AN ABSTRACT THESIS OF

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Computer software was developed for use in demonstrating mathematical topics in the classroom. Each program was written to be used as an enhancement for the teacher's usual teaching strategies, and not as a tutorial for the student. The programs were then used for demonstrating topics in college level mathematics courses and the students' reactions were tested with a questionnaire. The use of each program was discussed in detail, intended as a guide for their use. The results of the questionnaire suggest that the majority of the students felt the programs were a benefit and are in favor of the use of a computer in the mathematics classroom.

A STUDY OF THE USE OF MICROCOMPUTERS IN MATHEMATICS EDUCATION

A Thesis

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by

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CHAPTER I

INTRODUCTION

Introduction

When the writer of this paper was student teaching, his responsibilities included teaching two courses in computer programming. Most of those students were also enrolled in an algebra class that he taught. Coincidentally, at one point during the semester, the course content of the two courses centered around the graphing of conic sections. For instance, in the algebra class, the students were learning what effect changing certain numbers in the equations of conic sections had on their graphs. Simultaneously, the computer class was learning to graph the same type of equations on the monitor of a microcomputer, using computer graphics techniques.

As one might expect, the students realized quickly the ease with which their algebra problems in graphing might be accomplished using the programs written in their computer class. Very soon, they had not only mastered the graphing of conic sections but had begun to familiarize themselves with the graphs of other equations as well. They experimented with graphing many types of equations and functions, and in a relatively short time, they gained far more insight into the meaning of functions -- for example -- than they could have accomplished from a routine algebra assignment on graphing.

Statement of the Problem

It became apparent that using a computer to help teach students to graph equations could be very beneficial. The problem which this thesis addresses was to discuss the ramifications of creating and using microcomputer graphics programs in the teaching of mathematics courses. The content of this thesis will provide the secondary and college level classroom teacher with the programs, and the knowledge of how best to use those programs, needed for implementing computer assisted instruction into their normal course curriculum.

Importance of the Study

The use of a computer as a tool to help the classroom teacher present many of the topics of mathematics has many advantages. For example, it is customary for an instructor to teach the graphing of linear equations by drawing examples on the chalkboard. Although there is nothing wrong with that approach, most such drawings are at least a little crude, and, often, not extremely accurate. Using a computer to draw a line insures the line will be perfectly straight and go through exactly the points that it should. Also, the amount of time taken to accurately graph a linear equation on the chalkboard limits the number of examples that are possible for study in one class period. The computer will graph the equation very rapidly and precisely, allowing time to show many more examples and permitting much more practice. Another important advantage a teacher will gain by using a computer is the opportunity for more student input into the lesson. Having students give the equations to be used in the programs helps keep them involved and interested, and allows the teacher to more closely monitor how well they are learning. When a teacher uses the same teaching strategies every day, students can easily become bored. Thus, any chance an instructor has to vary his/her teaching style and teaching methods can provide a more interesting session. Students find the computer to be very intriguing. Hence, a topic or a course taught to them using the computer certainly serves to focus their interest. Once the usefulness of the computer in teaching is recognized, this study will provide the basis for implementing its use. With the programs contained in this thesis, and the discussions of how to best use them, mathematics teachers will have what is needed in order to bring a microcomputer into their classroom.

Procedures Followed

As the author began work on this study, considerable time was spent writing the graphics programs. It was important that each program be simple to use. Thus, each one was formatted to use a command menu. The title of each program was chosen to describe the general topic with which it was concerned. The command menu was designed to separate topics into more specific categories. Some programs might conceivably be used every day for a week, using a different choice of the menu each time.

Most of the graphics techniques used in the programs were fairly simple, allowing for other individuals to modify them to suit their particular needs. The author devised a method to allow for a function or equation to be changed, as many times as desired, during program execution. Each program was designed to maintain careful balance between versatility and a controlled number of input statements. It was found that, making the programs flexible enough to accept a wide variety of changes forced the user to deal with too many input statements, which slowed the presentation and dulled its effectiveness. The original function in each program, located on the disk, was determined to be the best function with which to begin the program.

After the programs were written, the author spent approximately one year using them in actual classroom situations. A questionnaire was devised, appropriate to the study, which was used to test student reactions to the use of the computer in the classroom. A form of the questionnaire was completed by each student who viewed one of the computer demonstrations. The information gathered through the use of this questionnaire was used to evaluate the usefulness of the graphics programs and is compiled in chapter three.

Limitations of the Study

The writer chose for this study graphing topics primarily related to the fields of Algebra, Trigonometry, and introductory Calculus. Obviously, such topics as three - dimensional graphing could have been considered, but in the spirit of confining discussion to the three mentioned fields, such topics were necessarily excluded.

Most of the programs contained herein, as was alluded to before, were tested before an actual class. Some, however, were not. The untested programs covered topics that the author had not had the opportunity to teach, either in his own classes or in those of other teachers. However, this is not considered a major limitation, and one should not construe that those programs were inferior, since each one was run many times by the author, using many different test modes.

The questionnaire, designed to test student reactions to the programs, was very brief and somewhat general. When class time must be used to complete questionnaires, there simply cannot be an excessive number of questions, and all questions must be designed so that answers are short. However, since the main thrust of this study was to develop the programs and explain their use, this is not considered a major limitation.

The thesis does not include any instruction as to the basic use of computer systems. It was assumed by the author that any materials needed for this type of instruction would be readily available, and was therefore excluded. Users of the programs are required to know only how to turn the computer on, load the program desired, then how to run it.

As a final limitation, it should be noted that these programs were written specifically for the IBM-PC and IBM-PCjr. As a result, the disk provided will not run on a computer that is not IBM compatible. In order to use the contained programs on any other computer, one must code in the programs from the listings provided, making any necessary changes in syntax.

Overview of the Thesis

Chapter one is a chapter that provides the reader an introduction to the study. The second chapter is a survey of literature related to the study of computer assisted instruction, and to computer graphics techniques in general. Chapter three contains the results of the questionnaire on program effectiveness, which was completed by every student who observed a demonstration using one of the programs. The actual statistics are listed, then an interpretation, followed by a sampling of comments written by the students pertaining to the use of the programs as a teaching aid.

In chapter four, a detailed discussion of each of the programs is presented. This discussion includes the standard method of presenting each topic, without the use of a computer, and leads into how the computer might be added to enhance that presentation. The chapter offers statements to provide the instructor with some idea as to when the best time is to present a computer demonstration, according to the usual order of the mathematics curriculum.

The fifth chapter provides the technical portion of the thesis. It is here that the actual programs, with step - by - step directions for their use, are discussed. The way in which input statements should be handled, and the best numbers and functions to enter in each program, are important topics in this discussion. The chapter deals with each program stored on the diskette separately. Included in the discussion of each program, are some examples of how text, equations, and input statements appear on the screen of the computer.

The sixth and final chapter is a summary of the thesis.

Conclusions are made, as well as any additional recommendations for instructors using these programs in their classrooms. In addition, some other topics that could be very nicely presented with the use of a computer are suggested for further study.

Following this chapter, is the bibliography, and then two appendixes. The first appendix contains figures that illustrate how the screen of the computer will appear at selected times during the execution of each program. The second appendix contains the program listings for each of the programs designed for this study and found on the provided diskette.

CHAPTER II

A SURVEY OF RELATED MATERIALS

Introduction

Throughout the available reading materials on Computer Assisted Instruction, several themes are most prevalent. Nearly all publications will devote some of the writing to dealing with the added student motivation that is possible with the use of a computer in the classroom. Also two very important topics that occur quite frequently are the changes, or lack of changes, in the teaching style and methods used by instructors who present topics with a microcomputer, and changes in the ways that students learn and retain new material. Any writing about Computer Assisted Instruction lists many new uses of the computer in the classroom; from using it as a gradebook, to using it to illustrate topics in front of a class. Another point of frequent discussion in the literature was the serious lack of available software written for use of the computer as a teaching aid. These are the topics that most often recur in reading on Computer Assisted Instruction, and are the topics of concern here.

Student Motivation

Every teacher, whether it be in mathematics or in any other subject, is very much concerned with student motivation. The very brightest student, who is capable of accomplishing a great deal, will often be satisfied to produce only minimum effort, if not motivated otherwise. The subject of student motivation is an important one when considering the use of Computer Assisted Instruction.

The 1984 Yearbook of the National Council of Teachers of Mathematics, entitled Computers in Mathematics Education, dealt with this subject a great deal. In this book, the computer was said to be a "provocative, motivating device" (4,4) that is available for use in today's mathematics classroom. It went on to say, "Students and teachers alike become excited and enthusiastic about the computer as a new tool in the classroom." (4,4)

One author compiled a list of the benefits that a mathematics teacher receives by integrating the use of computer software into classroom curriculum. Among those benefits is, "Computers can stimulate student learning." (4,13) The use of computer graphics, the level of active student participation, and the excitement caused by the challenge of gaming type programs, were given as some of the main contributing factors to the added motivation.

To understand the ways in which computer graphics can increase a student's level of motivation, one needs only to observe a class being presented a lesson with the use of moving graphics. For whatever individual reasons, every student watches the screen of the computer. Alfred Bork, in discussing the use of computer graphics in teaching, wrote, "Visuals have long been recognized as an important factor in motivating people. We cannot afford to neglect such motivation in student environments." (1,29) His point is not just that the computer would be beneficial to use, but that the computer is essential.

Later in his book, Mr. Bork mentions the difference between a passive and an active learning medium. A passive medium, such as reading, offers no student interaction and no real built in motivational factors. On the other hand, an active medium, such as a computer program, offers moving visuals, and quite often direct student interaction. He writes, "The first advantage, not always realized, is the level of interaction possible." (1,247)

Another factor in the motivation of students, was discovered by the author of this thesis, in the student responses to the questionnaire concerning the computer demonstrations. Many of the comments written by students who had been exposed to Computer Assisted Instruction, mentioned the fact that the computer represents a new and different method for them to learn, which captured their interest. Any time a teacher varies the method of presentation, the possible monotony of class pace can be broken, and the students are motivated by the very act of change. The following chapter includes comments illustrating this point.

Changes in Teaching

The use of a computer in the classroom can cause some changes in the methods used to teach mathematics. One article suggests that the use of software as an in-class teaching aid, will "create an environment for three - way interaction: teacher, students, and microcomputer." According to the article, software can, among other things, "enhance their (teachers) capabiities in presenting new material, and encourage them to use effective teaching strategies." (4,89)

There are some who feel that the increasing use of the computer as a teaching aid is somehow an attempt to replace the teacher. Of course, this is not the case, since classroom software is intended only to increase the capabilities of the teacher. Warning that the computer is not a panacea, an add-on, or a replacement for the teacher, one article suggests, "Despite the cries of those who fear that education will be dehumanized, the computer will, in fact, only enhance the role of the teacher." (4,13) One author had this to say, "Teaching is still teaching whether done by computer or any other device." (1,12)

As a discussion concerning the potential changes in teaching methods, the following comment from the opening article in the 1984 NCTM Yearbook is offered:

> Mathematics teachers stand at the threshold of a new era in education. Imagine how you might have felt if, as a teacher, you had learned of a technological breakthrough called a printing press, which would become an easy, inexpensive, reliable, comprehensive, graphical, dynamic instrument for transmitting information and stimulating learning. Computing technology constitutes such a revolution. (4,9)

Each of the adjectives given above for the printing press, of course, applies equally well to the microcomputer.

The same article suggests that the ways in which mathematics is taught may change, and that the computer will become an instructional tool that will "put the mathematics teacher into deep, personal interactions with the student." (4,5) Robert Burke, the author of <u>CAI Sourcebook</u>, offered this opinion, "I think CAI is one of the most elegant alternatives to traditional methods we have." (2,3)

It is a common theme throughout the reading on CAI, that the modern mathematics teacher needs to make some changes. The use of the computer as a teaching aid is not presented as a device to be used by those teachers who desire it, but instead as an essential tool that every teacher of mathematics needs to use, for the benefit of the student. "Mathematics teachers need to study state-of-the-art instructional computing -- from the point of view of both the student and the teacher; ... " is how one author worded it. (4,9) Another wrote, "If computers are to have a place in the mathematics classroom, then they should be used as teaching tools when appropriate for what is being taught." (4,135) In the journal for the NCTM, Mathematics Teacher, it was written that "Geometric concepts, including the use of computer graphics, should be integrated into entry-level courses wherever appropriate." (5,373) The same article reports that teachers should begin immediately to deal with the impact of the microcomputer on the content of all mathematics courses.

Changes in Learning

Also in the article just referred to, was a comment concerning the impact of the computer on the way that students learn. The author remarks, "Computers and other elements of information technology are changing in fundamental ways both the mathematics that it is important to learn, and the ways that mathematics, traditional or otherwise, can be learned." (5,373) Being able to see a concept presented with computer graphics has many advantages over traditional methods of teaching the same concept. Learners receive the same information, but in a different manner.

In Learning With Computers, it was written that computer graphics offers an alternate method of conveying information to the learner, "different from the manner information is conveyed through words or numbers." (1,29) The difference between the textbook and computer is very profound. When a topic is presented on a computer screen, the entire text or the complete picture does not need to be displayed at once. For example, in a trigonometry textbook, the diagram used to illustrate solving a triangle by the law of sines, when two triangles are possible, appears as the complete figure. The triangle is on the page, with side of known length that is opposite the known angle already appearing in two places, showing both triangles simultaneously. Customarily the instructor will explain how that side is allowed to "swing" through an arc, touching the base of the triangle whenever a triangle is possible. This arc is usually drawn on the diagram as well, in its entirety. Using a computer, the same diagram is drawn, but the students can actually see it being formed. The side is visually moved through the arc, showing precisely how it is possible to have more than one triangle, and then each triangle may be dealt with separately. The instructor controls when each step of the diagram is constructed, allowing for pauses and discussion. With the textbook drawing, "...there is no sense of information evolving in time to aid the student in understanding the phenomenon." (1,32)

The computer programs written to aid in the instruction of mathematics almost always parallel the conventional teaching strategies and textbook outline, but the information is given in a method more likely to facilitate student understanding. One researcher of the subject stated, "Students exposed to this type of computer - enhanced approach are likely to witness dynamic illustrations of traditionally developed theory and technique." (3,62)

It is often the case that students form a bias against using graphics techniques for the solution of certain mathematical problems, such as the solution of a system of two linear equations, or the zeroes of a polynomial function. The lack of accuracy in the graphs and the difficulty of creating the graph itself make this method of solution seem unworthy, and the students will often dismiss it before any real understanding of the concept is reached. "Computing technology offers promise of changing this style of operation to a bias in favor of graphics." (3,6)

Classroom Uses of the Computer

Programs are continually being created that allow more uses of the microcomputer in the classroom. A teacher can use a program to demonstrate an idea, which will generate examples, questions, and summaries. In the CAI Sourcebook, the statement is made that, "The machine can be made to produce such dislpays accurately, and flexibly." It goes on to say that, "The teacher needs to integrate such demonstrations into the overall development of instruction by stimulating, guiding, pacing, and directing." (2,3)

In 1980, Robert Taylor classified the instructional roles of the computer into three main categories -- tutor, tool, and tutee. (4,20) Most of the software that has been created has been for using the computer as a tutor for the individual student. This means the student needing additional help goes one on one with the computer, using a tutorial program. The computer as a teaching tool, which is the thrust of this thesis, involves the teacher using the computer in front of an entire class at one time, just as one might use the chalkboard. As a tutee, the computer is "taught" by the students. In other words, the students learn mathematics concepts by programming them into a computer.

In discussing the use of the computer for teaching calculus, Kathy Heid points out that graphing functions on a computer screen is the easiest, and most appealing enhancement of calculus teaching. She stated:

> With computer relief from the tedium of plotting and connecting (X,Y) points, the teacher in a classroom demonstration, or the student in an individual exploration, can focus on the interpretation of a curve's intercepts, asymptotes, slopes, or concavity. (3,62)

This author continues with a discussion of the opportunity the computer provides for increasing student understanding of the limit concept. The computer allows for a much more vivid demonstration of the epsilon - delta definition of the limit, a typically difficult topic for students to understand. (3,62)

The Lack of Software

As was alluded to before, most of the software that has been created for mathematics instruction, has been for the use of the computer as a tutor. The shortage of software for use of the computer as an in - class teaching aid is well documented. At a 1975 conference of the AMS/ MAA, Stephen Garland asked, "Why, given the great number of experiments using computing to teach calculus, and given the attention devoted to the subject in journals and conferences, hasn't there been more widespread acceptance and use of this approach to calculus ?" He cited that limited computing facilities, including the lack of suitable software, was a major factor. (3,61) In the NCTM Yearbook for 1984, it was stated that despite many proposals and much developmental effort, the methods of teaching mathematics in the secondary schools are remarkably similar to those of thirty years ago. (4,20) In another article in the same yearbook, it was written that, "Most of the software currently available for CAI in schools has been developed for use by a single student interacting with a single terminal." It goes on to say that software should be created for the teacher to use in the classroom, since "teachers remain the best source of student learning, irreplacable by any computer." (4,89)

CHAPTER III

RESULTS OF REACTION QUESTIONNAIRE

Introduction

In teaching these courses -- Introduction to Algebra, College Algebra, Trigonometry, and Calculus I -- and appearing as a guest in other Calculus classes, the writer presented a total of 21 computer demonstrations to enhance the topic being studied. For nearly every computer demonstration given, an accompanying questionnaire was distributed among those present. This questionnaire was designed to determine the overall reaction the students had concerning both the value of that particular demonstration, and the use of Computer Aided Instruction in general.

This chapter is a question - by - question treatment of the results of the questionnaire. Each question was to be answered in two ways, first by the participant writing a brief comment, then also by circling a number between one and five, with one being the lowest, and five being the highest. The results are listed by first giving the question, then the total number who responded to that question, the average of all the numerical answers, and a sample of the comments written.

The Questions

Question One. The first question the students were asked to answer dealt with their prior knowledge of the topic demonstrated to them. The answer given to this question had possible effects on the answer to the next three questions, so they are discussed with that concern. For the most part, unless a student was repeating a course, or was in some other way ahead of the rest of the class, prior knowledge was rated fairly low.

The question read as follows: Rate your prior knowledge of the subject being demonsrated. A total of 95 people responded to this question, and the average of the responses was 2.35. The indication is, as would be expected, that most students were learning the topic for the first time. As for comments written on the first question, if any were written, they were usually a list of background courses taken, or a statement in direct correlation to the number that was circled.

Question Two. The second question was as follows: Do you feel the demonstration gave you an adequate understanding of the subject, as opposed to the instructor using just the text? As was mentioned before, the way in which the students rated their prior knowledge of the subject seemed to have some effect on the way they answered this question. A total of 78 students gave a numerical answer to this question, and the overall average answer was 4.26.

The following is a breakdown of the averages of the answers for each of the five possible ratings for the first question. First, for those rating their prior knowledge as a 1, there were 26 who responded to this question, with 3.48 being the average. There were 18 students who rated themselves at 2, and then responded to the second question. The average of these 18 was 3.83, a value somewhat higher then the last. Of those who rated themselves at 3, there were 23 who answered question two, with the value for the average being 3.91. For a rating of 4, there were 11 students that responded, and their average response was

4.5. There were no students that rated themselves at level 5, who answered the second question. Obviously, there was a definite increase in the average response, as the rating for prior knowledge increased.

Of the comments that were written concerning this question (and the others), there were mixed reactions. The number of positive, negative, and indifferent comments directly relates to the average of the numerical results, but only a few of each are presented here.

There were not any decidely negative comments on this question, but one student remarked, "I saw no difference between the text and the demonstration." Another student wrote, "I think it helped me understand it, but by using the computer and the text, an even better understanding could be reached."

Others indicated that they preferred the use of the computer, by writing such comments as, "Moving pictures are always better than a textbook." Several students quoted an old saying, and wrote, "A picture is worth of thousand words!" Another positive reaction was, "A few minutes of demonstration can replace a week of wading through textbook examples."

<u>Question Three.</u> The third question essentially asked the same information as the second, but from a different viewpoint. The question was: Did the program, along with the instructors help, seem to do a good job of explaining the subject ? Instead of rating the understanding they received, the students were rating the performance of the program, and the averages were a little higher. Again, the results are given both as an overall

total, and by the response to the first question.

A total of 94 responses were given to this question, and the average of those responses was 4.35. If students rated their previous knowledge as the lowest, a 1, the average was 4.21, with 28 being in the category. With a rating of 2, there were 23 students, for an average of 4.39. Of those who rated themselves at level 3, there were 26 who responded to this question, giving an average of 4.19. For level four, the total responses was 14, and that average was 4.78. There were only three students who rated their prior knowledge at a 5, and their average answer to question three was 4.67. Unlike the second question, the results seemed to be independent of the students response to question one.

In response to this question, concerning the program on limits, one student had this to say, "Yes, but I was confused about where they got delta." Apparently, the instructor failed to give an explanation concerning how the computer calculated that value, and the student focused on that problem. Another negative comment was, "It was sort of complicated." It is unknown whether this person was referring to the topic, or the way it was presented on the computer, but either interpretation might imply dissatisfaction.

One student, who seemed to have no preference in the method that was used, commented, "Yes, although the text was also helpful to me." A more positive comment was, "Yes, it was more clearly illustrated." Two comments made reference to the input from the teacher, along with the computer program. The first wrote, "Yes, the chalkboard example before made the CRT (computer) equations very easy to follow." The other concerned the instructor's help during the demonstration, saying, "Yes, we needed certain questions answered as it was demonstrated."

Question Four. The fourth question on the questionnaire was: Try to rate your knowledge after the demonstration, as compared to before it. This question was intended to test the amount of improvement the students felt they achieved, and for the most part seemed to accomplish that. There were, however, a few cases where students were apparently confused, since they rated their knowledge after the demonstration as lower than before it. Hopefully the assumption can be made that these students circled the number that described their increase, and not their overall knowledge, since to assume otherwise would imply the demonstration subtracted from their knowledge. There were four students who answered in this manner, and fourteen that indicated their knowledge the same before as afterward.

For the group that rated their knowledge before as 1, there were 29 that responded to the fourth question, and the average of their answers was 3.83. Those that rated themselves as 2 before averaged a rating of 4.05 after the demonstration, with a total of 22 responding. Of the students that rated prior knowledge at level 3, 25 responded for an average post-rating of 3.80. A rating of 4 on the first question produced a group of 15 that responded, and the average was 4.27. Of the three people that rated their prior knowledge at 5, 2 responded to the fourth question. One of the answers was at level 3, while the other was at level 5, giving an average of 4. All together, there were 93 responses to the fourth question, and the overall average rating of their knowledge after the demonstration of 3.95, as compared to the overall rating of 2.35 before the demonstration.

Of the comments written, there were none that were completely negative; however, a few expressed concern that the demonstration alone was not enough. Among those comments was, "It helped me, but I need more hands on work, like with the exercises in the book." Another wrote,"Since it was such a new subject to me, I was still a bit confused."

Several students wrote comments concerning how the demonstration helped them. One such reaction, which concerned the Law of Sines program, was, "It helped me to actually watch the triangles form." Writing about a demonstration of parabolas using the Conics program, one student commented, "I learned how different parts of the equation change the thickness and whether the curved line goes (opens) up or down." Another student compared it to the text, saying, "I realize now what the book was trying to explain.

Question Five. From the fifth question on, the questions were designed to get student reactions to the use of Computer Aided Instruction in general, beginning with: Do you feel the use of the computer was interesting ? A total of 94 students gave numerical responses to this question, and the average response was 4.67. Of the five possible answers, two students answered with 3, and everyone else responded higher.

Of the comments written pertaining to this question, only one was negative. That response was, "Yes, but I'd rather see problems written out on the board." Of the other comments, several mentioned the difference in teaching technique. One such comment was, "It seemed to capture everyone's attention, opposed to listening to a teacher lecture." Another student wrote, "It was new and different." Similar to that was, "A little variety never hurts." One student liked seeing the concept presented on the computer, writing, "Yes, one can see the math at work this way." Another mentioned the computer in this way, "Yes, the graphics are very helpful."

Question Six. Question number six asked the students to react to the presence of the computer, asking: Did you feel alienated at all by the computer ? A total of 87 people offered a numerical answer, and the average answer was 1.69. There were six students that answered with a 4, but most answered with 1.

Some of the comments received on this question indicate there might have some confusion as to what was being asked. It was intended that they answer pertaining only to their feelings during the demonstration, but many seemed to answer with respect to computers in general. Typical of these comments were, "Yes, since I have no experience with them," and, "No, I've had two programming classes." One student had this remark, "Yes, it takes the instructor away from the class." A more positive comment was, "No, it's just an electric chalkboard." There were many answers such as, "A little bit," or, "No, not at all."

Question Seven. The seventh question was: Would you like to see a computer used in the classroom again, both in this class and classes you may have in the future ? The overall average answer was very positive, at 4.46. Of the 92 people responding, there were none that answered with a 1, and only two that answered with a 2.

Several of the comments received were nondecisive, such as, "It depends on the subject matter." One student indicated having no opinion, with, "It really doesn't make much difference to me." Others said they would like to see it used again, provided some conditions were met. Of those, one wrote, "Yes, in classes where it would help." Another was, "Yes, as long as there is a teacher."

There were two decidedly negative comments, one of which the meaning is not entirely clear. That comment read, "Not on a regular classroom atmosphere." The other negative reaction was, "No, I found it more distracting than helpful."

The positive comments included, "I can think of a lot of classes I've already taken where a little computer demonstration would be practical." Another was, "Yes, since computers are tools of the future." One student remarked that, "All classes should." Concerning the trend in teaching technique, one comment was, "I feel education is going that way."

