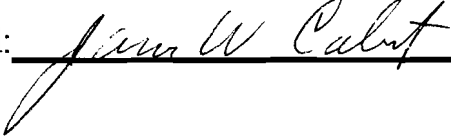


AN ABSTRACT OF THE THESIS OF

Zuming Gong for the **Master of Science** in **Physical Science, Physics**
presented in **May 1996**. Title: **A DESIGN OF AN UNDERGRADUATE**
EXPERIMENT TO MEASURE THE SPEED OF LIGHT.

Abstract approved: 

In 1957, an American Journal of Physics publication entitled “Laboratory Measurement of the Velocity of Light”, authors W.Parker Alford and Albert Gold state: “The method is based on the result that the Fourier transform of a pair of identical pulses yields a frequency spectrum with zeros which are simply related to the time separation of the pulses”^[6]. Their method consisted of light traveling along two different paths being detected by a single detector and the zeros of the Fourier transform being determined by directing the output of the detector through a short-wave radio input and tuning through a range of short-wave frequencies. The present work is to try to update this experiment by having the detector create a digital data file, and using fast Fourier transform techniques to locate the zeros. This report is about the use of computer (spreadsheet) modeling to evaluate the feasibility of the experiment and to determine the experimental parameters.

**A DESIGN OF AN UNDERGRADUATE EXPERIMENT
TO MEASURE THE SPEED OF LIGHT**

A Thesis
Presented to
the Division of Physical Sciences
EMPORIA STATE UNIVERSITY

In Partial Fulfillment of
the Requirements for the Degree
Master of Science

by
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May 1996

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A DESIGN OF AN UNDERGRADUATE EXPERIMENT TO MEASURE THE SPEED OF LIGHT

CHAPTER 1

INTRODUCTION

The speed of light, as one of the most fundamental of all physical constants, is important in a general physics course because it plays a basic role in electromagnetism as well as relativity, and because its measurement writes a significant and dramatic chapter in the story of the evolution of physics. The experimental design described here will give students the opportunity to perform the experiment and to review its significance in the history of physics.

The purpose of the experiment to be performed at ESU is to apply computer data processing to the basic measurement. The apparatus to be used in the experiment should be available in most physics laboratories, and performing the experiment will provide undergraduate students with the opportunity to learn something of the history of the measurement. The data processing will be based on the principle of the Fourier transform, and a FFT (fast Fourier transform) algorithm will be programmed into the computer. In addition, problems of communication between the measurement apparatus and computer will need to be identified.

CHAPTER 2

THE HISTORY OF MEASUREMENTS OF THE SPEED OF LIGHT

The history of the measurement of the speed of light is very interesting since so many scientists, scholars, and experts have worked on this subject. The history of their efforts has been recorded and provides a useful reference to similar activities today. In the early years, from the 1600's to 1940's, many famous scientists, such as Galileo, Roemer, Fizeau, Foucault, Michelson, and others, performed the experiment using various methods. In more recent years, from the 1950's to 1970's, experiments to measure the speed of light were performed at many universities. Those which have been published in the American Journal of Physics will be briefly reviewed for this chapter.

1. The Measurements in Early Years

The first person who tried to measure the speed of light was Galileo, and his effort in 1600 was not successful^[1]. Galileo and his assistant were separately stationed on two hilltops a long distance apart. By one sending a flash of light toward the other to observe and return a signal, Galileo hoped to measure the time difference. However, the speed of light was too fast to produce a measurable time difference.

In 1675, Roemer, a Danish astronomer using astronomical methods, became the first man to get a fairly accurate measurement of the speed of light^[2]. He found the eclipse period of a moon of the planet Jupiter to be different when the Earth was traveling toward Jupiter from that measured when the Earth was traveling away, and deduced that the difference was due to the finite speed of light. From his measurement, Roemer determined the speed of light to be 186,000 miles per second (2.99×10^8 m /s).

Little was done in the following years until the mid 1800's. Then, between the 1850's and 1940's, successful attempts to determine the speed of light were made by Fizeau, Foucault, Cornu, Young, Forbes, and Michelson. All of their experiments were performed in earthbound laboratories, rather than by using astronomical observations. The best known of these experiments were Fizeau's toothed wheel device and Foucault's rotating mirror device. Later experiments using their methods were more successful because of the improved apparatus and techniques. Of the later methods, that of Michelson and his coworkers using a rotating mirror was considered to be by far the most accurate.

2. The Measurements in Later Years

Many experimental activities for measuring the speed of light were reported in the American Journal of Physics from the 1950's to 1970's. The methods different universities adopted to perform the experiment provide extensive and valuable references to current research.

(1) Some methods used in the experiment to measure the speed of light

Experiments based on the principle of a rotating mirror were improved by several different experimenters. Other techniques included using pulsed or chopped light sources, and Kerr cell shutters^[2].

a) Rotating mirror

The method of rotating mirror consists of a mirror mounted on a motor rotating at high speed. The light beam is reflected from M to R and back to M, and then the returned light beam is seen at a slightly greater angle than that directly reflected back by M (see Figure 1).

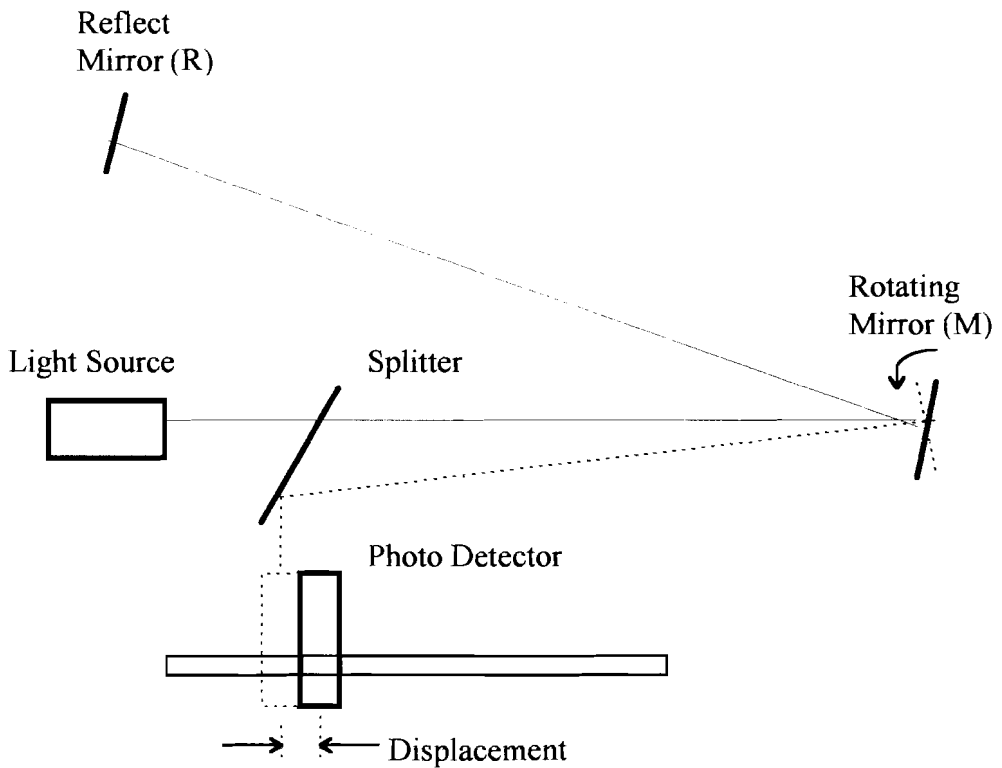


Figure 1

The position of the detector is displaced by this angular difference. This method is simple to understand, and not too difficult to perform. However, the accuracy is not high and is limited by the maximum possible speed of rotation of the motor. Also, the price of the apparatus is high, and the length of the required optical path seems to be too long.

b) Pulsed or chopped light source

A straightforward way to measure the speed of light is to use a pulsed light source such as light chopper, spark, stroboscope, or modulated laser. The time difference for light beam traveling two paths, long and short, as shown in Figure 2, is directly obtained, and the speed of light is calculated from the equation of $c = d/t$. The apparatus and optical principles employed in this method are similar to those of the rotating mirror.

c) Kerr cell shutter

The Kerr cell shutter consists of a small glass container containing sealed-in metal electrodes and filled with pure nitrobenzene^[3]. The Kerr cell shutter rotates the plane of the polarized light when a high voltage is applied, shutting off the light if the cell is mounted between two polarizing screens. The Kerr cell is capable of chopping a beam of light at extremely high frequencies. The scheme of this experiment is shown in Figure 3.

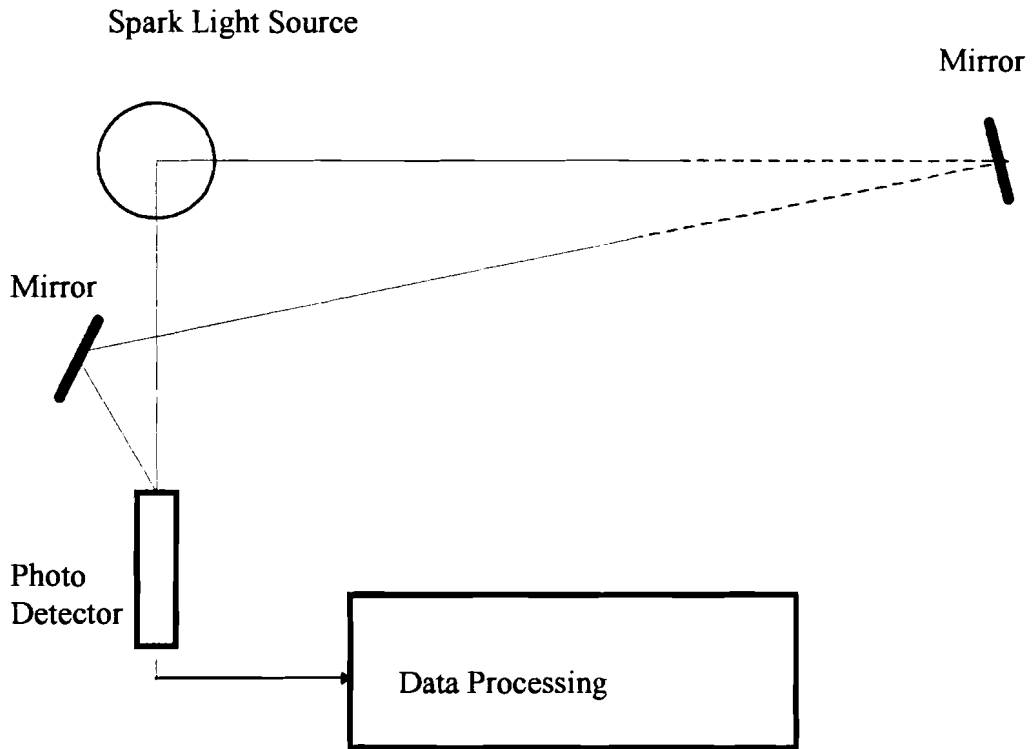


Figure 2

Using pulsed light source as a system for measuring the speed of light

The Kerr cell shutter is provided with an adjustable DC polarizing potential of several thousand volts upon which is superimposed a large rf voltage. Because of the rf voltage, one radio cycle transmits one light pulse of the rf frequency. The advantage of the Kerr cell over the rotating mirror is better

accuracy and shorter path length. However, the Kerr cell modulation seems to be cumbersome since it requires high voltages and involves circuits having large delay times. Also, the material used in the Kerr cell is hazardous, and not available to most laboratories.

