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The purpose of this thesis is to investigate the effect of motivation on learning computational estimation strategies by showing examples of estimation used as a life skill. The effect of motivation on students' attitude toward estimation and the correlation between mental computational abilities and computational estimation skills were also considered. Two sections of a general education mathematics course at a small university were involved in the study. One class served as the experimental group, receiving a lecture on the usefulness of estimation in daily living before receiving instruction on computational estimation strategies. The other class served as the control group, receiving instruction only on computational estimation strategies.

# AN INVESTIGATION INTO THE EFFECT OF MOTIVATION ON LEARNING COMPUTATIONAL ESTIMATION

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CHAPTER ONE

# The Problem and Related Literature

## Introduction

During the last decade, estimation has received a considerable amount of attention from mathematics educators. A major emphasis on estimation has been called for in recent recommendations for curricular changes in school mathematics. (Conference Board of the Mathematical Sciences, 1989; National Council of Supervisors of Mathematics, 1988; National Council of Teachers of Mathematics, 1989). Estimation is an essential mathematical skill that is used extensively in the real world. Recent surveys show that more than 80% of all adult usage of mathematics involve estimation (Reys, 1984). However, this is not yet reflected in the classroom. When asked to estimate, many students compute the exact answer, then round to make their answer appear to be an estimate (Reys, Rybolt, Bestgen, & Wyatt, 1982; Schoen, Friesen, Jarrett, & Urbatsch, 1981; Sowder & Wheeler, 1989). Other studies have shown that students perform worse when estimating the results of computational exercises than when finding the exact answer to the same exercises (Lindquist, Carpenter, Silver, & Matthews, 1983). These studies suggest that there is a strong emphasis placed on paper-and-pencil algorithms. With the advances and availability of hand-held calculators and computers, less emphasis should be placed on these algorithms and more emphasis should be placed on computational estimation. An interesting study by Reys, Bestgen, Rybolt, and Wyatt (1980) shows the dependence and trust being placed in calculators. For this study, a calculator was wired to give incorrect answers. Students and adults who were good estimators were told to check their estimates using the tainted calculator. The incorrect answer on the calculator was usually accepted as correct even by these good estimators, and the error was assumed to be in their own estimates. The use of the hand-held calculator has become so widespread, it occasionally has and can overshadow the importance of mental computation and student confidence in their own mental computational abilities. Although the calculator is an important part of mathematics, it should not take the place of computational estimation, mental computation, or even all paper-and-pencil algorithms. The relationship between these processes must be determined.

## Teaching Estimation in the Classroom

The recent recommendations for curricular changes in school mathematics by the Conference Board of the Mathematical Sciences (1989), the National Council of Supervisors of Mathematics (1988), and the National Council of Teachers of Mathematics (1989) show the importance of computational estimation in the mathematics classroom. Knowing that a topic needs to be taught and actually teaching it, however, requires concrete answers to questions like these: What specific skills should be taught? How should these skills be sequenced and merged into the existing mathematics curriculum? How should the topic be presented and practiced? (B. Reys, NCTM 1986). Barbara Reys reports that poor concept development of fractions, decimals, and percents inhibit the ability to estimate answers to problems containing these numbers (Arithmetic Teacher, 1986). National mathematics assessments found that only 24% of thirteen year-olds estimated 12/13 + 7/8 correctly and only 21% correctly estimated 3.04 x 5.3 from multiple choice test items (Suydam, 1984). The NCTM's Curriculum and Evaluation Standards offers a solution to this problem. "The study of computational estimation should be integrated with the study of concepts underlying whole numbers, fractions, decimals, and rational numbers so that these concepts can be constructed meaningfully by the learner." (1989) Developing and using computational estimation skills can also promote thinking skills and problem solving (Reys, 1984).

There are many different estimation procedures used by good computational estimators (Reys et al, 1982). Several general processes have been observed to build a basis for computational estimation. Three key processes are reformulation, translation, and compensation. Reformulation is the process of altering numerical data to produce a more mentally manageable form. This process leaves the structure of the problem intact. Translation is the process of changing the mathematical structure of the problem to a more mentally manageable form. This form is then used computationally to process the numerical data. Adjustments made to reflect numerical variation that come about in the reformulation or translation processes describe the compensation process. These processes are combined in many specific strategies. Barbara Reys describes some of these computational estimation strategies in <u>Estimation and Mental Computation</u> (NCTM, 1986). These include front-end and compensation, clustering, compatible numbers, rounding and range, and special numbers.

The front-end and compensation strategy focuses on the "front end", or the left-most digits, of a number. Because these are the most significant, they are the most important for forming an estimate. The operation is performed on the front end, and then compensation is made for the part of the numbers not considered in the front-end operation. For example, to estimate the sum of 436, 719, 284, and 562, the front-end operation would be $4 + 7 + 2 + 5 = 18$. These numbers are in the hundreds place, so the front-end operation produces the result of 1800. Compensation would then be made, estimating that 36 and 62 is about 100, and 19 and 84 is also about 100. Adding 1800 to the compensation of 200 would produce 2000 for an estimate of the sum of the four numbers. Front-end and compensation can be used for each of the four main operations.

The clustering strategy is suited for a particular type of problem. It is used when a group of numbers cluster around a common value. For example, 4823, 5371, 5089, 4694, and 5142 all cluster around 5000. Since there are five numbers, an estimated sum of these numbers would be $5000 \times 5 = 25{,}000$.

The compatible numbers strategy refers to a set of numbers that can be easily "fit together", or are easy to manipulate mentally. This strategy is primarily used when estimating division problems. When estimating 3388/7, some compatible sets would be 3500/7, 3200/8, and 4000/8. Some sets which are not compatible include 3000/7, 3400/7, and 3400/8.

Rounding and range is an efficient estimation strategy for multiplying two multidigit numbers. This involves the traditional procedure of rounding each number to the nearest power of ten determined by the left-most digit. Then multiplying these two rounded numbers produces an estimate. An example of this strategy follows for estimating the product of 2741 and 628. Rounding 2741 to the nearest thousand makes 3000, and rounding 628 to the nearest hundred makes 600. Then $3000 \times 600 = 1{,}800{,}000$ is an estimate for the product of 2741 and 628. The rounding strategy can also create a range in which the exact answer belongs. The lower limit to the range would be found by rounding both numbers down, and the upper limit would be found by rounding both numbers up. For example, the product of 465 and 87 is between 400 x 80 = 32,000 and 500 x 90 = 45,000.