Question Eight. The eighth, and final question, on the questionnaire, concerned using the computer as an out of class tutor. It read this way: How do you feel about being allowed time to use the computer alone, needing no prior computer experience, with a tutorial type program ? Again, the average response was above 4, at 4.25, with 87 students giving numerical answers. Four students answered this question with the lowest value of a 1, while two answered with a 2, and the rest answered higher.

A few of those that indicated they did not want to use the computer as a tutor, wrote comments pertaining to this. One such reaction was, "Computers are not teachers, they can't tell why something happens, just how." Some of the other negative comments indicated that the students involved misunderstood that no computer literacy would be required beforehand, writing, "I have no knowledge of computers," and, "I wouldn't know what to do."

One comment was neither for nor against the idea, saying, "Yes, but it's not really necessary." The rest of the comments were positive, of which a typical reaction was, "I would like that very much." Similar to that was, "Yes, I think it is a great idea and it helps." One student indicated that it would be a good idea for a study aid before tests, and another wanted it as an aid for homework, writing, "For out of class assignments, and possibly even in class."

Item Nine. The last item on the questionnaire was not a question, but simply asked for any additional comments and suggestions. Nearly every student who completed a questionnaire wrote some sort of a comment, and quite a few suggestions were made. The most common suggestion for improvement was typical of this comment, "It really needs to be used on a bigger screen." A standard computer monitor was used, in all cases, and for the larger classes, the students in the back apparently could not see well enough. There were no reactions indicating completely negative attitudes, but one student remarked, "I am not in favor of someone who does not know what they are doing using it as an instructional aid." This student may have been concerned that the computer was going to replace the mathematics teacher, which was also a concern of the student who wrote, "I felt it was a good demonstration, and it could be used as an aid to the instructor, but not as an instructor itself." One student made these observations, "I think the computer is a fantastic way to demonstrate mathematics. It can give examples quicker and with more accuracy (particularly with graphs) than the instructor can on the blackboard." Another very favorable comment was, "It's interesting to watch the computer draw different lines as different variables are substituted for X and Y coordinates and Y intercepts."

Summary

All of the results given above are summarized in the Table I on the following page. The answers to the first question, which was to rate prior knowledge of the subject, are used to categorize the results of the next three items, as was done in the preceding narrative.

TABLE I

SUMMARY OF NUMERICAL RESPONSES TO QUESTIONNAIRE

Questionnaire Item		Number Responding	Average Response
Question 1			
Overall		95	2.35
Question 2			
Rating on	1	26	3.48
item	2	18	3.83
# 1	3	23	3.91
	4	11	4.50
	5	0	----
Overall		78	4.26
Question 3			
Rating on	1	28	4.21
item	2	23	4.39
# 1	3	26	4.19
	4	14	4.78
	5	3	4.67
Overall		94	4.35
Question 4			
Rating on	1	29	3.83
item	2	22	4.05
# 1	3	25	3.80
	4	15	4.27
	5	2	4.00
Overall		93	3.95
Question 5			
Overall		94	4.67
Question 6			
Overall		87	1.69
Question 7			
Overall		92	4.46
Question 8			
Overall		87	4.25

CHAPTER IV

OVERVIEW OF USING THE PROGRAMS IN THE CLASSROOM

Introduction

One of the important advantages a teacher gains from using a computer to present a lesson is a little variety in the standard classroom presentation. This can not only be important to the student, but to the instructor as well, since teachers are subject to the problems of repetition also. It should be stressed, however, that using these programs will not induce any radical change in teaching strategies, so an instructor new to them will not be forced to rethink old lesson plans. Instead, the computer will be used as a tool to enhance the standard presentation of a topic, and will allow the teacher to present the material in basically the same method, and at the same time, as would normally be done.

In this chapter, each of the programs will be discussed from the standpoint of the topic involved, and how the program will relate to the usual presentation of that topic. The standard method of presenting a new topic to students is the main thrust of this chapter, showing how the computer might be used to enhance that presentation, as opposed to changing it. The chapter will not be a step - by - step approach to implementing the programs, as that will be done in the next chapter, but instead will describe how the instructor can most easily fit the computer demonstrations into his/her normal curriculum and teaching style.

In order to induce a grouping of these programs, they have been separated into three main categories. The first of these categories contains all programs pertaining to topics taught in the algebra sequence, such as graphing lines, parabolas, and functions. The second category groups the programs written for trigonometry, and includes such topics as graphing the six trigonometric functions, the law of sines, and polar coordinates and equations. The final category contains the programs that deal with topics taught somewhere in the calculus series, most notable of which are the programs on limits, derivatives, and integration. The remaining part of this chapter, then, is separated as described into these three categories. Each of the three sections will then be subdivided for consideration of each program separately. The sections come in the order that was listed above, and begins with the programs written for algebra.

Algebra Programs

Included in the category of programs written for algebra, are three programs. The first of these is the program entitled "Lines," which begins as an introduction to graphing on the Cartesian Coordinate system. The second program is "Conics," which is used to teach graphing each of the five conic sections. The third program, called "Function," will graph more complicated functions.

Point Plotting. The first step in teaching algebra students to graph equations in the X-Y plane, is defining this plane for them. They will most likely be familiar with the number line, which is simply a one-dimensional graph. So, it is customary to begin the discussion there. Explaining how the Cartesian Coordinate system is nothing more than two perpendicular number lines, where the second line adds the second dimension, gives them the definition in familiar terms. Of course, if a point is to be plotted in two dimensions, it must have two coordinates, one for the horizontal component and one for the vertical. By this time in the lesson, most teachers will have the axes drawn on the board, and begin giving examples of how to plot points. Letting the students suggest coordinates, or telling where a point with given coordinates will be, is a good way get them involved.

All of the above strategies can be accomplished in a very similar manner, using the program "Lines." The first choice on the menu of this program, is designed to introduce point plotting. It is recommended that the teacher introduce the topic on the chalkboard first, drawing the axes and discussing them. Then, run the program, and use it to show where points with certain coordinates are placed. The computer draws the X and Y axes, labels them, then draws a grid on the screen very similiar to graph paper. Next, the user will enter the coordinates of a point, and the point is plotted on the grid. It is very effective to have one student offer the coordinates of a point, and another predict exactly where that point will be, by pointing at the screen. Then, the instructor types the coordinates, presses enter, and the point will appear. This method allows for considerable student involvement, and helps them enjoy the lesson.

Graphing Linear Equations by the Point Plotting Method. For the most part, when the students feel comfortable with the X-Y plane, and do well with graphing points, the next step is to graph basic linear equations by arbitrarily picking values for X, and computing the corresponding Y value, producing an ordered pair. Several of these ordered pairs are graphed, then connected with a straight line. The author's recommendation is to teach this lesson, and the following one on finding the X and Y intercepts, by using the chalkboard and a straight edge. The computer is too fast to effectivly illustrate the process of picking values and calculating ordered pairs. Once the students learn to graph by these two methods, they are ready to learn the concept of slope, which will lead to the remaining methods for graphing linear equations.

Slope. When beginning the lesson on slope, the students need to be given the basic concepts of rise over run, positive versus negative slope, and zero versus undefined slope. Once this is done, the program can be used right away. As it runs, the first thing the computer does is to draw the X-Y plane, with the grid. It then pauses and asks the user to input the first of two points. When it is entered the program graphs it, and the second point is requested. After graphing that one, it begins constructing the right triangle formed by the two points and the lines of the grid. Starting with the vertical leg first, it draws a unit at a time, fairly slowly, so that the viewers can count as it goes. When the triangle is complete, it will ask for the correct slope to be entered. If an incorrect slope is entered, the program erases the answer and asks for it again. This continues until it receives the correct slope, at which time it asks if the user wishes to try another two points. It is beneficial to do several examples of each possible type of slope -- positive, negative, zero, and undefined -- so the students become well aquainted with the differences.

Slope Intercept Form of a Line. Once the students have mastered slope and the concept of intercepts, they are of course ready to graph linear equations in the slope - intercept form. The third choice on the menu of the Lines program first allows the user to input an equation. Then, the graph of the line appears on the grid. This program needs no lead in, and can be used to introduce the concepts involved. The linear equation originally in the program is a good one to begin with, and is graphed by defaulting through the input statements. The instructor can then have them discover, by counting, the slope of the line, and its Y-intercept. If the connection between the slope and intercept on the graph, and the numbers in the equation, is not made, change the equation slightly and graph another. Usually, by that time the students will see how the graph and the equation relate, and they are ready to answer questions relating to these concepts. A good way to use the program at this time is to type an equation, and then have the students give the slope and intercept, before it is graphed. Next, after several examples of this kind, the teacher might have individuals give the equation for a line with a certain slope and intercept.

Point Slope Form of a Line. The fourth and final choice on the menu of this program will allow the user to graph lines in the two-point form. The students typically enjoy this program a great deal, since it is designed with some of the qualities of a game. What it does is to ramdomly select two points, and place them on the graph. Then, the students must give the proper equation that will draw a line connecting those two points. The instructor types in an equation, which has been suggested by the students, and the computer graphs it. It is obvious at that point whether the equation was correct or not, and the program will ask if it was. If "Y" is entered, it assumes the line was correct, and will then graph two more points. If, however, the line was not correct, the program asks for another try at it. The computer requires that the equation is entered as:

$$Y = m (X - h) + k ,$$

and not as:

$$(Y - k) = m (X - h) ,$$

so the students need to be aware of how to use both forms.

Parabolas. The next of the Algebra programs will enable one to graph the conic sections, and typically the first of these to be dealt with is the parabola. The usual teaching strategy for parabolas is to begin by using the point plotting method. Before using the computer to graph parabolas, it is best to start at the chalkboard, and graph Y = X^2. Once it is on the board, tell the students what it is called, and how all parabolas are simply variations of that one. It is helpful to graph, by hand, a few more parabolas that have some of the variations, and even have the students graph one at their desks. When this is done, the conic sections program will offer the best way for them to practice. The first choice on the menu of this program will allow one to graph parabolas, using the general form of the equation:

$$Y = a(X - h)^2 + k ,$$

and allowing the user to make changes in the three constants a, h, and k. The use of the program will teach them to graph parabolas in this form, without having to use the point plotting method, since they can see that each time the vertex of the parabola is at (h,k), and also discover how the number a affects whether it opens up or down and how "thick" it is. Again, it works very well to have individuals give the equation of a parabola that has its vertex at a given point, and has characteristics of being wide or narrow, opening up or down.

Circles. After parabolas, the next concept studied is circles. As with most of the programs, it is best to introduce the topic on the board, using the point-plotting method, starting with something simple like:

$$X^2 + Y^2 = 4.$$

Usually, this is when the teacher will relate the circle to the distance formula, and then derive the center-radius form of the equation. Once the students are somewhat familiar with this type of equation, they are ready for the computer demonstration. The second choice on the menu of the conic sections program is "Circles," and when it is selected, it will allow the user to change the values of h, k, and r, in the following equation:

$$(X - h)^2 + (Y - k)^2 = r^2 .$$

The resulting circle is graphed, and then the program waits for further changes.

Ellipses. The ellipse part of the program is treated in basically the same method; starting on the board, introducing the ellipse, then moving to the computer. The program uses an equation of the form that follows:

X^2 / a^2 + Y^2 / b^2 = 1,

with the user entering the values for a and b. Many times there will be a student who asks if all ellipses have their center at the origin, which of course they do not. The capability to move the center is obtained by making that choice when the ellipse menu selection has been made. Its equation is of the form :

(X - h)^2 / a^2 + (Y - k)^2 / b^2 = 1 .

The user will be asked which form of the equation is desired, and the program uses whichever is requested. For the above form, values for the four numbers, h, k, a, and b, must be entered by the user.

Hyperbolas. The final choice on the menu is used to graph the one remaining class of conic sections -- hyperbolas. This portion of the conic sections program will have two sub choices, allowing first the Y term to be the negative one, then the X term. Both use an equation of the following form:

X^2 / a^2 - Y^2 / b^2 = 1 ,

except the second one has a negative sign before the X term, and a positive sign before the Y term. Most students, when first faced with this type of equation, will still be thinking in terms of the ellipse. Using the point-plotting method is a good way to illustrate the difference, because the existence of asymptotes becomes apparent, and at the same time the students can see how there cannot exist a Y-intercept or X-intercept, depending on the equation. The standard technique for graphing a hyperbola of this form involves drawing a rectangle, centered at the origin, with the horizontal sides running from -a to a, and the vertical sides running from -b to b. The asymptotes of the hyperbola are the extended diagonals of this rectangle, which are dotted in. Then, the hyperbola is drawn in accordingly. Since the students are taught to graph hyperbolas with this method, the program was written so that it graphs the rectangle, draws the aymptotes, and then graphs the curve itself.

Trigonometry Programs

The next set of programs is designed for use in front of a trigonometry class, and covers three main topics. The first of these programs illustrates, with moving graphics, the Law of Sines. The second program graphs variations of the six trigonometric functions, and has a menu choice for each. Finally, the third program deals with polar coordinates and the graphing of polar equations.

Law of Sines. Trigonometry textbooks vary greatly as to the order in which topics are presented, so the law of sines program is to be used whenever the course deals with solving triangles. Usually, textbook writers discuss the solution of right triangles, and move from there to study triangles that may be solved using the law of sines. Of these triangles, there are two types. The first, which is the simplest, and the first choice on the menu, is a triangle where two angles and the length of a side opposite one of those angles are given. Then, the missing angle and the length of each of the two missing sides must be found. When first faced with the prospect of "solving" this triangle, many students do not understand that there can only be one triangle possible, and it should be explained to them that the given angles and side force the triangle to have a certain shape, unique from any other.

After the law of sines has been proved, the class should be ready to view the demonstration using the law of sines program. The program will choose one of the unknown sides to be the base of the triangle, and places it horizontally on the bottom of the screen. The other unknown side will be connected, with one of the given angles, to the left end of the base (the other end has yet to be determined). The side with known length is placed at the right edge of the screen, with its end touching the base at the specified angle. When the teacher presses enter, the side on the right moves, keeping the correct angle with the base, until its top end comes in contact with the side extending up from the left. At that point the only possible triangle has been formed, and the program pauses in execution. Across the bottom of the screen is written the law of sines formula, with a missing side left as a variable. The students then compute the correct answer for this side and the teacher enters it. If the answer is within certain tolerances, the value is placed next to the appropriate side on the drawing. Then the above procedure is repeated for the other missing side. If an answer is incorrect, the entry is erased and asked for again. The last of the missing pieces, the third angle, must also be entered, and once it is correctly given, the triangle is declared solved. Whenever the program pauses with a question, every student should be encouraged to calculate the answer, and then when a concensus is reached, the teacher enters the value.

The second choice on the menu of the Law of Sines program demonstrates the case of two given sides, and a given angle that is opposite one of those sides. Before the program is run, it helps if the students understand that with this information, there may be none, one, or two possible triangles. The program is effective at showing why this is true, but students need to be aware of the different cases before running the program. As with the previous case of the law of sines, the side of unknown length is placed across the bottom as the base. To be consistent with the diagram found in nearly all textbooks, the program then places one of the given sides on the left endpoint of the base, with the one given angle between them. The third side, also of known length, is connected to the top end of that side, at an arbitrary angle that prevents it from touching the base. On the instructor's command, this third side swings slowly down until its free end touches the base. When this occurs, the computer beeps, leaves a side (and thus a triangle), and continues its arc until it again touches the base. Once the arc has been completed, a question will appear asking how many triangles were possible. When the correct answer is given, the class solves each possible triangle, one at a time. The program ceases execution when all possible triangles are solved.

Graphing Trigonometric Functions. The trigonometry curriculum includes several topics dealing with the graphs of the six trigonometric functions. At this point in the curriculum, it is customary to spend some time using the point - plotting method. Trigonometry students should be accustomed to graphing, but often they need to review, and the process of determining ordered pairs will provide that review. Once they have seen the graphs of the sine and cosine functions, the next step usually involves changing the amplitudes, the periods, and finally causing phase shifts, of these basic functions. These topics may either be introduced on the board, then demonstrated on the computer, or introduced on the computer first.

The computer program for graphing trigonometric functions does a fine job of presenting these changes, and allows for quick demonstration of many examples. The program, entitled "Triggraf", has a separate choice on the menu for each of the trigonometric functions. It is suggested that they be presented in the order they appear on the menu -- sine, cosine, tangent, cotangent, secant, and cosecant -- beginning with amplitude changes in the sine and cosine waves. After these have been satisfactorily learned, use the sine wave to illustrate period changes. Next, phase shifts are presented. Finally, combinations of all three are used.

Before the tangent function is graphed using the computer, the students should be reviewed as to what an asymptote is, and when one will occur in a graph, so they are prepared for the dotted asymptotes that the program creates on the screen. The secant and cosecant functions are normally studied last, once the students are already familiar with period changes and phase shifts.

Polar Coordinates and Equations. Since the polar coordinate system will be, for the most part, completely new to the students, it is best to begin this topic on the chalkboard. It is also beneficial to distribute copies of polar graph paper, and then explain how a point with coordinates that express a radius and an angle are plotted on it. The program titled "Polar" can be run following just a brief introduction of the coordinate system, using the first of the two menu choices. This choice asks for the coordinates of a point to be given, and plots that point on a polar coordinate graph created on the screen. An effective way of using this part of the program is to ask individuals to give the coordinates for a point that will be graphed at a specific place on the screen, perhaps a spot where the teacher is pointing. This program will also accept negative angles and negative radii, which should not be neglected in the demonstration.

When the students have learned how to plot points in polar form, they are ready to be introduced to polar equations. The usual teaching strategy, as expected, is to graph one or more by hand on the board, using the point - plotting method. By having the students graph an equation on the paper that was distributed, a teacher can be sure they understand what is involved in the process. After that has been done, the second portion of the polar program may be run, which allows one to graph many polar equations. First, the program allows the equation to be entered.

Then, it graphs that equation on the same system it used for plotting points. Since graphing polar equations by hand is very time consuming, students learn what general shapes the different equations produce much faster when the computer is used.

Calculus Programs

The last of the three groups of programs consists of those written for the three main concepts of first semester Calculus. The first of these programs is "Limits," and uses various techniques to present that topic. Second is the program titled "Derivatv", which teaches the derivative as the slope of the tangent line to a curve at a given point. The third program, "Integral," teaches upper and lower Riemann sums, and defines integration in those terms. Also, this program has the capability to graph an illustration of the trapezoidal rule, a method used to approximate the definite integral.

Limits. This program is designed to be used as an introduction of the topic, and can be used with very little prediscussion from the teacher. The menu on the program has three choices, the first of which is an explanation of limits using moving graphics. The user is told that the function in the program has a value of 3 when X = 5. Then the axes are drawn, and a dotted horizontal line is drawn, at a height of 3, from the Y axis to the line defined by X = 5, with a dotted vertical line drawn from there to the X axis. The computer begins graphing the function, from both ends of the screen, which causes the curve to close in on the point (5,3). As this is happening, two small arrows close in, at the same rate, to 5 on the X axis, while two others converge to 3 on the Y axis. When the two arrows along the Y axis get within .1 unit of 3, the program pauses to compute how close the arrows along the X axis had come to the 5. At that time the teacher should introduce the traditional epsilon - delta process of defining the concept of "limit." By explaining that, if no matter how close one wished to bring the arrows in on the Y value, they could do so by bringing the arrows close enough to the given X value, then that particular Y value must be the limit of the function as X approaches the given value, the teacher prepares the students nicely for the formal definition of the limit.

Not until after the students have been given the epsilon-delta definition of the limit, should the second choice on the menu be used. This choice graphs an entered function, asks at what value of X the user wishes to test for a limit, then asks for a value for epsilon. When that has been entered, the program draws the customary two horizontal lines from the Y axis to the curve, at an epsilon's distance on each side of the functional value for the given X. Next, it draws the two vertical lines from the points where these horizontal lines associated with epsilon intersected the curve, down to the X axis. The program computes the distances from the given X value to these two vertical lines, and assigns delta the value of the smaller one. At that point the user may enter another value for epsilon, and the old lines are erased and new ones drawn to fit the new epsilon value. This may be done as many times as is desired.

The last choice on the menu constructs the table often found in textbooks, that lists X values and their corresponding functional values, as X approaches the given value. It allows X to approach from both the right and the left simutaneously, which illustrates whether the function values converge on the limit from both sides.

Derivatives. The program entitled "Derivatv" was designed to follow the usual teaching method for presenting the derivative of a function. This method involves establishing a set value for X, followed by trying to find the slope of the tangent line to the curve at the point (X,f(X)). Since the slope of any secant line through the given points can be determined, a secant line is drawn through the point (X,f(X)) and another point on the curve, called (X+h,f(X+h)). The slope of this secant line is determined to be (f(X+h) - f(X)) divided by (X+h - X). In order to bring the secant line down into the position of a tangent line, the point (X+h,f(X+h)) must be moved closer to the set point (X,f(X)), which can be accomplished by making the value of h smaller. If h is allowed to approach a value of zero, then the point (X+h,f(X+h)) will approach the given point, and the secant line will approach the position of the actual tangent line. Thus, by taking the limit of the slope of the secant line as h approaches 0, the slope of the tangent line is obtained. Then, this is expanded to become the definition of the derivative of the function at X. In the spirit of adhering to this strategy for teaching derivatives, the program graphs a function, then draws a secant line through the selected point and an arbitrary point on the curve, somewhere near the right edge of the screen. This line is erased, as another is drawn through a slightly closer point, and this is continued until the given point is attained. At that point, the secant line becomes the tangent, and is left in place. Its slope is computed and given as the value of the derivative of the function at the given value of X.

Integration. The typical teaching strategy for introducing the integral, is to consider the region bounded by a continuous function, the X axis, and two vertical lines that intersect the functions graph. The program "Integral" is intended to be a complete introduction, and the first choice on the menu starts at the very beginning. A parabola, opening down, is graphed, and the region between the curve, its X intercepts, and the X axis, is considered. First, the curve is enclosed by a rectangle, and the area is given. Then, the interval from the left intercept to the right is partitioned into three equal parts, and three rectangles are drawn, using the maximum height of the curve in each subinterval as the height of the rectangle. The areas of these three rectangles are summed, and the total printed. When this stage is completed, the teacher will explain that this new sum is closer to the actual area under the curve, than was the first single rectangle. Next, the computer repeats the above with ten partitions of the interval, and the sum of the areas is given. As the next step, the computer allows the user to determine the number of partitions, which the students are eager to offer. The program will draw the rectangles for any number of partitions desired, but for a partition of more than about 200, the execution is very slow. Finally, the computer uses a partition of 200, and graphs the rectangles for that. While the program is running, the instructor can discuss the concept of Upper and Lower Riemann sums, and how they differ from Riemann sums in general. Also, when the program completes the graphing of 200 subintervals, the teacher may consider the concept of the limit as the number of partitions approaches infinity. This is then given as one of the definitions of the integral of a function.

The second choice on the menu of the program is used to change the function under study. Different from the format of most of the programs, this is done because the user will not want to enter a new function for each of the menu choices. The third and fourth choices graph upper and lower Reimann sums, respectively, for the function entered using choice two on the menu. Choice five lists a table of partitions, with their corresponding Riemann sum values and trapezoidal rule value, starting with a partition of 2 and going to 140. The final choice on the menu is used for the approximate integration technique of the Trapezoidal Rule. This choice may not be used until later in the course than the others, since most textbooks present approximate integration as the last section of the integrals chapter. In order to use the trapezoidal rule part of the program, the students need to be reviewed on what trapezoids are, and how the trapezoidal rule formula is derived. The program graphs a given function, over a given interval, accepts the number of partitions, then draws the trapezoids on the graph. Then, the areas are summed and the sum is printed.

CHAPTER V

SPECIFICS IN THE USE OF EACH PROGRAM

Introduction

In order to best implement the programs contained in this thesis, some additional ideas on the most effective way to present each, are needed. For instance, with many of the programs, the user is allowed to enter any number desired as a coefficient in an equation. Although this is a very important characteristic, in extreme cases it could lead to "bad examples" that do a poor job of illustrating the point. If an equation is entered that causes the graph to be beyond the scope of the screen, obviously the students will not benefit. Therefore, the main emphasis of this chapter is to provide information on the most useful numbers and equations to enter when a particular program is in execution. Also, this chapter will give detailed instructions for the use of each program. The chapter is divided into seperate sections, one for each of the programs, and each section includes the above mentioned suggestions, as well as any further ideas for class participation not given in the previous chapter.

Items Common to All Programs

The diskette provided is formatted using PC-DOS version 2.1, and if used on a compatible computer, is designed to be self-booting. In other words, all the user is required to do in order to use any of the programs, is to insert the diskette into the computer and turn it on. The autoexec file on the disk automatically runs a program called "prompt," which prints a list of the programs, with explanations, and asks the user to type the name of the desired program. When this is done, the desired program is automatically run. If, however, the computer being used does not recognize the DOS version, it should be turned on with the appropriate DOS disk, put into BASICA, and then the desired program should be loaded and run.

Every one of the programs has several input statements, which allow the user to enter information into the program. This information must, however, be entered in a very precise method, set by the programmer, and explained each time by the displayed "prompt." A prompt is a message printed on the screen, whenever the program pauses for an input statement, which explains very briefly what is to be entered, and the format that must be used to enter it. In most cases, the prompt will be very short, with the format given as variables. If only one number is to be entered, then no format is given, and the user simply types the number and presses enter. If two or more numbers are needed, the format is displayed, followed by a question mark. For example, if the coordinates of a point are needed, the prompt would be this:

```
X,Y ? ,
```

telling the user it expects the X coordinate, a comma, and then the Y coordinate. If information is entered incorrectly, the computer will reject it, displaying the message "Redo From Start." Often, this message will cause the screen to scroll up a line, which in turn causes the coordinate system on the screen to be out of phase with where the program expects it to be. Thus, anything plotted thereafter will not appear in the correct place. If this happens, the program should be stopped, and returned to the command menu to start over. This is most easily accomplished by performing a Control-Break, done on the IBM-PC by holding down the Control key, marked CTRL, and pressing the BREAK key (which is also marked with Scroll Lock). When the Break message is visible on the screen, RUN the program again, making the same menu choice as before.