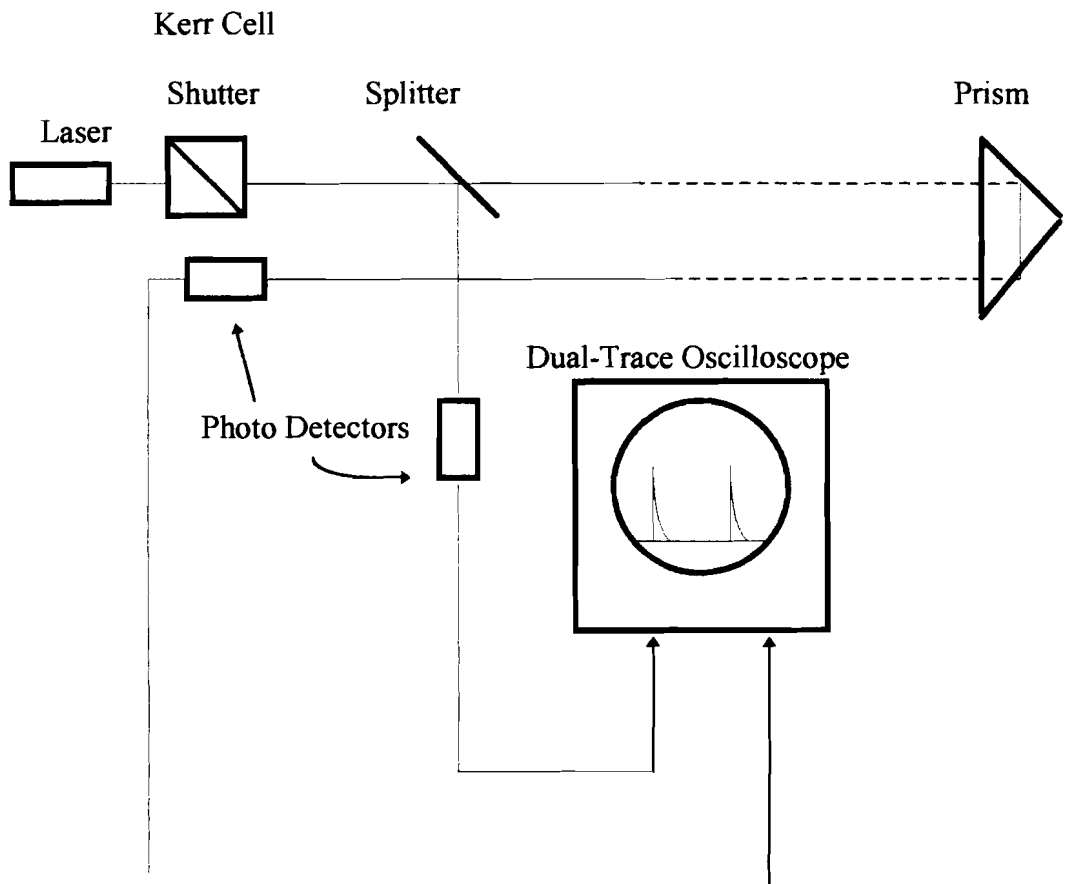


Figure 3

Using the Kerr cell as a system for measuring the speed of light

(2). Overview of the experiments in later years

In the thirty years, from the 1950's to the late 1970's, many papers describing the measurement of the speed of light were published in the American Journal of Physics. Some of these experimental activities were reviewed in order to decide what kind of experiment will be performed at Emporia State University.

In 1954, Bucknell University designed an experiment to measure the speed of light for which the method of the Kerr cell shutter was used^[4]. The experiment was designed for undergraduate instruction with emphasis on simplicity rather than precision. The length of the optical path was only 3.742 m, and the result of the speed of light measurement was $2.976 \pm 0.006 \times 10^8$ m/s. The Kerr cell method was considered for use at Emporia State University, but it was rejected because of the high voltage required (several kV at high frequencies), and because of the explosive nature of the medium.

In the next year, 1955, students at the Massachusetts Institute of Technology worked on the measurement of the speed of light using the indirect method of measuring the electric and magnetic constants^[5]. The reported result of this work was $2.976 \pm 0.050 \times 10^8$ m/s. Students elsewhere can perform the calculation with the known values of ϵ_0 and μ_0 , but such an activity is not really considered to be an experiment.

In 1957, the University of Rochester presented a paper about the measurement of the speed of light^[6]. Their experiment used a pulsed light source to simplify the optical system. In addition, the experimental information was processed by using the Fourier transform. Since the frequencies in the Fourier transform spectrum were related to the real time, the selected frequencies were used to obtain the delay time τ , and hence to obtain the speed of light c . The length of the optical path for this experiment was 64.2 m, and the accuracy was within 1%.

In the same year, 1957, the Leybold Company in New York manufactured a special apparatus for measuring the speed of light by means of a rotating mirror. The device, with slight modification, was similar to the experiment originally performed by Foucault. By rotating a mirror at the high speed of 30,000 rpm and folding the optical path many times, the length of the room needed to perform this experiment was only around 10 meters, and the accuracy was about 5%.

In 1964, the Leybold apparatus for measuring the speed of light was improved by Ohio Wesleyan University^[7]. The accuracy of the experiment was improved to 1 % by using a detector mounted on the carriage of a microscope. As a result of the improvement the speed of light was found to be $2.995 \pm 0.015 \times 10^8$ m /s.

In 1967, the University of Waterloo made a measurement of the speed of light with what was called a pseudo-thermal source^[8]. The experiment was

based on the scheme of the experiment performed at the University of Rochester. The optical system, the receiver, and data processing were the same, but the light source was different. A laser and a chopper driven by a motor were combined to replace the spark light source. The result of this change in the experiment was reported to be $3.00 \pm 0.20 \times 10^8$ m/s.

In 1968, the University of Puget Sound designed an undergraduate experiment to measure the speed of light using the method of a pulsed light source^[9]. The optical system was as in earlier attempts. A stroboscope and a telescope were combined to transmit a parallel beam of pulsed light, and an oscilloscope was used directly to observe the slight delay time of the two pulses. The optical path was 320 m, and the reported result of the experiment was a speed of $2.8 \pm 0.10 \times 10^8$ m/s.

In 1969, the University of Missouri at Rolla improved the method of using a Kerr cell by incorporating a solid-state, electro-optical light source^[10]. A data processing technique called Phase-Shift Analysis was used where the delay time was measured to obtain the speed of light. The result of this improvement was $2.937 \pm 0.066 \times 10^8$ m/s.

In 1969, a method used for measuring the speed of light was developed at Washington University using a pulsed light source^[11]. The pulse, generated by a flip-flop circuit, was input into one channel of a dual-trace oscilloscope as a reference signal, and simultaneously triggered a diode light pulser directly toward a detector whose output was fed into the other channel of the

coupled to it through a long light path were used as a regenerative feedback loop. When the light source was switched on, a beam of light traveled to a distant mirror and was returned to the detector located adjacent to the source. The detector response caused the light source to be turned off. A short time later, determined by speed of light and the distance to the return mirror, the detector output vanished and the light source went back on. The optical path of this device was 60 m and yielded the result of $2.996 \pm 0.010 \times 10^8$ m/s.

In 1972, the University of Arkansas improved on the method of the rotating mirror to perform an experiment on the measurement of the speed of light^[16]. The improvement was to use a moveable photo-diode detector which was displaced due to the slightly different angle of reflection observed by using the rotating mirror. The path length was 38 m, and the reported result was $3.00 \pm 0.02 \times 10^8$ m/s.

CHAPTER 3

THE DESIGN OF AN EXPERIMENT TO MEASURE THE SPEED OF LIGHT AT EMPORIA STATE UNIVERSITY

After reviewing the history of the measurement of the speed of light, particularly the experiments performed at several universities in the later years, it was considered to be important to continue to develop this activity and to establish a new experiment combining traditional methods and modern technology. After comparing the various methods used at each university, the experiment at the University of Rochester in 1957 was chosen as the basic concept in this design of the experiment. It was chosen because of its use of the Fourier transform and the possibility of using a computer model in the analysis.

1. Theory and Principle

Since the basic theory of this experiment is the Fourier transform, its conception and application must be understood if the experiment is to be successful. The fundamental elements of Fourier analysis are the Fourier series (FS), the Fourier transform (FT), the discrete Fourier transform (DFT), and the fast Fourier transform (FFT).

(1) The basic concepts of Fourier transform

The Fourier transform is a principal analysis tool in many of today's scientific challenges. The essence of the Fourier series of a waveform is to decompose or separate the waveform into a sum of sinusoids of different frequencies^[17]. The Fourier transform is derived from the Fourier series. In the Fourier series, any function can be expressed in the periodic form,

$$f(t) = (a_0/2) + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt$$

or in the complex form,

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{int}$$

where $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(nt) dt,$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(nt) dt,$$

and $c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) e^{-int} dt$

Here, a_n , b_n , and c_n are the coefficients of the infinite number of frequency terms presented in the series. As an example, Figure 4 and Figure 5 are

pictorial representations of the Fourier series square wave (in time) which displays each amplitude and frequency of the associated sinusoids (in frequencies).

The square wave can be decomposed in Fourier series as

$$f(t) = \frac{2}{\pi} \left(\frac{\sin t}{1} + \frac{\sin 3t}{3} + \frac{\sin 5t}{5} + \dots \right)$$
$$= \frac{2}{\pi} \sum_{n=1,3,5}^{\infty} \frac{\sin nt}{n}$$

If $n = 1,3,5$ in the summation, the composed wave approaches a square wave as shown in Figure 4.

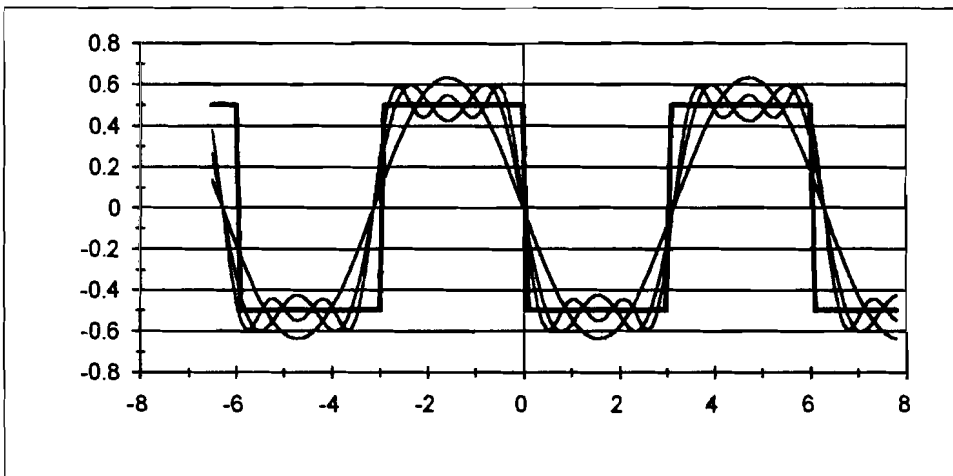


Figure 4

Square wave and composition of its series

The frequency points in the composite square wave can be clearly displayed by the spectrum as shown in Figure 5(a) and Figure 5(b).

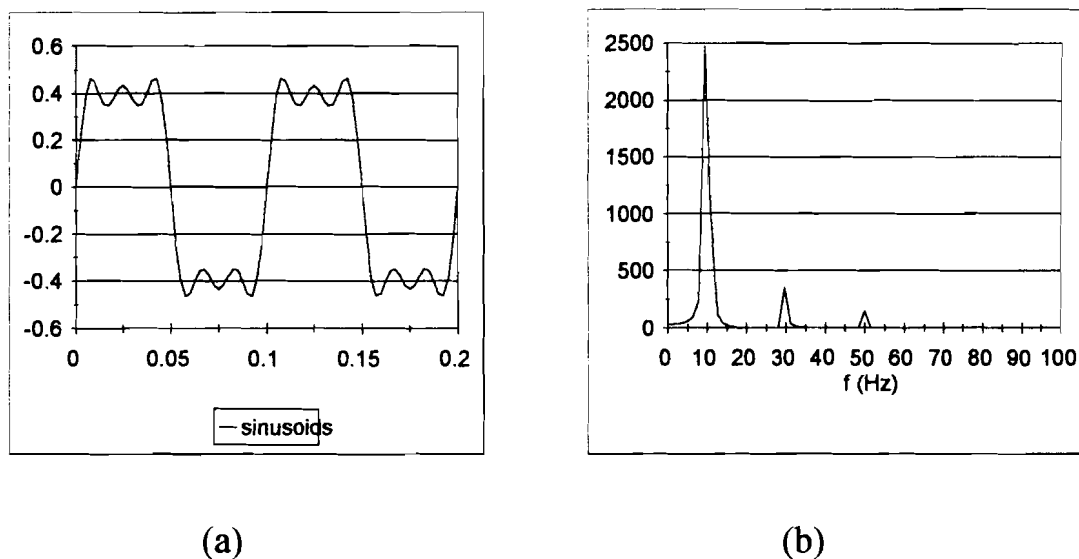


Figure 5
The composite square wave and its frequency spectrum

The Fourier transform is very closely related to the Fourier series^[18]. The series representation is useful in the region of the infinite interval $(-\infty, \infty)$ if the function is periodic, while Fourier transforms are employed in describing nonperiodic functions on the finite interval $(-T, T)$. Assuming the substitution $t \rightarrow \pi t / T$, then the complex form becomes

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi t/T}$$

where
$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) e^{-in\pi t/T} dt$$

The discrete frequencies appearing in the summations are identified as

$$\omega = \frac{n\pi}{T},$$

and the differences between successive frequencies as

$$\Delta \omega = \frac{\pi}{T}$$

After inserting the $\Delta \omega$ into the series, taking the limit as $T \rightarrow \infty$, and letting the term $n \Delta \omega$ become the continuous variable ω , the integrals defined as Fourier transforms can be derived. They are

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(\omega) e^{i\omega t} d\omega$$

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt$$

The function $g(\omega)$ represents the envelope of the intensities of the various frequencies, where $f(t)$ is the actual function as it appears in time. The function $g(\omega)$ is defined to be the Fourier transform of $f(t)$. Its application is to decompose or separate any waveform into a sum of sinusoids of different frequencies.