The special numbers strategy involves fractions, decimals, and percents which are close to "special" values that are easy to compute mentally. For example, in the problem 7/8 + 12/13, each fraction is close to 1. Therefore, an estimate of this sum might be 1 + 1 = 2. Also .48 x 36 could be estimated as 1/2 x 36 = 18, and 9.67% of 193 could be estimated as 10% of 193 which is 19.3. This strategy could be taught along with the development of fraction, decimal, and percentage concepts, which might help students better understand these concepts.

A variety of computational estimation strategies need to be taught as well as the ability to select an appropriate strategy to fit the context of the problem. When strategies other than rounding are taught, the performance of students increases significantly (Reys, Trafton, Reys, & Zawojewski, 1984). Research has also shown that unless specific strategies are taught, few students will develop them on their own (Reys, 1985).

## Evaluation of Estimation Skills

Testing is an essential component of the learning process. Students often believe that what is important to learn is what they are tested on. Testing estimation skills has been one of the major road-blocks in incorporating estimation in the classroom. Standardized tests as well as teacher-made tests often do not test estimation. Many students compute exact answers and round these answers to reflect an estimate on estimation test items (Reys et al., 1982; Schoen et al., 1981; Sowder et al., 1989). Some examination procedures have been found to test computational estimation skills, but only one estimation strategy is necessary to complete these examinations. One study used open-ended questions and limited the time for each problem to only a few seconds. This forced computational estimation to be used, but about 70% of the 6th and 8th grade students tested used only the rounding estimation strategy (Schoen, Blume, & Hart, 1987).

Developing examinations which test a variety of computational estimation strategies is an important component of teaching estimation skills. One test item format is having the students decide whether an exact answer is above or below a given reference number. Other formats involve multiple choice test items. Multiple choice test questions can be used in several different formats. The standard multiple choice test gives the choice between one correct estimate and other estimates which reflect frequent errors. Some research suggests that students often do not produce a direct estimate when given this type of test item, but rather consider each choice individually and decide whether to accept or reject it (Reys et al., 1980). Another multiple choice format gives the choice between estimates of different orders of magnitude. This format is particularly appropriate for multiplication and division estimation, but for estimation with other operations, other formats should be used. A third multiple choice format involves choosing an interval in which the exact answer belongs. This format could test more than one estimation strategy if care is taken to set up the intervals where a correct estimate is the endpoint of two intervals. Therefore, another strategy must be used to determine which of these two intervals contain the exact answer.

Limiting the amount of time is another important component of testing estimation skills. Allowing only a few seconds more for each question dramatically changes the performance on estimation tests (Reys, 1984). Robert Reys believes that it is better to give too little time than too much time on estimation examinations. He suggests that too much time simply reduces the examination to paper-and-pencil computation, whereas too little time might produce wild guessing (1988).

## The Legitimacy of Estimation

When am I ever going to use this? This is one of the most commonly asked questions in the mathematics classrooms at any grade level. Often students are much more willing to learn if they know they will be using the skills being taught. One study showed that good computational estimators were uncertain when or how their estimation skills were developed, but they suggested they learned these skills on their own because they were necessary (Reys et al., 1980). Establishing an estimation mind-set is a primary component in the instruction of estimation. The NCTM Standards recommend that students should learn what is meant by an estimate, when it is appropriate to estimate, and how close an estimate should be in a certain situation (1989). Encouraging students to use estimation might help them view it as a legitimate part of mathematics. Another way to help establish the legitimacy of estimation might be to show the array of estimates found in newspapers and magazines. Research has shown that students are more successful in computational estimation for a problem written in context than for pure computation (Reys et al., 1980). Applications in daily living might also help to show the usefulness of estimation as a life skill.

## The Effect of Motivation on Learning

Motivating students to learn is an important objective for many instructors. There is an abundance of literature showing how this might be accomplished. However, little research directly correlating motivation with performance had been conducted up to 1978 (Keller, Kelly, & Dodge, 1978). The same has held true in recent years. One mathematical study did find that intrinsic motivation to study mathematics is a significant positive predictor of student development of space relations (Iben, 1988). Another study, involving history students, investigated the effect of student selection of instructional materials (Rogers, 1990). The study concluded that the performance of students improves when they see a purpose for history instruction. This thesis lays the preliminary ground work for the investigation of this conclusion in the mathematics classroom. What will be the effect of motivation on learning computational estimation strategies?

# CHAPTER TWO

## Description of the Study

### Hypotheses

The purpose of the study is to provide a pilot for further investigation into the effect of motivation on students learning computational estimation strategies by showing examples of estimation used as a life skill. Computational estimation is defined as the process of computing a reasonable answer suitable for solving problems in everyday living by finding an approximation of the exact value.

H1: Students in the experimental group, given proper motivation before teaching computational estimation strategies, and students in the control group, receiving no motivation, will show no differences in improvement of computational estimation skill.

H2: There will be no correlation between mental computational abilities and computational estimation skills in either group.

H3: Students in both groups will demonstrate the same attitudes toward the importance of computational estimation as a life skill.

## Sample

The study compared an experimental group with a control group. The subjects were undergraduate college students, mainly freshmen, who were enrolled in two sections of Principles of Mathematics, a general education course. Tables 1 and 2 show the composition of the two classes.

Table 1

Male and Female Composition of Classes

| | **Control** | **Experimental** |
|---|---|---|
| Male | 7 | 10 |
| Female | 16 | 11 |

Table 2

Classification

| | **Control** | **Experimental** |
|---|---|---|
| Freshman | 19 | 11 |
| Sophomore | 3 | 6 |
| Junior | 0 | 3 |
| Senior | 1 | 1 |

Students were already enrolled in their course sections before any information was revealed about their involvement in a study. Treatments were randomly assigned to the two classes. The control group met at 10:00 a.m. and the experimental group met at 1:00 p.m. Both classes met for fifty minute periods on Mondays, Wednesdays, and Fridays each week.