All of the programs use one of two methods for inputting a new function or equation. Most common among the programs is the use of the EDIT command within the program, which stops program execution and displays the line containing the function, together with its line number. When this method is used, something similar to the following is printed on the screen:

```
1000 Y = 2*X - 1
GOTO 550
```

The cursor will be positioned under the left digit in the line number, in this case 1000. The user simply moves the cursor past the equals sign, and types in the desired right side of the equation. It is important to note that the symbol * must be used for multiplication, the symbol / for division, and the symbol ^2 for exponentiation (in this case squaring). Since the program is not running while these changes are being made, it must be re-started once the new function has been entered. This is accomplished by sending the program to the next line, in this example 550, through the use of a GOTO statement. The GOTO has been printed in a position where all the user must do is type in the equation, then press enter, putting the cursor on the GOTO line. By pressing enter a second time, the GOTO is executed, and the program begins execution at the point where it had stopped. The other method used by some of the programs, is to simply use input statements. The general form of the equation is printed on the screen, with letters such as a, b, h, or k for numbers the user is allowed to change. Below the equation is printed an input prompt, asking for the values that are be assigned to the letters. When they are correctly entered, the equation containing the letters is erased, and replaced by the same equation with the numbers that were entered. The following is an example, using the equation of an ellipse:

```
X^2 / a^2  +  Y^2 / b^2  =  1

a,b ?
```

After the user enters the values to be assigned to a and b, the equation is rewritten with those values.

For the computer graphics in most of the programs, the computer is put into "high resolution" graphics mode. If the machine on which the programs are run does not have the capability of color graphics in this mode, then some program editing must be done. Somewhere near the start of each of the programs will be two statements appearing as follows:

```
30 CLEAR,,,32768 : SCREEN 6 : COLOR 3,4
40 'SCREEN 2
```

For the IBM-PCjr, the first statement clears the graphics memory for use, and turns on Screen 6, the high resolution color screen. On the IBM-PC, the Clear statement is not recognized, and therefore must be taken out. In line 40 above, there is an apostrophe before the word SCREEN. This symbol signals the computer to ignore that line, so if using the program on an IBM-PC, insert an apostrophe before the word CLEAR in line 30, and delete the one in line 40. Once this is done, the program will run, and all graphics will be in black and white.

The Program LINES

The program entitled "Lines" is designed to aid in the instruction of graphing in the Cartesian Coordinate system, beginning with plotting points, and ending with linear equations in the point - slope form. An example of what the screen looks like during the Slope portion of the program is given as figure 1-A in appendix A, and figure 1-B shows how the screen appears during the Point - Slope portion.

The opening menu of the program is as follows:

```
          COMMAND MENU

  1. Point Plotting

  2. Slope

  3. Lines in the Slope - Intercept form

  4. Lines in the Point - Slope form

  5. Terminate Program


  ENTER YOUR CHOICE ?
```

Point Plotting. The first choice, Point Plotting, constructs a grid that extends from -8 to 8 along the X axis, and from -5 to 5 along the Y axis. In the lower left hand corner of the screen is the prompt; X,Y ? asking for the coordinates of a point to be entered. Any coordinates within the scope of the grid may be entered, and are subsequently plotted as a small circle at the proper spot. Points graphed on the outer edges of the grid do not show up as complete circles, since part of the point is "off" the screen. If a point entered is off the screen, it will simply not appear on the graph, and the program continues. The program will accept any numerical value as a coordinate, positive or negative, such as 1.5, but not 3/2.

Slope. The second choice on the menu teaches the concept of slope, by having the user enter any two points to be used as the endpoints of a line segment. The prompts for entering these points are:

```
Point 1: X,Y ?
Point 2: X,Y ?
```

The coordinates for these points are restricted to integers, so numbers like 4.5 and 3/2 are not allowed. Once the points are entered and graphed on the screen, the line segment joining them is drawn, and then the right triangle, formed by this segment and the grid, is slowly constructed. The vertical leg is drawn first, a unit at a time, then the horizontal leg. When the triangle is complete, the following prompt appears:

```
Slope ?
```

The user is expected to enter the correct slope. Since several formats for the slope are possible, the following instructions are pertinent. If the slope is zero, a 0 is entered. If the slope is undefined, the user enters UND . Finally, if the slope is neither of those two, then it must be entered as a fraction.

However many units are contained in the vertical leg, followed by / , then the number of units in the horizontal leg, is the format used for this case. Very important here is that even if the slope is a reducible fraction, it still must be entered as the exact units of vertical change over the exact units of horizontal change. For instance, if the right triangle constructed has a vertical leg of 4 units, and a horizontal leg of 2 units, the program will consider a slope of 2 as incorrect. Instead, 4/2 must be entered.

Slope Intercept-Form. The linear equation that is in the program when it is loaded is Y = 1/2 * X + 1. It is suggested that this equation is graphed first, by defaulting through the input operation with two presses of the ENTER key. When the line is graphed, the teacher might stop and have the students compute its slope and Y - intercept, then see how they relate to the equation. The program then asks if another is desired, which would be answered by typing Y for Yes. This program uses the first method discussed before, which stops the program and allows the user to edit the equation. Any previous lines are not erased before a new one is graphed, allowing for visual comparisons. If the screen becomes cluttered, clear the screen by returning to the command menu and making the same choice again.

Any equation entered is accepted, so care must be taken to insure that the graph appears on the screen. Since the Y axis goes only from -5 to 5, it is best stay within those bounds, although parts of some lines with Y-intercepts beyond this will appear, possibly across a corner of the screen. No vertical answers that is was correct, the computer will not detect this and assume it was correct. In the event that this happens, simply request another problem and go on from there.

It is important that the students recognize that the two equations:

$$Y = m * (X - h) + k \quad \text{and} \quad Y - k = m * (X - h)$$

are equivalent, since the first of these is required by the program.

The Program CONICS

"Conics" is a program written for the purpose of graphing each of the conic sections using the general form of the equation of each. The scale on the axes for each part of the program is from -16 to 16 for X values, and from -10 to 10 for Y values. If, at any time, numbers are entered that exceed this scale, a part or all of the graph will be missing from the screen. Figures 2-A and 2-B illustrate how the screen appears after an ellipse and a hyperbola have been graphed, respectively.

All of the portions of this program use the second method discussed above for changing the equation being graphed, which is by entering the values for certain parameters in the equation.

With the exception of hyperbolas, all the conic section equations are graphed without erasing any previously graphed equations. This is to allow for comparison of several graphs, but occasionally the screen will need to be cleared before another is graphed. The screen can be cleared in the same method as mentioned for the program "Lines," by returning to the menu and making the same choice as before. The menu for this program appears on the screen as follows:

```
              COMMAND MENU

        1. Graphing Parabolas

        2. Graphing Circles

        3. Graphing Ellipses

        4. Graphing Hyperbolas

        5. Terminate Program


        ENTER YOUR CHOICE ?
```

Parabolas. The parabolas portion of the program begins by creating a coordinate system very similar to the one used by the Lines program, except for the extended scale of the conics program. Then, in the lower left corner of the screen, the general form of the equation of a parabola is written, with the input prompt below that. The form of parabolic equation used in this program is as follows:

Y = a * (X - h)^2 + k .

The users are to enter values for the three parameters, a, h, and k, and when they have been given values, the general equation is rewritten with the respective values in place of the a, h, and k. Then, the parabola is graphed. Any values may be entered, but it is best to keep the vertex of the parabola away from the sides of the screen so that a suitable portion of the graph is visible. When the instructor is demonstrating how the value of a effects the parabola, using a few extreme values is effective. Very large numbers for a make the parabola close in almost to a straight line, while very small numbers, such as .01, cause the parabola to be very wide. The teacher should always have the students compare the picture on the screen with the equation, to be sure they realize where the three numbers a, h, and k are used, and what effect each has on the graph.

Circles. The circles portion of the program is used in very much the same method as for graphing parabolas. The equation that is used for graphing circle is the center - radius form, as follows:

(X - h)^2 + (Y - k)^2 = r^2 .

The three values missing from the equation are entered, according to the prompt, and the equation is then written with the numbers instead of the letters. After that has been done, the circle is graphed, and the user is asked if another is desired. As with the parabolas, any values may be entered, but if the center is too close to an edge to accommodate the radius, part of the circle will be missing. This is not a particularly undesrirable occurence, as long as the center does appear on the screen, with enough of the circle to show its shape. The teacher should take care to point out where the three inputted numbers appeared in the equation, and how each effected the graph of the circle.

Ellipses. When the third choice on the menu of the conics program is selected, the following will appear on the screen:

```
Select the form of elliptic equation desired.

1.  X^2 / a^2  +  Y^2 / b^2  =  1

2.  (X - h)^2 / a^2  +  (Y - k)^2 / b^2  =  1

CHOICE ?
```

The first choice is the form most often taught first, and when it is selected, the axes are drawn, and the ellipse is drawn with its center at the origin, and the characteristics set by a and b. When the graph is complete, the program asks:

```
Another ?
```

If the user enters an N, the program then asks:

```
Other form ?
```

If this question is answered with a Y, the other form of equation is offered for graphing. The other form listed above allows the center of the ellipse to be moved from the origin, to the point (h,k).

<u>Hyperbolas.</u> As with ellipses, the hyperbola portion of the program allows for a choice of two types of equations. The first thing appearing on the screen is:

```
 Select the form of hyperbolic equation
     1.    X^2 / a^2  -  Y^2 / b^2  =  1
     2.  - X^2 / a^2  +  Y^2 / b^2  =  1
     CHOICE ?
```

If the first choice is made, the user may graph hyperbolas that open right and left, while the second choice allows one to graph hyperbolas opening up and down. For whichever choice is made, the corresponding equation is used, and the values of a and b are entered. The values entered should be small enough that at least a small part of the hyperbola will fit on the screen. When the values are entered, they are placed in the equation, and the graph is drawn. First the rectangle and asymptotes (discussed in the last chapter) are drawn as dotted lines, then the curve is graphed.

For illustrating how the graph of the hyperbola relates to its asymptotes, a good example is to choose the second form of the equation, causing the hyperbola to open up and down, then enter a 3 for the value of a, and 1 for the value of b.

The Program FUNCTION

The program "Function" differs from most of the others in the way it was designed. There is no openning menu for this program, so the first thing done is to enter the function. The method of stopping the program and editting the function line is used in this program, and once it has been entered, some questions are asked of the user about the function. The first of these questions is as follows:

Is the function continuous ?

If this question is answered with Y for yes, the program continues. If, however, there are discontinuities, the user should answer with N for no, in which case the following question is asked:

Are there any intervals where the function is undefined ?

If the answer is Y, the left and right endpoints of the interval where the function is not defined, are then entered. If, for instance, the left endpoint is negative infinity, then it should be entered as any X coordinate past the left edge of the screen, which the user will determine to be either -16, -10, -5, or -1. The next question appearing on the screen, whether an interval was entered before or not, is as follows:

Number of points where undefined ?

If the answer is 0, the program goes to the next stage. An answer of a positive integer, however, will allow the user to enter the X values of all the points where their function is undefined. The reason that the prompts concerning undefined values are needed, is to avoid error messages such as "Division by zero" and "Illegal function call" (which occurs, for instance, if the computer attempts to compute the square root of a negative number).

When all discontinuities have been entered, the user then decides the scale with which the axes will be drawn. That is done in the following manner:

```
          Choose the scale for the X axis.
       1.  -16 to 16
       2.  -10 to 10
       3.   -5 to 5
       4.   -1 to 1
       CHOICE ?
```

These numbers indicate the values on the X axis that will be at the edges of the screen. The values on the Y axis are according to the length of a unit, as determined by the above choice, however, they are not the same as for the X axis. Because of the dimensions of the screen, the number at the top edge will be smaller then the number at the right edge of the screen.

The scale is chosen according to the function entered, by where its intercepts, asymptotes, and other points of interest are located. As an example, the function defined by:

Y = .01 * (X + 9) * (X + 3) * (X - 1)

would require either the scale -16 to 16, or -10 to 10, in order to illustrate all the X intercepts. The function:

Y = X * SIN (1/X)

is most interesting near the origin, so the scale -1 to 1 would be chosen.

Figure 3 in the appendix A shows an example of a function graphed using this program. The scale was chosen as -10 to 10, and discontinuties were entered as -4 and 4. The function entered was:

Y = (X + 4)^2 / (X - 16)^2

After the function has been graphed, the user is asked first if a change of the scale is desired, and if so, the program branches back to that set of choices. If no change is desired, the user is asked if a new function is to be graphed. An answer of Y starts the program over, while an answer of N ends it.

<u>The Program SINELAW</u>

The program to introduce the law of sines, entitled "Sinelaw," uses low resolution graphics instead of high. This allows color graphics to be used even on the IBM-PC, and has the effect of making the text on the screen twice as large. When the program is run, the first thing that appears on the screen is the command menu, which is as follows:

```
     COMMAND MENU

 1.  Two Angles and a side opposite one of the angles.
 2.  Two sides and an angle opposite one of the sides.
 3.  Terminate program
```

The first choice demonstrates the case of the law of sines most often taught first. When it is selected, the prompts for entering the two known angles and the known side are placed on the screen. They are as follows:

1st ANGLE ?

2nd ANGLE ?

SIDE ?

There is no labeling of these quantities as Alpha, Beta, side a, and so on, during the prompt, since this can cause confusion for the students when textbook examples use a different label for a particular item. The program always considers the first angle entered as Alpha, the second as Beta, and the entered side as a. Thus, any examples should be entered with that in mind.

For the following description of what appears on the screen, refer to figures 4-A and 4-B. The unknown side c is placed horizontally across the screen, from one edge to the other, and the triangle is constructed above it. Side b, the other unknown side, is connected to the left end of side c, with the given angle Alpha between them. This side will extend to the top of the screen until its length has been determined. The side of known length, a, is connected to the other end of side c, with the angle Beta between them. When the teacher presses enter, the side c is shortened, bringing the known side a along, at the proper angle, until the top end of a touches side b. At that point, side b is shortened to the proper length, and all the pieces of the newly formed triangle are labeled. Known pieces are given their respective values, while unknowns are labeled with letters. All unknown sides are labeled with small letters, a through c, and unknown angles are given capital letters, A through C. When the triangle has been properly labeled, the following appears below the triangle:

```
    b              ( a )
_________   =  ______________          b = ?

sin (beta)       sin (alpha)
```

All variables listed in parentheses are known values, and their numerical value is printed in those places. The students are to compute the value of side b, round off to one decimal place, and tell the teacher what to enter. If the value entered is incorrect, the computer beeps, erases it, redisplays the prompt. When the correct value is given, the letter b on the triangle is replaced by its value, and then the bottom of the screen is cleared. Next, the following is written:

```
        Angle C ?
```

As soon as the correct angle is given, it is placed on the triangle in place of the letter C. The law of sines is then written again, just as before, only using the letter c and the angle computed for C. When the correct answer is supplied, it is also placed on the triangle, and then the program declares the triangle solved.

In order for all of the triangle to fit on the screen, it is not possible to use any obtuse angles, and sides should be kept less than 12 units in length. If it is calculated that an entered triangle will not fit on the screen, a message indicating so is printed, and the value input prompts are redisplayed. The permissable lengths of the given side depend on the angles entered, but 12 is a good guideline. If a right triangle is to be displayed, enter 90 degrees for the first angle; then enter the second angle. The entered side, for this case, may be fairly long, but again depends on the value of the second angle.

The second case when the law of sines is used occurs when two side lengths, and the measure of an angle opposite one of those sides, are the known pieces of the triangle. Depending on the values of the known parts, there might be none, one, or two triangles possible. This concept is difficult for the students, so a teacher might elect to introduce it before using the program. It seems to work well, however, to use the program to introduce the concept, choosing first a case where no triangles are possible, then one where one triangle is possible, and finally a case where two are possible. When the second menu choice is made, the following prompts are printed first:

```
Angle ?
1st Side ?
2nd Side ?
```

After these values have been entered, the screen is cleared, and the computer begins to construct the triangle. Refer to figure 5-A for an illustration of the following description, and to figure 5-B for the next stage of the construction of the triangle.

As before, the unknown side is placed across the bottom of the screen, from left edge to right edge. This side will be designated as c. The second side entered, which is designated as b, is connected to the left edge of c, using the given angle A. At its top end, the remaining side, a, is connected with an arbitrary angle. This side will extend to the right, almost horizontally, so that its other end does not touch side c.

When the teacher presses the enter key, the second stage of the construction begins. Side a begins to swing, from its point of contact with side b, until it gone through an arc sufficient to take it past the vertex at angle A. If, at any point during its arc, the free end of side a touches side c, the computer beeps and leaves a side in place, drawn in blue. The user is then asked how many triangles were possible, which is determined by how many blue lines are on the screen. If the program is used to develop the concept of how many are possible, the teacher should lead the students to decide why that particular number of triangles occurred. When the correct answer for the number of triangles has been entered, the program redraws the first blue line in white, erases the other (if applicable), and labels the triangle as before. The triangle is then solved exactly as was done with the first case, having the user supply the values of the missing parts. If two triangles were possible, after the first is solved, it is erased and the second one drawn. When all possible triangles are solved, the program returns to the menu.

Again, entered values should be limited in order to keep the triangle on the screen. For illustrating the three cases possible, some recommendations for values are given. The first case, where no triangles are possible, is well demonstrated by entering 30 for the known angle, 4 for the first known side, and

10 for the second known side. These values will cause the side that is swung to never come in contact with the bottom side, showing that it is not possible to construct a triangle of those dimensions. The second case occurs when the side opposite the known angle is exactly equal to the altitude of the triangle to the horizontal base. This produces a right triangle, and the swinging side will touch the bottom side at the very bottom of its arc, and only at that one position. Good values to use for this case are 30 degrees for the known angle, 5 for the first side entered, and 10 for the second side. These values will form a 30 - 60 - 90 right triangle. Any values may be used for the third case, two triangles, as long as the triangle fits on the screen, and the first side length entered is smaller than the second, and longer than the height of the triangle. Values such as 55 degrees for the angle, 7 for the first side, and 9 for the second side, work very well. Then, to produce the last case, where the side opposite the entered angle (entered first) is longer than the other entered side. In this case, the arc created by the swinging of the side intersects the bottom side in one point, then again on the line extended from the bottom side, outside the triangle. A good way of illustrating this case is to use the same values as for the previous case, except reverse the values of the two sides that are entered. Enter 55 for the angle, 9 for the first side, and 7 for the second side.

The Program TRIGGRAF

The program entitled "Triggraf" is used to graph the six trigonometric functions. When it is run, the following command menu appears on the screen.

```
                  COMMAND MENU

    1.  Graph of Sine Function

    2.  Graph of Cosine Function

    3.  Graph of Tangent Function

    4.  Graph of Cotangent Function

    5.  Graph of Secant Function

    6.  Graph of Cosecant Function

    7.  Terminate Program

    CHOICE ?
```

For the first three choices, the program uses the edit command to stop the program and allow the function to be changed. The next three choices, the Cotangent, Secant, and Cosecant, all use input statements for the parameters a, b, and c, in the following general equation:

Y = a * TRIG(b * (X - c)).

When any of the six choices are entered, the first thing to appear on the screen are directions on how to enter the function to be graphed. Then, when ready, the instructor presses enter to move into the input procedure. Once the equation has been entered, the axes are drawn and the equation is graphed. An example of the screen output for this program, using the cosecant function, is given as figure 6 in appendix A.

All of the equations will accept the word PI as part of the phase shift parameter. For example (X - PI/2), where this was either typed into the equation, or PI/2 was entered for the phase shift parameter, would be recognized as valid. The scale on the X axis is from negative PI to four times PI, and on the Y axis is from -5 to 5. Entering an amplitude larger than 5 will result in part of the graph being off the screen.

For all of the trigonometric functions that apply, asymptotes are drawn as dotted vertical lines. For graphing the secant and cosecant functions, the value of the parameter a should not exceed 4, since the scale on the Y axis only goes to 5, and very little of the graph would be visible.

The Program POLAR

The program "Polar" is used to plot points, and graph equations, on a polar coordinate system. The command menu for this program appears as follows:

```
          COMMAND MENU

  1.  Plot Points with Polar Coordinates

  2.  Graph Polar Equations

  3.  Exit Program


  CHOICE ?
```

When the choice is made to plot points, the polar coordinate system is created on the screen, then the user is asked to enter the coordinates, in polar form, of a point to be plotted. The scale on the graph extends to a radius of 11, and lines are drawn at 0, 30, 45, 60, 90, degrees, and their multiples. Either of the coordinates may be entered as a positive or negative value. Once the point has been graphed, the program asks if another is desired. If so, the Point prompt is cleared and redisplayed, but any previous points are left on the graph.

When the second menu choice, for graphing polar equations, is selected, the first message appearing on the screen is:

Does your equation contain a square root ?

If the answer is N, the screen is cleared and the equation is entered by the user. Also, the user is asked to enter the last value to be used for the angle theta, as a multiple of PI. The prompt for this is as follows:

Theta begins as 0. Enter stopping point as PI, 2PI, 3PI, ... ?

If, however, the answer to the square root question is Y, the the equation must be entered without the square root, after a variable called PSQR. The user types in what PSQR is to be. Then, when the program is running, the value of PSQR is computed for varying values of Theta. That value is then tested to determine if it is negative. If not, the value is used, and its square root is computed. For example, in order to graph an equation such as:

P = SQR (sin (Theta)) ,

the user would be asked to enter the equation without the square root, after PSQR, as follows:

PSQR = sin (THETA) .

Equations such as this look the best if a large coeffecient is used before the SIN, such as 20, 30, or 40. This makes the graph large enough to be seen easily by the students. Figure 7 in the appendix A shows the graph of an Archimedes Spiral, using this program. The figure does not appear perfectly round, which is caused by the printer, as the polar coordinate system on the screen appears circular.

For most polar equations, the stopping point for Theta should be 2PI, since the graph will repeat after that. For any of the spirals, however, values of up to 8PI might be used, causing the graph to have half that many complete spirals.

The Program LIMITS

The program "Limits" is intended to be used for an introduction to limits. The command menu appears as follows:

```
              COMMAND MENU
    1.  Explanation of Limits
    2.  Input a new Function
    3.  Graph Function with Epsilon - Delta lines
    4.  Table of Function Values as X Approaches some Number
    5.  Exit Program
    CHOICE ?
```

The first choice, an explanation of limits, gives a brief introduction of what is meant by the limit of a function, then uses moving graphics to present an example. Figure 8-A shows how the screen appears after the first example has been graphed. The program writes on the screen that X values will be chosen closer and closer to 5, and that the value of the function at 5 is 3 (f(5) = 3). The function being used is:

```
         Y = 1/20 * (X - 1) * (X - 6) * (X - 10)  +  2
```

Then, when the teacher presses enter, the screen is cleared, the axes are drawn, and dotted lines are drawn from the point (0,3) to (5,3), and from (5,0) to (5,3). The program then begins graphing the function, moving from both X = 1 and X = 7, in toward the point (5,3). As the different values for X are used, arrows are moved along just below the X axis, toward the number 5. When the functional value gets sufficiently close to 5 from both directions, two arrows are moved along the Y axis, indicating the Y value, toward the number 3. The process stops when the arrows along the Y axis are within .1 of the value 3, which has brought the arrows along the X axis to within .1 of 5. The customary epsilon and delta lines are drawn from the axes to the graph, and the distances between the lines are printed.

Once the first example has been completed, the function is changed (by the program) to :

```
Y = 1/20 * (X - 1)*(X - 6)*(X - 10) + 2.5     if X not equal to 5
               Y = 3                          if X = 5
```

This is the same function as before, except it has been shifted up .5 units. The functional value at X = 5 remains the same.

Then, the process of graphing the function is repeated; however this time it is not possible to bring the functional value to within a distance of .1 of 3, so it is determined that 3 cannot be the limit of the function, as X approaches 5.

The second choice on the menu is used to change the function in the program. The reason for having a separate menu choice for this purpose is so the user does not have to deal with entering a new function every time another menu choice is made. Since the main emphasis of the demonstration is not the graph of the function, but rather to illustrate the epsilon - delta definition of the limit, it is conceivable that one may never wish to change the original function. The original function, which is :

Y = sqr (X) + 1,

is very good for showing the way delta depends on the chosen epsilon. There are, however, other functions that work well in the program also, to demonstrate some special cases.

Choice two on the menu must be chosen before choices 3 and 4 will be allowed. When it runs, the user is asked to input the desired function. Due to difficulties in constructing the epsilon and delta lines, the user is restricted to the first quadrant, so only functions that will appear in that quadrant should be entered.

When the third choice is selected, the user is first asked to enter the value of X where the limit is to be tested. Then, the equation entered during choice two will be graphed over the interval [0,14], and at the top of the screen will appear this prompt:

EPSILON ?