In most measurements, the data sampled from a signal is not a continuous curve but rather is discrete. In the range of discrete data from a signal, the quantity t is not continuous but rather is incremented as $m \Delta t$, where $m = 0, 1, 2, \dots, N-1$. After inserting the $m \Delta t$ into the Fourier transform, the processing of the discrete Fourier transform (DFT) may be numerically performed according to

$$f(m \Delta t) = \frac{1}{N} \sum_{n=0}^{N-1} g(n \Delta \omega) e^{i 2 \pi m n / N}$$

$$g(n \Delta \omega) = \frac{1}{N} \sum_{m=0}^{N-1} f(m \Delta t) e^{-i 2 \pi m n / N}$$

The transform $g(n \Delta \omega)$ displays the frequency spectrum obtained from the original signal $f(m \Delta t)$.

Although the DFT has many advantages, it is a computationally intensive operation. With N data points, only one point in ω space will be produced as the result of N operations. It is necessary to perform N summations to evaluate all the $g(n \Delta \omega)$, for a total of N^2 operations.

In order to reduce the number of computations, the fast Fourier transform was conceived from the particular properties of the transform^[18]. The scheme was to separate the summation into two parts according to the N; one is the even part, the other is the odd part. After deriving the rearranged summations, the number of operations was reduced from N^2 to $N \log_2 N$. As a result, the speed of the calculations of the FFT is much faster than those of the discrete Fourier transform.

(2). The application of the Fourier transform to the experiment

The proposed method for measuring the speed of light at ESU is to use a stroboscope to illuminate a photomultiplier via a short path and simultaneously via a long path. The sum of the two signals are to be output from a detector, digitized and processed by Fast Fourier Transform (FFT) techniques using a computer model to obtain a frequency spectrum. The theory of the Fourier transform applied to a pair of identical signals follows from the consideration of the transform of two identical pulses slightly separated in time. For a single pulse $E(t)$, the Fourier transform is

$$G(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} E(t) e^{-i\omega t} dt$$

Assuming that two such pulses are separated by a time τ , the Fourier transform of the composite signal is:

$$\begin{aligned}
H(\omega) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} [E(t) + E(t - \tau)] e^{-i\omega t} dt \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} [E(t) e^{-i\omega t} dt + E(t - \tau) e^{-i\omega t} dt] \\
&= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{+\infty} E(t) e^{-i\omega t} dt + \int_{-\infty}^{+\infty} E(t - \tau) e^{-i\omega t} dt \right] \\
&= \frac{1}{\sqrt{2\pi}} \left[G(\omega) + \int_{-\infty}^{+\infty} E(t - \tau) e^{-i\omega t} dt \right]
\end{aligned}$$

Changing the variable to perform the second integral:

$$u = t - \tau$$

$$du = dt$$

Now:

$$\begin{aligned}
H(\omega) &= \frac{1}{\sqrt{2\pi}} \left[G(\omega) + \int_{-\infty}^{+\infty} E(u) e^{-i\omega(u+\tau)} du \right] \\
&= \frac{1}{\sqrt{2\pi}} \left[G(\omega) + e^{-i\omega\tau} \cdot \int_{-\infty}^{+\infty} E(u) e^{-i\omega u} du \right] \\
&= \frac{1}{\sqrt{2\pi}} \left[G(\omega) + G(\omega) \cdot e^{-i\omega\tau} \right] \\
&= \frac{1}{\sqrt{2\pi}} G(\omega) (1 + e^{-i\omega\tau}) \\
&= \frac{1}{\sqrt{2\pi}} G(\omega) 2 e^{-i\omega\tau/2} [(e^{i\omega\tau/2} + e^{-i\omega\tau/2}) / 2] \\
&= \frac{2}{\sqrt{2\pi}} \cdot G(\omega) \cdot e^{-i\omega\tau/2} \cdot \cos(\omega\tau/2)
\end{aligned}$$

when $\omega \tau / 2 = (2n+1)\pi/2$, $\cos(\omega \tau / 2) = 0$. Consequently, the Fourier transform $H(\omega)$ has zeros at these values. Since τ is the delay time between the two pulses, and since the angular frequency ω is

$$\omega_n = 2\pi f_n = (2n+1)\frac{\pi}{\tau}$$

then the frequencies at which the zeros occur will be

$$f_n = \frac{2n+1}{2\tau}.$$

So $\tau = (2n+1) / 2 f_n$

$$c = \frac{d}{\tau} = 2d f_n / (2n+1)$$

where c is the speed of light, d is the path difference, $n = 0, 1, 2, 3, \dots$, and f_n is the frequency values at the zero (or minimum) points in the FFT spectrum. Thus the speed c can be calculated from the two parameters d and f_n provided that n is known. If n is not known, two consecutive zeros (or minima) are required to determine the speed c . Thus

$$\begin{aligned} f_{n+1} - f_n &= \frac{2(n+1)+1}{2\tau} - \frac{2n+1}{2\tau} \\ &= \frac{2n+3-2n-1}{2\tau} \\ &= 1/\tau \end{aligned}$$

thus $\tau = 1 / (f_{n+1} - f_n)$.

$$c = \frac{d}{\tau} = d * (f_{n+1} - f_n).$$

This shows that the delay time between pulses may be determined from the zeros (or minima) of the frequencies, and it becomes very easy to calculate the speed of light by using the length of optical path and the two consecutive frequencies in the FFT spectrum.

2. The design of the experiment

The experimental method at the University of Rochester was based upon the result that the Fourier transform of a pair of identical pulses yielded a frequency spectrum with zeros which were simply related to the time of separation of the pulses. The principal pieces of apparatus required for the experiment were a spark light source, a photomultiplier and high-voltage power supply, and a short-wave radio receiver as shown in Figure 6.

The photomultiplier received the two signals from the light source and input them into a short-wave radio receiver. The zero points in the frequency spectrum were obtained by detecting minimum loudness at particular frequencies as the receiver was tuned through a range of short-wave frequencies. The speed of the data processing was slow, and the precision of the measurement limited by that of the detection system.

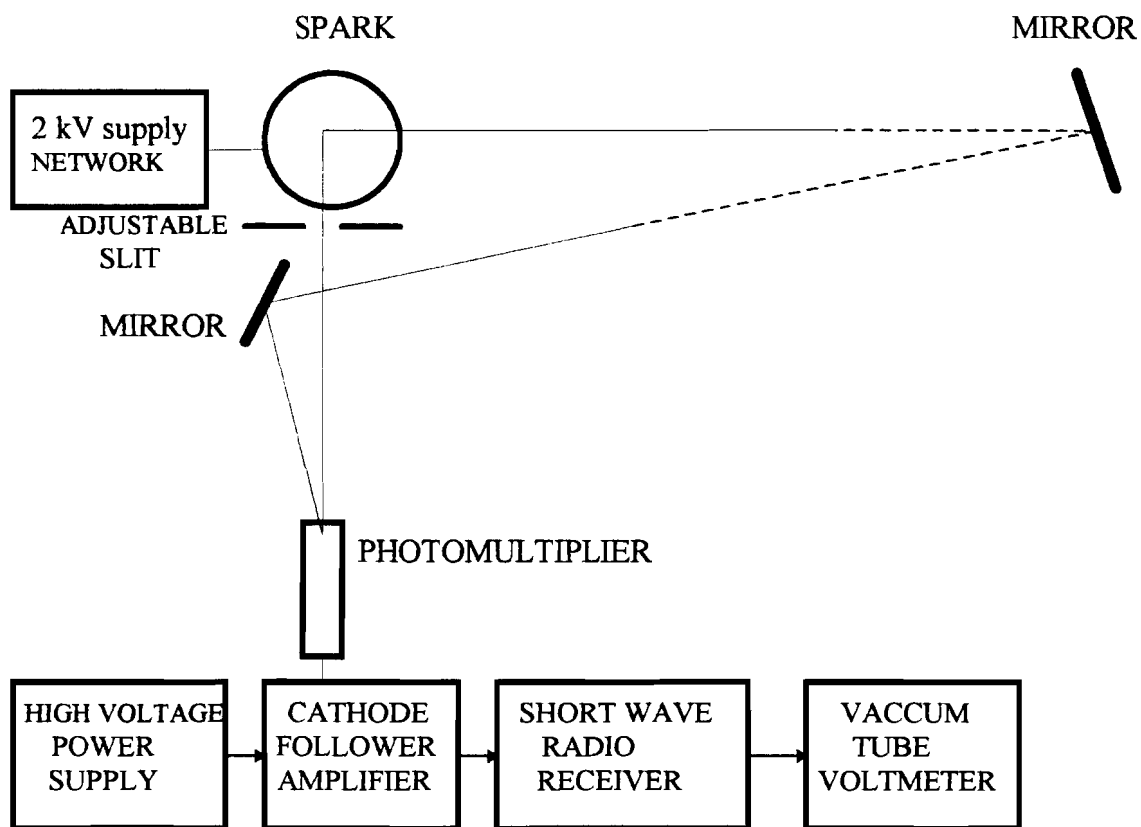


Figure 6

In order to adopt the same idea employed in the experiment at the University of Rochester and incorporate the experimental apparatus available in the physics laboratory at ESU, some changes to the experiment would be required. Figure 7 shows the details.

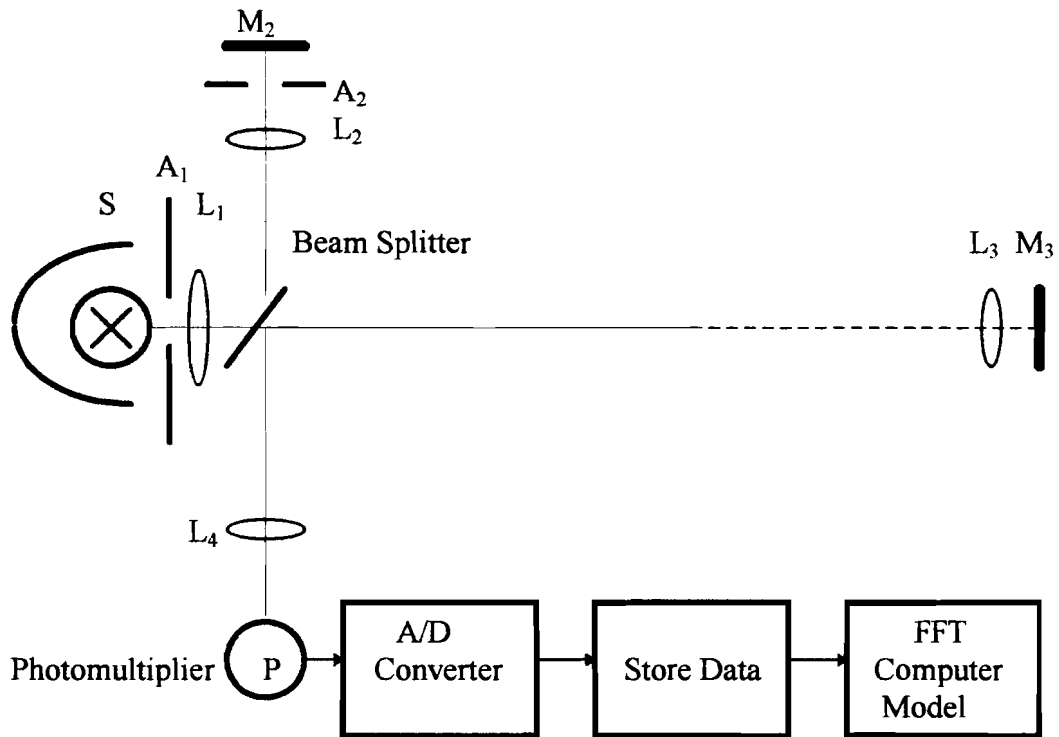


Figure 7

First, a stroboscope instead of the spark will be used as a light source. The stroboscope light source can offer a signal of sharper exponential form than that of a spark, and the flash frequency is easily adjusted to meet the requirements of the experiment. Second, the final output from the device will be a digital signal instead of an analog one. Finally, data processing will be performed by a computer model using the FFT rather than by listening for relative minima.

3. Apparatus and a Diagram of the Optical System

The design of the measurement system is shown in Figure 7 on the previous page. A portable stroboscope will be used as a pulsed light source. The light beam will be separated by a beam splitter into two paths. In one, the short path, the light beam will be reflected by mirror 2. The other is a long path in which the light beam will be reflected by mirror 3. A slit will be available to adjust the intensity of the pulse in the short path to be the same as that of the pulse in the long path. The pairs of identical signals, arriving at slightly different times, will be received by a photo detector. The output from the detector will be input into an analog-digital converter, and then sampled by a computer system to perform the data processing required in the fast Fourier transform.

