C4*cosh(_c[2]^(1/2)*y)+
          _C2*cosh(_c[1]^(1/2)*x)*_C6*cos((_c[1]+_c[2])^(1/2)*z)*
          _C3*sinh(_c[2]^(1/2)*y)+_C2*cosh(_c[1]^(1/2)*x)*
          _C6*cos((_c[1]+_c[2])^(1/2)*z)*_C4*cosh(_c[2]^(1/2)*y)
```

38a.

```
>> k=maple('pdsolve','diff(u(x,y,z),x,x)+ diff(u(x,y,z),y,y) +
diff(u(x,y,z),z,z)=-f(x,y,z)','u(x,y,z)')

k =     ''
```

b.

```
>> k=maple('pdsolve','diff(u(x,y,z),x,x)+diff(u(x,y,z),y,y) +
diff(u(x,y,z),z,z) = -1','u(x,y,z)','build')

k =
u(x,y,z) = -1/2*x^2*_c[2]-1/2*x^2*_c[3]-1/2*x^2+
      _C1*x+_C2+1/2*_c[2]*y^2+_C3*y+_C4+1/2*_c[3]*z^2+_C5*z+_C6
```

c.

```
>> k=maple('pdsolve','diff(u(x,y,z),x,x) + diff(u(x,y,z),y,y) +
diff(u(x,y,z),z,z) = -1/(x^2+y^2+z^2)','u(x,y,z)')

k =     ''
```

d.

```
>> k=maple('pdsolve','diff(u(x,y),x,x)+diff(u(x,y),y,y) =-
1/(x^2+y^2)','u(x,y)')

k =     ''
```

39.

```
>> S = dsolve('Df = f+g+2*h', 'Dg = f+2*g+h', 'Dh=2*f+g+h')

S =
    f: [1x1 sym]
    g: [1x1 sym]
    h: [1x1 sym]

>> pretty(S.f)

  1/6 C1 exp(t) + 1/3 C1 exp(4 t) + 1/2 C1 exp(-t) + 1/3 C2 exp(4 t)
     - 1/3 C2 exp(t) + 1/3 C3 exp(4 t) + 1/6 C3 exp(t) - 1/2 C3 exp(-t)

>> pretty(S.g)

  1/3 C1 exp(4 t) - 1/3 C1 exp(t) + 1/3 C2 exp(4 t) + 2/3 C2 exp(t)
     + 1/3 C3 exp(4 t) - 1/3 C3 exp(t)

>> pretty(S.h)

  1/3 C1 exp(4 t) + 1/6 C1 exp(t) - 1/2 C1 exp(-t) + 1/3 C2 exp(4 t)
   -  1/3 C2 exp(t) + 1/6 C3 exp(t) + 1/3 C3 exp(4 t) + 1/2 C3 exp(-t)
```

40.

```
>> S = dsolve('Df = 2*f-5*g+csc(t)', 'Dg = f-2*g+sec(t)')

S =
    f: [1x1 sym]
    g: [1x1 sym]

>> pretty(S.f)

  C1 cos(t) + 2sin(t) C1 - 5sin(t) C2 + cos(t)log(sin(t)) - 2cos(t) t
         - 5 cos(t) log(cos(t)) + 2 sin(t) log(sin(t)) - 4 sin(t) t

>> pretty(S.g)

  sin(t) C1 + C2 cos(t) - 2 sin(t) C2 + sin(t) log(sin(t)) - 2 sin(t) t
   -  sin(t) log(cos(t)) - 2 cos(t) log(cos(t))
```

41.

```
>> S = dsolve('Df = 2*f', 'Dg = -2*f+g-2*h', 'Dh=f+3*h')

S =
    f: [1x1 sym]
    g: [1x1 sym]
    h: [1x1 sym]

>> pretty(S.f)
                                        exp(2 t) C1
>> pretty(S.g)
        C1 exp(t) - C1 exp(3 t) + C2 exp(t) + C3 exp(t) - exp(3 t) C3
>> pretty(S.h)
                         C1 exp(3 t) - exp(2 t) C1 + exp(3 t) C3
```

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