The value entered for epsilon must be restricted to less than 1 and greater than 0. If any other values are entered, a message indicating the acceptable values is printed and the prompt is redisplayed. It is recommended that a fairly large value, such as .8, be used at first. Then, smaller values may be used later. When an acceptable value has been given, the function is graphed and the epsilon - delta lines are drawn surrounding the specified value of X and its corresponding value for Y. The value for delta is computed, using the smaller of the two distances from X to the vertical lines drawn. A third delta line, in blue, is drawn at a distance of delta from X on the side where the larger distance is, so it is shown that values for X must be chosen from within a delta's distance of the given point, in order to assure that f(X) is within an epsilon's distance of the limit.

The values of epsilon and delta are printed at the bottom of the screen, along with the following prompt:

Another value for EPSILON (return to stop) ?

If the instructor wishes to choose a different epsilon, it is entered, and the process is repeated. If another is not wanted, the user simply presses the enter key. When this is done, the user is asked if another value for X is desired. If the answer is Y, the program branches back to the beginning of choice three. If not, it returns to the command menu.

For an example of a limit demonstration using the third menu choice, refer to figure 8-B in appendix A.

The fourth choice on the menu displays a table of function values, as X approaches a given value. This value is entered first, using the same prompt as for choice three. When it has been entered, the following table is constructed:

```
From the left           |           From the right
                        |
   X          f(X)      |     f(X)              X
________________________|_________________________
   6            5       |      6               10
                        .
                        .
                        .
```

The values for X begin at two units from the given value, and each new increment is half of the last one. The last value to be used is 0.000015 from X on either side. The values for f(X) are listed on the middle of the table, so that comparisons of the values from the left and right are easily seen.

To demonstrate a function whose limit does not exist at some given value, using something similar to,

Y = 1 / (X - 3),

and testing for a limit at 3, will cause the final values in the table to be very far apart. The function:

Y = (X^2 - 9) / (X - 3)

will demonstrate a function that has a limit at X = 3, but no functional value at X = 3.

The Program DERIVATV

The program to help teach the second main topic in Introductory Calculus, derivatives, is entitled "Derivatv." The menu for this program will appear on the screen as follows:

```
        COMMAND MENU

  1.  Explanation of Derivatives

  2.  Graph a Function and its Derivative Function

  3.  List a Table of Function Values and First
      Derivative values over a given interval.

  4.  Exit Program

  CHOICE ?
```

The first choice, the explanation of derivatives, uses the standard method for introducing the topic. The graph of:

Y = -1/4 * (X - 4)^2 + 4

is drawn, and a secant line through the curve is drawn from the point (3,3.75) to the point (6,3). This is illustrated in figure 9-A in the appendix A.

The teacher then explains that the way to find a tangent line to the curve at the point (3,f(3)) is to bring the other point where the secant intersects the curve closer and closer to the given point. The slope of the tangent line is found by taking the limit of the slope of the secant, as the distance of the moving point from the given point approaches zero. On the screen it is written this way:

The limit of (f(3+h) - f(3)) / ((3+h) - 3) as h goes to 0, gives the exact slope of the tangent line at (3,f(3)).

When ready, the teacher presses enter, and a secant is drawn to a point with an X coordinate .25 units closer to 3, and the old secant is erased. This continues until the X coordinate of the moving point reaches 3, and the tangent line is left in place. For each line drawn, its slope is calculated and printed in the top right corner of the screen. The final value of this slope is the slope of the tangent line, and the value of the derivative of the function.

The second menu choice first allows the user to change the present function, then choose the interval over which that function will be graphed. The scale of the axes is determined by the length of the interval, so any interval may be entered. When the interval has been entered, the screen is cleared, the axes are drawn, and the two curves are graphed, putting the left endpoint of the inputted interval at the left edge of the screen, and the right endpoint at the right edge of the screen. The two curves are graphed separately, starting with the derivative function. After it has been completed, the teacher presses enter to start the graphing of the function itself. It works very well for practice to have the students predict the intervals where the function will be decreasing or increasing, and the points where relative extremum and points of inflection will occur, based on the behavior of the derivative function. This gives the students understanding as to how positive, negative, and zero values of the first derivative will later be used to help graph more complicated functions.

Figure 9-B gives an example of the graph created, using a function similar to the second equation listed below.

The following is a list of functions, together with the appropriate intervals for graphing, that work very well with this portion of the program.

```
Y = SIN (X)                                   over   [ -3,7]

Y = 1/100 * (X + 3) * (X - 1) * (X - 5)       over   [-5,10]
     (the  1/100 is needed to keep the height of the graph within
      the limits of the screen)

Y = 2 * SIN (X) + .5*X                        over   [0,10]

Y = 1/2 * X + 3                               over   [-5,5]
      (this shows the constant derivative function)

Y = ABS(X - 3)                                over   [0,5]
      (this shows the discontinuity in the derivative function at
       a point where there can be no tangent line)

Y = ((X - 4)^2)^.333                          over   [0,10]
      (The  powers  here  are how fractional  exponents  must  be
       entered. This also has a discontinuous derivative)
```

There are, of course, many other functions that are very good for illustrating this point, so some experimenting can be done. Higher degree polynomials can be created by adding more binomial factors to the second equation listed above. This will, however, usually require that the leading coefficient be decreased in size. Otherwise, the graph will be beyond the scope of the screen in several places.

The third choice on the menu creates a table of values for the function and its first derivative for varying values of X. The first thing to appear on the screen after this selection has been made is the prompt:

Do you want to change the function ?

If the answer is Y, the user may input a new function in the same way as was done with the prior portion of the program. If another function is not desired, the function used is the last used during the second menu choice. After the function has been entered, the following prompt appears on the screen:

Enter the interval for the X values: A,B ?

Any interval may be entered, and there will always be ten values used from that interval. The values used are computed from the length of the interval, at an increment of one tenth the interval length. The table is intended to be used by first entering a fairly large interval, then continually smaller intervals that contain points where the first derivative has a desired value. When the first interval has been entered and the table is completed, the following prompt is displayed at the bottom of the screen:

Another interval ?

If the answer is Y, the program is branched back to the prompt for entering the interval. An answer of N returns the program to the command menu.

Due to the limited capability of the computer to handle large numbers of decimal places, when the interval used becomes less than .02 in length, many of the values for the derivative are slightly inaccurrate. For this reason, if an interval smaller than this is entered, the program will not accept it, and another is asked for. Usually an interval that is at least .02 units in length is sufficient to find the values of X where the first derivative is equal to zero, or any other desired value.

The Program INTEGRAL

The program "Integral" is very similar to "Derivatv" in its presentation and the way it is used. The following is the way the command menu will be on the screen:

```
        COMMAND MENU

  1.  Explanation of Intergation, using Riemann Sums

  2.  Input a Function and Interval of Integration

  3.  Graph Upper Sums

  4.  Graph Lower Sums

  5.  Graph Trapezoidal Rule

  6.  List Values for Upper Sums, Lower sums, and Trapezoidal
      Rule, Using Increasing Numbers of Subintervals

  7.  Exit Program

    CHOICE ?
```

The first choice is a good first time introduction to finding the area under a curve by using Riemann sums, leading to the definition of integration. When the choice is selected, text is written on the screen explaining the function that will be graphed, the interval over which it will be graphed, and how the area under that curve is to be estimated. A teacher using the program in front of a class will probably not want to read all the text to the students, but it does provide good notes for what should be explained prior to continuing. The equation that is graphed as an example is:

$$Y = -1/4 * (X - 4)^2 + 4 ,$$

which has X intercepts at 0 and 8. The interval used is the interval between the intercepts. The text first explains that a way to estimate the area between the curve and the X axis is to enclose the curve in a rectangle, the height of which is equal to the maximum height of the function, and the base of which goes from 0 to 8. The area of this rectangle is computed to be 32, so it is known that the desired area is less than 32. This example is graphed on the screen. Then, when ready, the teacher presses enter to move on to the next explanation. The next set of text explains that if three rectangles were drawn, enclosing the one third of the curve each, a better approximation would result. This example is then drawn, and the sum of the areas of the three rectangles is printed. The process is repeated with ten rectangles, and the concept of a partition, in this case regular, of the interval [0,8] is introduced. Figure 10-A in appendix A shows how the screen appears at this point in the demonstration.

After the sum using a partition of ten subintervals is calculated, the program allows the teacher to input a value for the next partition. The students will benefit from being allowed to suggest this value, and usually will offer a number between 50 and 80. These are very good values to use, at this point, since the graphic display of the Riemann sums will show that the sum computed is a very good approximation. Finally, the computer will graph an example, using a partition of 200, and the printed result is accurate to within 3 decimal places of the actual integral. The concept of taking the limit of the Riemann sums as the number of subintervals in the sequence of regular partitions is allowed to approach infinity is introduced then, and the definition of the integral is developed from that. If, when the instructor asks the students to give the number of subintervals in a partition, a student offers a number larger than 100 to 120, the instructor should probably suggest that they choose a smaller number, so that the computer's use of 200 subintervals is not redundant. Also, using a number larger than about 300 causes the program to run so slowly that much of the interest is lost.

The second choice on the menu is used to input a new function and to define the interval over which the integration will be performed. By using a separate menu selection for this purpose, the teacher does not need to be concerned with entering a function, even when a new one is not desired. For the most part, the best functions to use are those that stay above the X axis for most of the defined interval. Most of the screen is devoted to quadrants I and II. Thus, if the curve goes very far below the X axis, part of it will be absent from view. The scale of the axes is determined from the length of the interval entered, so horizontally, any function will fit. If the curve does go above the top edge of the screen, or below the bottom edge, the rectangles will be cut off. The values calculated for the areas, however, will still be accurate.

The following is a list of a few functions, and the intervals to use with them, that work well for this program.

```
Y = 1/6 * (X - 5)^2 + 1                       over   [0,10]

Y = 1/6 * (X - 5)^2 - 2                       over   [0,10]
         (this puts the vertex of the parabola below the X  axis,
          and demonstrates negative portions of the area sums)

Y = 1/10 * (X + 3) * (X) * (X - 3)  +  4   over   [-5,5]

Y = 2 * SIN ( 1/2*X )                          over   [-5,5]
```

The third and fourth menu choices are used to illustrate Upper and Lower Riemann sums, respectively. When chosen, the first thing that must be entered is the number of subintervals in a regular partition. When the prompt for this entry has been answered, the screen is cleared, and the curve is graphed over the previously defined interval. As soon as the curve is graphed, the appropriate Riemann rectangles are drawn, either using the maximum height in each subinterval, or the minimum height, depending on the choice. The sum of the areas of the rectangles is computed, and printed across the bottom of the screen. The user is then asked if a different number of subintervals is desired. If the answer is N, the program returns to the command menu. An answer of Y starts that part of the program over.

The fifth choice will graph and estimate the integral of the given function using the trapezoidal rule. Any of the above functions will work with this choice, but those with sharp bends will make the trapezoids more obvious. Also, if a partition of 20 or more subintervals is used, the tops of the trapezoids tend to blend with the curve itself, and are hard to see. Starting with a small partition, such as with 5 subintervals, then moving to the large ones, will show that the trapezoidal rule produces accurate results much faster than Riemann sums. Figure 10-B illustrates what appears on the screen after using the Trapezoidal Rule portion of the program.

Choice number six on the menu computes upper and lower Riemann sums, and areas by the trapezoidal rule, for varying numbers of subintervals. Beginning with a partition of 2 subintervals, the table prints values for selected numbers of subintervals, and ends when a partition is reached large enough that the values of the upper and lower sums are within two decimal places of each other. The viewers can see that the computed values for the three sums approach each other, as the number of subintervals increases. The last values printed are all equal, since they are rounded off to two decimal places. It should be explained to the students that this is a result of the computer rounding the values off, and that the values would never actually be equal. The table illustrates very well that of these methods of approximate integration, the trapezoidal rule is more accurate than the Riemann Sum approach.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary of Thesis

The purpose of this thesis was to study the use, and effect, of a microcomputer in the mathematics classroom. The increasing development of this type of technology, leads to changes in the traditional methods of teaching mathematics. There are, of course, many uses for the computer in the mathematics classroom, but the emphasis of this thesis was on the use of the computer as an in class teaching aid, a tool for the teacher to use, just as one might use a film strip or the chalkboard.

The first chapter was an introduction to the thesis. In it was discussed how the idea for the thesis originated, the importance of such a study, the procedures used to study the problem, and the limitations of the study. Also given in the chapter was an overview of the thesis.

Chapter two was a presentation of what other authors have written concerning the field of Computer Aided Instruction. Intended mainly to prove a case for the use of CAI, quotations were included from notable works on the subject. Used in the study were several works from the National Council of Teachers of Mathematics, including the 1984 Yearbook for that organization, which was entitled, Computers in Mathematics Education. Also used were several books written on the subject, including Learning With Computers, by Alfred Bork, and CAI Sourcebook, by Robert Burke.

The third chapter offered the reactions of the students that took part in the study. These students had all witnessed the use of a program written by the author, and had completed a questionnaire which asked for their opinion on several items. They were asked their reaction to the particular program they observed, their reaction to CAI in general, and any suggestions they might have. The answers to the questionnaire were both written comments, and a numerical response from 1 to 5. The numerical responses were totalled and averaged, and samples of the written comments were listed.

Chapter four treated the subject of using the programs in the classroom, from the aspect of how a mathematics teacher could most easily incorporate them into the normal curriculum of the course. The usual method of introducing a concept to a class was given. Then, the ways in which each program could be used in conjunction with that were discussed.

The fifth chapter was a detailed look at the use of each program, and could best be described as a user's guide to the programs stored on the diskette. Descriptions of how the teacher should enter values, what values and equations are the best to use, and explanations of what appears on the screen at different times during the execution of each program, were the important parts of the discussion. References were made to appendix A, which contains printouts of the screen during the execution of the programs. These figures were intended to help in interpreting the descriptions given in chapter five.

The sixth and final chapter summarizes the thesis, and gives conclusions and recommendations for those intending to use the programs offered.

Following the sixth chapter are a bibliography and two appendixes. The first appendix, as was mentioned above, contains printouts from the screen that the reader may wish to refer to. The second appendix contains the listings of the actual programs, as they appear on the diskette. These are included as an aid to those wishing to expand the programs to include other topics, or to modify the programs to more closely fit their individual classroom needs.

Conclusions

During the course of this study, several conclusions became obvious. The following is a list of conclusions made by the author, concerning the study.

1. When introducing algebra students to the Cartesian coordinate system, and particularly to point plotting, it is best to use chalkboard examples. Through conversations with students, and reactions written on the student questionnaires, it became apparent to the author that the point plotting part of the program "Lines" spent more time than necessary teaching a subject that is quickly and easily learned.

2. If the program "Conics" is used to teach the graphing of conic sections, with no other explanations being offered by the instructor, the students will miss a very important point. The fact that these curves result from the intersection of a plane with a cone should be taught, and usually is. But

beyond that, it is very easy for the instructor to neglect the locus of points definition of each. Knowing that a parabola is formed by the locus of points equidistant from a point and a line, is an important way for students to relate geometry and algebra, and adds a point of interest to the lesson. The program does nothing to illustrate this point; therefore, by itself, it is not enough.

3. After demonstrating the concept of limits to a class that had never been exposed to it before, and then again to a Calculus II class, it was shown that the use of moving graphics made possible by the computer, gave the students a better understanding of the concept. Too often the epsilon-delta definition of limits is never quite understood, and computer graphics help to alleviate this problem. The beginning level classes seemed to have satisfactory understanding of the definition following the demonstration, while an alarming number of the Calculus II students seemed to be learning it for the first time. Many of these students commented afterward that they now understood precisely what the definition meant, as opposed to only vague understanding prior to the demonstration.

4. Although the use of CAI designed for use in front of an entire class is quite often favorable, it became apparent from the questionnaires that the small computer terminal that was used was often not adequate. The determining factor was the size of the class involved. In the larger

classes, the students in the back of the room could not see enough detail on the screen to understand the topic very well. Despite encouragement to move forward in the room, it was difficult for some to see, and they invariably lost interest. For small classes, there was no problem, but if only the standard monitor size of 13 inches is available, then the viewing capabilities of the entire class should be a concern.

5. From the reading of other works on CAI, it can be concluded that there are many mathematics teachers who are pushing to make the use of the computer for teaching mathematics standard practice. Certainly there are those who feel the computer is not a necessity, and is to be used only if the instructor desires it. This is not the recurring theme, however, in the works reviewed. Instead, it is the belief of those authors, and the author of this thesis, that the computer allows the instructor to do a better job of teaching, and that mathematics teachers cannot afford to ignore it.

6. Apparent from the comments supplied by students on the questionnaires, was the fact that students themselves believe CAI is both important and effective. The high average response of 4.67 (out of 5), on the question asking if they thought that the use of the computer was interesting, indicates that a much higher level of student interest is achieved through the use of CAI. It was common for the author of this thesis to be asked by students when the

computer was going to be used again, indicating they were anxious for that, and that they preferred to be taught with the teacher using CAI.

7. Despite efforts to provide a multitude of error traps in the programs, it is still very easy for a user of the programs to enter something incorrectly. To completely error proof the programs, one would be required to spend much time, and add a multitude of lines to each. It seemed plausible to assume that those using the programs could learn easily enough how to avoid these problems, and the extra effort was not necessary.

Recommendations

Initially in this section, recommendations relating to the above conclusions are made, where appropriate. Pertaining to the second conclusion, concerning the use of the program "Conics," it is recommended that the teacher introduce conic sections with solid, three-dimensional visual aids, and with the locus of points definitions, then proceed to the graphing of the algebraic equations. It seems to help the students with the graphing techniques if they have these concepts first.

In relation to the third conclusion, as to the effectiveness of the program limits, it is also recommended that students are taught a strong intuitive definition of limits, before they are ever exposed to the epsilon-delta definition. Too often students develop the idea that the limit definition is beyond their comprehension, when in fact it is a rather straightfoward concept.

Concerning the fourth conclusion, that the small size of a standard computer monitor causes viewing problems in large classes, the solutions to this problem are varied. Most desirable, but unfortunately not always an option, is to use a larger monitor. These are expensive, so schools do not typically own them. Microcomputers may be connected to large color television sets, with the use of special adapter cables which are relatively inexpensive, if the television has that capability. Much simpler, and usually effective, is to use a seating arrangement that abandons the desks, and crowds the students together near the computer. This uncomfortable position does not need to be maintained for a long period of time, and is well tolerated by the students.

The seventh conclusion, dealing with the lack of error traps, warrants some further recommendations. First, if a teacher can afford the time to run a program through a few times, in order to become accustomed to the procedures before presenting it to a class, the problem virtually disappears. Also, by carefully fol- lowing the directions given in chapter five of this thesis, a teacher can avoid most errors. Then, if a problem does occur in front of a class, knowing how to get out of the program, and get it restarted quickly, will minimize any embarassment. This too should be practiced, and the methods for the procedure are given in the fifth chapter also.

For a teacher wishing to expand the number of topics which may be presented using a micromcomputer, the following recommendations are made. First, if one has no real experience with graphics programs, it is suggested that the program listings in the third appendix be studied. The methods used in those programs are fairly simple graphics commands, and the logic involved should be straightforward. It would be easiest to create a new program by using one of the existing programs as a skeleton, editing where needed. The actual main program for nearly all of them is merely a menu driven series of subroutine calls, so by writing additional subroutines, one could expand the possibilities of any given program.

One recommendation for expansion concerns the program "Lines." The addition of a subroutine that would graph a system of two linear equations, mark the intersection point, and supply the coordinates of that point, would not be a major task. By using the same method of inputting equations, and the same method of graphing those lines, the existing program could serve as a model.

A similar subroutine could be added to the program "Conics," which would graph any two equations, linear or not, and graphically solve the system. The structure of such a subroutine would be nearly identical to the one for a linear system, mentioned above.

Recommendations For Further Studies

Any one of the three main program categories studied in this thesis -- algebra, trigonometry, or calculus -- could be considered as a source for providing materials for related studies. As an example, should a person desire to do a study emphasizing only trigonometry, a program could be written for the solution of triangles using the law of cosines, which would be very similar to the program "Sinelaw." Furthermore, all of the trigonometric function definitions relating to the side lengths of right triangles could be presented nicely through a graphics approach.

It should be noted that the graphics in this study were all two-dimensional. A related study using three-dimensional graphics could certainly be of benefit to students dealing with the solid, analytical geometry concepts encountered in Calculus III.

A. BOOKS

1. Bork, Alfred M. *Learning With Computers.* Bedford, Mass.: Digital Press, 1981.

2. Burke, Robert L. *CAI Sourcebook.* Englewood Cliffs, N.J.:

3. Fey, James T. (ed.). *Computing and Mathematics. The Impact on Secondary School Curricula.* College Park, MD.: Univ. of Maryland, 1984

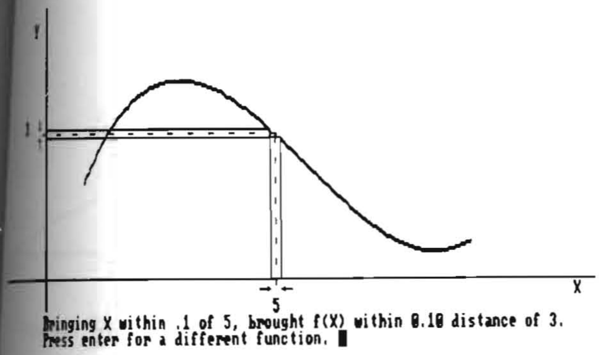
B. PUBLICATIONS OF LEARNED SOCITIES

4. National Council of Teachers of Mathematics. *1984 Yearbook, Computers in Mathematics Education.* edited by Viggo Hansen and Marilyn Zweng. Reston, VA.: Reston, 1984

5. Alpers, Donald J. "New Directions in Two-Year College Mathematics," *Mathematics Teacher.* National Council of Teachers of Mathematics. Reston, VA.: Vol. 28, No. 5, May, 1985

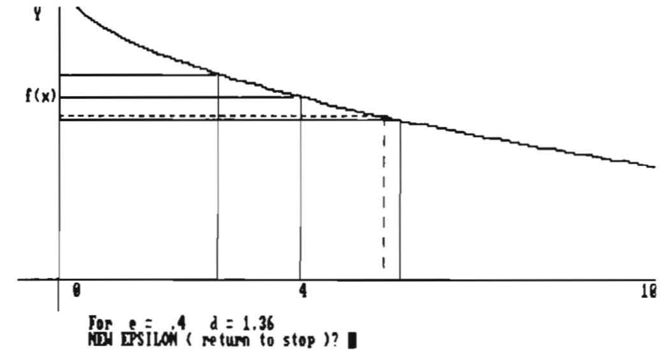
APPENDIX A

FIGURES ILLUSTRATING SCREEN OUTPUT



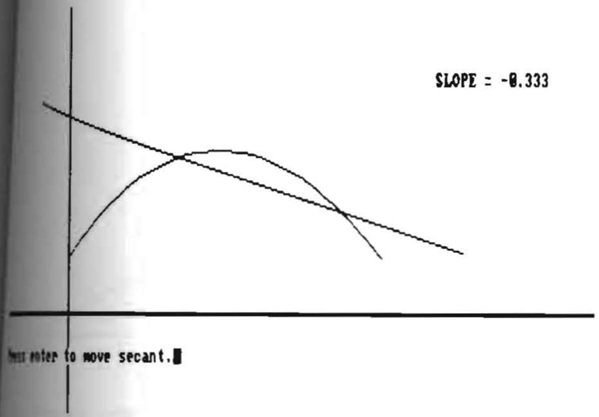
Bringing X within .1 of 5, brought f(X) within 0.10 distance of 3.
Press enter for a different function. █

Fig. 8-A



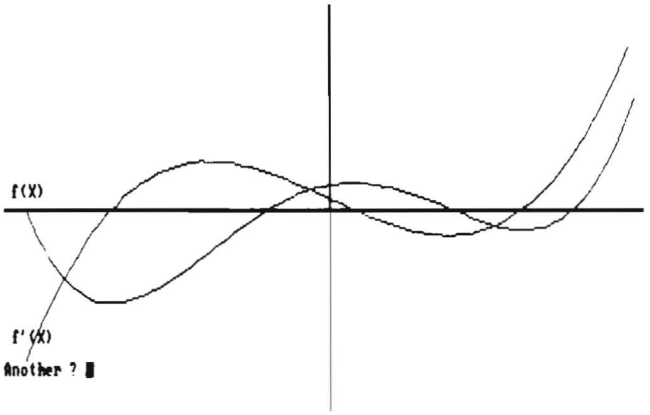
For e = .4 d = 1.36
NEW EPSILON (return to stop)? █