4. Computer Models to Determine the Experimental Parameters

There are many considerations necessary in order to obtain the appropriate parameters and to determine how they are related to each other. The experiment may be successful if the range of the parameters are previously determined. In order to avoid the tedious adjustments in the experiment to measure the speed of light, four important parameters have been identified. These are the shape of the pulse, the lowest possible density of sampling data in a pulse, the smallest possible delay time, and the duration of the pulse. The analysis presented here were all produced using the FFT program built into the Quatro Pro^[19] spreadsheet.

(1). Comparison of the shape of light pulse

The shape of the pulse is one of the important parameters affecting the FFT spectrum. Two kinds of pulses were considered. One was a Gaussian curve as might be generated by rotating a mirror or using a chopper. The other was an exponential curve as would be created by a stroboscope or spark light source. After comparing the two kinds of pulses using the FFT computer model, it was determined that the Gaussian pulse required a much longer delay time than the exponential pulse to obtain zero points in the frequency spectrum. In other words, a longer optical path would be required in using the chopper or rotating mirror than in using a stroboscope, making the stroboscope the better choice. The significance of this step was to use the computer model analysis to make the best selection of light sources. The figures, from Figure 8 to Figure 11, show the results obtained in comparing the two kinds of pulses obtained by using the FFT computer model under the same conditions, $1 \mu\text{s}$ delay time and 4096 points of sampling data.

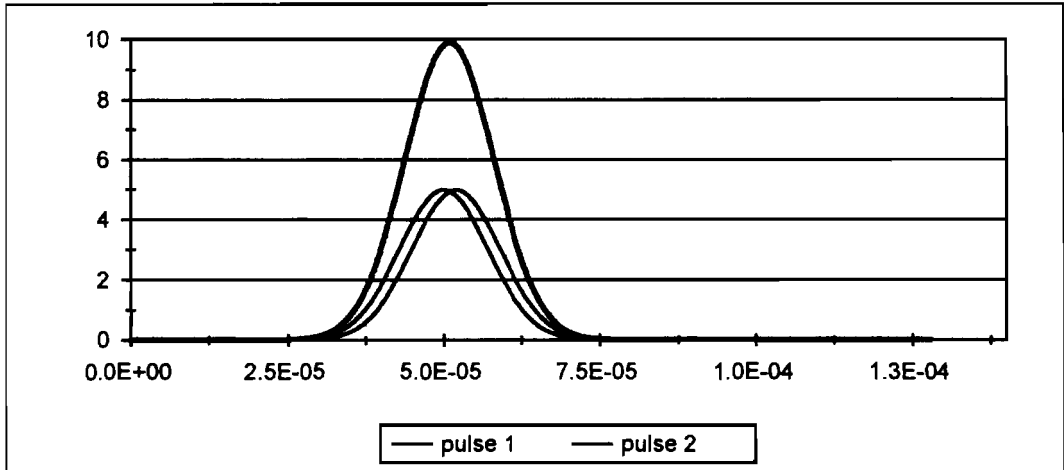


Figure 8

Gaussian pulses with 1 μ s delay time and 4096 data points

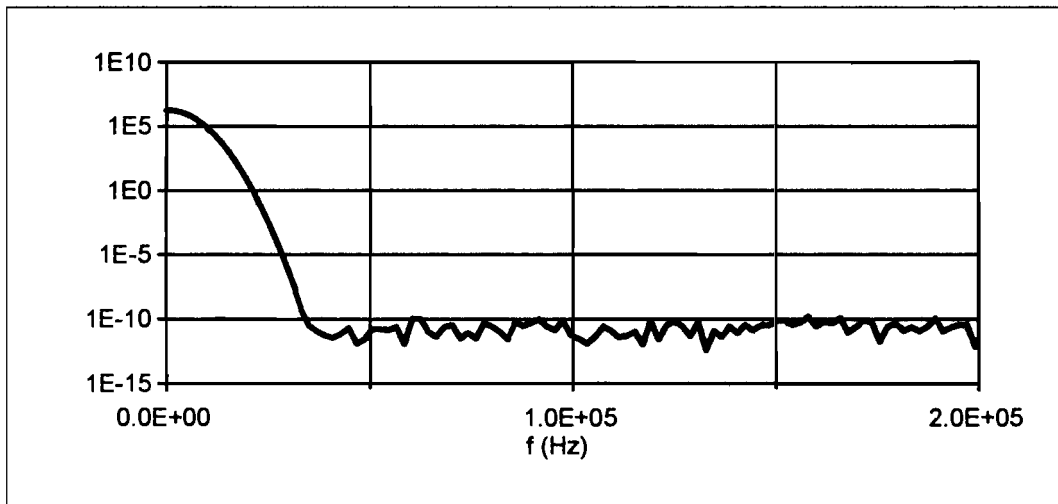


Figure 9

The frequency spectrum from Gaussian pulses above

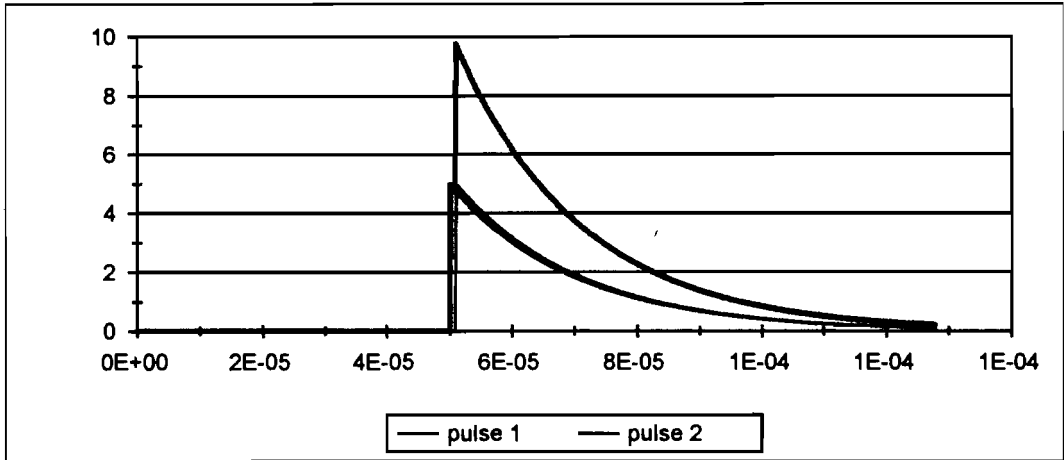


Figure 10

The exponential pulses with 1 μ s delay time and 4096 data points

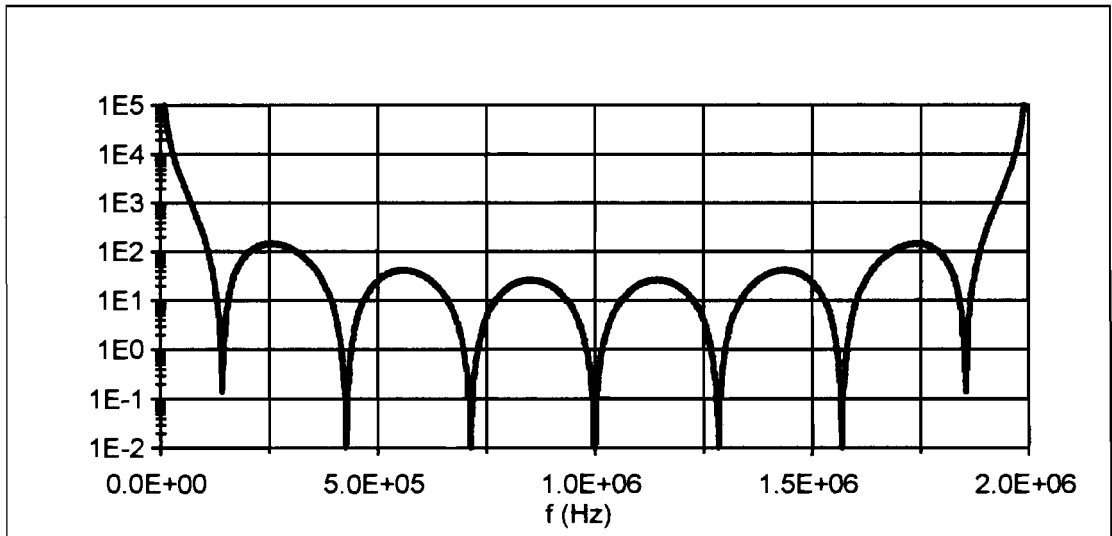


Figure 11

The frequency spectrum from the exponential pulses above

(2) Determination of the lowest possible density of sampling data in a given duration of the pulse

The density of sampling data refers to the number of data points which must be collected to be used in the FFT. From theory, the larger the number of points that are sampled, the better the FFT will display the details of the frequency spectrum. If the number of sampled points is less than the required minimum number, the FFT spectrum will lose some information about the zeros (or minima). While more information is available by sampling more points, a larger number of points requires faster and more sophisticated equipment. This problem of how many points are required can be solved by the analysis of the density of sampling data from the FFT computer model. It is the intent of this study to determine the fewest possible number of points that will allow a reasonable resolution for a given delay time. The results obtained from various density of sampling data by the FFT computer model are shown in Figure 12 through Figure 25.

A pair of identical pulses separated a slight delay time and their composite pulse are described in Figure 12.

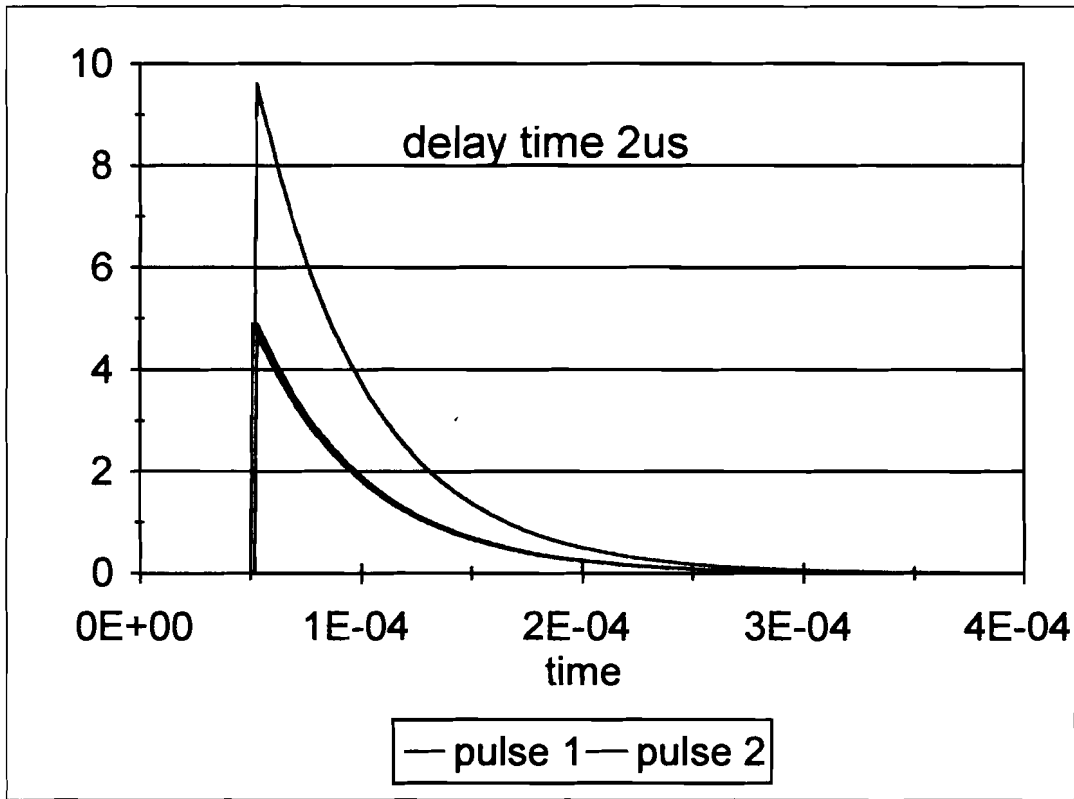


Figure 12

2 μ s of delay time from two exponential pulses

With the density of sampling data is 512 points, the various frequency spectra are displayed by different delay times in Figure 13 and Figure 14.