## Description of the Course

Principles of Mathematics is a course designed to develop an awareness of how mathematical skills and processes have relevance to other disciplines. Topics such as computational estimation, problem solving, sets, logic, algebra, geometry, measurement estimation, probability, and statistics are taught and related to problems in various disciplines, showing the usefulness of mathematics as a life skill.

## Instructor

Both classes were taught by the investigator. He has been a graduate teaching assistant at Emporia State University for the past two years and has taught Principles of Mathematics in the semester prior to this study.

The instruction of Principles of Mathematics is supervised by Dr. Connie Schrock, the investigator's thesis advisor. Dr. Schrock completed her Ph.D. in Mathematics Education at Kansas State University in 1989. She taught high school mathematics for seven years prior to completing her advanced degree. Dr. Schrock is currently in charge of the secondary mathematics education program at Emporia State University. She also develops inservice workshops for mathematics teachers and is involved in curriculum development for mathematics.

Instructional Procedures

The experimental class was presented a motivational lecture on estimation. The lecture began by explaining what is meant by estimation in a general sense, that it is a process of computing a reasonable answer without calculating the exact answer. Then examples of estimates were shown in newspaper articles. These included statistical and computational estimates. A discussion concerning the difference between rounded numbers and estimates followed, using numbers from the newspaper articles as examples. Rounded numbers were defined to come from rounding exact figures to numbers which were more readable in the articles. Many students are confused when the distinction between rounding as a computational estimation strategy and rounding as a reporting tool for clarity is not made. The remainder of the lecture illustrated uses for computational estimation as a life skill. Situations included comparing the cost of two items to determine which was the better buy, determining the amount of a tip to leave for a waiter or waitress at a restaurant, and determining the sale price of an item advertised as a certain percentage off the original price. A multiple page worksheet constructed by the investigator was assigned to this class. This worksheet was designed to increase the awareness of estimation used in daily living. The first page asked students to decide if estimation would be sufficient in certain situations. Some of these situations were presented in a manner that the answer depended on information not available to the student. This was designed to stimulate discussion by the students about additional conditions placed on these situations in which estimation would be sufficient and additional conditions in which estimation would not be sufficient. No supported answer was incorrect. The second page contained problems for the students to select the most sensible answer. This was intended to alert students that estimates need to be reasonable. The third page included a newspaper article in which students were to distinguish between estimates, exact numbers, and rounded figures. A copy of this assignment is located in Appendix A.

During the next class period, the experimental group and the control group were both presented a lecture on computational estimation strategies. The strategies explained were front-end and compensation, clustering, compatible numbers, rounding and range, and special numbers. These strategies are discussed in Chapter 1 of this thesis on pages 4-6. A summary of these computational estimation strategies was distributed to every student in both classes. This handout is located in Appendix B. Another worksheet was assigned for homework. This assignment asked students to determine how many digits would be in the answer of certain problems, to choose the closest number to the exact answer from four numbers, and to specify given fractions as close to either 0, 1/2, or 1. The remainder of the worksheet were problems to estimate the answer using the strategies presented in the lecture. A copy of this assignment is included in Appendix A, pages 37-41.

Experimental Design

The independent variable was the exposure to motivation. The dependent variables were scores on the various tests. For the analysis of the first hypothesis, a quasi-experimental Nonequivalent Control Group Design, illustrated in Figure 1, was used. In the diagram, the O's represent observations made from both groups. The X represents the experimental treatment, and the dashed line represents nonequivalent groups due to the inability to randomly assign students to each group. Internal validity of the experiment is affected by factors which, by themselves, could produce changes which might be mistaken for results of the treatment. Internal validity in this design is weakened because the design does not have pre-experimental sampling equivalence. This means that the lack of random assignment of subjects to the two groups weakens the reliability of the experiment. The experimenter was able to randomly assign the experimental treatment between the two groups which was made by the selection of a random number before the pilot study began. Although this design shows some weakness in internal validity, it yields higher reliability in external validity (Campbell & Stanley, 1963). External validity is the degree to which the results of a study can be generalized beyond the present conditions of testing. This design is reliable in external validity because the two groups were subjected to identical external factors.

Figure 1

Nonequivalent Control Group Design

| | | | |
|---|---|---|---|
| Experimental Group | O | X | O |
| Control Group | O | | O |

For the second hypothesis, the investigator found it necessary to use a pre-experimental design, Static-Group Comparison. In this design, measurement was made by using only a posttest for the two groups. A specific design was not employed for the preliminary investigation of the third hypothesis. The chi square test of homogeneity was used to compare attitudes toward estimation between the two groups.

## Instruments

The computational estimation test used for the Pretest

Posttest measurement was written by the investigator and given as a pilot to other mathematics graduate teaching assistants at Emporia State University. The examination contained 20 open-ended questions, in which 10 were computational questions and 10 questions were written in context. The test was administered using an overhead projector. Each problem was placed on its own overhead transparency. See Appendix C, page 47, for the format of transparencies used in this examination. The students were allowed to view the computational problems for 10 seconds and the problems written in context for 15 seconds. The time between questions was approximately 5 seconds allowing the students to record their answers and the investigator to change transparencies on the overhead projector. Before the examination began, two sample questions were presented to help the students understand the examination process and to allow them to adjust their seating arrangement for optimal viewing of the projection screen. The pretest and the posttest differed by rewriting 10 questions while the other 10 remained the same on both tests. These tests are located in Appendix C, pages 48 and 50. The examinations were graded using a range of correct estimates for each problem established by the investigator prior to the examination. These ranges were established by using many different computational estimation strategies which would produce acceptable estimates. A list of these ranges are found in

Appendix C, pages 49 and 51.

Mental computational abilities were tested using two types of examinations. Both were prepared by the investigator. One examination was given orally and contained 10 open-ended questions, all computational. Each problem was stated twice. Occasionally a problem would be stated a third time upon request of the students. The students were allowed as much time as needed to answer each problem. A list of problems used in this examination is found in Appendix C, page 52. The second examination was a written test, also containing 10 open-ended computational problems. Again the students were allowed as much time as needed to complete the examination. See Appendix C, page 53, for a copy of this test.