Fig. 8-B


SLOPE = -0.333

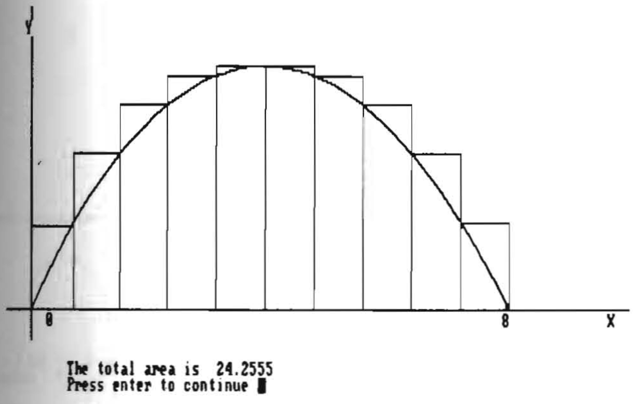
Press enter to move secant.█

Fig. 9-A



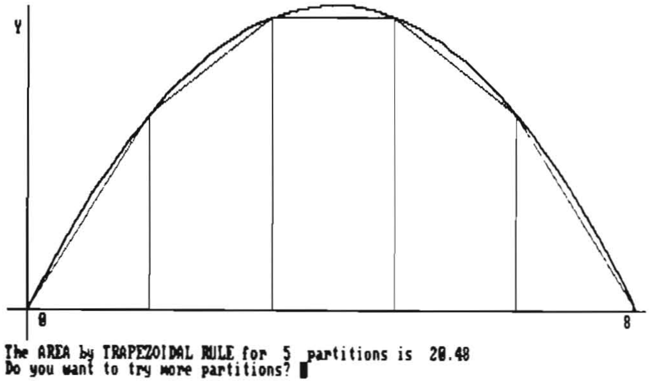
Another ? █

Fig. 9-B



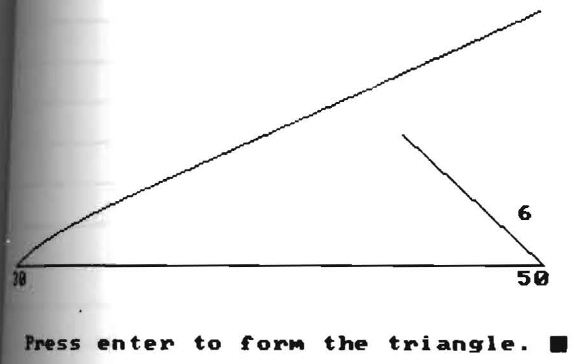
The total area is 24.2555
Press enter to continue █

Fig. 10-A



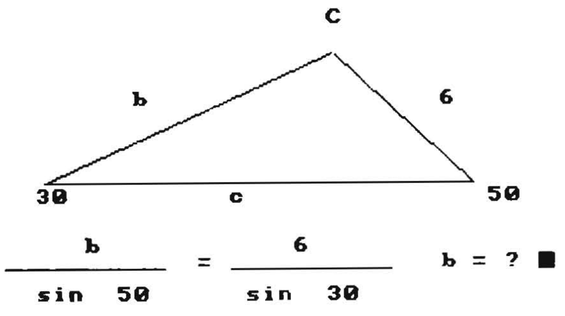
The AREA by TRAPEZOIDAL RULE for 5 partitions is 20.48
Do you want to try more partitions? █

Fig. 10-B



Press enter to form the triangle. ■

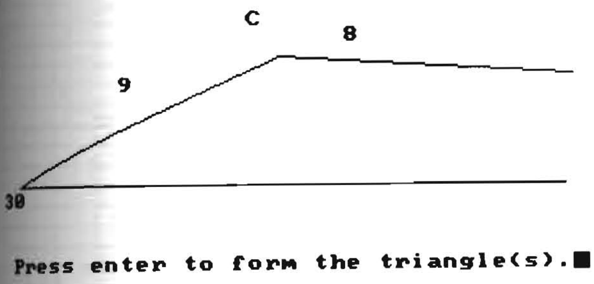
Fig. 4-A



$$\frac{b}{\sin 50} = \frac{6}{\sin 30}$$

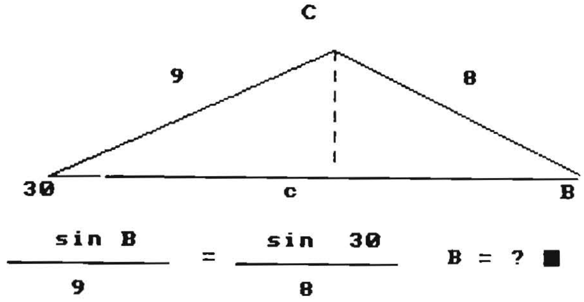
b = ? ■

Fig. 4-B



Press enter to form the triangle(s).■

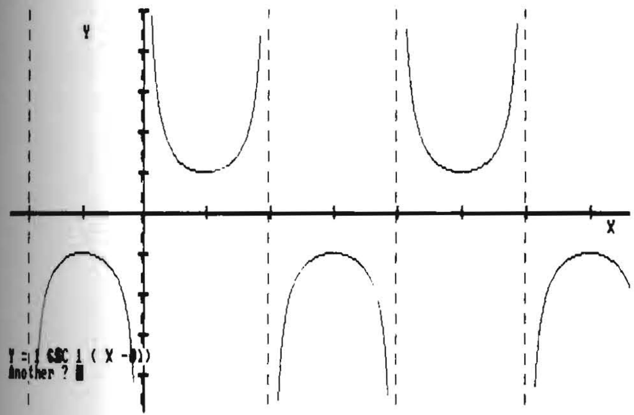
Fig. 5-A



$$\frac{\sin B}{9} = \frac{\sin 30}{8}$$

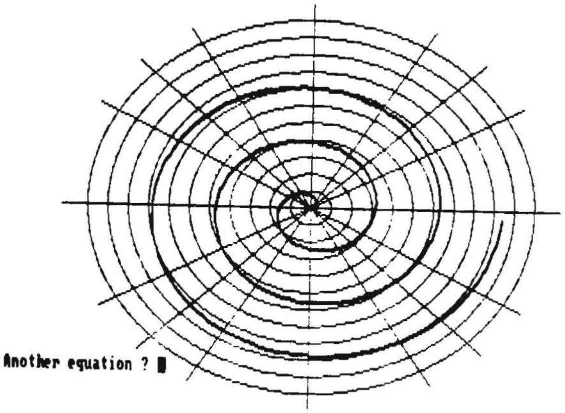
B = ? ■

Fig. 5-B



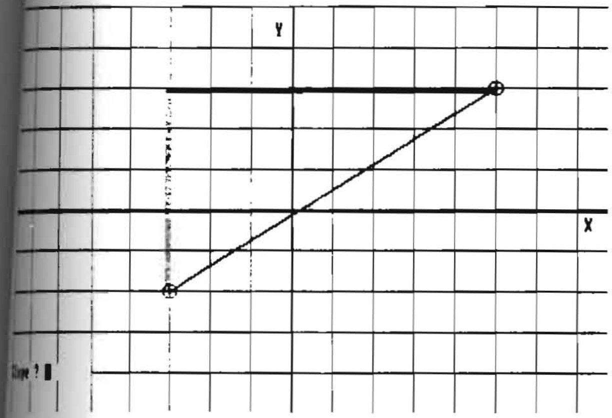
Y = 1 CSC 1 (X -0))
Another ? ■

Fig. 6



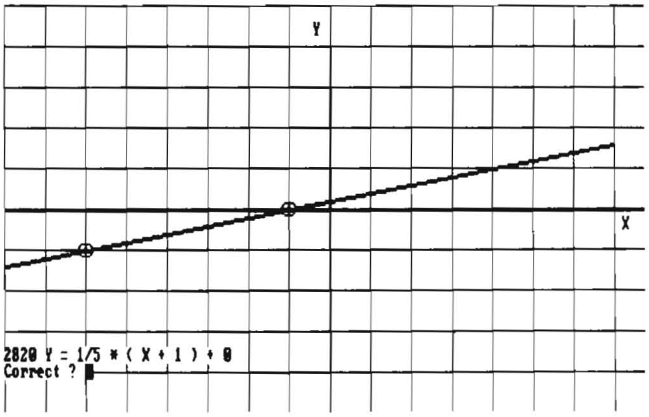
Another equation ? ■

Fig. 7



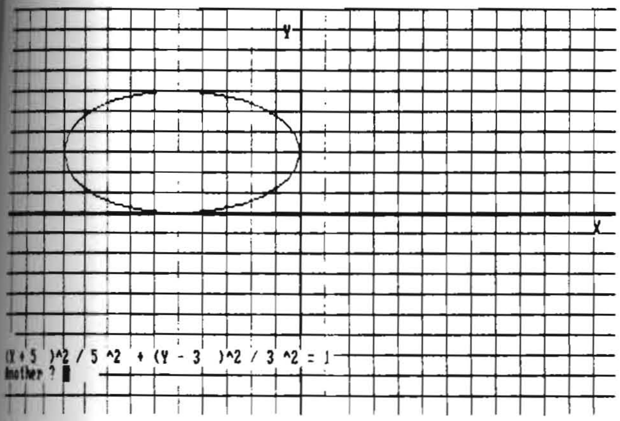
```
Slope ? █
```

Fig. 1-A



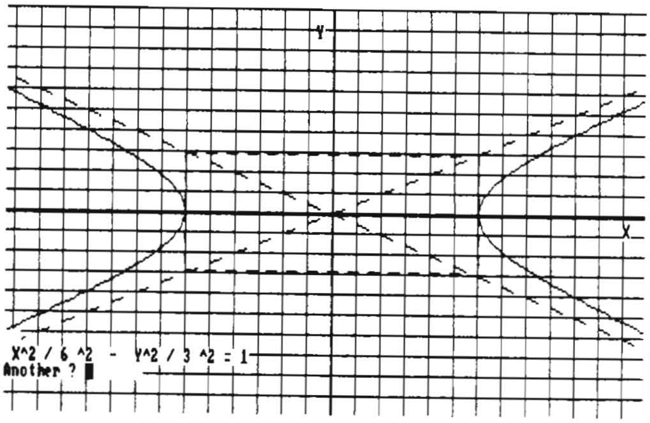
```
2820 Y = 1/5 * ( X + 1 ) + 0
Correct ? █
```

Fig. 1-B



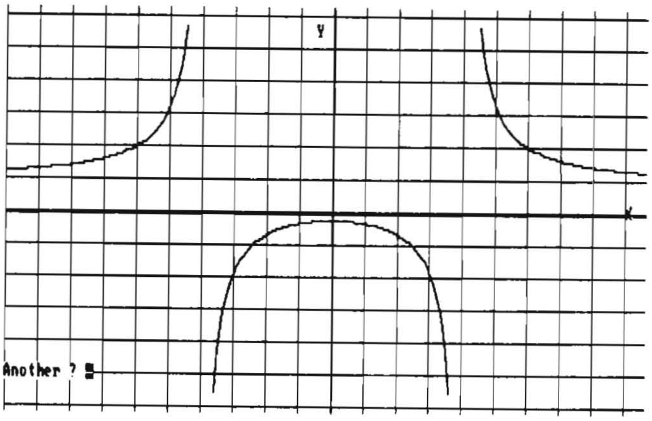
```
(X + 5  )^2 / 5 ^2  + (Y - 3  )^2 / 3 ^2 = 1
Another ? █
```