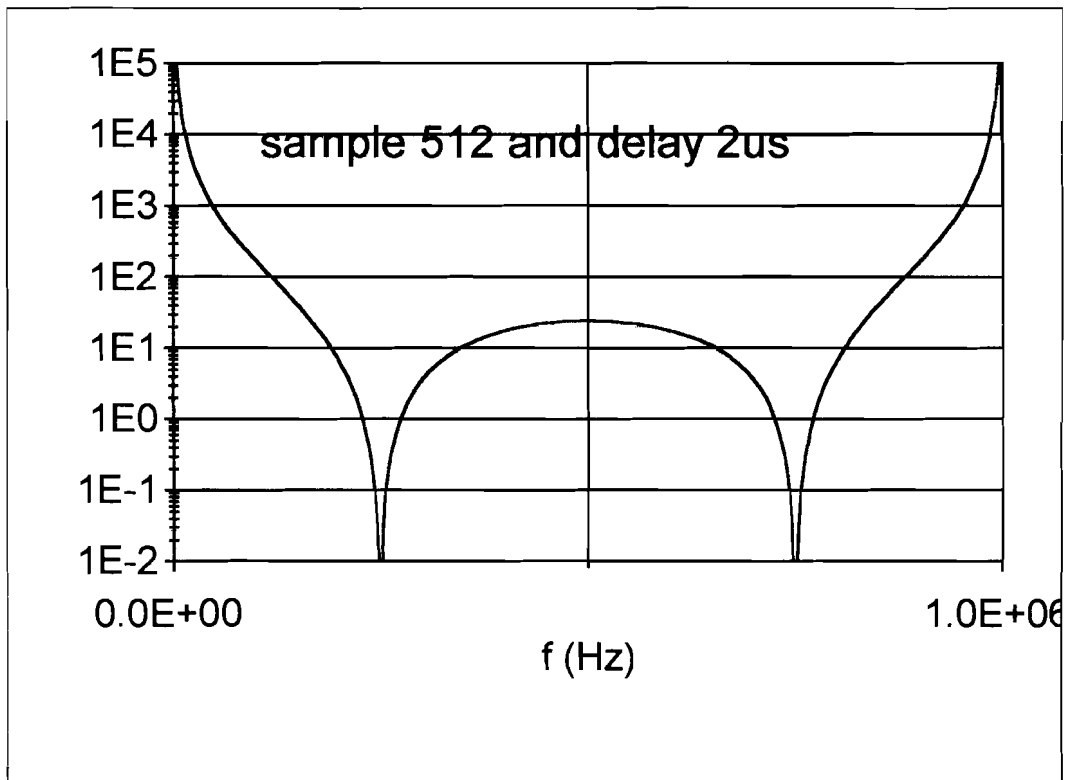


Figure 13

The spectrum from 512 points of sampling data and $2\mu s$ delay time

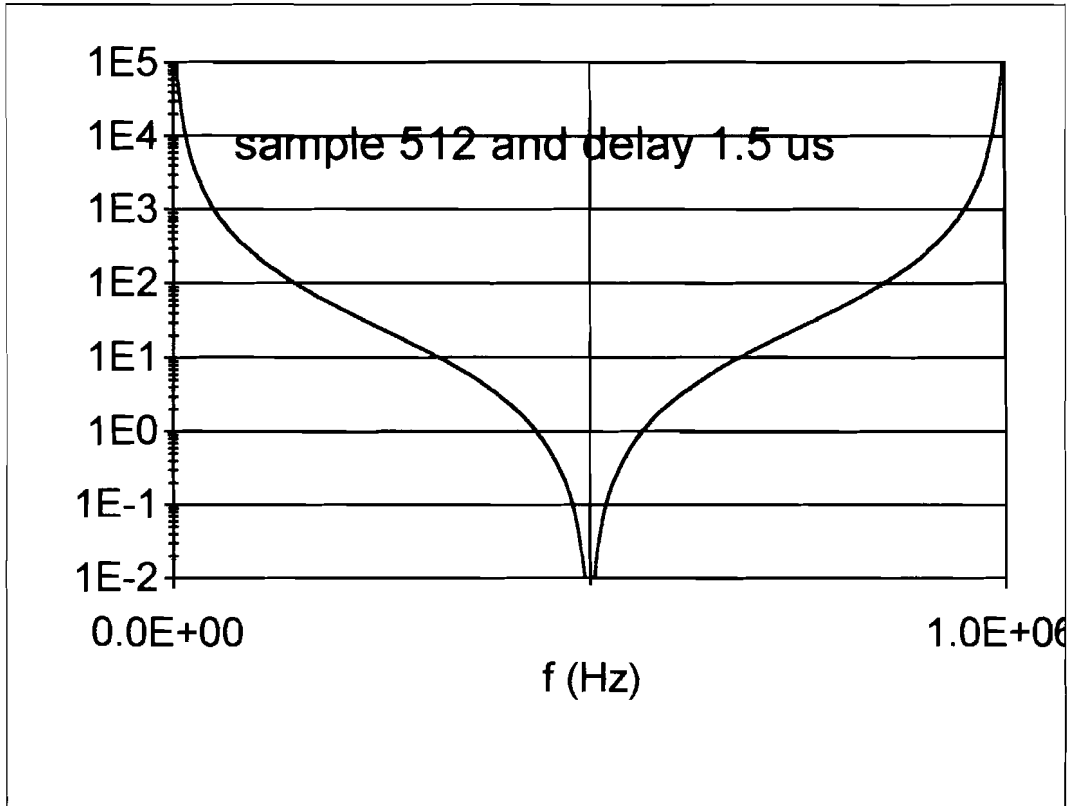


Figure 14

The spectrum from 512 points of sampling data and 1.5 μ s delay time

With the density of sampling data is 1024 points, the various frequency spectra are displayed by different delay times in Figure 15 through Figure 17.

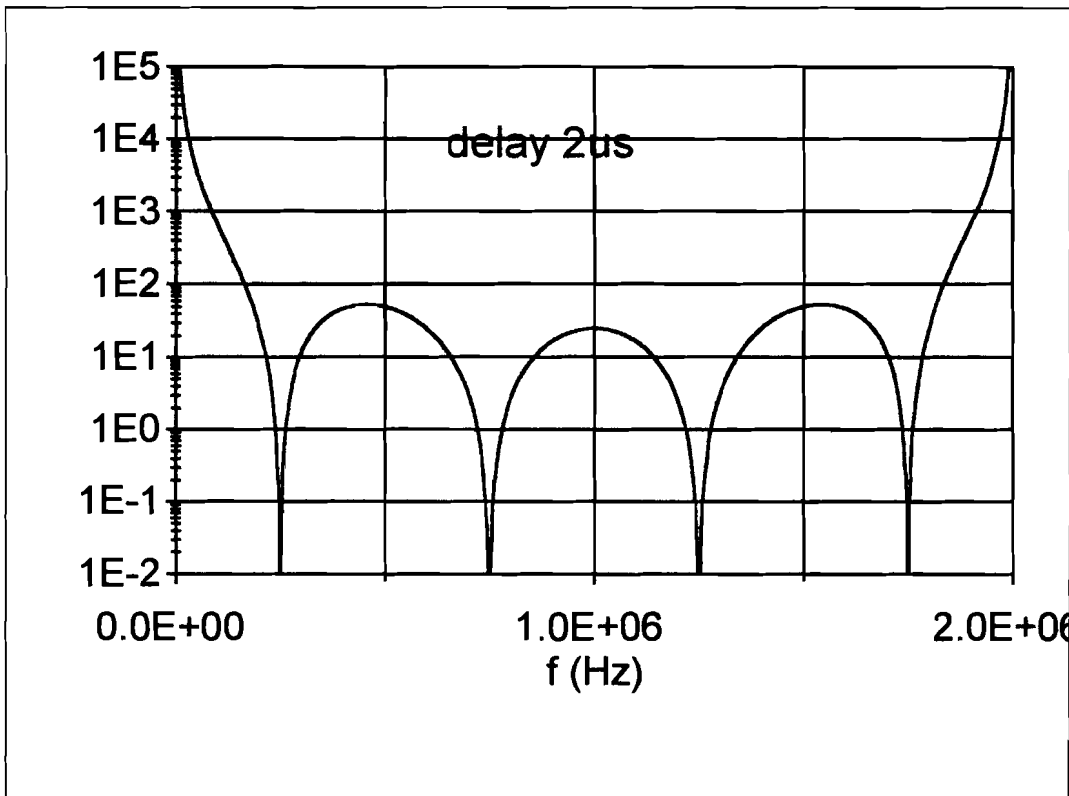


Figure 15

The spectrum from 1024 points of sampling data and 2 μ s delay time

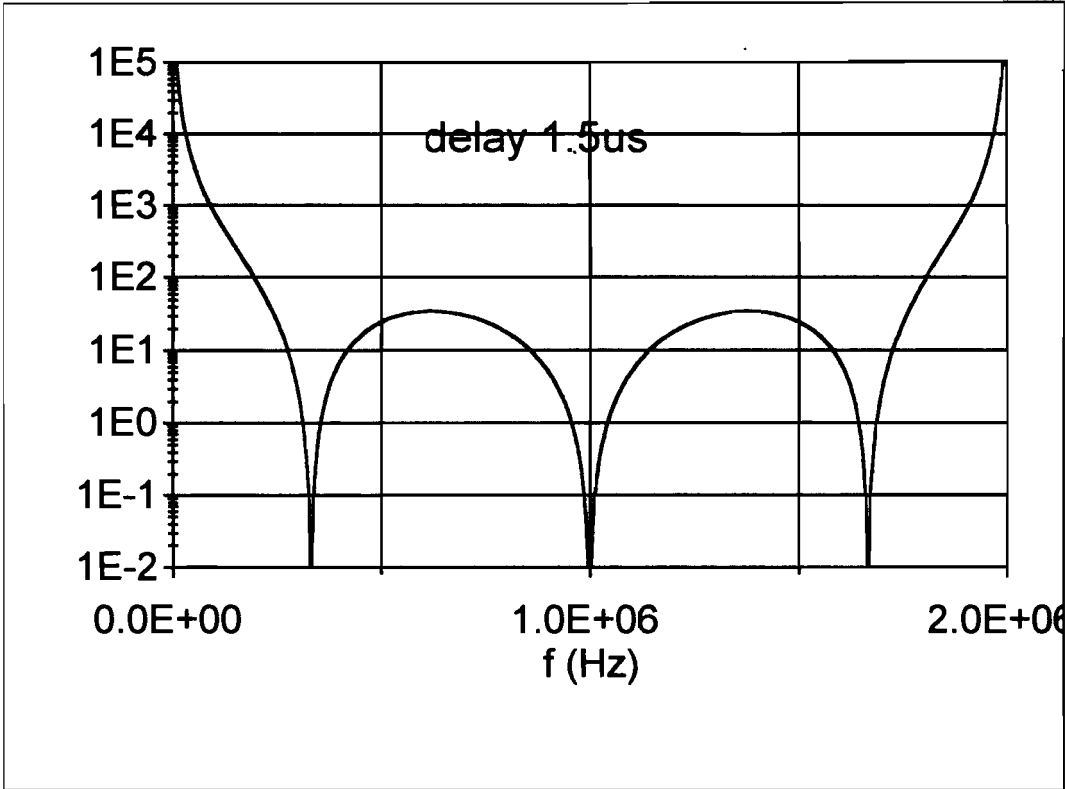


Figure 16

The spectrum from 1024 points of sampling data and 1.5 μ s delay time

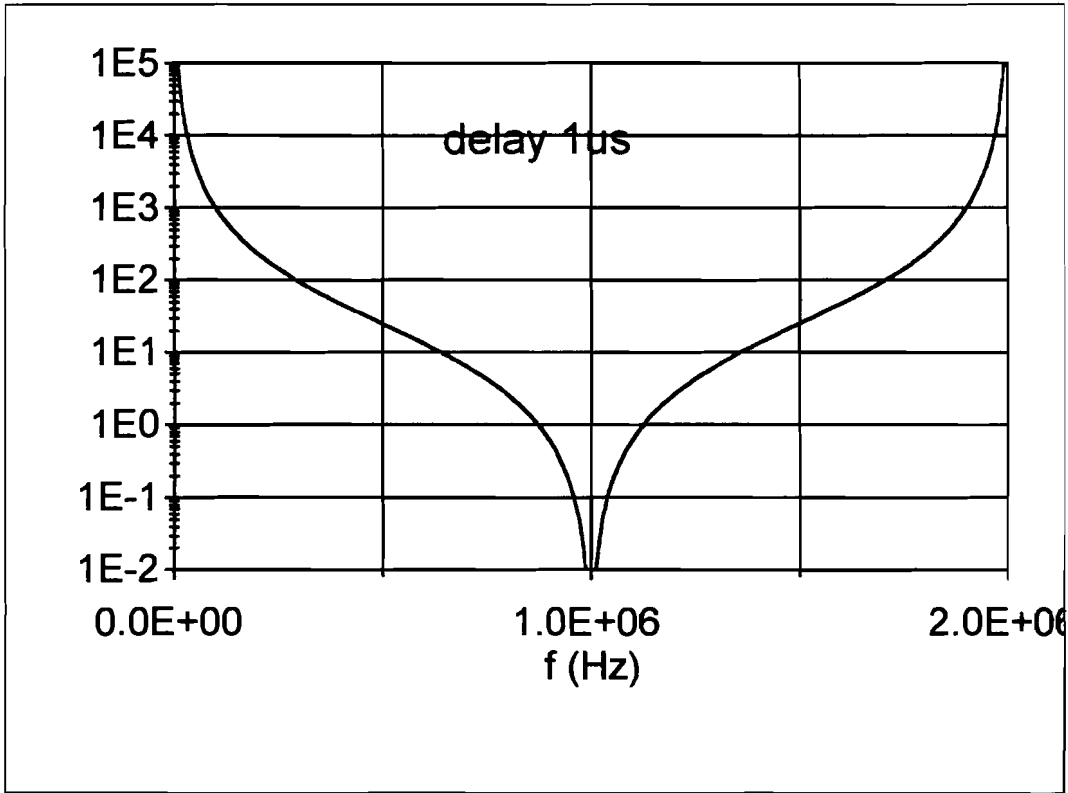


Figure 17

The spectrum from 1024 points of sampling data and 1 μ s delay time

With the density of sampling data is 2048 points, the various frequency spectra are displayed by different delay times in Figure 18 through Figure 21.