## Survey

The experimental and control groups were given a survey at the conclusion of the study which is included in Appendix D. Students were asked their opinions about computational estimation as a life skill, confidence in their own estimation abilities, and whether or not estimation strategies should be taught in the mathematics classroom. The results of this survey are located in Appendix E.

# CHAPTER THREE

## Results

The level of significance for this pilot study was set at $\alpha = .05$.

### Pretest Posttest Estimation Examination

The estimation examinations were used to measure the difference in computational estimation ability of each student from the beginning to the end of the study. Each examination had a total of 20 points possible. Table 3 reports the descriptive statistics for the four sets of data.

Table 3

Descriptive Statistics for the Estimation Scores

| | **Sample Size** | **Mean** | **STD DEV** | **Range** |
|---|---|---|---|---|
| Control Pre | 19 | 6.9474 | 3.0817 | 0-11 |
| Control Post | 22 | 7.1364 | 3.9315 | 0-15 |
| Experimental Pre | 19 | 8.9474 | 3.0088 | 2-13 |
| Experimental Post | 17 | 11.3529 | 2.7826 | 7-17 |

To test the first hypothesis, a repeated measures analysis of variance was performed. These results are summarized in Table 4. A significant effect was noted in the interaction between the groups and the trials. This indicates that the null hypothesis is to be rejected, which implies the experimental treatment may have had a positive effect on computational estimation performance.

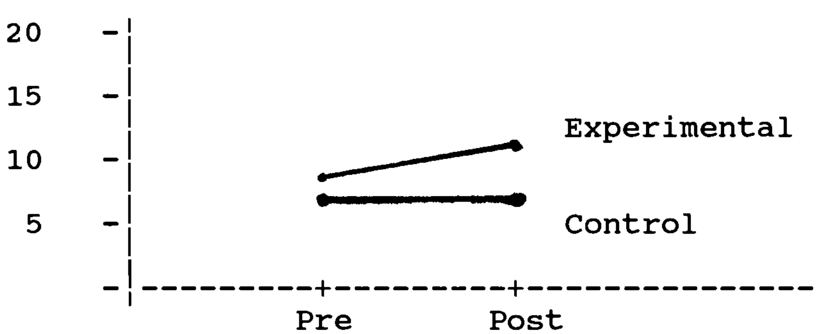
Table 4

Interaction of Estimation Scores

| Source | DF | SS | MS | F | PR>F |
|---|---|---|---|---|---|
| Group | 1 | 182.7672 | 182.7672 | 11.07 | .0018 |
| Subject(Group) | 42 | 693.7013 | 16.5167 | | |
| Trial | 1 | 22.0605 | 22.0605 | 7.71 | .0089 |
| Group X Trial | 1 | 22.2727 | 22.2727 | 7.79 | .0092 |
| Subject X Trial(Group) | 31 | 88.6667 | 2.8602 | | |

Figure 2

Interaction of Estimation Scores




By examining Figure 2, the interaction between the groups and the trials can be investigated. It appears that no change occurred in the control group from the pretest to the posttest. Although the experimental group had a somewhat higher mean on the pretest, it appears to have improved estimations skills from the beginning to the end of the study. These observations suggest that the experimental treatment may have had some effect in the improvement of the experimental group.

## Mental Computation Examination

Mental computational skills were measured using an oral test and a written test given after the posttest examination. The results of these two tests were compared to results of the estimation posttest to examine any association between mental computational abilities, as measured by the oral and written tests, and computational estimation skills, as measured by the estimation posttest. The Pearson correlation coefficient, shown in Table 5, is used to test the second hypothesis. A signifcant difference from 0 appears in the control group correlation between the posttest and the written test. However, no significant difference from 0 is noted in the other correlations. Therefore, the null hypothesis is accepted for hypothesis 2.

Table 5

Correlations Between Mental Computation and Computational Estimation Scores

| | Pearson Corr. Coeff. | Prob>\|R\| |
|---|---|---|
| Experimental Group | | |
| Posttest to Oral Test | .40374 | .1713 |
| Posttest to Written Test | .28211 | .3285 |
| Control Group | | |
| Posttest to Oral Test | .35100 | .1532 |
| Posttest to Written Test | .56931 | .0137 |

## Survey

The survey was used to measure the attitudes of the students toward computational estimation. It was distributed at the end of the study. Table 6 displays the questions contained in the survey. A Likert scale was used to measure the responses for each question which ranged from 1, representing strong disagreement, to 5 representing strong agreement. The averages from each question for each group are presented in Table 7. A complete summary of the survey is located in Appendix E.

Table 6

Questions from Survey of Attitudes About Estimation

1. Estimation is an important life skill.
2. I use more estimation in every day life than any other math skill.
3. After being taught estimation strategies, I feel more comfortable estimating answers.
4. I do not believe students should be taught to estimate.
5. Anyone can make good estimates without any training.

Table 7

Averages for Survey Results

| Question | Experimental | Control |
|---|---|---|
| 1 | 4.13 | 3.42 |
| 2 | 4.25 | 3.11 |
| 3 | 3.63 | 3.37 |
| 4 | 1.94 | 2.26 |
| 5 | 2.44 | 2.84 |

A comparison of the averages shows the experimental class more favorable toward computational estimation, as measured from the first three questions. A notable difference is seen in question 2, implying that the experimental group views estimation as a useful mathematical skill more positively than the control group. From the last two questions of the survey, the experimental class indicates more support for the instruction of computational estimation.

The chi square test of homogeneity was performed to further analyze the survey data. Table 8 displays the results of this test. A significant difference is indicated in question 2. This significance, along with the differences in the means, implies that the experimental group views estimation as a useful mathematical skill more positively than the control group. Although no other questions show significance in the chi square test, some interesting observations can be made from examining the individual results of the survey. These results are located in Appendix E. In question 1 all students in the experimental group perceive estimation to be an important life skill, but only 58% of the control group believe this. Both groups consider the teaching of estimation to be important. In question 4, only 2 students, one from each group, do not believe that students should be taught to estimate.