Fig. 2-A



```
 X^2 / 6 ^2  -  Y^2 / 3 ^2 = 1
Another ? █
```

Fig. 2-B



```
Another ? █
```

Fig. 3

APPENDIX B

PROGRAM LISTINGS

```
10 '**********   GRAPHING LINES        MIKE MOSIER   ***********
20 '
30 '                    STORED UNDER:   LINES
40 '                       JUNE 15, 1985
50 '
60 CLEAR,,,32768! : SCREEN 6 : COLOR 3,4
70 'SCREEN 2
80 CLS
90 LOCATE 3,15:PRINT "PROGRAM LINES
100 LOCATE 5,15:PRINT "COMMAND MENU
110 LOCATE 8,10:PRINT "1. Point Plotting
120 LOCATE 10,10:PRINT "2. Slope
130 LOCATE 12,10:PRINT "3. Lines in the Slope - Intercept form
140 LOCATE 14,10:PRINT "4. Lines in the Two Point form
150 LOCATE 16,10:PRINT "5. Terminate Program
160 '
170 LOCATE 22,10:INPUT "ENTER YOUR CHOICE ", CMD
180       ' this is an error trap for their choice!
190 WHILE CMD <> FIX(CMD) OR CMD < 1 OR CMD > 5
200    LOCATE 24,10:PRINT "Please enter a 1, 2, 3, 4, or 5"
210    GOTO 170
220 WEND
230 '
240 WHILE CMD <> 5
250    ON CMD GOSUB 330, 470, 920, 1220
260    LOCATE 23,1 :INPUT "Press enter to return to the command menu. ",B$
270    GOTO 80
280 WEND
290 '
300 SCREEN 0
310 END
320 '
330 '**************   SUBROUTINE TO PLOT POINTS     *************
340 CLS
350 GOSUB 1820 'to draw the Cartesian Plane
360 LOCATE 23,1 : INPUT "X,Y ";X,Y
370 '
380 PX = 320 + 40 * X  :  PY = 100 - 20 * Y 'figure pixels and plot
390 CIRCLE (PX,PY),2 : CIRCLE (PX,PY),7      ' the point
400 '
410 LOCATE 23,1 : PRINT "                     "
420 LOCATE 23,1 : INPUT "Another ";A$
430 LOCATE 23,1 : PRINT "                "
440 IF A$ = "Y" OR A$ = "y" THEN 360
450 RETURN
460 '
470 '******************   SUBROUTINE TO EXPLAIN SLOPE   ***************
480 CLS
490 GOSUB 1820 'to draw the graph
500 LOCATE 23,1 : INPUT "Point 1 ";X1,Y1 'enter the two points
510 LOCATE 23,9 : PRINT "        "
520 LOCATE 23,1 : INPUT "Point 2 ";X2,Y2
530 LOCATE 23,9 : PRINT "        "
540 IF X1 > X2 THEN SWAP X1,X2 : SWAP Y1,Y2 'leftmost point should be X1,Y1
550 '
```

```
560 PX1 = 320 + X1 * 40 : PY1 = 100 - Y1 * 20 'figure the pixels
570 PX2 = 320 + X2 * 40 : PY2 = 100 - Y2 * 20 'and plot the two points
580 CIRCLE (PX1,PY1),2  : CIRCLE (PX1,PY1),7
590 CIRCLE (PX2,PY2),2  : CIRCLE (PX2,PY2),7
600 LINE (PX1,PY1)-(PX2,PY2)  :  LINE (PX1+1,PY1)-(PX2+1,PY2) 'connect them
610 '
620 IF Y2 < Y1 THEN SIGN = 1  ELSE  SIGN = -1 'determines pos. or neg. slope
630 '
640 FOR J = 1 TO 500 : NEXT J                        'all J loops are time eaters
650 '
660 FOR I = PY1 TO (PY2-SIGN*20) STEP  SIGN*20 'loop to draw vertical leg
670    LINE (PX1-2,I)-(PX1-2,I+SIGN*20)         'of the triangle
680    LINE (PX1-4,I)-(PX1-4,I+SIGN*20)
690    FOR J = 1 TO 500 : NEXT J
700 NEXT I
710 FOR I = PX1 TO PX2-40 STEP 40     'loop to draw horizontal leg of triangle
720    LINE (I,PY2+1)-(I+40,PY2+1)
730    LINE (I,PY2+2)-(I+40,PY2+2)
740    FOR J = 1 TO 500 : NEXT J
750 NEXT I
760 '
770 LOCATE 23,1 : PRINT "                "
780 DELTAY = Y2 - Y1   :    DELTAX = X2 - X1
790 IF DELTAY = 0 THEN 850
800 IF DELTAX = 0 THEN 870
810 IF DELTAX > 9 THEN NUMLEN = 2 ELSE NUMLEN = 1 'no. of digits to check
820 '
830 LOCATE 23,1 : INPUT "Slope ";M$
840 IF VAL(LEFT$(M$,2)) = DELTAY  AND VAL(RIGHT$(M$,NUMLEN)) = DELTAX THEN 900
          ELSE LOCATE 23,1 : PRINT "         " : GOTO 830
850 LOCATE 23,1 : INPUT "Slope ";M$
860 IF M$ = "0" THEN 900  ELSE LOCATE 23,1 : PRINT "         " : GOTO 850
870 LOCATE 23,1 : INPUT "Slope ";M$
880 IF M$ = "UND" OR M$ = "und"  THEN 900 ELSE LOCATE 23,1 : PRINT "           "
:          GOTO 870 'to let them try again
890 '
900 LOCATE 22,1 : INPUT "Another ";A$
910 IF A$ = "Y" OR A$ = "y"  THEN 470  ELSE RETURN
920 '*****************  SUBROUTINE TO GRAPH SLOPE - INTERCEPT FORMS  ********
930 CLS
940 GOSUB 1820       ' to draw graph
950 GOSUB 1110       ' to input new equation
960 X = -7           ' initial value for X
970 GOSUB 1170       ' figure Y for that X
980 PX1 = 320 + X * 40 : PY1 = 100 - Y * 20 'convert point to pixels
990 '
1000 FOR X = -6 TO 7 STEP .5 ' loop to draw the line
1010    GOSUB 1170 ' to figure Y
1020    PX2 = 320 + X * 40 : PY2 = 100 - Y * 20 'convert point to pixels
1030    IF PY2 > 199 OR PY2 < 1 THEN 1060 'since it's off the screen
1040    LINE (PX1,PY1)-(PX2,PY2)          'draw the line through the points
1050    LINE (PX1,PY1+1)-(PX2,PY2+1)
1060    PX1 = PX2 : PY1 = PY2
1070 NEXT X
1080 LOCATE 23,1 : INPUT "Another ";A$
1090 IF A$ ="Y" OR A$ = "y"  THEN GOTO 950
1100 GOTO 260
1110 '       SUBROUTINE TO INPUT EQUATION
```

```
1120 '
1130 LOCATE 23,1 : PRINT "GOTO 1150       "
1140 LOCATE 22,1 : EDIT 1190
1150 GOSUB 960
1160 '
1170 '      SUBROUTINE TO FIGURE FUNCTIONAL VALUES
1180 '
1190 Y =  1/2*X
1200 RETURN
1210 '
1220 '*****************  SUBROUTINE TO GRAPH TWO - POINT FORMS  ********
1230 '
1240 CLS
1250 GOSUB 1820 'to draw graph
1260 ' the following statement turns on the random number generator and uses
1270 RANDOMIZE VAL(RIGHT$(TIME$,2)) + VAL(MID$(TIME$,4,2)) 'the clock for seed
1280 XVAL1 = INT( RND * 10 ) ' pull out a value, decide if pos. or neg., then
1290 IF XVAL1 / 2 = INT (XVAL1 / 2)  THEN SIGN = 1  ELSE   SIGN = -1 'make sure
1300 IF XVAL1 > 7 THEN 1280                                  'it fits on screen
1310 X1 = XVAL1 * SIGN
1320 '
1330 'the next three blocks repeat procedure for other coordinates
1340 YVAL1 = INT( RND * 10 )
1350 IF YVAL1 / 2 = INT (YVAL1 / 2)  THEN SIGN = -1  ELSE   SIGN =  1
1360 IF YVAL1 > 4 THEN 1340
1370 Y1 = YVAL1 * SIGN
1380 '
1390 XVAL2 = INT( RND * 10 )
1400 IF XVAL2 / 2 = INT (XVAL2 / 2)  THEN SIGN = -1  ELSE   SIGN =  1
1410 IF XVAL2 > 7 THEN 1390
1420 X2 = XVAL2 * SIGN
1430 YVAL2 = INT( RND * 10 )
1440 IF YVAL2 / 2 = INT (YVAL2 / 2)  THEN SIGN =  1  ELSE   SIGN = -1
1450 IF YVAL2 > 4 THEN 1430
1460 Y2 = YVAL2 * SIGN
1470 '
1480 PX1 = 320 + 40 * X1 :  PY1 = 100 - 20 * Y1        ' these plot the two
1490 CIRCLE (PX1,PY1),2 : CIRCLE (PX1,PY1),7 .         ' random points
1500 PX2 = 320 + 40 * X2 :  PY2 = 100 - 20 * Y2
1510 CIRCLE (PX2,PY2),2 : CIRCLE (PX2,PY2),7
1520 '
1530 LOCATE 23,1 : PRINT "GOTO 1550" ' let user input equation
1540 LOCATE 22,1 : EDIT 1790
1550 '
1560 X = -8        ' draw the line that was entered
1570 GOSUB 1770
1580 PX1 = 320 + 40 * X   :    PY1 = 100 - 20 * Y
1590 IF PY1 > 199 OR PY1 < 1 THEN X = X + .1 : GOTO 1570 'make sure it's on the
1600 '
1610 FOR X = -7 TO 7 STEP .5                                  'screen
1620    GOSUB 1770
1630    PX2 = 320 + 40 * X   :    PY2 = 100 - 20 * Y
1640    IF PY2 > 199 OR PY2 < 1 THEN 1670
1650    LINE (PX1,PY1) - (PX2,PY2)
1660    LINE (PX1,PY1+1)-(PX2,PY2+1)
```

```
1670    PX1 = PX2 : PY1 = PY2 'move the points up
1680 NEXT X
1690 '
1700 LOCATE 23,1 : INPUT "Correct "; A$
1710 LOCATE 23,11: PRINT " "
1720 IF A$ <> "Y" AND A$ <> "y" THEN GOTO 1530
1730 LOCATE 23,11: PRINT " "
1740 LOCATE 23,1 : INPUT "Another "; A$
1750 IF A$ = "Y" OR  A$ = "y" THEN GOTO 1230
1760 GOTO 260
1770 '       SUBROUTINE WITH THE EQUATION
1780 '
1790 Y =   M * ( X - H ) + K
1800 RETURN
1810 '
1820 '***************  SUBROUTINE TO DRAW GRAPH  ******************
1830 '
1840 'SCREEN 2
1850 '           first draw the axes
1860 LINE (0,100)-(640,100)      :      LINE (0,101)-(640,101)
1870 LINE (320,0)-(320,200)      :      LINE (321,0)-(321,200)
1880 LOCATE 14,77 : PRINT "X"    :      LOCATE 2,39 : PRINT "Y"
1890 '
1900 '          loop to draw the vertical grid
1910 FOR I = 0 TO 640 STEP 40
1920    LINE (I,0)-(I,200),2
1930 NEXT I
1940 '
1950 '          loop to draw the horizontal grid
1960 FOR I = 0 TO 200 STEP 20
1970    LINE (0,I)-(640,I),2
1980 NEXT I
1990 '
2000 RETURN
```

```
10 '**********  CONIC SECTIONS       MIKE MOSIER  ***********
20 '
30 '
40 '                   STORED UNDER:   CONICS
50 '                       JUNE 30, 1985
60 '
70  CLEAR,,,32768!  :   SCREEN 6  :  COLOR 3,4
80 'SCREEN 2
90 CLS
100 LOCATE 3,12:PRINT "THE PROGRAM CONICS"
110 LOCATE 5,15:PRINT "COMMAND MENU
120 LOCATE  8,10:PRINT "1. Graphing Parabolas
130 LOCATE 10,10:PRINT "2. Graphing Circles
140 LOCATE 12,10:PRINT "3. Graphing Ellipses
150 LOCATE 14,10:PRINT "4. Graphing Hyperbolas
160 LOCATE 16,10:PRINT "5. Terminate Program
170 '
180 LOCATE 22,10:INPUT "ENTER YOUR CHOICE ", CMD
190 WHILE CMD <> FIX(CMD) OR CMD < 1 OR CMD > 5
200    LOCATE 24,10:PRINT "Please enter a 1, 2, 3, 4, or 5"
210    GOTO 180
220 WEND
230 '
240 WHILE CMD <> 5
250    ON CMD GOSUB 320, 630, 1030, 1570
260    LOCATE 23,1 :INPUT "Press enter to return to the command menu. ",B$
270    GOTO 90
280 WEND
290 SCREEN 0
300 END
310 '
320 '**********  SUBROUTINE TO GRAPH PARABOLAS  ***************
330 '
340 GOSUB 2270 ' to draw the X - Y plane
350 '
360 LOCATE 22,1 : PRINT "                                    "
370 LOCATE 22,1 : PRINT "Y = a ( X - h )^2 + k"
380 LOCATE 23,1 : INPUT "a,h,k ";A,H,K
390 IF H < 0 THEN SIGN1$ = "( X +"  ELSE SIGN1$ = "( X -"  'these lines print
400 IF K < 0 THEN SIGN2$ = "- "  ELSE SIGN2$ = "+ "        'out the equation
410 LOCATE 22,1 : PRINT "Y = ";A;SIGN1$;ABS(H);")^2 ";SIGN2$;ABS(K)
420 '
430 X = -15 : Y = A*(X-H)^2 + K ' set the initial value for X, and figure Y
440 PX1 = 320 + 24*X    :    PY1 = 100 - 10*Y       'figure the first pixels
450 '
460 '       loop to graph the parabola
470 FOR X = -15 TO 15 STEP .3
480    Y = A * ( X - H )^2 + K
490    PX2 = 320 + 24*X      :      PY2 = 100 - 10*Y
500    '      test to see if the point is on the screen
510    IF PY2 > 199 OR PY2 < 0 THEN 530
520    LINE (PX1,PY1)-(PX2,PY2)  :  LINE (PX1+1,PY1)-(PX2+1,PY2)
530    PX1 = PX2   :   PY1 = PY2     ' moves old point up to new point position
540 NEXT X
550 '
```

```
560 '       See if they want to graph another
570 LOCATE 23,1 : PRINT "                        "
580 LOCATE 23,1 : INPUT "Another ";A$
590 IF A$ = "y" OR A$ = "Y" THEN LOCATE 23,1 : PRINT "              " : GOTO 360
600 '
610 RETURN
620 '
630 '************  SUBROUTINE TO GRAPH CIRCLES  ***************
640 '
650 GOSUB 2270 'to draw the X - Y plane
660 '
670 LOCATE 22,1 : PRINT "                                            "
680 LOCATE 21,1 : PRINT "                    "
690 ' get the equation of the circle
700 LOCATE 22,1 : PRINT "(X - h)^2 + (Y - k)^2 = r^2"
710 LOCATE 23,1 : INPUT "h,k,r "; H,K,R
720 LOCATE 23,1 : PRINT "                    "
730 IF R < 0 THEN LOCATE 21,1 : PRINT "NO CIRCLE !! " :GOTO 970
740 IF H < 0 THEN FIRST$ = "(X +" ELSE FIRST$ = "(X -"      'next three lines
750 IF K < 0 THEN SECND$ = "(Y +" ELSE SECND$ = "(Y -"      'print it back out
760 LOCATE 22,1 : PRINT FIRST$;ABS(H);")^2 + ";SECND$;ABS(K);")^2 =";R;"^2"
770 '
780    XCENTER = 320 + 24*H  :  YCENTER = 100 - 10*K            'finds the
                                                            pixels of the center
790 CIRCLE (XCENTER,YCENTER),2                                ' mark the center
800 PX1 = XCENTER - R *24  :  PY1 = YCENTER : PY2 = YCENTER     'sets first set
                                                                of pixels
810 '      begin graphing the circle
820 FOR X = -R TO R STEP .15
830    SQ = R^2 - X^2
840    Y = SQR ( SQ )
850    PX2 = XCENTER + 24*X : PY3 = YCENTER - 10 * Y : PY4 = YCENTER +10 * Y
860    IF PY3 > 0   THEN LINE (PX1,PY1)-(PX2,PY3)
870    IF PY4 < 200 THEN LINE (PX1,PY2)-(PX2,PY4)
880    PX1 = PX2 : PY1 = PY3 : PY2 = PY4
890 NEXT X
900 X = R
910 SQ = R^2 - X^2
920 Y = SQR ( SQ )
930 PX2 = XCENTER + 24*X : PY3 = YCENTER - 10 * Y : PY4 = YCENTER +10 * Y
940 IF PY3 > 0   THEN LINE (PX1,PY1)-(PX2,PY3)
950 IF PY4 < 200 THEN LINE (PX1,PY2)-(PX2,PY4)
960 '
970 '     see if they want to graph another
980 LOCATE 23,1 : INPUT "Another ";A$
990 LOCATE 23,1 : PRINT "                   "
1000 IF A$ = "Y" OR A$ = "y" THEN 660
1010 RETURN
1020 '
1030 '************  SUBROUTINE TO GRAPH ELLIPSES  ***************
1040 '
1050  CLS : XCENTER = 320 : YCENTER = 100
1060  LOCATE 10,15 : PRINT "Choose which equation you want. "
1070  LOCATE 12,15 : PRINT "1.   X^2 / a^2  +  Y^2 / b^2  = 1
1080  LOCATE 14,15 : PRINT "2. (X - h)^2 / a^2  +  (Y - k)^2 / b^2  = 1
1090  LOCATE 17,15 : INPUT "CHOICE ";CHC
1100  IF CHC = 2 THEN 1210
1110  GOSUB 2270
```

```
1120 XCENTER = 320 : YCENTER = 100
1130 LOCATE 22,1 : PRINT "                                        "
1140 LOCATE 22,1 : PRINT "X^2 / a^2  + Y^2 / b^2 = 1"
1150 LOCATE 23,1 : PRINT "                         "
1160 LOCATE 23,1 : INPUT "a,b "; A,B
1170 LOCATE 21,1 : PRINT "                    "
1180 IF A < 0 OR B < 0 THEN LOCATE 21,1 : PRINT "Positives only." : GOTO 1150
1190 LOCATE 22,1 : PRINT " X^2 /"A;"^2  +  Y^2 /";B;"^2 = 1"
1200 GOTO 1360
1210 GOSUB 2270
1220 '
1230 LOCATE 22,1 : PRINT "                                        "
1240 LOCATE 22,1 : PRINT "(X - h)^2 / a^2  + (Y - k)^2 / b^2 = 1"
1250 LOCATE 23,5 : PRINT "          "
1260 LOCATE 23,1 : INPUT "h,k "; H,K
1270 LOCATE 23,1 : PRINT "                "
1280 LOCATE 23,1 : INPUT "a,b "; A,B
1290 LOCATE 21,1 : PRINT "                   "
1300 LOCATE 23,1 : PRINT "                "
1310 IF A < 0 OR B < 0 THEN LOCATE 21,1 : PRINT "Positives only." : GOTO 1270
1320 IF H < 0 THEN FRST$ = "(X +" ELSE FRST$ = "(X -"
1330 IF K < 0 THEN SCND$ = "(Y +" ELSE SCND$ = "(Y -"
1340 LOCATE 22,1 : PRINT FRST$;ABS(H);" )^2 /"A;"^2  + ";SCND$;ABS(K);" )^2 /";B
;"^2 = 1
1350 XCENTER = 320 + 24*H : YCENTER = 100 - 10*K
1360 PX1 = XCENTER - 24*A : PY1 = YCENTER :  PY2 = PY1
1370 '     begin graphing the ellipse
1380 FOR X = -15 TO 15 STEP .2
1390    SQ = B^2 - X^2 * B^2/A^2
1400    IF SQ < 0 THEN 1470
1410    Y = SQR ( SQ )
1420    '
1430    PX2 = XCENTER + 24*X  :  PY3 = YCENTER - 10*Y : PY4 = YCENTER + 10*Y
1440    IF PY4 > 199 OR PY3 < 0 THEN 1460
1450    LINE (PX1,PY1)-(PX2,PY3)  :  LINE (PX1,PY2)-(PX2,PY4)
1460    PX1 = PX2 : PY1 = PY3 : PY2 = PY4
1470 NEXT X
1480 '
1490 '    see if they want to graph another
1500 LOCATE 23,8 : PRINT "        "
1510 LOCATE 23,1 : INPUT "Another ";A$
1520 IF A$ = "Y" OR A$ = "y" THEN IF CHC = 1 THEN 1120 ELSE 1220
1530 LOCATE 23,1 : INPUT "Other form ";A$
1540 IF A$ = "Y" OR A$ = "y" THEN IF CHC = 2 THEN 1110 ELSE 1210
1550 RETURN
1560 '
1570 '************  SUBROUTINE TO GRAPH HYPERBOLAS ***************
1580 '
1590 CLS
1600 YSIGN = -1
1610 LOCATE 10,15 : PRINT "Choose which equation you want. "
1620 LOCATE 12,15 : PRINT "1.  X^2 / a^2  -  Y^2 / b^2  = 1
1630 LOCATE 14,15 : PRINT "2. -X^2 / a^2  +  Y^2 / b^2  = 1
1640 LOCATE 17,15 : INPUT "CHOICE ";CHC
1650 IF CHC = 2 THEN YSIGN =  1 : GOTO 1760
1660 GOSUB 2270 'to draw the X - Y plane
```

```
1670 ' get the equation of the HYPERBOLA
1680 LOCATE 22,1 : PRINT "                    "
1690 LOCATE 22,1 : PRINT "X^2 / a^2  - Y^2 / b^2 = 1"
1700 LOCATE 23,1 : PRINT "            "
1710 LOCATE 23,1 : INPUT "a,b "; A,B
1720 LOCATE 21,1 : PRINT "           "
1730 IF A <= 0 OR B <= 0 THEN LOCATE 21,1 : PRINT "Positives only." : GOTO 1700
1740 LOCATE 22,1 : PRINT " X^2 /"A;"^2  -  Y^2 /";B;"^2 = 1"
1750 GOTO 1850
1760 '
1770 GOSUB 2270
1780 LOCATE 22,1 : PRINT "                    "
1790 LOCATE 22,1 : PRINT "- X^2 / a^2  + Y^2 / b^2 = 1"
1800 LOCATE 23,1 : PRINT "            "
1810 LOCATE 23,1 : INPUT "a,b "; A,B
1820 LOCATE 21,1 : PRINT "           "
1830 IF A <= 0 OR B <= 0 THEN LOCATE 21,1 : PRINT "Positives only." : GOTO 1800
1840 LOCATE 22,1 : PRINT "- X^2 /"A;"^2  +  Y^2 /";B;"^2 = 1"
1850 RBOX = 320 + 24*A :LBOX = 640 - RBOX :TBOX = 100 - 10*B :BBOX = 100 + 10*B
1860 FOR I = LBOX TO RBOX-10 STEP 20
1870   LINE (I,TBOX+1)-(I+10,TBOX+1)    : LINE (I,BBOX-1)-(I+10,BBOX-1)
1880 NEXT I
1890 FOR J = TBOX TO BBOX-5 STEP 10
1900   LINE (RBOX-1,J)-(RBOX-1,J+5)    : LINE (LBOX+1,J)-(LBOX+1,J+5)
1910 NEXT J
1920 M = ( 2 * B )/( 2 * A )          'slope of the asymptote
1930 FOR X = -15 TO 15                'following loops draw asymptotes
1940   Y = M*X
1950   PX1 = 320 + 24*X : PY1 = 100 - 10*Y
1960   X2 = X+.5 : Y2 = M*X2
1970   PX2 = 320 + 24*X2 : PY2 = 100 - 10*Y2
1980   LINE (PX1,PY1)-(PX2,PY2)
1990 NEXT X
2000 FOR X = -15 TO 15
2010   Y = -M*X
2020   PX1 = 320 + 24*X : PY1 = 100 - 10*Y
2030   X2 = X+.5 : Y2 = -M*X2
2040   PX2 = 320 + 24*X2 : PY2 = 100 - 10*Y2
2050   LINE (PX1,PY1)-(PX2,PY2)
2060 NEXT X
2070 PX1 = 320 - 24*15 :  BPY1 = 210  :   TPY1 = -10
2080 '      begin graphing the hyperbola
2090 FOR X = -15 TO 15 STEP .2
2100   IF YSIGN = -1 THEN IF X > -A AND X < A  THEN TPY2 = 100 : BPY2 = 100 :
                  PX2 = 320 + 24*A :  GOTO 2180
2110   SQ = B^2*X^2/A^2 + YSIGN * B^2
2120   IF SQ < 0 THEN 2180
2130   Y = SQR ( SQ )
2140   '
2150   PX2 = 320 + 24*X : BPY2 = 100 + 10*Y : TPY2 = 100 - 10*Y
2160   IF BPY2 > 199 OR TPY2 < 0 THEN 2180
2170   LINE (PX1,TPY1)-(PX2,TPY2)  :  LINE (PX1,BPY1)-(PX2,BPY2)
2180   PX1 = PX2 : TPY1 = TPY2 : BPY1 = BPY2
2190 NEXT X
2200 '
2210 '    see if they want to graph another
```

```
2220 LOCATE 23,1 : PRINT "                  "
2230 LOCATE 23,1 : INPUT "Another ";A$
2240 IF A$ = "Y" OR A$ = "y" THEN 1590
2250 RETURN
2260 '
2270 '*************   SUBROUTINE TO DRAW GRAPH PAPER   **************
2280 '
2290 CLS         ' draw the axes first
2300 LINE (0,100)-(640,100)         :       LINE (0,101)-(640,101)
2310 LINE (320,0)-(320,200)         :       LINE (321,0)-(321,200)
2320 LOCATE 14,77 : PRINT "X"       :       LOCATE 2,39 : PRINT "Y"
2330 '
2340 '             loop to draw the vertical grid
2350 FOR I = 8  TO 640 STEP 24
2360    LINE (I,0)-(I,200),2
2370 NEXT I
2380 '
2390 '             loop to draw the horizontal grid
2400 FOR I = 0 TO 200 STEP 10
2410    LINE (0,I)-(640,I),2
2420 NEXT I
2430 '
2440 LOCATE 14,77 : PRINT "X"       :       LOCATE 2,39 : PRINT "Y"
2450 '
2460 RETURN
```

```
10 '**********  FUNCTIONS              MIKE MOSIER  *************
20 '
30 '              STORED UNDER : FUNCTION
40 '                MAY 15, 1984
50 '
60 CLS
70  CLEAR,,,32768! : SCREEN 6 : COLOR 3,4
80 'SCREEN 2
90 A = -20 : B = -20 : P(1) = -20  'flag values for points of discont
100 '
110 LOCATE 10,1 : PRINT "Move the cursor to the right of the equals and make cha
nges.
120 LOCATE 11,1 : PRINT "Then, press enter twice."
130 LOCATE 15,1 : PRINT "GOTO 150"
140 LOCATE 14,1 : EDIT 650
150 LOCATE 17,1 : INPUT "Is f(X) continuous ";A$
160 IF A$ = "Y" OR  A$ = "y" THEN 270
170 LOCATE 17,1 : INPUT "Any intervals where f(X) is undefined ";A$
180 LOCATE 17,1 : PRINT "                                         "
190 IF A$ <>"Y" AND A$ <>"y" THEN 210
200 LOCATE 17,1 : INPUT "a,b ";A,B
210 LOCATE 17,1 : INPUT "Number of points where undefined ";N
220 LOCATE 17,1 : PRINT "                                         "
230 FOR I = 1 TO N
240    LOCATE 16,1 : PRINT "Point ";I : LOCATE 16,9 : INPUT P(I)
250    LOCATE 16,9 : PRINT "       "
260 NEXT I
270 CLS
280 LOCATE 10,20 : PRINT "Choose the scale for the X axis."
290 LOCATE 12,25 :PRINT "1. -16 to 16
300 LOCATE 14,25 :PRINT "2. -10 to 10
310 LOCATE 16,25 :PRINT "3.  -5 to 5
320 LOCATE 18,25 :PRINT "4.  -1 to 1
330 LOCATE 20,25 :INPUT "CHOICE ";CHOICE
340 '
350 IF CHOICE = 1 THEN SCALE = 20 : X = -16  : INC = .2   'INC is the increment
360 IF CHOICE = 2 THEN SCALE = 32 : X = -10  : INC = .1   'for the graphing loop
370 IF CHOICE = 3 THEN SCALE = 58 : X = -5   : INC = .05 'SCALE is the pixels
380 IF CHOICE = 4 THEN SCALE = 198: X = -1   : INC = .005'per unit X
390 '
400 CLS
410 GOSUB 690 'to create the coordinate system
420 '
430 '       begin graphing the function
440 IF X >= A AND X <= B  THEN PX1 = 320+SCALE*B : X = B 'since discont.
450 GOSUB 640
460 PX1 = 320 + SCALE * X   :   PY1 = 100 - SCALE/2 * Y 'first set of pixels
470 XX = X 'store initial X
480 K = 1
490 FOR X = XX TO -XX  STEP INC
500    IF X >= A AND X <= B  THEN PX1 = 320+SCALE*B : GOTO 580 'check for dis-
510    IF P(K) = X THEN : K = K + 1 : GOTO 580               'continuities
520    GOSUB 640
530    PX2 = 320 + SCALE * X : PY2 = 100 - SCALE/2 * Y
540    IF PY2 > 199 OR PY2 < 0 THEN 570
550    IF PY1 > 199 OR PY1 < 0 THEN 570
```

```
560    LINE (PX1,PY1)-(PX2,PY2) : LINE (PX1+1,PY1)-(PX2+1,PY2)
570    PX1 = PX2 : PY1 = PY2
580 NEXT X
590 '
600 LOCATE 23,1 : INPUT "Another ";A$
610 IF A$ = "Y" OR A$ = "y" THEN GOTO 10
620 CLS
630 END
640 '******  SUBROUTINE CONTAINING THE FUNCTION  ********
650 Y =  (X^2 + 4 ) / (X^2 -16)
660 RETURN
670 '
680 '
690 '***********  SUBROUTINE TO DRAW THE AXES  ********
700 '
710 LINE (0,100)-(640,100)  :  LINE (0,101)-(640,101)
720 LINE (320,0)-(320,200)  :  LINE (321,0)-(321,200)
730 LOCATE 2,39 : PRINT "Y" :  LOCATE 13,77 : PRINT "X"
740 FOR I = 0 TO 320 STEP SCALE