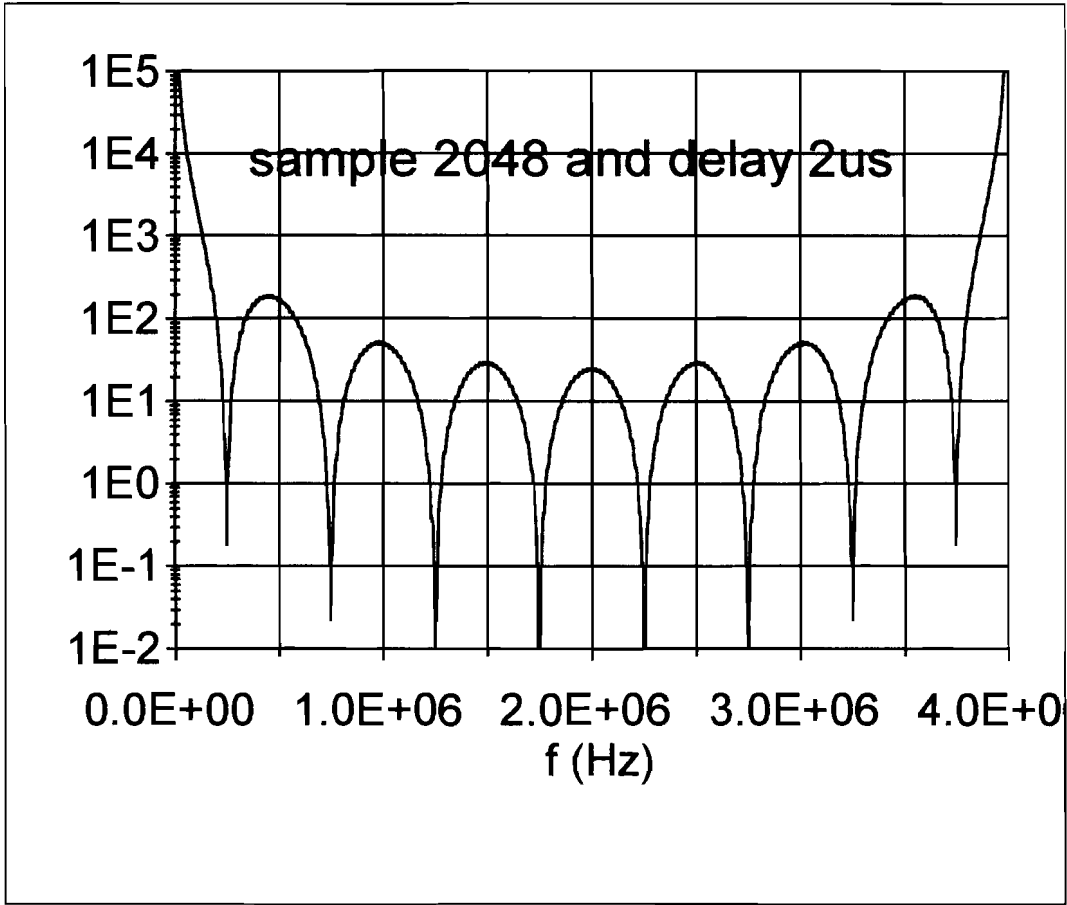


Figure 18

The spectrum from 2048 points of sampling data and 2 μ s delay time

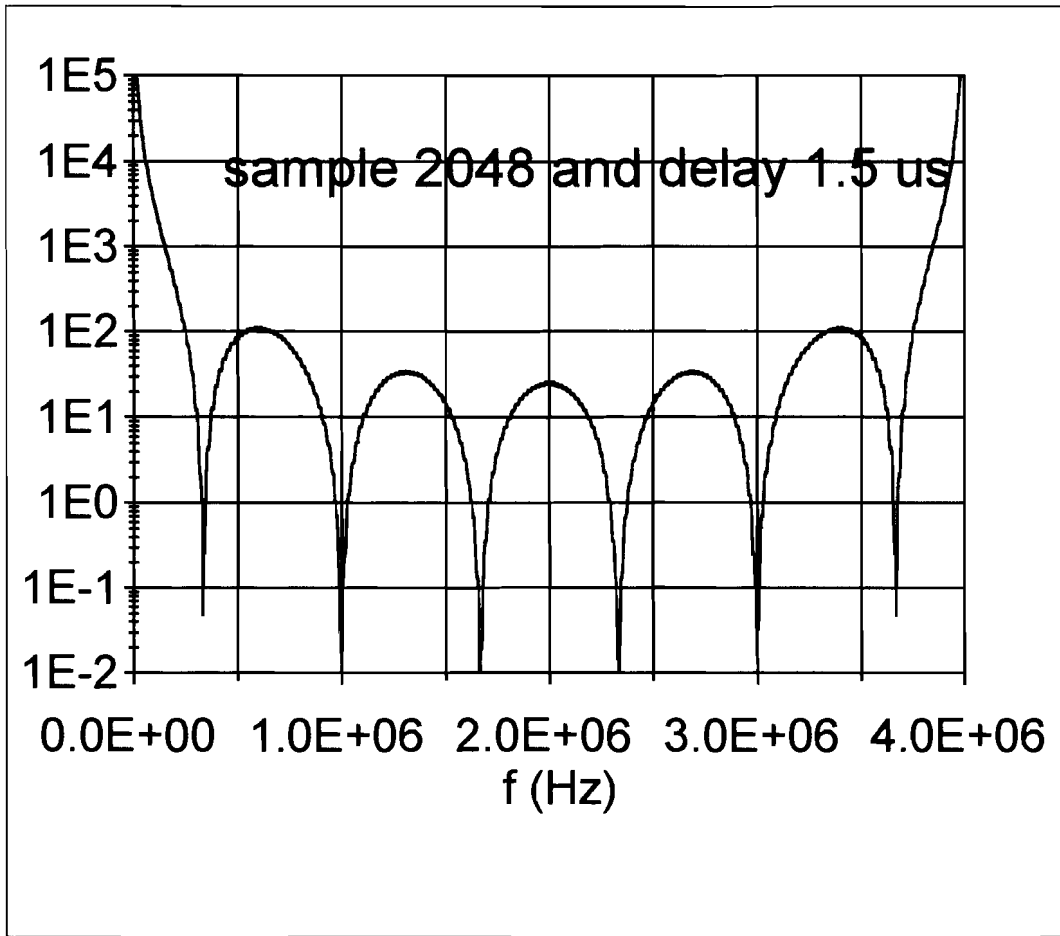


Figure 19

The spectrum from 2048 points of sampling data and 1.5 μ s delay time

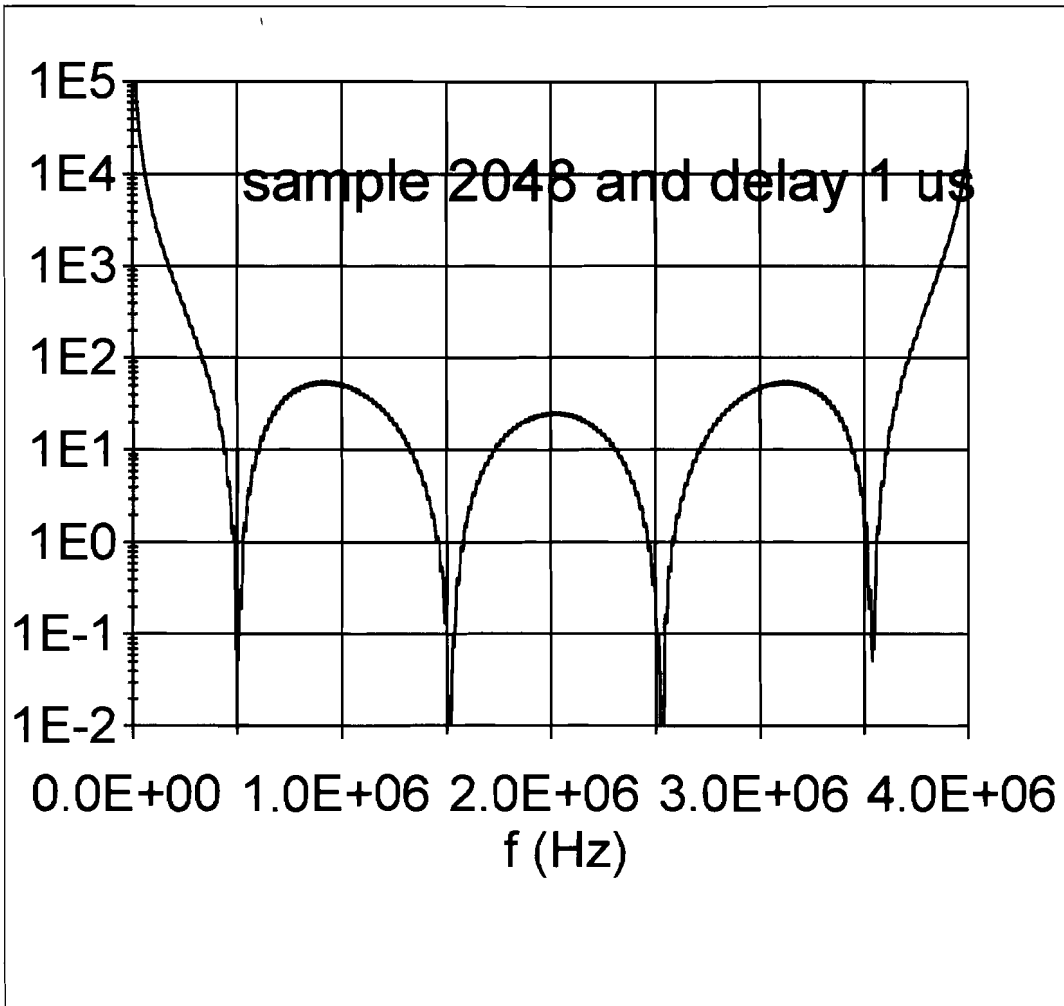


Figure 20

The spectrum from 2048 points of sampling data and 1 μ s delay time

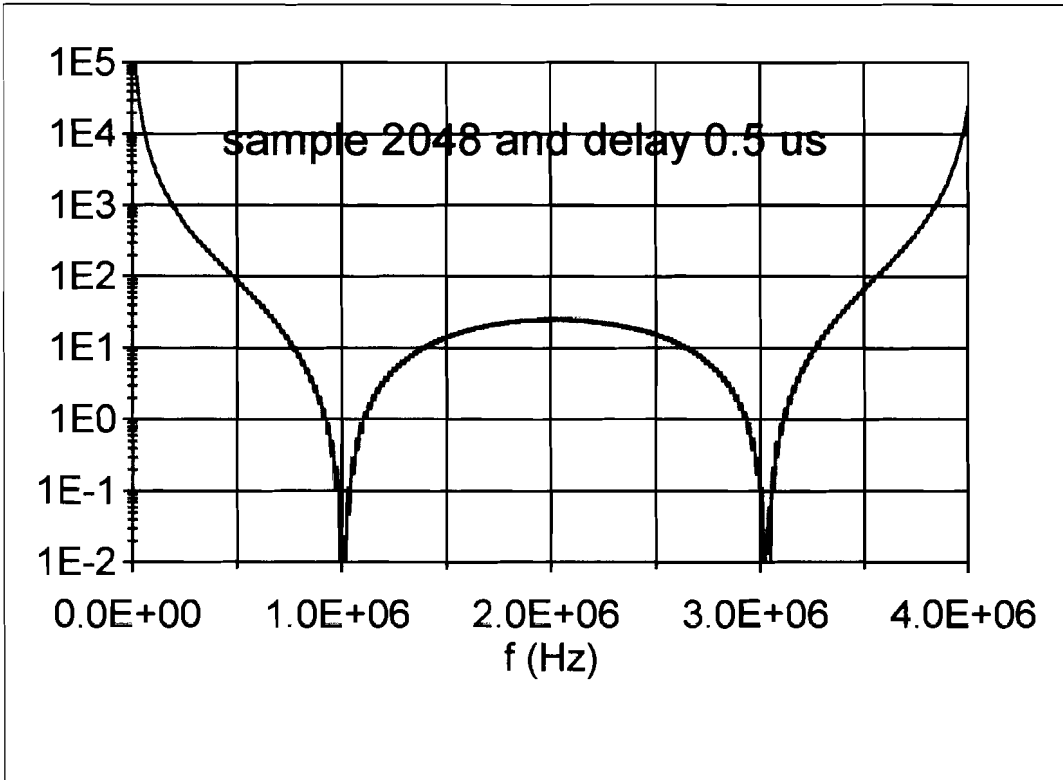


Figure 21

The spectrum from 2048 points of sampling data and 0.5 μ s delay time

With the density of sampling data is 4096 points, the various frequency spectra are displayed by different delay times in Figure 22 through Figure 25.