Table 8

Comparisons of Survey Results

| **Question** | **DF** | **Chi Square** | **p** |
|---|---|---|---|
| 1 | 4 | 8.808 | .066 |
| 2 | 4 | 11.897 | .018 |
| 3 | 4 | 1.804 | .406 |
| 4 | 4 | 4.424 | .352 |
| 5 | 4 | 2.303 | .680 |

Confounding Factors

Factors other than the experimental treatment which might have an effect on the results of an experiment are present in every experiment. One of these confounding factors for the testing of hypothesis 1 might be the male and female composition of the classes. The experimental class has only one more female than male, but the control group has over twice as many females as males. Many studies have documented the mathematical advantage of males over females (Fennema & Carpenter, 1981; NAEP, 1983; Meyer, 1989). This, too, might contribute to the success of the experimental class. Another factor might be the classification of students in each class. The control group has 19 freshmen, 3 sophomores, and 1 senior, but the experimental group has only 11 freshmen, along with 6 sophomores, 3 juniors, and 1 senior. A third factor might be the differences in sample size from the pretest to the posttest. Not all students involved in this study completed each of the examinations used to test the first hypothesis. The difference in instructional time might also be a confounding factor. The experimental class was exposed to estimation for two class periods, while the control class only spent one class period on estimation. One more factor might be the difference in the pretest mean. The experimental class showed a higher computational estimation ability than the control class at the beginning of the study. An assumption might be made that better students seem to strive for more success. However, analysis of the scattergrams in Figure 3, illustrating the relationship of the pretest scores to the difference between the pretest and posttest scores, shows that this is not the case. Improvement exhibited in the experimental class was associated with students of low to medium computational estimation skills measured by the pretest. The students of higher estimation abilities showed little improvement. The control class, on the other hand, exhibited little difference between pretest and posttest scores. This analysis favors the experimental treatment for the improvement of the experimental class.

Figure 3

Comparison of Pretest Scores to the Difference in Scores from Pretest to Posttest

Control group

```
D          |
i        6 |
f          |
f        4 |
e   i      |       x       x x   x
r   n    2 |           x   x
e          |         x
n   S      x-+-+-x-+-+-x-+-+-+-+-x-+-+-+-+-+-+-+-+-+-
c   c      |       x 5 x      10 x      15        20
e   o   -2 |       x       x x x
    r      |
    e   -4 |
    s      |           Pretest Scores
```

Experimental group

```
D          |   x
i        6 |
f          |           x x   x   x
f        4 |               x
e   i      |                 x
r   n    2 |           x   x
e          |                   x     x
n   S       -+-+-+-+-+-+-+-x-+-+-+-+-+-+-+-+-+-+-+-+-
c   c      |         5     x  10     x  15        20
e   o   -2 |
    r      |                       x
    e   -4 |
    s      |           Pretest Scores
```

Confounding factors for the testing of hypothesis 2 might include the different methods of testing computational estimation abilities and mental computation skills. Each problem in the estimation examination was presented on an overhead projector individually for a prescribed amount of time. Mental computation skills were tested with an oral examination in which the students did not see the problem, and with a written examination. The students were allowed as much time as needed for both mental computation examinations. Another factor might be the differences in sample size from the posttest to the mental computation examinations. Again, not all students involved in this study completed each of the examinations used to test the second hypothesis.

For the third hypothesis, one confounding factor could be the possibility of the experimental class having a more positive attitude toward computational estimation than the control class at the beginning of the study. Another factor might be that estimation was the only topic included in the survey. The students in the experimental group might have been more inclined to report an attitude in which they speculated the investigator would be interested instead of a true representation of the attitudes of those students.

# CHAPTER FOUR

## Recommendations

### Summary

The purpose of this study was to provide a pilot for further investigation into the effect of motivation on students learning computational estimation by illustrating uses of estimation in daily living. The subjects were undergraduate students enrolled in two sections of a general education course in mathematics. One section was randomly selected to be the experimental group while the other section served as the control. The experimental section was given a fifty minute lecture on the uses of estimation as a life skill. The following class period they received instruction on the procedures of different computational estimation strategies. The control group received instruction only on the procedures of computational estimation strategies. Both classes were instructed by the investigator.

### Recommended Changes

This study provides a pilot for future investigation into the effect of motivation to enhance the learning of computational estimation. Some points should be considered in future studies based on this pilot. Mental computational abilities should be measured prior to the treatment using the same testing format as the computational estimation examination. Surveys in attitude toward a number of mathematical topics should be conducted at the beginning and at the end of the course. This would increase the reliability of the attitudes toward estimation because estimation would not be singled out. Also a change needs to be made on the computational estimation examination. Problem 1 for both the pretest and the posttest needs to be stated as an estimation problem, such as: What is about half of 3533? The second page for the motivational homework assignment, found in Appendix A, page 35, could use some modification on the choices of sensible answers. Almost all of the most sensible answers are the middle values of the three choices. Care should be taken in creating answers in which this does not occur.

## Recommendations for Teaching Computational Estimation

Teaching estimation in a mathematics classroom can be very challenging. First of all, many students view mathematics as always requiring exact answers. Estimation is a legitimate part of mathematics with extensive use in daily living. Effective instruction in computational estimation should include the meaning of estimation and the explanation that it is not computing exact answers. Also care should be taken to use the correct notation when dealing with computational estimation. This should help students to see the difference between exact and estimated answers. A distinction should be made between rounding as a computational estimation strategy and rounding as a reporting tool for clarity. Students should also be taught to determine when computational estimation is appropriate and whether answers to certain problems are sensible. Another recommendation is incorporating illustrations of estimation used in daily living into teaching computational estimation strategies.

Estimation is one of the most useful mathematical skills in daily living. Effective methods of teaching estimation must be developed. This study provides a pilot for future studies into the effect of using motivation to enhance the learning of computational estimation. Further investigation into this approach and others is necessary before any conclusions can be made about effective methods for teaching computational estimation.

# APPENDIX A

## Homework Assignments

Estimation Homework #1                    Name____________________

Read carefully!! For problems 1-10, answer yes if an estimation would be good enough and answer no if you would need a more accurate answer.