750    LINE (320+I,0)-(320+I,200) :  LINE (320-I,0)-(320-I,200)
760 NEXT I
770 '
780 FOR I = 0 TO 100 STEP SCALE/2
790    LINE (0,100+I)-(640,100+I) : LINE (0,100-I)-(640,100-I)
800 NEXT I
810 '
820 RETURN
```

```
10 '********** TRIGONOMETRY, LAW OF SINES  ***********
20 '                    MIKE MOSIER
30 '
40 SCREEN 1 : CLS
50 COLOR 4,7
60 CLS
70 LOCATE 6,10 : PRINT "PROGRAM SINELAW"
80 LOCATE 10,15 : PRINT "CHOICES
90 LOCATE 12,5  : PRINT "1. TWO ANGLES AND A SIDE
100 LOCATE 13,5  : PRINT "2. TWO SIDES AND AN OPPOSITE ANGLE
110 LOCATE 14,5  : PRINT "3. TERMINATE PROGRAM
120 LOCATE 18,5  : INPUT "ENTER CHOICE ", CHOICE
130 '
140 WHILE CHOICE <> 3
150    ON CHOICE GOSUB 210,1250
160    GOTO 60
170 WEND
180 SCREEN 0 : WIDTH 80
190 END
200 '
210 '********  SUBROUTINE TO DRAW 2 ANGLES & A SIDE  ********
220 '
230 PI = 3.1415927#
240 CLS
250 LOCATE 5,15: PRINT "   "                'input user's data
260 LOCATE 5,5 : INPUT "1st ANGLE ", ALPHA
270 IF ALPHA > 90  THEN  LOCATE 6,5 : PRINT "Must be <= 90" : GOTO 250
280 LOCATE 7,5 : INPUT "2nd ANGLE ", BETA
290 LOCATE 9,5 : INPUT "SIDE      ", SIDEA
300 '
310 CLS
320 LINE(20,125)-(300,125)   'draw side c
330 '
340 ALPH2 = 90 - ALPHA        'complement of ALPHA for finding XB below
350 AL2RAD = ALPH2 * (PI / 180) : ALRAD = ALPHA * (PI / 180) 'find all angles
360 BETA2 = 90 - BETA                                        'and convert to radians
370 BETRAD = BETA * (PI / 180)  : BET2RD = BETA2 * (PI / 180)
380 GAMMA = 180 - ( ALPHA + BETA ) : GAMRAD = GAMMA * (PI / 180)
390 '
400 SIDEB = SIN( BETRAD ) * ( SIDEA / SIN ( ALRAD ))
410 SIDEC = SIN( GAMRAD ) * ( SIDEA / SIN ( ALRAD ))
420 YB = SIDEB*SIN(ALRAD)  :  PYB = 125 - 14*YB     'XB,YB are coordinates of
430 XB = SIDEB*SIN(AL2RAD) :  PXB = 20 + 18*XB      'top vertex
440 IF PYB > 5 AND PXB < 310 AND PXB > 10 THEN 460 ELSE LOCATE 10,15 : PRINT "Tr
iangle won't fit on the screen."
450 LOCATE 12,15 : INPUT "Please press enter.",A$ : GOTO 240
460 TXB = (125/14) * TAN(AL2RAD)  : PTXB = 18*TXB +20  'current pixels for top
470 LINE (PTXB,0)-(PXB,PYB) : LINE (PXB,PYB)-(20,125)  'end of side b
480 LOCATE 17,2 :PRINT ALPHA
490 YA = SIDEA*SIN(BETRAD) : XA = SIDEA*SIN(BET2RD)
500 PXA = 300 - XA*18  :   PYA = 125 - 14*YA  'pixels for top of side A
510 IF PXA > PXB THEN 530 ELSE LOCATE 10,15 : PRINT "Triangle won't fit on the s
creen."
520 LOCATE 12,15 : INPUT "Please press enter.",A$ : GOTO 240
530 LINE (300,125)-(PXA,PYA) 'draw side a
540 '
550 CTEN = CINT(10*(SIDEA / SIN(ALRAD) *  SIN(GAMRAD)))
```

```
560 SIDEC = CTEN / 10     'this avoids internal round off errors that exist
570 PC = INT(SIDEC * 18 + 20)    ' PC is pixel for right end of SIDEC
580 LOCATE 17,36 : PRINT BETA
590 LOCATE 13,36 : PRINT SIDEA
600 LOCATE 21,5  : INPUT "Press enter to form the triangle. ",A$
610 LOCATE 21,5  : PRINT "                                    "
620 LOCATE 17,36 : PRINT "    "
630 LOCATE 13,36 : PRINT "    "
640 '   the following loop shortens bottom side until it equals SIDEC
650 FOR I = 0 TO 200 STEP 1
660    IF 300-I = PC THEN GOTO 710
670    LINE (300-I,125)-(PXA,PYA),0
680    LINE (300-I-1,125) - (PXA-1,PYA)
690    PXA = PXA - 1    'moves top of side a into new position
700 NEXT I
710 '
720 LINE (PXB,PYB)-(PTXB,0),0 'erase uneeded portion of side b
730 '
740 MIDX = (PXA + 300 - I) / 2  : MIDY = (125 + PYA) / 2   'begin labelling
750 LOCATE .12*MIDY,.12*MIDX+3 : PRINT SIDEA               'the triangle
760 '
770 PXC = .12*((PC+20)/2)
780 IF PXC < 1 THEN PXC = 1 ELSE IF PXC > 36 THEN  PXC = 36
790 LOCATE 17,PXC : PRINT "c"
800 LOCATE 17,.12*PC+2 : PRINT BETA
810 LOCATE 22,28
820 '
830 LOCATE .12*PYB-1,.12*PXB+1 : PRINT "C"
840 LCBX = .12*((PXB+20)/2)-2 : LCBY = .12*((125+PYB)/2)
850 IF LCBX < 1 THEN LCBX = 1 : IF LCBY < 1 THEN LCBY = 1
860 LOCATE LCBY,LCBX : PRINT "b"
870 '
880 'following block prints the law of sines for side b
890 LOCATE 20,6 : PRINT "b           "; SIDEA
900 LOCATE 21,1 : PRINT "__________  = __________"
910 LOCATE 23,1 : PRINT "  sin ";BETA;"     sin ";ALPHA
920 '
930 LOCATE 21,27 : PRINT "         "
940 LOCATE 21,28 : INPUT "b = ";B
950 IF 10*B <> CINT(10*((SIDEA*SIN(BETRAD)) / SIN(ALRAD)))  THEN GOTO 930
960     ' if answer for B is correct
970 IF LCBX-2 < 0 THEN LCBX = 3
980 LOCATE LCBY,LCBX-2 : PRINT B  'place value on the triangle
990 LOCATE 20,6 : PRINT "                     "
1000 LOCATE 21,1 : PRINT "                             "
1010 LOCATE 23,1 : PRINT "                              "
1020 LOCATE 21,28: PRINT "          "
1030 '
1040 LOCATE 21,24 : PRINT "        "
1050 LOCATE 21,10 : INPUT "Angle C = "; G
1060 IF G <> GAMMA THEN GOTO 1040
1070 LOCATE .12*PYB-1,.12*PXB+1 : PRINT G
1080 LOCATE 21,10 : PRINT "                    "
1090 LOCATE 20,6 : PRINT "c           ";SIDEA
1100 LOCATE 21,1 : PRINT "___________  = __________"
1110 LOCATE 23,1 : PRINT "  sin ";GAMMA;"     sin ";ALPHA
```

```
1120 '
1130 LOCATE 21,30 : PRINT "         "
1140 LOCATE 21,28 : INPUT "c = ";C
1150 IF C <> SIDEC THEN GOTO 1130
1160 '
1170 LOCATE 17,PXC : PRINT USING " ##.#";SIDEC
1180 LOCATE 20,6 : PRINT "                     "
1190 LOCATE 21,1 : PRINT "                             "
1200 LOCATE 23,1 : PRINT "                              "
1210 LOCATE 21,28: PRINT "          "
1220 LOCATE 21,5 : PRINT "The triangle is solved !"
1230 LOCATE 23,10 : INPUT "Press enter for menu. ",A$
1240 RETURN
1250 '************  SUBROUTINE FOR  2  SIDES AND AN ANGLE  *************
1260 '
1270 CLS : PI = 3.1415927# : SUB = 0
1280 LOCATE 10,10 : INPUT "ALPHA  = ",ALPHA
1290 LOCATE 12,10 : INPUT "SIDE a = ",SIDEA
1300 LOCATE 14,10 : INPUT "SIDE b = ", SIDEB
1310 CLS
1320 LINE (20,125) - (300,125)
1330 ALPHZ = 90 - ALPHA
1340 ALZRAD = ALPHZ * (PI / 180) : ALRAD = ALPHA * (PI / 180)
1350 SINBTRD = SIN(ALRAD) * SIDEB / SIDEA  :  GOSUB 2480 'to compute BETA
1360 GAMMA = 180 - ( ALPHA + BETA ) : GAMRAD = GAMMA * PI/180
1370 SIDEC = SIN(GAMRAD) * SIDEA / SIN(ALRAD)
1380 HEIGHT = SIN(ALRAD) * ( SIDEB )
1390 PYB = 125 - (14*HEIGHT)
1400 PXB = 20 + (18* SIN(ALZRAD) * SIDEB)
1410 LINE(20,125)-(PXB,PYB)
1420 LOCATE 17,1 : PRINT ALPHA
1430 LCBX = .12*(PXB+20)/2 - 1 : LCBY = .12*(PYB+125) / 2 - 1
1440 LOCATE LCBY,LCBX : PRINT SIDEB
1450 LCAX = .12*(PXB)+5 : LCAY = .12*(PYB)
1460 LOCATE LCAY,LCAX : PRINT SIDEA
1470 LCCY = .12*PYB-1 : IF LCCY < 1 THEN LCCY = 1
1480 LCCX = .12*PXB
1490 LOCATE LCCY,LCCX : PRINT "C"
1500 '
1510 PYA1 = PYB + (14 * SIN(.08) * SIDEA)
1520 PXA1 = PXB + (18 * SIN( PI/2 - .08 ) * SIDEA)
1530 LINE (PXB,PYB)-(PXA1,PYA1)
1540 LOCATE 21,3 : INPUT "Press enter to form the triangle(s).",A$
1550 LOCATE 21,3 : PRINT "                                      "
1560 LOCATE LCAY,LCAX : PRINT "  "                              "
1570 FOR G = .1 TO 2.6 STEP .02   'loop increments the angle of side a off hor.
1580    PYA2 = PYB + (14 * SIN(G) * SIDEA)  'figure new endpoints for side a
1590    PXA2 = PXB + (18 * SIN( PI/2 - G ) * SIDEA)
1600    LINE (PXB,PYB) - (PXA1,PYA1),0      'erase old side and draw new one
1610    LINE (PXB,PYB) - (PXA2,PYA2)
1620    LINE (20,125) - (300,125)         ' fill in accidentally erased points
1630    IF ABS(BETRD - G) >= .01  AND ABS((PI-BETRD) - G ) >= .01  THEN 1700
1640    '   if not, form a triangle
1650    SUB = SUB + 1 'sub counts the number of triangles
1660    IF SIDEA = SIDEB AND SUB > 1 THEN SUB = 1 : GOTO 1700 'isosceles tri.
1670    IF SIDEA > SIDEB AND SUB > 1 THEN SUB = 1 : GOTO 1700 'side A > side B
```

```
1680    BEEP                   'store the pixels for end of side a in arrays
1690    LINE (PXB,PYB)-(PXA1,PYA1),2 : PYA(SUB) = PYA1 : PXA(SUB) = PXA1
1700    PXA1 = PXA2 : PYA1 = PYA2    'delete old value with new
1710    IF PYA(SUB) <> 0 THEN LINE (PXB,PYB)-(PXA(SUB),PYA(SUB)),2
1720 NEXT G
1730 '
1740 LOCATE 20,1 : INPUT "How many triangles are possible ";NUM
1750 IF NUM <> SUB THEN BEEP : GOTO 1740
1760 IF NUM=0 THEN LOCATE 22,10:PRINT "CORRECT !!":FOR I=1 TO 1000:NEXT :RETURN
1770 LOCATE 20,1 : PRINT "                                       "
1780 LOCATE 20,1 : INPUT "Press enter to look at first triangle. ",A$
1790 LOCATE 20,1 : PRINT "                                        "
1800 K = 1
1810 ' next line erases end af arc and second blue line
1820 LINE (PXB,PYB)-(PXA1,PYA1),0 : LINE (PXB,PYB)-(PXA(2),PYA(2)),0
1830 LINE(20,125)-(PXB,PYB) ' redraw side B
1840 LINE (PXB,PYB)-(PXA(1),PYA(1))
1850 LINE (PXA(1),125)-(300,125),0 'erases extra part of bottom side
1860 FOR I = PYB TO 120 STEP 10 : LINE (PXB,I)-(PXB,I+5),2 : NEXT I'draws height

1870 '
1880 LOCATE 17,1 : PRINT ALPHA     'label the triangle
1890 LOCATE 17,.12*PXA(K)+1 : PRINT "B"
1900 LCAX = .12*(PXB+PXA(K))/2 + 2 : LCAY = .12*(PYB+125)/2 -1
1910 LOCATE LCAY,LCAX : PRINT SIDEA
1920 PXC = .12*(PXA(K)+20)/2 : LOCATE 17,PXC : PRINT "c"
1930 LOCATE 20,4 : PRINT "sin B         sin ";ALPHA
1940 '   begin solving the triangle
1950 LOCATE 21,1 : PRINT "__________  = __________"
1960 LOCATE 23,1 : PRINT "    ";SIDEB;"           ";SIDEA
1970 '
1980 LOCATE 21,30 : PRINT "         "
1990 LOCATE 21,28 : INPUT "B = ";B
2000 IF B <> BETA THEN BEEP: GOTO 1980
2010 LOCATE 17,.12*PXA(K)+1 : PRINT BETA
2020 '
2030 LOCATE 20,4 : PRINT "                       "
2040 LOCATE 21,1 : PRINT "                        "
2050 LOCATE 23,1 : PRINT "                        "
2060 LOCATE 21,28 : PRINT "            "
2070 '
2080 LOCATE 21,14 : PRINT "         "
2090 LOCATE 21,5 : INPUT "Angle C = ";C
2100 IF CINT(10*C) <> CINT(10*GAMMA) THEN BEEP : GOTO 2080
2110 LOCATE LCCY,LCCX : PRINT C
2120 '
2130 LOCATE 21,5 : PRINT "              "
2140 '
2150 LOCATE 20,4 : PRINT "  c             ";SIDEA
2160 LOCATE 21,1 : PRINT "__________  = __________"
2170 LOCATE 23,1 : PRINT " sin ";C  ;"     SIN ";ALPHA
2180 '
2190 LOCATE 21,30 : PRINT "         "
2200 LOCATE 21,28 : INPUT "c = ";SDCEE
2210 SIDEC = CINT( 10 * SIN(GAMRAD) * SIDEA / SIN(ALRAD)) / 10
2220 IF CINT(10*SDCEE) <> CINT(10*SIDEC) THEN BEEP: GOTO 2190
2230 LOCATE 17,PXC : PRINT USING "##.#";SIDEC
```

```
2240 LOCATE 20,4 : PRINT "                        "
2250 LOCATE 21,1 : PRINT "                         "
2260 LOCATE 23,1 : PRINT "                         "
2270 LOCATE 21,28 : PRINT "             "
2280 '
2290 LOCATE 21,5 : PRINT "This triangle is solved !"
2300 IF SUB = 1 THEN LOCATE 23,5 : INPUT "Press enter for command menu.",A$
     : RETURN
2310 IF K = 2 THEN LOCATE 23,5 : INPUT "Press enter for command menu.",A$
   : RETURN
2320 LOCATE 23,5 : INPUT "Press enter for the second one.",A$
2330 '
2340 K = 2    ' signals that second triangle is solved
2350 LOCATE 17,.12*PXA(1)+1: PRINT "     "
2360 LOCATE LCCY,LCCX : PRINT "       "
2370 LOCATE 17,PXC : PRINT "       "
2380 LOCATE LCAY,LCAX : PRINT "       "
2390 LINE (PXB,PYB)-(PXA(1),PYA(1)),0
2400 LINE (PXB,PYB)-(PXA(2),PYA(2))
2410 LOCATE 21,5 : PRINT "                          "
2420 LOCATE 23,5 : PRINT "                                   "
2430 '
2440 BETA = 180 - BETA : GAMMA = 180 - (ALPHA + BETA) 'compute data for 2nd tri.

2450 BTRD = BETA * PI / 180  :  GAMRAD = GAMMA * PI / 180
2460 GOTO 1890
2470 RETURN
2480 '****************  SUBROUTINE TO FIGURE INVERSE SINE  ****************
2490 INC = .01     'the closer the SINE is to 1, the more accurate the increment
2500 LOCATE 10,5 : PRINT "Computing, please stand by ..."
2510 IF SINBTRD >= .7  THEN B = .77   : INC = .007    'must be
2520 IF SINBTRD > 1 THEN BETRD = 5   : GOTO 2880
2530 IF SINBTRD >= .8  THEN B = .92   : INC = .006
2540 IF SINBTRD >= .95 THEN B = 1.25  : INC = .003
2550 IF SINBTRD >= .99 THEN B = 1.41  : INC = .0008
2560 IF SINBTRD >= .9999 THEN B = 1.54  : INC = .0001
2570 '
2580 'each successive loop narrows in the angle whose sine is equal to SINBTRD
2590 FOR I = B TO 1.58 STEP .01
2600    IF ABS( SIN(I) - SINBTRD ) < 5*INC THEN GOTO 2620
2610 NEXT I
2620 IF ABS(SIN(I) - SINBTRD) < 1E-08 THEN BETRD = I : GOTO 2860
2630 FOR J = I-.05 TO I+.1  STEP .005
2640    IF ABS( SIN(J) - SINBTRD ) < INC       THEN GOTO 2660
2650 NEXT J
2660 IF ABS(SIN(J) - SINBTRD) < 1E-08 THEN BETRD = J : GOTO 2860
2670 FOR K = J-.01  TO J+.05   STEP .001
2680    IF ABS( SIN(K) - SINBTRD ) < INC/5  THEN 2700
2690 NEXT K
2700 IF ABS(SIN(K) - SINBTRD) < 1E-08 THEN BETRD = K : GOTO 2880
2710 FOR L = K-.005  TO K+.01   STEP .0005
2720    IF ABS( SIN(L) - SINBTRD ) < INC/10   THEN 2740
2730 NEXT L
2740 IF ABS(SIN(L) - SINBTRD) < 1E-08 THEN BETRD = L : GOTO 2880
2750 FOR M = L-.001  TO L+8.999999E-03  STEP .0001
2760    IF ABS( SIN(M) - SINBTRD ) < INC/50  THEN 2780
2770 NEXT M
2780 IF ABS(SIN(M) - SINBTRD) < 1E-08 THEN BETRD = M : GOTO 2880
2790 FOR N = M-.0005  TO M +.007  STEP .00005
```

```
2800    IF ABS( SIN(N) - SINBTRD ) < INC/500 THEN 2820
2810 NEXT N
2820 IF ABS(SIN(N) - SINBTRD) < 1E-08 THEN BETRD = N : GOTO 2880
2830 FOR P = N-.0001  TO N +.002  STEP .00001
2840    IF ABS( SIN(P) - SINBTRD ) < INC/1000 THEN 2860
2850 NEXT P
2860 '
2870 BETRD = P
2880 BETA = BETRD * (180/3.1415927#)
2890 BETA = CINT(BETA*10) / 10
2900 '
2910 LOCATE 10,5 : PRINT "                                "
2920 RETURN
```

```
10 '**********  GRAPHING TRIG FUNCTIONS    MIKE MOSIER  ***********
20 '
30 '
40 '                    STORED UNDER:   TRIGRAF
50 '                        JUNE 15, 1985
60 '
70 CLEAR,,,32768! : SCREEN 6 : COLOR 3,4
80 'SCREEN 2
90 CLS
100 LOCATE 15,10:INPUT "Please press CAPS LOCK if not previously done, then ENTE
R. ",A$
110 CLS
120 LOCATE 5,15:PRINT "COMMAND MENU
130 LOCATE 8,10:PRINT "1. Graph of Sine Function
140 LOCATE 10,10:PRINT "2. Graph of Cosine Function
150 LOCATE 12,10:PRINT "3. Graph of Tangent Function
160 LOCATE 14,10:PRINT "4. Graph of Cotangent Function
170 LOCATE 16,10:PRINT "5. Graph of Secant Function
180 LOCATE 18,10:PRINT "6. Graph of Cosecant Function
190 LOCATE 20,10:PRINT "7. Terminate Program
200 '
210 LOCATE 22,10:INPUT "ENTER YOUR CHOICE ", CMD
220        ' this is an error trap for their choice!
230 WHILE CMD <> FIX(CMD) OR CMD < 1 OR CMD > 7
240    LOCATE 24,10:PRINT "Please enter a 1, 2, 3, 4, 5, 6, or 7
250    GOTO 210
260 WEND
270 '
280 WHILE CMD <> 7
290    ON CMD GOSUB 370,710,1040,1460,2050,2630
300    LOCATE 23,1 :INPUT "Press enter to return to the command menu. ",B$
310    GOTO 110
320 WEND
330 '
340 SCREEN 0
350 END
360 '
370  '***********   SUBROUTINE TO GRAPH SINE WAVES  *************
380 '
390 CLS
400 LOCATE 10,10 : PRINT "In general, we'll be graphing equations of the
410 LOCATE 11,10 : PRINT "form   Y = a SIN b ( X - c )
420 LOCATE 13,10 : PRINT "When you see the number 690, followed by an equation,
430 LOCATE 14,10 : PRINT "please EDIT that EQUATION as desired, then press enter
 twice.
440 LOCATE 17,10 : INPUT "Press enter to continue. ",A$
450 '
460 CLS : GOSUB 3200
470 LOCATE 23,1 : PRINT "GOTO 490         "
480 LOCATE 22,1 : EDIT 690
490 LOCATE 23,1 : PRINT "                           "
500 PI = 3.1415927#
510 '
520 '     begin graphing their equation
530 X = -PI :  GOSUB 690 'to figure the Y value for the initial X of -PI
540 PX1 = 150 + 40 * X    :  PY1 = 100 - 20 * Y   'compute first PIXELS
550 FOR X = -PI TO 3*PI STEP .05
```

```
560    GOSUB 690
570    PX2 = 150 + 40*X  :  PY2 = 100 - 20 * Y
580    IF PY2 > 199 OR PY2 < 0 THEN GOTO 600
590    LINE (PX1,PY1)-(PX2,PY2)
600    PX1 = PX2  :  PY1 = PY2
610 NEXT X
620 '
630 LOCATE 23,1 : PRINT "                          "
640 LOCATE 23,1 : INPUT "Another ";A$
650 IF A$ = "Y" OR A$ = "y"  THEN 460
660 GOTO 300 'can't exit subroutine with RETURN after the above program break
670 '
680 '   subroutine containing the equation
690 Y = 1  * SIN( 1 * ( X + 0     ))
700 RETURN
710  '***********   SUBROUTINE TO GRAPH COSINE WAVES   *************
720 '
730 CLS
740 LOCATE 10,10 : PRINT "In general, we'll be graphing equations of the
750 LOCATE 11,10 : PRINT "form   Y = a COS b ( X - c )
760 LOCATE 13,10 : PRINT "When you see the number 1010, followed by an equation,

770 LOCATE 14,10 : PRINT "please EDIT that EQUATION as desired, then press enter
 twice.
780 LOCATE 17,10 : INPUT "Press enter to continue. ",A$
790 '
800 CLS : GOSUB 3200 'to draw the axes
810 LOCATE 23,1 : PRINT "GOTO 830          "
820 LOCATE 22,1 : EDIT 1020
830 LOCATE 23,1 : PRINT "                                   "
840 PI = 3.1415927#
850 '      begin graphing their equation
860 X = -PI :  GOSUB 1020 'to figure the Y value
870 PX1 = 150 + 40 * X   :  PY1 = 100 - 20 * Y
880 FOR X = -PI TO 4*PI STEP .1
890    GOSUB 1020
900    PX2 = 150 + 40*X  :  PY2 = 100 - 20 * Y
910    IF PY2 > 199 OR PY2 < 0 THEN GOTO 930
920    LINE (PX1,PY1)-(PX2,PY2)
930    PX1 = PX2  :  PY1 = PY2
940 NEXT X
950 '
960 LOCATE 23,1 : PRINT "                          "
970 LOCATE 23,1 : INPUT "Another ";A$
980 IF A$ = "Y" OR A$ = "y"  THEN 800
990 '
1000 GOTO 300
1010 ' subroutine to contain the function
1020 Y =  1 * COS( 1 * ( X - 0     ))
1030 RETURN
1040  '***********   SUBROUTINE TO GRAPH TANGENT FUNCTION   *************
1050 '
1060 CLS
1070 LOCATE 10,10 : PRINT "In general, we'll be graphing equations of the
1080 LOCATE 11,10 : PRINT "form   Y = a TAN b ( X - c )
1090 LOCATE 13,10 : PRINT "When you see the number 1340, followed by an equation
,
1100 LOCATE 14,10 : PRINT "please EDIT that EQUATION as desired, then press ente
r twice.
1110 LOCATE 17,10 : INPUT "Press enter to continue. ",A$
1120 '
```

```
1130 CLS : GOSUB 3200
1140 LOCATE 23,1 : PRINT "GOTO 1160         "
1150 LOCATE 22,1 : EDIT 1360
1160 LOCATE 23,1 : PRINT "                             "
1170 PI = 3.1415927#
1180 '     begin graphing their equation
1190 X = -PI :  GOSUB 1360 'to figure the Y value
1200 PX1 = 150 + 40 * X    :  PY1 = 100 - 20 * Y
1210 FOR X = -PI TO 4*PI STEP .05
1220    GOSUB 1360
1230    IF ABS(Y) > 15  THEN GOSUB 1380 ' to construct an asymptote
1240    PX2 = 150 + 40*X   :  PY2 = 100 - 20 * Y
1250    IF PY2 > 199 OR PY2 < 0 THEN GOTO 1270
1260    LINE (PX1,PY1)-(PX2,PY2)
1270    PX1 = PX2   :   PY1 = PY2
1280    COUNT = 0
1290 NEXT X
1300 '
1310 LOCATE 23,1 : PRINT "                  "
1320 LOCATE 23,1 : INPUT "Another ";A$
1330 IF A$ = "Y" OR A$ = "y"  THEN 1130
1340 GOTO 300
1350 '
1360 Y =  1 * TAN( 3 * ( X - PI/2 ))
1370 RETURN
1380 '    Asymptote Subroutine
1390 COUNT = COUNT + 1 : IF COUNT > 1 THEN RETURN
1400 FOR I = 0 TO 200 STEP 10
1410    PX = 150 + 40*X
1420    LINE (PX,I)-(PX,I+5)
1430 NEXT I
1440 RETURN
1450 '
1460  '***********   SUBROUTINE TO GRAPH COTANGENT FUNCTION  ************
1470 '
1480 CLS
1490 LOCATE 10,10 : PRINT "In general, we'll be graphing equations of the
1500 LOCATE 11,10 : PRINT "form   Y = a COT b ( X - c )
1510 LOCATE 13,10 : PRINT "When you see a,b ? enter those two numbers, with the
comma.
1520 LOCATE 14,10 : PRINT "When you see c ? enter the value of c, then press ent
er.
1530 LOCATE 15,10 : PRINT "When entering c, use PI anyplace that value is desire
d."
1540 LOCATE 16,10 : PRINT "For example;  c ? -PI/2 would be accepted. "
1550 LOCATE 18,10 : INPUT "Press enter to continue. ",A$
1560 '
1570 CLS : GOSUB 3200
1580 SIGN = 1 : PI = 3.1415927#
1590 LOCATE 22,1 : PRINT "Y = a COT b( X - c )
1600 LOCATE 23,1 : INPUT "a,b ";A,B
1610 LOCATE 23,1 : PRINT "                  "
1620 LOCATE 23,1 : INPUT "c ";C$
1630 LOCATE 23,1 : PRINT "                  "
1640 '
1650 IF C$ = "0" THEN C = 0 : GOTO 1700       'the following block determines
1660 IF LEFT$(C$,1) = "-" THEN SIGN = -1      '    the shift from the string C$
1670 IF C$ = "PI" OR C$ = "-PI" THEN GOTO 1690
```

```
1680 C = SIGN * PI / (VAL(RIGHT$(C$,1))) : GOTO 1700
1690 C = PI * SIGN
1700 IF SIGN = -1 THEN SIGN$ = "( X + "  ELSE SIGN$ = "( X - "
1710 LNGTH = LEN(C$) : IF SIGN = -1 THEN LNGTH = LNGTH - 1
1720 LOCATE 22,1 : PRINT "Y =";A;"COT";B;SIGN$;RIGHT$(C$,LNGTH);")"
1730 '
1740 '      begin graphing their equation
1750 X = - PI             'initialize X, then get first pixel point
1760 COT = TAN ( B * ( X - C )) : IF COT = 0 THEN X = X + .1 : GOTO 1760
1770 Y = A* 1 / COT
1780 PX1 = 150 + 40 * X    :  PY1 = 100 - 20 * Y
1790 FOR X = -PI TO 4*PI STEP .08
1800    IF TAN ( B*(X-C)) = 0 THEN 1890
1810    Y = A * ( 1 / TAN ( B * ( X - C )))
1820    IF ABS(Y) > 15  THEN COUNT = COUNT + 1 :GOSUB 1980 ' to draw asymptote
1830    PX2 = 150 + 40*X   :  PY2 = 100 - 20 * Y
1840    IF PY2 > 199 OR PY2 < 0 THEN GOTO 1880
1850    IF PY1 > 199 OR PY1 < 0 THEN 1880
1860    LINE (PX1,PY1)-(PX2,PY2)
1870    COUNT = 0
1880    PX1 = PX2  :  PY1 = PY2
1890 NEXT X
1900 '
1910 LOCATE 23,1 : PRINT "                              "