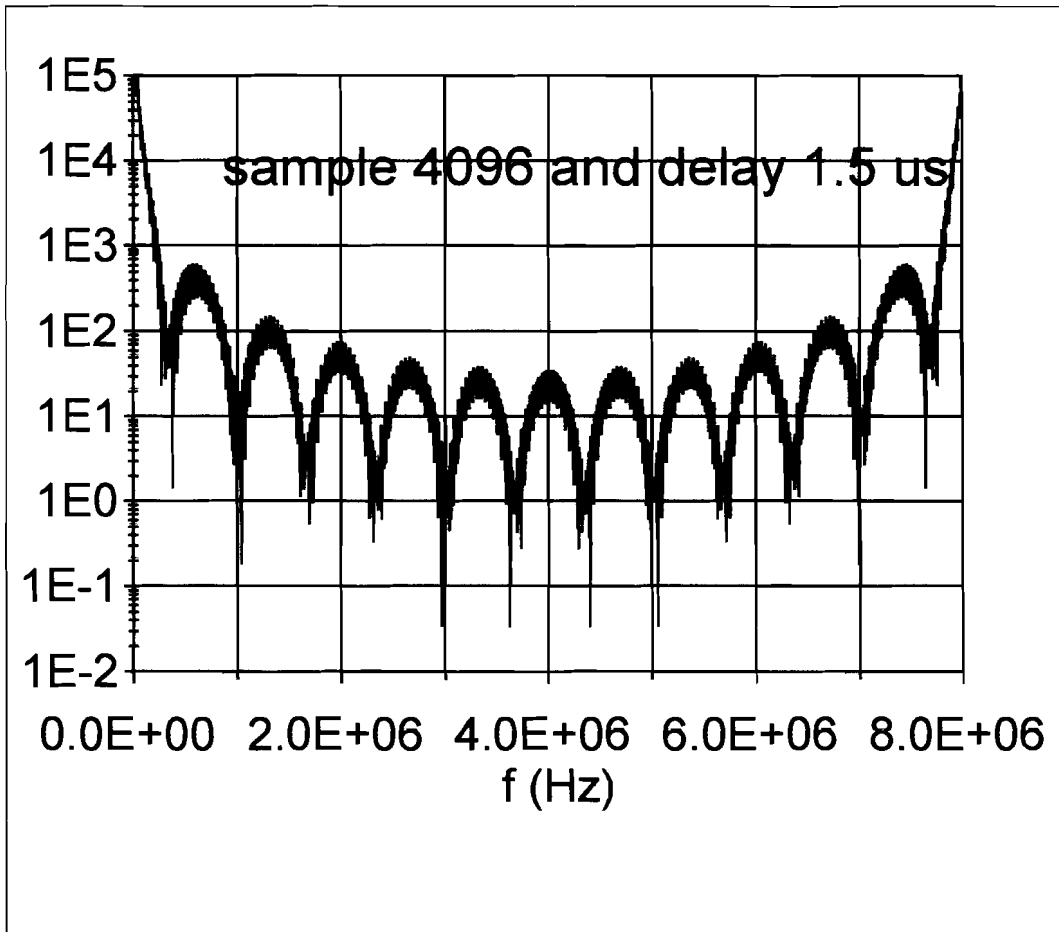


Figure 22

The spectrum from 4096 points of sampling data and 1.5 μ s delay time

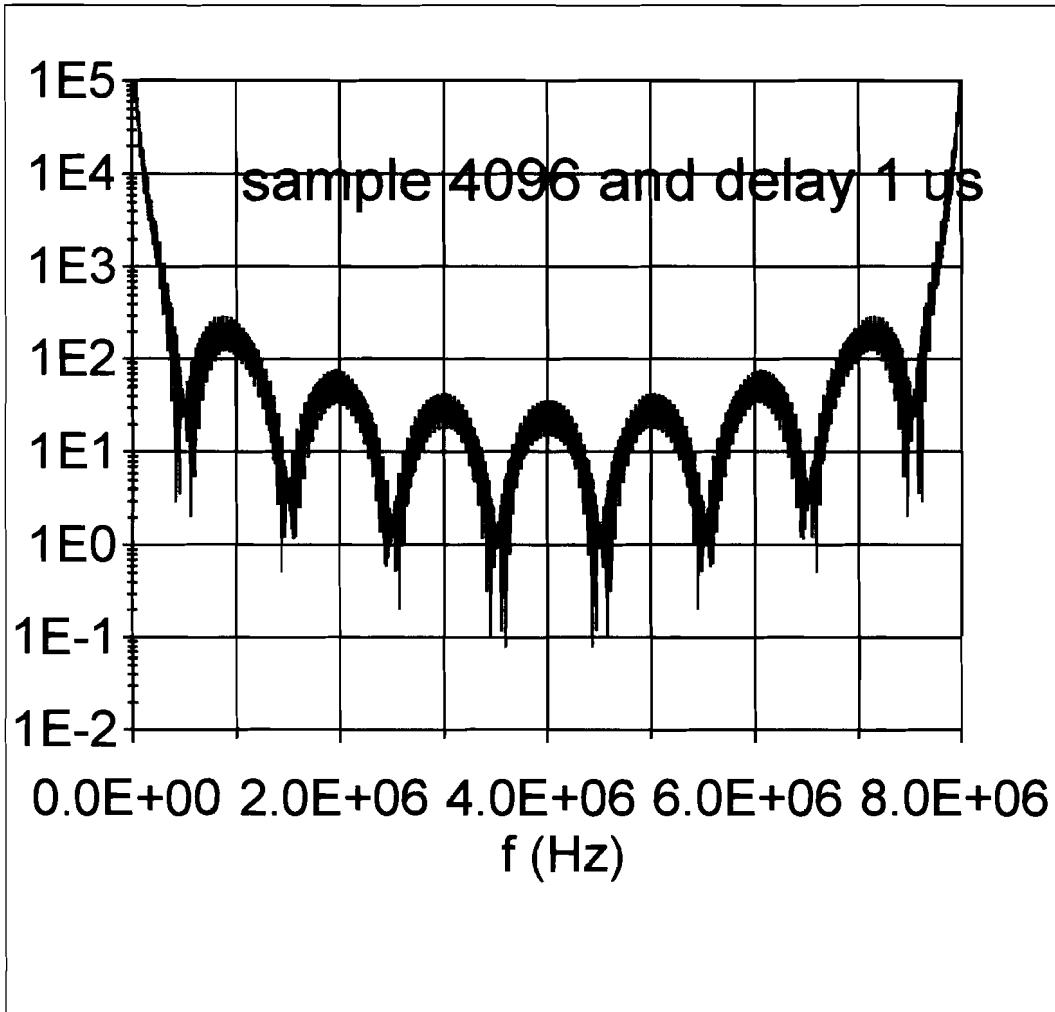


Figure 23

The spectrum from 4096 points of sampling data and 1 μ s delay time

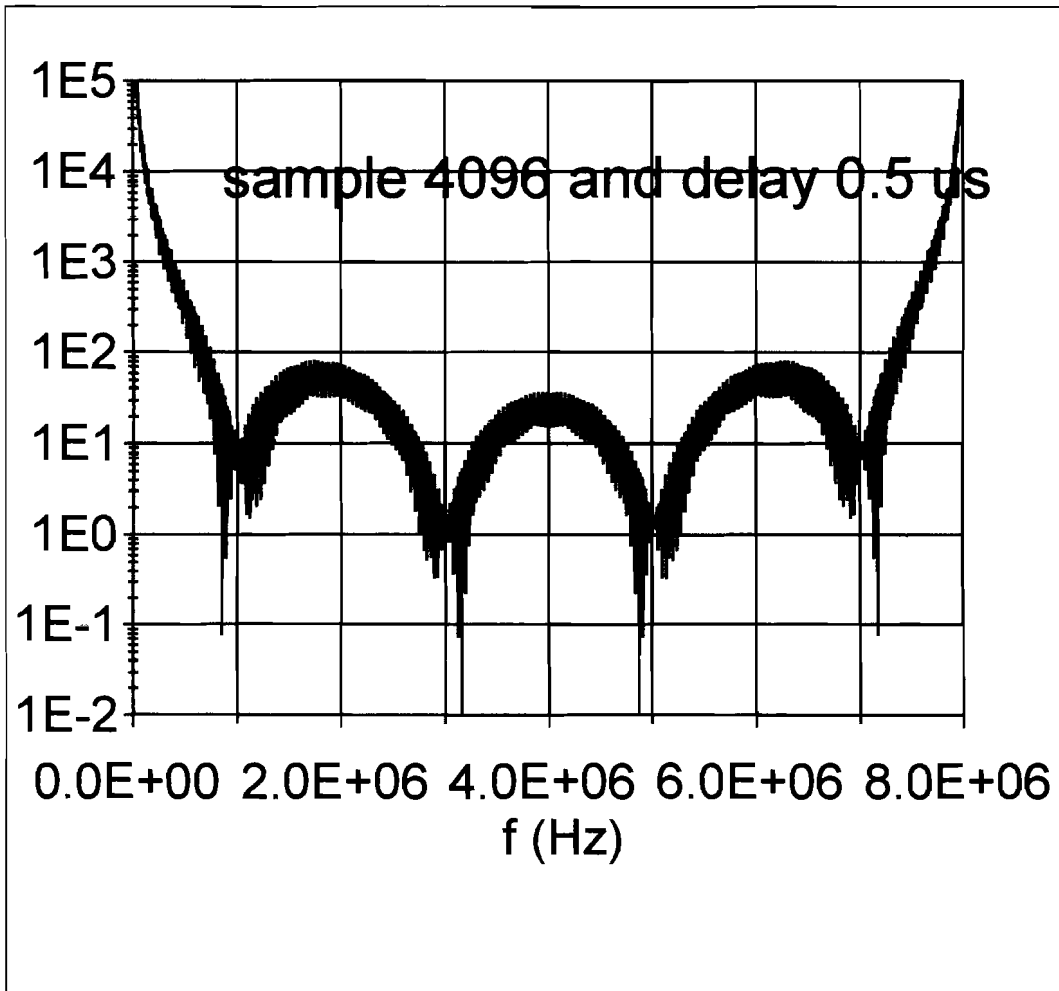


Figure 24

The spectrum from 4096 points of sampling data and 0.5 μ s delay time

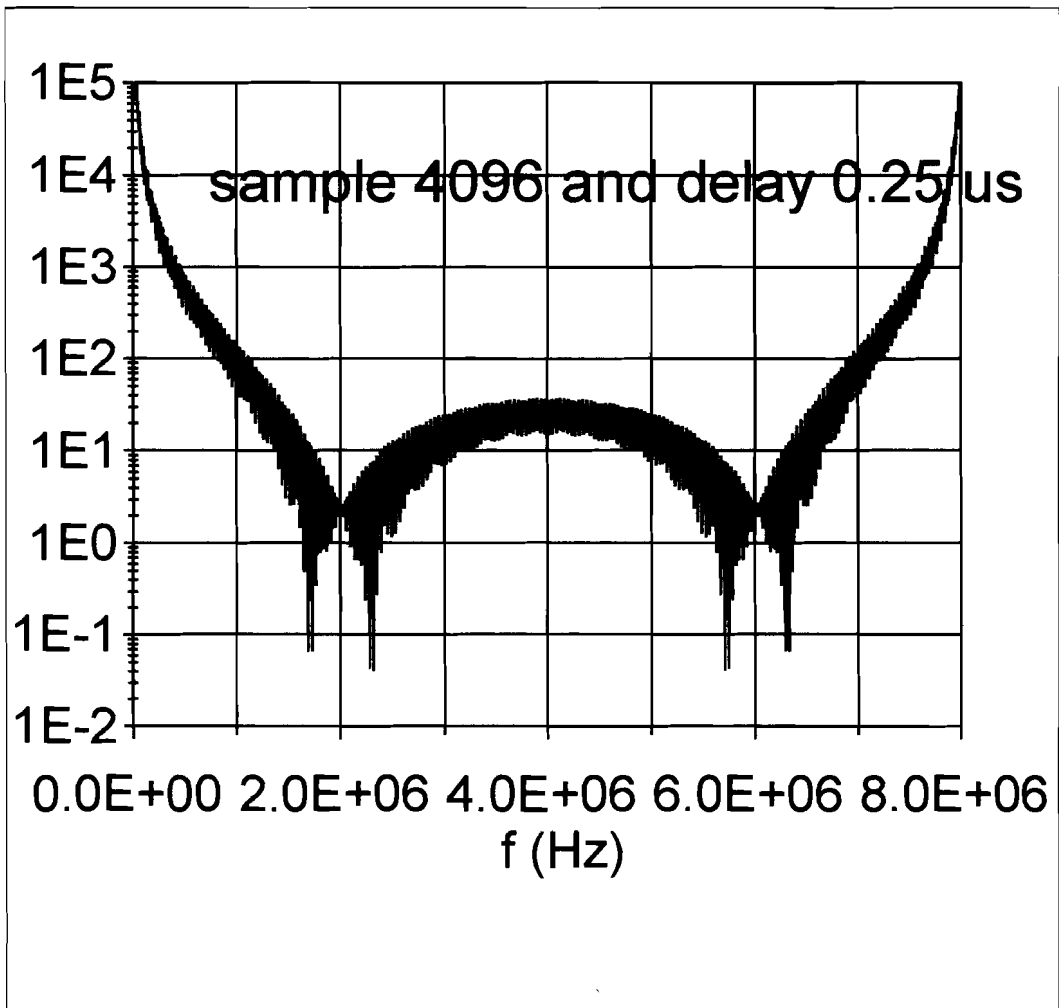


Figure 25

The spectrum from 4096 points of sampling data and 0.25 μ s delay time

(3) Relationship between the smallest possible delay time and the optical length

One objective of the experiment was to make the optical path length as short as possible. Since a short optical path results in a small delay time, it was important to learn what will be the shortest possible delay time for a given sampling rate. If the optical path length is shorter than the required length, the delay time between two pulses will be too small, resulting in the FFT computer model not being able to recognize the separated pulses and not being able to display the zeros in its frequency spectrum. Thus, the smallest delay time may be determined using the FFT computer model by putting in the previously determined parameters of the shape of the pulse and the density of sampling data.

The smallest possible delay time found using an exponential pulse was obtained by the FFT computer model in various densities of sampling data. In references to Figure 12 through Figure 25, when the density of sampling data in a pulse was 512 points, the smallest possible delay time which would show two minima in the frequency spectrum was $2 \mu s$. Thus, a delay time of less than the limiting value of $2 \mu s$ would not be able to display a useable frequency spectrum through the FFT computer model. If the density of sampling data of a pulse was 1024 points, the smallest possible delay time would be $1.5 \mu s$; if 2048, the smallest possible delay time would be $1 \mu s$; if 4096, , the smallest possible delay time would be $0.25 \mu s$.

The distance is easy to calculate by the formula:

$$d = c \times \tau$$

Where, d is the distance of the optical path, c is the speed of light, and τ is the delay time. If the speed of light is considered as 3×10^8 m, and τ is assumed to be $0.5 \mu\text{s}$, then the distance should be 150 meters. With one reflection, the optical path is 75 m. Conversely, the optical path distance can be chosen, and from that the delay time and consequently the sampling rate are determined. Thus starting with the dimensions of the space available, it will be possible to determine the requirements of the detector, the A/D converter, and the data storage system

(4) The effect of the duration of a pulse

The duration of a pulse refers to the time when the pulse is active. By looking at this parameter in the FFT computer model, nothing changed in the frequency spectrum when the duration of a pulse was changed under the same density of sampling data. These results are illustrated in Figure 26 through Figure 29.