1. After eating at a restaurant, you want to tip 15% of the bill. How much would you need?

2. A worker is mixing cement to pour the foundation of a house. How much cement will he need?

3. You are writing a paper for English class. How much paper will you need?

4. Bob and Sue both caught large fish in the fishing contest. Which one has the largest fish?

5. You are at McDonalds and have ordered a Big Mac, fries, and a coke. How much do you pay the cashier?

6. When washing your dinner dishes, how much dishwashing soap do you use?

7. The family has decided to carpet the living room. When purchasing the carpet, how many square yards should be bought?

8. Your boss figures up what you made this month. How much do you get paid?

9. Fred is driving to Kansas City. Does the tank have enough gas to get there?

10. Five of your friends are coming over for supper Friday night. How many hamburgers should you make?

11. Think about what you did today. List three situations where you used estimation. These must be different from the situations discussed in class.

12. Think about all the different times you perform calculations. What percentage of the time do you use

estimation: pencil and paper:

calculator: mental computation (finding the exact answer in your head):

Percentages should add up to 100.

For problems 13 - 18, circle the most sensible answer.

13. The gas mileage you might obtain with a compact car is

A. 12 mpg B. 89 mpg C. 28 mpg

14. The price of a six-pack of cola is

A. $.75 B. $1.99 C. $4.50

15. The starting salary for a new college graduate is

A. $5800/yr. B. $18,000/yr. C. $53,000/yr.

16. A pair of shoes costs

A. $45.00 B. $250.00 C. $3.50

17. A family of four went to the movies and spent

A. $12.00 B. $2.50 C. $50.00

18. My bookbag can hold

A. 15 books B. 40 books C. 3 books

# Poachers wrecking plan to cut ivory trade

Associated Press

LAUSANNE, Switzerland — A meeting on wildlife trade opened Monday with the admission that a plan to control ivory sales is not working and African elephant populations are suffering an alarming decline.

Prince Bernhard of the Netherlands told 800 delegates and observers at a 14-day session of the Convention on International Trade in Endangered Species that supporters of the ivory quota system "must admit that it has failed to control the illegal ivory trade."

Bernhard, president of the World Wide Fund for Nature, warned delegates of alarming census reports. "You must give benefit of the doubt to the survival of the species," he said.

The convention issued an infraction report detailing how African nations, especially Zaire and Gabon, had flouted ivory wildlife trade restrictions.

The report also singled out Argentina, Spain, Japan and Thailand, among others, for failing to comply with convention regulations on other species. It mentioned only the most flagrant abuses, it said, indicating widespread problems in policing international trade.

A separate report dealt with ivory, the heart of the wildlife trade issue.

Delegates from 103 convention nations must decide whether to leave elephants under their present designation as threatened species or list them as an endangered species, which would force a ban on ivory trade. The convention's rulings are not legally binding agreements; they rely on international pressure for enforcement.

The United States, Canada and the 12-nation European Economic Community halted ivory imports in June.

Zimbabwe, South Africa and Botswana argue that careful management had increased their elephant herds, entitling them to sell tusks to finance their game reserves.

Zimbabwe says it earns $9 million a year by selling ivory from herds estimated at 52,000. Ivory fetches more than $150 a pound on world markets.

But East African states say herds have been reduced continent-wide from 1.3 million in a decade to just more than 600,000 and can only be protected by a ban that applies to the entire continent.

Environmentalists say poachers kill at least 70,000 elephants a year, and that poorer nations such as Kenya do not have the resources to stop well-armed poachers, many of whom come from neighboring countries.

Use the above newspaper article to complete the following.

1. Underline 3 numbers which are reported exactly.

2. Circle 2 numbers which may have been rounded to simplify the information.

3. Place a square box around 3 different estimates.

4. What is the difference between a number which is estimated and a number that has been rounded?

Estimation Strategies Name______________________________

READ DIRECTIONS CAREFULLY!!

1. Decide how many digits there should be in the answer.

   a) 241 + 87 + 539

   b) 4000 - 3257

   c) 42 x 39

   d) 6 x 362

   e) $9\overline{)4257}$

2. Circle the number to the right of the problem which is closest to the answer.

| | | | | |
|---|---|---|---|---|
| a) $.8\overline{)623.7}$ | 8 | 80 | 800 | 8000 |
| b) 180 x .48 | 9 | 90 | 900 | 9000 |
| c) $.28\overline{)8.875}$ | 3 | 30 | 300 | 3000 |

3. Place the fractions $\frac{4}{5}$, $\frac{1}{14}$, $\frac{9}{11}$, $\frac{11}{19}$, $\frac{2}{23}$, $\frac{6}{14}$, $\frac{13}{16}$, and $\frac{5}{9}$ with the appropriate statement.

   Close to 0:

   Close to $\frac{1}{2}$ :

   Close to 1:

4. Use the front-end and compensation strategy to estimate the answer. SHOW EACH STEP!!

   a) 6 x 462

   b) 4616 + 1829 + 2178 + 1356

c) $3\frac{5}{7} + \frac{3}{11} + 6\frac{2}{3} + 2\frac{4}{13}$

5. Use the clustering strategy to estimate the answer. SHOW EACH STEP!!

a) 42,687 + 36,951 + 38,427 + 41,874 + 43,218

b) 1.123 + 1.347 + .875 + 1.073 + .926 + .828

c) 268 + 241 + 257 + 235

6. Use the compatible numbers strategy to estimate the answer. SHOW EACH STEP!!

a) $3.8\overline{)23,193}$

b) 826 + 342 + 198 + 964 + 621

c) $6\overline{)503}$

7. Use the rounding and range strategy to estimate the answer. SHOW EACH STEP!!

a) 47 x 18

b) 82 x 39

c) 31 x 63

8. Find "nice" numbers to work with and estimate the answer. SHOW EACH STEP!!

a) $\frac{79}{81}$ of 16

b) .48 x 170

c) 1.17% of 534

9. Estimate 17.37 + 21.68 + 22.46 + 18.21 using the strategies described and SHOW EACH STEP!!

a) front-end and compensation

b) clustering

c) rounding

In problems 10-19, (a) Identify the strategy you choose, and (b) Estimate the answer (SHOW EACH STEP!!)

10. There are 2,773,822 people in Yokohama, Japan. Chicago has a population of 3,005,072 and Paris, France has a population of about 2,150,000. What would you estimate to be the total number of people in these three cities?

11. The company policy where you work states that you will be reimbursed for the business use of your car at $.21 per mile. About how much can you expect to be reimbursed for the round-trip mileage if your supervisor has asked you to drive a client to the airport which is 95 miles away?

12. A grocery store has vegetable soup on sale at a price of six cans for a dollar. How much is one can?

13. Five students got together to walk to raise money for a charity. Their distances were 3.5 miles, 2.8 miles, 5.1 miles, 4.2 miles, and 4.7 miles. Estimate the total amount of miles walked.

14. Fred and Larry went out to eat. They ordered two salads that cost $1.36 each. Fred had steak and shrimp for $11.56 and Larry ordered chicken for $8.44. They both ordered ice cream for dessert that cost $1.75 each. If they leave a $7 tip and need to pay sales tax of 4.5%, what is an estimate of the total bill?

15. Mark found a summer job working as a carpenter's apprentice. He is guaranteed 30 hours per week and an hourly wage of $6.35. If Mark can work from June through August, what will his income be?

16. A homeowner decides to line both sides of a 55-foot sidewalk with landscaping timbers. The timbers are sold only in 8-foot lengths. How many 8-foot timbers are needed to line the sidewalk?

17. In a hostile takeover of a corporation, only 27% of the 1628 employees remained with the corporation. What was the number of employees that remained?

18. You are contracted for a roofing job. You estimate that you need about 4,800 roofing nails. According to a handbook, the roofing nails you need count out at 189 nails per pound. Estimate how many pounds of nails you need.

19. Mary and Jon are playing a video game. They decide to play three games and the person with the highest score will win. Mary scored 634, 278, and 511. Jon had 520, 295, and 610. Which do you expect to be the winner?

# APPENDIX B

## Supplementary Instructional Material

## ESTIMATION STRATEGIES

1. Front-end and Compensation
2. Clustering
3. Compatible Numbers
4. Rounding and Range
5. Special Numbers ("Nice")

### 1. Front-end and Compensation

a) Sheri picked up several items for her family at the grocery store. They cost \$1.26, \$4.79, \$.99, \$1.37, and \$2.58. What should she estimate as her total at the check-out line without sales tax?

Focus on the left most digits and then adjust the result.

| | | | |
|---|---|---|---|
| Front: | 1 + 4 + 1 + 2 | | |
| Compensate: | .26 + .79 | 1 | |
| | .99 | 1 | 3 |
| | .37 + .58 | 1 | |

Estimate: 8 + 3 = 11

She should expect to pay about \$11.

b) Estimate 4 x 736

Front: 4 x 700 = 2800
Compensate: 4 x 36 is about 120

Estimate: 2800 + 120 = 2920

### 2. Clustering

a) A movie theater had 3,423 people attend Monday, 2,681 on Tuesday, 2,298 on Wednesday, 3,865 on Thursday, and 4,106 on Friday. Approximately how many attended the theater during the week?

First estimate the average, a number that each of the values "cluster" around. Then multiply by the number of values.

| | |
|---|---|
| Estimated Average: | 3000 |
| Number of Values: | 5 |
| | 15000 |

Approximately 15,000 went to the movie theater last week.

b) Joe buys a variety of snacks for the long trip. The snacks cost \$.89, \$.73, \$.65, \$.81, \$.97, \$.71, and \$.77. What should he estimate these snacks to cost?

| | |
|---|---|
| Estimated Average: | \$.80 |
| Number of Values: | 7 |
| | \$5.60 |

Joe should expect to pay about \$5.60 plus tax.

3. <u>Compatible Numbers</u>

a) Mrs. Lyon went out to buy seven video games to give as Christmas presents. Her total bill came to \$33.88. When her husband asked what she spent on each gift, she gave him an estimate. What would be a good estimate?

Compatible numbers refer to numbers that can easily "fit together", numbers that are easy to manipulate.

| | Compatible | Not Compatible |
|---|---|---|
| 7 )33.88 | | |
| | 7 )35 | 7 )30 |
| | 8 )32 | 7 )34 |
| | 8 )40 | 8 )33 |

She could have estimated each gift cost \$5.

b) Estimate 27 + 49 + 38 + 65 + 56 + 81

Look for numbers that add up close to 100.

27 + 81 is around 100
49 + 56 is around 100
38 + 65 is around 100

The sum is about 300.

4. <u>Rounding and Range</u>

Estimate 23 x 78

Round the numbers to ones which are easier to use.

Over Estimate: 30 x 80 = 2400 Round up both numbers.
Under Estimate: 20 x 70 = 1400 Round both numbers down.
23 x 78 is less than 2400, but more than 1400.

Better Estimate: 20 x 80 = 1600 Round one up and one down.

5. <u>Special Numbers ("Nice")</u>

a) Bob lives 11 blocks east of campus and 9 blocks north of the mall. He discovered that it is 1 mile from his apartment to campus and also 1 mile from his apartment to the mall. In the morning he jogs three blocks towards campus, then turns around and jogs home. In the afternoon he jogs two blocks toward the mall, then jogs home. How far does he jog everyday?

Morning: jogs 6 blocks total with 11 blocks being 1 mile
jogs 6/11 of a mile

Afternoon: jogs 4 blocks total with 9 blocks being 1 mile
jogs 4/9 of a mile

Everyday: 6/11 + 4/9

Find a "nice", easy number to work with that is to each.

6/11 is close to 1/2
4/9 is close to 1/2

Estimate: 1/2 + 1/2 = 1

Bob jogs about 1 mile every day.

b) Estimate 18 x .34

.34 is close to 1/3

Estimate: 18 x 1/3 = 6

c) What is 9.84% of 816?

9.84% is close to 10%

Estimate: 10% of 816 = 81.6

# APPENDIX C

## Examination Instruments

## Number 4

A car traveled 317 miles on 15.7 gallons of gas. Estimate the miles per gallon.

# Estimation Pretest

1. What is half of 3533?

2. A six-pack of pop costs $1.75. Estimate the cost of one can.

3. Estimate the sum: 12321 + 12321 + 12321 + 12321

4. A car traveled 317 miles on 15.7 gallons of gas. Estimate the miles per gallon.

5. Your bill for dinner is $8.72. You pay with a $20 bill. Approximately how much money do you get back?

6. Estimate $.11\overline{)96.8}$

7. Your dinner bill is $21.65. The waitress deserves a 15% tip. How much do you leave for her?

8. A recipe calls for 2 1/3 cups flour for a loaf of bread and 2 1/2 cups flour for a cake. Approximately how much flour do you need to bake 2 loaves of bread and 3 cakes?