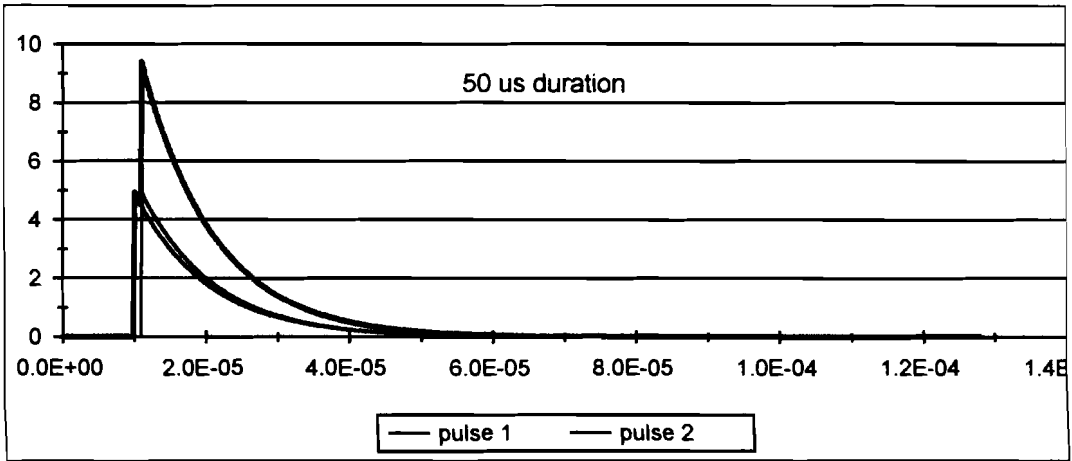


Figure 26
50 μ s duration of a pulse

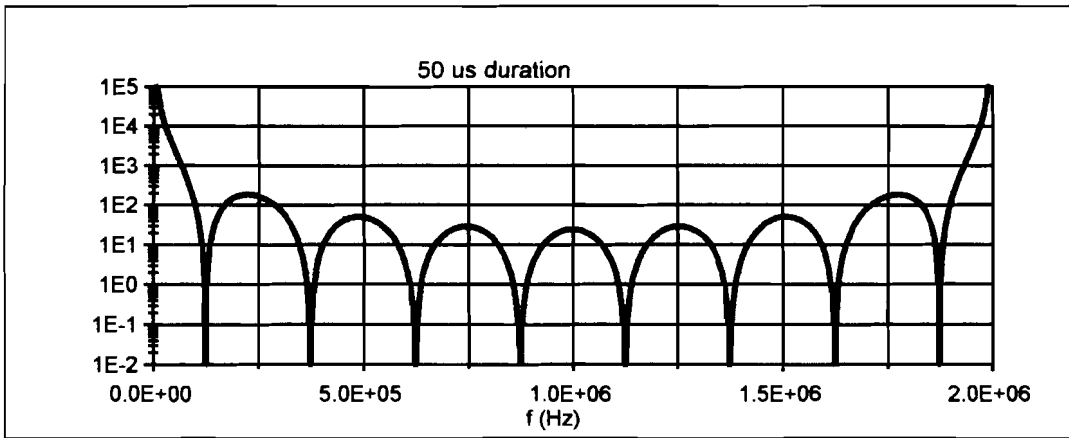


Figure 27
The frequency spectrum from the 50 μ s duration of a pulse

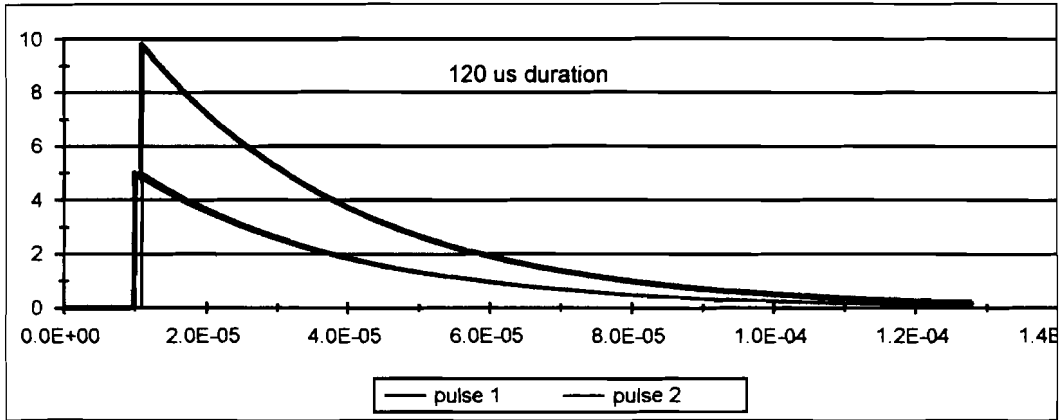


Figure 28
120 μ s duration of a pulse

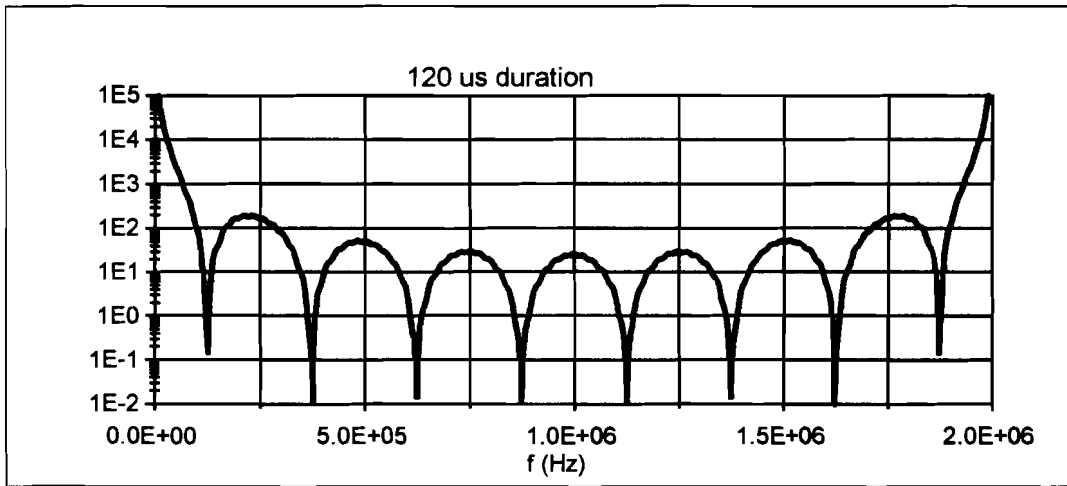


Figure 29
The frequency spectrum from 120 μ s duration of a pulse

Although it is unnecessary to require a short duration of an exponential pulse, this parameter should still be considered to be important among the properties of the experimental apparatus. If the time of the duration of a pulse is calculated from the density of sampling data and the limit of the high frequency of the storage device, then the flash frequency of the stroboscope can be chosen. If the limit of the high frequency of sampling data of the storage device is 20 MHz and the density is 4096, then

$$\begin{aligned} T &= \text{density of sampling data} / \text{the frequency limit of the storage device} \\ &= 4096 / 20 \text{ MHz} \\ &= 204.8 \mu\text{s} \end{aligned}$$

The flash frequency of the stroboscope is equal to $1/T$, around 5000 Hz.

(5) Summary of the parameter analysis

Four parameters are fundamental and necessary for the design of this experiment. Searching and determining the range of these parameters by using computer models can help the experiment to be successful. The actual parameters will depend upon the highest data processing rate that it is possible to achieve in the laboratory and that will be the subject of another investigation.

CHAPTER 4

CONCLUSION

Through the review of the experiments to measure the speed of light, many materials and references have been collected to determine whether or not it is feasible to continue with the current experimental design. It is important to retain the traditional concepts while improving the experiment by applying modern technology.

As one of the very powerful mathematics tools, the Fourier transform is widely applied in solving practical problems, especially signal problems. This tool, programmed in computer spreadsheets, is now proposed to be used in the experiment to measure the speed of light at ESU in order to determine the important parameters of this experiment and their relationship to each other. After determining these parameters, the experiment might be performed in a reasonable way and with successful results.

The principle and apparatus in the experiment were generally designed for use in the undergraduate physics laboratory. It is hoped that undergraduate students who perform the experiment to measure the speed of light may extend their knowledge by searching and reviewing the techniques from the past, and at the same time, practice the application of Fourier analysis.

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Zuming Gong

Signature of Author

May 8, 1996

Date

*A Design of an Undergraduate Experiment
to Measure the Speed of Light*

Title of Thesis

Doug Cooper

**Signature of Graduate Office
Staff Member**

May 8, 1996

Date received