9. Estimate 12/13 + 7/8

10. Estimate 86,934 - 23,758

11. A certain fabric costs $3.89 per yard. You need 6 1/4 yards. Approximately how much will this cost?

12. A television regularly costs $489.95. It is now on sale for 20% off. Estimate the sale price of the TV.

13. Estimate 478 x .24

14. A professional football player makes $786,425 in one year. He plays 16 games in that year. Approximately how much does he earn per game?

15. Estimate 7 1/9 - 2 5/8

16. A college is mailing 2137 letters. Each letter requires a 29 cent stamp. Approximate the cost of postage.

17. Estimate 4 1/3 + 1 3/4 + 5/6 + 2 4/9

18. Estimate $32\overline{)652,132}$

19. Estimate 32,841 x 493

20. Estimate the total cost of four items costing $27.69, $12.82, $21.21, and $33.42.

# Range of Correct Estimates for Estimation Pretest

1. 1700 - 1800
2. $.25 - $.40
3. 48,000 - 50,000
4. 19 - 21
5. $11 - $12
6. 800 - 1000
7. $3 - $4
8. 10 - 13
9. 1 3/4 - 2
10. 60,000 - 70,000
11. $24 - $25
12. $370 - $400
13. 100 - 120
14. 40,000 - 60,000
15. 4 - 5
16. $500 - $700
17. 9 - 10
18. 19,000 - 21,000
19. 15,000,000 - 17,000,000
20. $90 - $100

Estimation Posttest

1. What is half of 7573?

2. A six-pack of pop costs $1.75. Estimate the cost of one can.

3. Estimate the sum: 94251 + 86792 + 90163 + 88711

4. A car traveled 317 miles on 15.7 gallons of gas. Estimate the miles per gallon.

5. Your bill for dinner is $36.27. You pay with a $50 bill. Approximately how much money do you get back?

6. Estimate $.11\overline{)96.8}$

7. Your dinner bill is $21.65. The waitress deserves a 15% tip. How much do you leave for her?

8. A recipe calls for 2 1/3 cups flour for a loaf of bread and 2 1/2 cups flour for a cake. Approximately how much flour do you need to bake 2 loaves of bread and 3 cakes?

9. Estimate 6/13 + 4/9

10. Estimate 86,934 - 23,758

11. A certain fabric costs $2.87 per yard. You need 4 1/3 yards. Approximately how much will this cost?

12. A television regularly costs $489.95. It is now on sale for 20% off. Estimate the sale price of the TV.

13. Estimate 418 x .34

14. A professional football player makes $786,425 in one year. He plays 16 games in that year. Approximately how much does he earn per game?

15. Estimate 4 1/8 - 1 7/11

16. A college is mailing 2137 letters. Each letter requires a 29 cent stamp. Approximate the cost of postage.

17. Estimate 2 4/7 + 2/3 + 3 5/12 + 4 3/4

18. Estimate $26\overline{)542,841}$

19. Estimate 44,289 x 289

20. Estimate the total cost of four items costing $27.69, $12.82, $21.21, and $33.42.

## Range of Correct Estimates for Estimation Posttest

1. 3700 - 3800
2. $.25 - $.40
3. 350,000 - 360,000
4. 19 - 21
5. 13 - 14
6. 800 - 1000
7. $3 - $4
8. 10 - 13
9. 3/4 - 1
10. 60,000 - 70,000
11. 12 - 13
12. $370 - $400
13. 120 - 160
14. 40,000 - 60,000
15. 2 - 3
16. $500 - $700
17. 11 - 12
18. 20,000 - 21,000
19. 12,000,000 - 15,000,000
20. $90 - $100

## Mental Computation Oral Test

1. 800 + 900
2. 70 x 300
3. 630,000 ÷ 9
4. 5400 - 3800
5. 4,300,000 ÷ .1
6. 800 x 9000
7. 40,000 x 600
8. 30,000 + 80,000
9. 150,000 ÷ 50
10. 300,000 x .8

Mental Computation Worksheet Name______________________________

DO THE WORK IN YOUR HEAD!!

1. 700,000 x 40 = ______________________________

2. $340\,\overline{)6800}$ = ______________________________

3. 500,000 + 700,000 = ______________________________

4. $200\,\overline{)56{,}000}$ = ______________________________

5. 3,200,000 - 1,900,000 = ______________________________

6. 9000 x .6 = ______________________________

7. $.05\,\overline{)3500}$ = ______________________________

8. 4300 + 2800 = ______________________________

9. .04 x .08 = ______________________________

10. $600\,\overline{)42}$ = ______________________________

# APPENDIX D

## Survey Instrument

Estimation Survey Name___________________________

Please respond to the following statements on the scale provided. The scale goes from 1 to 5 with 1 being strong disagreement and 5 being strong agreement.

SCALE

| Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |

1. Estimation is an important life skill.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |

2. I use more estimation in every day life than any other math skill.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |

3. After being taught estimation strategies, I feel more comfortable estimating answers.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |

4. I do not believe students should be taught to estimate.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |

5. Anyone can make good estimates without any training.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |

# APPENDIX E

## Results of Survey Instrument

## Results of Survey of Attitudes Toward Estimation

**Response Scale**

| Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |

| **Question** | **Group** | **Response** 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 1 | Experimental | 0 | 0 | 0 | 14 | 2 |
| | Control | 1 | 2 | 5 | 10 | 1 |
| 2 | Experimental | 0 | 0 | 2 | 8 | 6 |
| | Control | 2 | 7 | 0 | 7 | 3 |
| 3 | Experimental | 0 | 0 | 6 | 10 | 0 |
| | Control | 2 | 0 | 6 | 11 | 0 |
| 4 | Experimental | 4 | 10 | 1 | 1 | 0 |
| | Control | 2 | 12 | 4 | 0 | 1 |
| 5 | Experimental | 2 | 8 | 3 | 3 | 0 |
| | Control | 1 | 7 | 6 | 4 | 1 |

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