1920 LOCATE 23,1 : INPUT "Another ";A$
1930 IF A$ = "Y" OR A$ = "y"  THEN 1570
1940 GOTO 300
1950 '
1960 Y =  1 / TAN( 1 * ( X - 0     ))
1970 RETURN
1980 '     Asymptote Subroutine
1990 IF COUNT > 1 THEN RETURN
2000 FOR I = 0 TO 200 STEP 10
2010    PX = 150 + 40*X
2020    LINE (PX,I)-(PX,I+5)
2030 NEXT I
2040 RETURN
2050  '***********   SUBROUTINE TO GRAPH SECANT FUNCTION   *************
2060 '
2070 CLS
2080 LOCATE 10,10 : PRINT "In general, we'll be graphing equations of the
2090 LOCATE 11,10 : PRINT "form    Y = a SEC b ( X - c )
2100 LOCATE 13,10 : PRINT "When you see a,b ? enter those two numbers, with the
comma.
2110 LOCATE 14,10 : PRINT "When you see c ? enter the value of c, then press ent
er.
2120 LOCATE 15,10 : PRINT "When entering c, use PI anyplace that value is desire
d."
2130 LOCATE 16,10 : PRINT "For example;  c ? -PI/2 would be accepted. "
2140 LOCATE 18,10 : INPUT "Press enter to continue. ",A$
2150 '
2160 CLS : GOSUB 3200
2170 SIGN = 1 : PI = 3.1415927#
2180 LOCATE 22,1 : PRINT "Y = a SEC b( X - c )
2190 LOCATE 23,1 : INPUT "a,b ";A,B
2200 LOCATE 23,1 : PRINT "                          "
2210 LOCATE 23,1 : INPUT "c ";C$
2220 LOCATE 23,1 : PRINT "                          "
```

```
2230 '
2240 IF C$ = "Ø" THEN C = Ø : GOTO 229Ø        'the following block determines
225Ø IF LEFT$(C$,1) = "-" THEN SIGN = -1        '   the shift from the string C$
226Ø IF C$ = "PI" OR C$ = "-PI" THEN GOTO 228Ø
227Ø C = SIGN * PI / (VAL(RIGHT$(C$,1))) : GOTO 229Ø
228Ø C = PI * SIGN
229Ø IF SIGN = -1 THEN SIGN$ = "( X + "  ELSE SIGN$ = "( X - "
23ØØ LNGTH = LEN(C$) : IF SIGN = -1 THEN LNGTH = LNGTH - 1
231Ø LOCATE 22,1 : PRINT "Y =";A;"SEC";B;SIGN$;RIGHT$(C$,LNGTH);")"
232Ø '
233Ø '     begin graphing their equation
234Ø X = - PI          'initialize X, then get first pixel point
235Ø CSE = COS ( B * ( X - C )) : IF CSE = Ø THEN X = X + .1 : GOTO 235Ø
236Ø Y = A* 1 / CSE
237Ø PX1 = 15Ø + 4Ø * X   :  PY1 = 1ØØ - 2Ø * Y
238Ø FOR X = -PI TO 4*PI STEP .Ø5
239Ø    IF COS ( B*(X-C)) = Ø THEN 248Ø
24ØØ    Y = A * ( 1 / COS ( B * ( X - C )))
241Ø    IF ABS(Y) > 18  THEN COUNT = COUNT + 1 :GOSUB 256Ø ' to draw asymptote
242Ø    PX2 = 15Ø + 4Ø*X  :  PY2 = 1ØØ - 2Ø * Y
243Ø    IF PY2 > 199 OR PY2 < Ø THEN GOTO 247Ø
244Ø    IF PY1 > 199 OR PY1 < Ø THEN 247Ø
245Ø    LINE (PX1,PY1)-(PX2,PY2)
246Ø    COUNT = Ø
247Ø    PX1 = PX2  :  PY1 = PY2
248Ø NEXT X
249Ø '
25ØØ LOCATE 23,1 : PRINT "                   "
251Ø LOCATE 23,1 : INPUT "Another ";A$
252Ø IF A$ = "Y" OR A$ = "y"  THEN 216Ø
253Ø GOTO 3ØØ
254Ø '
255Ø RETURN
256Ø '    Asymptote Subroutine
257Ø IF COUNT > 1 THEN RETURN
258Ø FOR I = Ø TO 2ØØ STEP 1Ø
259Ø    PX = 15Ø + 4Ø*X
26ØØ    LINE (PX,I)-(PX,I+5)
261Ø NEXT I
262Ø RETURN
263Ø  '***********   SUBROUTINE TO GRAPH COSECANT FUNCTION  *************
264Ø '
265Ø CLS
266Ø LOCATE 1Ø,1Ø : PRINT "In general, we'll be graphing equations of the
267Ø LOCATE 11,1Ø : PRINT "form   Y = a CSC b ( X - c )
268Ø LOCATE 13,1Ø : PRINT "When you see a,b ? enter those two numbers, with the
comma.
269Ø LOCATE 14,1Ø : PRINT "When you see c ? enter the value of c, then press ent
er.
27ØØ LOCATE 15,1Ø : PRINT "When entering c, use PI anyplace that value is desire
d."
271Ø LOCATE 16,1Ø : PRINT "For example;  c ? -PI/2 would be accepted. "
272Ø LOCATE 18,1Ø : INPUT "Press enter to continue. ",A$
273Ø '
274Ø CLS : GOSUB 32ØØ
275Ø SIGN = 1 : PI = 3.1415927#
276Ø LOCATE 22,1 : PRINT "Y = a CSC b( X - c )
277Ø LOCATE 23,1 : INPUT "a,b ";A,B
```

```
2780 LOCATE 23,1 : PRINT "                           "
2790 LOCATE 23,1 : INPUT "c ";C$
2800 LOCATE 23,1 : PRINT "                           "
2810 '
2820 IF C$ = "0" THEN C = 0 : GOTO 2870         'the following block determines
2830 IF LEFT$(C$,1) = "-" THEN SIGN = -1        '   the shift from the string C$
2840 IF C$ = "PI" OR C$ = "-PI" THEN GOTO 2860
2850 C = SIGN * PI / (VAL(RIGHT$(C$,1))) : GOTO 2870
2860 C = PI * SIGN
2870 IF SIGN = -1 THEN SIGN$ = "( X + "  ELSE SIGN$ = "( X - "
2880 LNGTH = LEN(C$) : IF SIGN = -1 THEN LNGTH = LNGTH - 1
2890 LOCATE 22,1 : PRINT "Y =";A;"CSC";B;SIGN$;RIGHT$(C$,LNGTH);")"
2900 '      begin graphing their equation
2910 X = - PI            'initialize X, then get first pixel point
2920 SNE = SIN ( B * ( X - C )) : IF SNE = 0 THEN X = X + .1 : GOTO 2920
2930 Y = A* 1 / SNE
2940 PX1 = 150 + 40 * X    :  PY1 = 100 - 20 * Y
2950 FOR X = -PI TO 4*PI STEP .05
2960    IF SIN ( B*(X-C)) = 0 THEN 3050
2970    Y = A * ( 1 / SIN ( B * ( X - C )))
2980    IF ABS(Y) > 18  THEN COUNT = COUNT + 1 :GOSUB 3130 ' to draw asymptote
2990    PX2 = 150 + 40*X  :  PY2 = 100 - 20 * Y
3000    IF PY2 > 199 OR PY2 < 0 THEN GOTO 3040
3010    IF PY1 > 199 OR PY1 < 0 THEN 3040
3020    LINE (PX1,PY1)-(PX2,PY2)
3030    COUNT = 0
3040    PX1 = PX2  :  PY1 = PY2
3050 NEXT X
3060 '
3070 LOCATE 23,1 : PRINT "                            "
3080 LOCATE 23,1 : INPUT "Another ";A$
3090 IF A$ = "Y" OR A$ = "y"  THEN 2740
3100 GOTO 300
3110 '
3120 RETURN
3130 '     Asymptote Subroutine
3140 IF COUNT > 1 THEN RETURN
3150 FOR I = 0 TO 200 STEP 10
3160    PX = 150 + 40*X
3170    LINE (PX,I)-(PX,I+5)
3180 NEXT I
3190 RETURN
3200 '***************  SUBROUTINE TO DRAW GRAPH  *****************
3210 '
3220 'SCREEN 2
3230 '             first draw the axes
3240 LINE (0,100)-(640,100)     :     LINE (0,101)-(640,101)
3250 LINE (150,0)-(150,200)     :     LINE (151,0)-(151,200)
3260 LOCATE 14,77 : PRINT "X"   :     LOCATE 2,18 : PRINT "Y"
3270 '
3280 '          mark off units of PI / 2  on the X axis
3290 PI = 3.1415927#
3300 FOR X = -(PI) TO (4*PI) STEP (PI/2)
3310    I = 150 + 40*X
3320    LINE (I,97)-(I,103)   :   LINE (I+1,97)-(I+1,103)
```

```
3330 NEXT X
3340 '
3350 '      mark units of 1 on the Y axis
3360 FOR Y = - 5 TO 5
3370    I = 100 - 20*Y
3380     LINE (145,I)-(155,I)    :    LINE (145,I+1)-(155,I+1)
3390  NEXT Y
3400 '
3410 RETURN
3420 '
```

```
10 '********** POLAR COORDINATES PROGRAM **************
20 '               BY MIKE MOSIER
30 '
40 '            STORED UNDER:   POLAR
50 '                JUNE  1985
60 '
70 CLEAR,,,32768!  :  SCREEN 6 : COLOR 3,4
80 'SCREEN 2
90 '
100 CLS
110 '    create the menu
120 LOCATE 10,30 : PRINT "COMMAND MENU"
130 LOCATE 12,20 : PRINT "1.   PLOT INDIVIDUAL POINTS IN POLAR COORDINATES"
140 LOCATE 14,20 : PRINT "2.   GRAPH A FUNCTION IN POLAR COORDINATES"
150 LOCATE 16,20 : PRINT "3.   EXIT PROGRAM
160 '
170 LOCATE 20,20 : INPUT "Enter your choice. ",CHOICE
180 WHILE CHOICE <> 3
190    ON CHOICE GOSUB 250, 450
200    GOTO 60
210 WEND
220 '
230 SCREEN 0 : CLS
240 END
250 '********  SUBROUTINE TO PLOT POINTS  ***********
260 '
270 CLS
280 GOSUB 890
290 LOCATE 21,1 : PRINT "ENTER POINT"
300 LOCATE 22,3 : PRINT "(   ,   )"
310 LOCATE 22,4 : INPUT "",P,THETA
320 LOCATE 21,1 : PRINT "           "
330 LOCATE 22,3 : PRINT "        "
340 LOCATE 22,4 : PRINT "         "
350 THET = THETA * PI/180
360 X = P * COS (THET)  :  Y = P * SIN (THET)
370 PX1 = 300 + 20 * X  :  PY1 = 100 - 8.399999 * Y
380 CIRCLE(PX1,PY1),2 : CIRCLE(PX1,PY1),3 : CIRCLE(PX1,PY1),5
390 '
400 LOCATE 22,1 : INPUT "Another point ";A$
410 LOCATE 22,1 : PRINT "                    "
420 IF A$ = "Y" OR A$ = "y" THEN 290
430 '
440 RETURN
450 '***********  SUBROUTINE TO GRAPH POLAR FUNCTIONS  *********8
460 '
470 CLS
480 LOCATE 10,1 : INPUT "Will your equation include a square root "; ROOT$
490 IF ROOT$ = "Y" OR ROOT$ = "y" THEN 540
500 LOCATE 10,1 : PRINT "Make any desired change in the following equation"
510 LOCATE 11,1 : PRINT "then hit ENTER TWICE. ( or default through )
520 LOCATE 16,1 : PRINT "GOTO 590"
530 LOCATE 15,1 : EDIT 830
540 CLS : LOCATE 10,1 : PRINT "Let PSQR be equal to P squared, and type in"
550 LOCATE 12,1 : PRINT "your equation. "
```

```
56Ø LOCATE 15,1 : PRINT "GOTO 58Ø "
57Ø LOCATE 14,1 : EDIT 85Ø
58Ø ROOT$ = "Y"
59Ø CLS:LOCATE 15,1 : INPUT "Theta will begin at Ø.  Enter stopping point as PI,
 2PI, 3PI, ... "; DONE$
6ØØ IF DONE$ = "PI" OR DONE$ = "pi" THEN DONE = 3.1415927# : GOTO 62Ø
61Ø DONE = VAL(LEFT$(DONE$,1))* 3.1415927#
62Ø GOSUB 89Ø
63Ø GOSUB 8ØØ
64Ø PX1 = 3ØØ + 2Ø*P   :  PY1 = 1ØØ
65Ø FOR THETA = Ø TO DONE  STEP  .Ø5
66Ø    GOSUB 8ØØ
67Ø    X = P * COS (THETA)  :  Y = P * SIN (THETA)
68Ø    PX2 = 3ØØ + 2Ø * X   :  PY2 = 1ØØ - 8.399999 * Y
69Ø    LINE ( PX1,PY1) - ( PX2,PY2 )
7ØØ    LINE ( PX1,PY1+1) - ( PX2,PY2+1 )
71Ø    LINE ( PX1+1,PY1+1) - ( PX2+1,PY2+1 )
72Ø    PX1 = PX2  :  PY1 = PY2
73Ø NEXT THETA
74Ø '
75Ø LOCATE 23,1 : INPUT "Another equation ";A$
76Ø LOCATE 23,1 : PRINT "                          "
77Ø IF A$ = "Y" OR A$ = "y" THEN GOSUB 45Ø
          ELSE  GOTO 6Ø
78Ø '
79Ø '
8ØØ '**********   SUBROUTINE TO GIVE FUNCTIONAL VALUES
81Ø '
82Ø IF ROOT$ = "Y" OR ROOT$ = "y" THEN 85Ø
83Ø P = 1Ø*SIN (3*THETA)
84Ø RETURN
85Ø PSQR = -4Ø* COS ( 3*THETA)
86Ø IF PSQR < Ø THEN P = Ø : RETURN
87Ø P = SQR( PSQR )
88Ø RETURN
89Ø '************  POLAR COORDINATES GRAPH PAPER  ***************
9ØØ '
91Ø PI = 3.1415927#
92Ø CLS
93Ø '
94Ø FOR I = Ø TO 22Ø  STEP 2Ø
95Ø    CIRCLE(3ØØ,1ØØ),I,2
96Ø NEXT I
97Ø '
98Ø LINE (55,1ØØ)-(545,1ØØ),2
99Ø LINE (3ØØ,Ø )-(3ØØ,2ØØ),2
1ØØØ  '
1Ø1Ø FOR ANGLE = Ø TO 36Ø STEP 3Ø
1Ø2Ø    ANG = ANGLE * PI/18Ø
1Ø3Ø    X = 12 * COS(ANG)  :  Y = 12 * SIN(ANG)
1Ø4Ø    PX1 = 3ØØ + 2Ø*X : PY1 = 1ØØ -  8.399999 * Y
1Ø5Ø    LINE ( PX1,PY1 ) - ( 3ØØ,1ØØ),2
1Ø6Ø NEXT ANGLE
1Ø7Ø '
1Ø8Ø FOR ANGLE = 45 TO 315 STEP 45
1Ø9Ø    ANG = ANGLE * PI/18Ø
11ØØ    X = 12 * COS(ANG)  :  Y = 12 * SIN(ANG)
111Ø    PX1 = 3ØØ + 2Ø*X : PY1 = 1ØØ -  8.399999 * Y
112Ø    LINE ( PX1,PY1 ) - ( 3ØØ,1ØØ),2
113Ø NEXT ANGLE
114Ø '
115Ø RETURN
```

```
10 '**********  LIMITS INSTRUCTION    MIKE MOSIER  ***********
20 '
30 '                       STORED UNDER:   LIMITS
40 '                          JULY 30, 1984
50 '
60 CLEAR,,,32768! : SCREEN 6 : COLOR 3,4
70 'SCREEN 2
80 CLS
90 LOCATE 3,15:PRINT "PROGRAM LIMITS
100 LOCATE 5,15:PRINT "COMMAND MENU  (Note that choice 2. must be made before 3
or 4)
110 LOCATE 8,10:PRINT "1. Explanation of Limits
120 LOCATE 10,10:PRINT "2. Input a Function
130 LOCATE 12,10:PRINT "3. Draw and Figure an Epsilon for a given Delta
140 LOCATE 14,10:PRINT "4. List a table of values as X approaches given number
150 LOCATE 16,10:PRINT "5. Terminate Program
160 '
170 LOCATE 22,10:INPUT "ENTER YOUR CHOICE ", CMD
180 WHILE CMD > 2 AND CMD < 5
190    IF TRAP = 2 THEN 230
200    LOCATE 24,10:PRINT "You must choose 2. before 3 or 4"
210    GOTO 170
220 WEND
230 WHILE CMD <> FIX(CMD) OR CMD < 1 OR CMD > 5
240    LOCATE 24,10:PRINT "Please enter a 1, 2, 3, 4, or 5"
250    GOTO 170
260 WEND
270 '
280 WHILE CMD <> 5
290    ON CMD GOSUB 370, 1180, 1420, 2330
300    LOCATE 23,10:INPUT "Press enter to return to the command menu. ",B$
310    GOTO 80
320 WEND
330 '
340 SCREEN 0
350 END
360 '
370 '**************  SUBROUTINE TO GIVE EXPLANATION  *************
380 '
390 CLS : COUNT = 0 : VRTSHFT = 2
400 LOCATE 5,10: PRINT "When talking about the limit of a function,"
410 LOCATE 6,10: PRINT "we mean that as we get closer to some value of X,"
420 LOCATE 7,10: PRINT "the function gets closer to some specific value.
430 LOCATE 9,10: PRINT "This 'specific value ', if it exists, is called the"
440 LOCATE 10,10: PRINT "LIMIT of the Function, or f(x), as x approaches"
450 LOCATE 11,10: PRINT "a given value.  In the following example, we'll let"
460 LOCATE 12,10: PRINT "X get closer and closer to 5, knowing that f(5) = 3."
470 LOCATE 14,10: PRINT "If the graph closes in on 3, as x closes in on 5, then
480 LOCATE 15,10: PRINT "3 must be the limit of f(x) as x approaches 5."
490 LOCATE 20,10: INPUT "Press enter for an example. ",A$
500 '
510 CLS
520 COUNT = COUNT + 1                    'count times through this block
530 LINE (1,150) - (630,150)             'draw the axes and label them
540 LINE (75,1) - (75,165)
550 LINE (76,1) - (76,165)
```

```
560 PX3 = .12 * 75   :LOCATE 2,PX3:PRINT "Y"
570 LOCATE 20,77:PRINT "X"
580 '
590 X = 1    : GOSUB 1160 'to figure Y
600 PX1 = 75 + 48 * X  :  PY1 = 150 - 24* Y
610 X = 9   : GOSUB 1160
620 PX4 = 75 + 48 * X  :  PY4 = 150 - 24* Y
630 HASH = 75 + 48*5                          'pixel for the dotted line to X
640 PX3 = .125 * HASH :LOCATE 21,PX3:PRINT 5
650 LIM = 150 - 24*3                          'pixel for the dotted line to y
660 PX3 = .13  * LIM :LOCATE PX3,5 : PRINT 3
670 FOR I = 150 TO LIM   STEP -10 : LINE(HASH,I)-(HASH,I+3),1 : NEXT I
680 FOR I = 75 TO HASH  STEP  20 : LINE(I-5,LIM)-(I,LIM),1 : NEXT I
690 PSET(HASH,LIM):PSET(HASH+1,LIM):PSET(HASH+1,LIM+1):PSET(HASH,LIM+1)
700 '
710 LOCATE 23,15 : INPUT "Press enter to continue.",A$
720 LOCATE 23,15 : PRINT "                              "
730 FOR I = 0 TO 3.9 STEP .05
740    X = 1 + I : GOSUB 1160                 '} approach XL from the left
750    PX2 = 75 + 48 * X  :  PY2 = 150 - 24 * Y
760    IF I < 2 THEN 810
770    LINE(65,PY1)-(65,PY2)                  '}  these draw the y-arrows
780    LINE(65,PY1-10)-(65,PY1-5),0           '}
790    PRESET(63,PY1-1) : PRESET(67,PY1-1)    '}
800    PSET(63,PY2-1) : PSET(67,PY2-1)        '}
810    PRESET(PX1-3,154) : PRESET(PX1-3,156)  '}  these draw the x-arrows
820    PSET(PX2-3,154) : PSET(PX2-3,156)      '}
830    LINE (PX1-15,155)-(PX1-10,155),0       '}
840    LINE(PX1,155)-(PX2,155)                '}
850 '
860    LINE(PX1,PY1)-(PX2,PY2)                '}  now drawing the graph
870    LINE(PX1,PY1+1)-(PX2,PY2+1)
880    PX1 = PX2 :PY1 = PY2                   '}  prepare for the next I
890    '
900    X = 9 - I : GOSUB 1160                 '} approach XL from the right
910    PX3 = 75 + 48 * X  :  PY3 = 150 - 24 * Y
920    IF I < 2 THEN 970
930    LINE(65,PY4)-(65,PY3)                  '} documentation applies as
940    LINE(65,PY4+10)-(65,PY4+5),0           '}   above
950    PRESET(63,PY4+1) : PRESET(67,PY4+1)
960    PSET(63,PY3+1) : PSET(67,PY3+1)
970    PRESET(PX4+3,154) : PRESET(PX4+3,156)
980    PSET(PX3+3,154) : PSET(PX3+3,156)
990    LINE(PX4+15,155)-(PX4+10,155),0
1000    LINE(PX4,PY4)-(PX3,PY3)
1010    LINE(PX4,PY4+1)-(PX3,PY3+1)
1020    LINE(PX4,155)-(PX3,155)
1030    PX4 = PX3  :  PY4 = PY3
1040    '
1050 NEXT I
1060 LINE (75,PY1)-(PX1,PY1),1
1070 LINE (75,PY3)-(PX3,PY3),1
1080 LINE (PX1,150)-(PX1,PY1),1
1090 LINE (PX3,150)-(PX3,PY3),1
1100 E = ABS( Y - 3 )
1110 LOCATE 22,10 : PRINT "Bringing X within .1 of 5, brought f(X) within "
```

```
1120 LOCATE 22,57 : PRINT USING "#.##";E;:PRINT " distance of 3."
1130 VRTSHFT = 2.5 : IF COUNT = 2 THEN GOTO 1150
1140 LOCATE 23,10 : INPUT "Press enter for a different function.",A$ : GOTO 510
1150 RETURN
1160 Y = 1/20 * (X-1) * (X-6) * (X-10) + VRTSHFT
1170 RETURN
1180 '************  SUBROUTINE TO INPUT FUNCTION AND DECIDE AXES  ******
1190 '
1200 CLS
1210 LOCATE 5,10:PRINT "I have written on the screen below, the number
1220 LOCATE 6,10:PRINT "2790, followed by Y = ( some function of X)
1230 LOCATE 8,10:PRINT "The cursor is at the start of that line, so if
1240 LOCATE 9,10:PRINT "that's not the function you want, move the cursor
1250 LOCATE 10,10:PRINT "to the right of the equals sign, and type your
1260 LOCATE 11,10:PRINT "function, then hit ENTER twice."
1270 LOCATE 16,10:PRINT "GOTO 1300 "
1280 '
1290 LOCATE 15,10:EDIT 2930
1300 CLS:LOCATE 10,10 : PRINT "Do you want to enter an interval for the graph.
1310 LOCATE 11,10 : INPUT "Enter Y for yes, or press ENTER to default to [ 0,10
]. ",A$
1320 IF A$ = "" THEN A = 0 : B = 10 : GOTO 1360
1330 CLS
1340 LOCATE 11,10:PRINT "Enter the Left and Right endpoints or your interval
1350 LOCATE 12,10:INPUT "as A,B ",A,B
1360 A = 0 : B = 10 : SCALE = 600 / ( B - A )
1370 YAXE = 640 - B*SCALE                             ' pixel the Y-Axis is at
1380 '
1390 TRAP = 2  'this is an error trap making sure the user picks 2 before 3-4
1400 '
1410 GOTO 300
1420 '**************  SUBROUTINE TO DRAW EPSILON - DELTA STUFF *************
1430 CLS
1440 LOCATE  8,10:INPUT "At what value of X do you want to test for a limit"; XL

1450 CLS
1460 GOSUB 2570 'to draw the graph
1470 LOCATE 2,15:PRINT "               "   ' to clear the line first
1480 LOCATE 2,10:INPUT "EPSILON = "; EPSLN
1490 LOCATE 2,10:PRINT "                 "
1500 '
1510 WHILE EPSLN < 0 OR EPSLN > 1      'error trap
1520   LOCATE 2,20:PRINT "PLEASE KEEP IT BETWEEN  0 AND 1 !!"
1530   GOTO 1470
1540 WEND
1550 '
1560 '                                    XL and YL are X & Y at the limit
1570 X = XL : GOSUB 2920 : YL = Y      ' find YL for the following statements
1580 PEPUP = 150 - SCALE/2 * (YL + EPSLN)      'pixel for upper epsilon line
1590 PEPDO = 150 - SCALE/2 * (YL - EPSLN)      'pixel for lower epsilon line
1600 PLIM = 150 - SCALE/2 * YL                 'pixel for limit epsilon line
1610 '
1620 X = X + .5 : GOSUB 2920
1630 IF Y > YL THEN INC = -1
1640 PY1 = PLIM  :  X = XL
1650 WHILE  ABS(PY1 - PEPUP) > 1
1660   IF INC THEN X = X + .04 ELSE X = X - .04
1670   GOSUB 2920 'to figure Y for this X
```

```
1680    PY1 = 15Ø - SCALE/2 * Y
1690    IF PY1 < Ø OR PY1 > 15Ø THEN X = Ø  : PY1 = PEPUP
1700 WEND
1710 DELT1 = (X - XL)     'this will be the delta to the right of X
1720 X = XL               'set X back to the middle
1730  '
1740 PY1 = PLIM
1750 WHILE ABS( PY1 - PEPDO ) >  1
1760    IF INC THEN X = X - .Ø4 ELSE X = X + .Ø4
1770 '  IF  PY1 < PEPDØ THEN PY1 = PEPDO : GOTO 1820
1780    GOSUB 2920 'to figure Y for this X
1790 '   IF INC THEN IF Y < PREDY THEN DELT2 = Ø : GOTO 1830
1800    PY1 = 15Ø - SCALE/2 * Y
1810    IF PY1 > 15Ø OR PY1 < Ø THEN X = Ø  : PY1 = PEPDO
1820 WEND
1830 DELT2 = (XL - X)     'this will be the delta to the left of X
1840 IF ABS(DELT1) <= ABS(DELT2)  THEN DELTA = ABS(DELT1)
                                  ELSE DELTA = ABS(DELT2) 'take the small
1850 '
1860 '         Now find the functional values for ( X - d ) & ( X + d )
1870 X = XL - DELT2
1880 GOSUB 2920
1890 PX1 = YAXE + SCALE * X : PY1 = 15Ø - SCALE/2 * Y
1900 X = XL + DELT1
1910 GOSUB 292Ø
1920 PX2 = YAXE + SCALE * X : PY2 = 15Ø - SCALE/2 * Y
1930 PXL = YAXE + SCALE * XL: PYL = 15Ø - SCALE/2 * YL
1940 LINE (PXL,15Ø) - (PXL,PYL)            'drawing the LIMIT lines
1950 LINE (PXL-8,PYL) - (YAXE,PYL)
1960 LX3 = .125 * PXL
1970 LOCATE 2Ø,LX3: PRINT XL
1980 '
1990 LINE (PX1-8,PY1) - (YAXE,PY1)         'drawing the EPSILON lines
2000 LINE (PX2-8,PY2) - (YAXE,PY2)
2010 LX3 = .125 * PYL       : LX4 = .125 * YAXE - 1
2020 LOCATE LX3,LX4-2 : PRINT "f(x)
2030 '
2040 X = XL + DELTA : GOSUB 292Ø
2050 PX3 = YAXE + SCALE * X : PY3 = 15Ø - SCALE/2 * Y
2060 X = XL - DELTA : GOSUB 2920
2070 PX4 = YAXE + SCALE * X : PY4 = 15Ø - SCALE/2 * Y
2080 FOR I = PY3 TO 145 STEP 1Ø : LINE (PX3,I) - (PX3,I+5) : NEXT I
2090 FOR J = PY4 TO 145 STEP 1Ø : LINE (PX4,J) - (PX4,J+5) : NEXT J
2100 LINE (PX1,15Ø) - (PX1,PEPDO)           'drawing the DELTA lines
2110 LINE (PX2,15Ø) - (PX2,PEPUP)
2120 FOR I = YAXE TO PX3 STEP 1Ø : LINE(I,PY3)-(I+5,PY3) : NEXT I
2130 FOR J = YAXE TO PX4 STEP 1Ø : LINE(J,PY4)-(J+5,PY4) : NEXT J
2140 '
2150 LOCATE 22,1Ø:PRINT "For  e = ";EPSLN;: PRINT USING "  d = "+"#.##"; DELTA
2160 LOCATE 23,4Ø : PRINT "        "
2170 LOCATE 23,1Ø:INPUT "NEW EPSILON ( return to stop )"; EPSLN
2180 '
2190 IF EPSLN = Ø THEN 23ØØ
2200   LINE (PX1,15Ø) - (PX1,PY1),Ø
2210   LINE (PX4,15Ø) - (PX4,PY4),Ø
2220   LINE (PX3,15Ø) - (PX3,PY3),Ø           'erasing delta lines
2230   LINE (PX2,15Ø) - (PX2,PY2),Ø           'erasing delta lines
```

```
2240   LINE (PX3-8,PY3) - (YAXE,PY3),0
2250   LINE (PX1-8,PY1) - (YAXE,PY1),0
2260   LINE (PX4-8,PY4) - (YAXE,PY4),0          'erasing epsilon lines
2270   LINE (PX2-8,PY2) - (YAXE,PY2),0     'erasing epsilon lines
2280   GOTO 1500
2290 '
2300 LOCATE 23,10:INPUT "Do you want to pick another value for X ";A$
2310 IF MID$(A$,1,1) = "Y" OR MID$(A$,1,1) = "y" THEN 1420
2320 RETURN
2330 '************** SUBROUTINE TO LIST VALUES AS X IS APPROACHED ****
2340 '
2350 CLS
2360 LOCATE 10,10:PRINT "At what value of X do you want to test for a limit ?
2370 LOCATE 12,20:INPUT "X = ",XL
2380 CLS
2390 PRINT "    FROM THE LEFT";TAB(35) "    FROM THE RIGHT"
2400 PRINT
2410 PRINT " X            f(X)";TAB(25)"¦";TAB(35)"¦";" f(X)            X"
2420 PRINT "_______________________ ¦";TAB(35)"¦";"____________________________
2430 '
2440 I = 1
2450 WHILE (XL + I) >= (XL + 9.999999E-06)
2460    LEFT = XL - I
2470    RIGHT = XL + I
2480    X = LEFT : GOSUB 2920 'to find Y for this X
2490    PRINT LEFT,Y;TAB(25)"¦";TAB(35)"¦";
2500    X = RIGHT  :GOSUB 2920
2510    PRINT Y; TAB(50) RIGHT
2520    I = I/2
2530 WEND
2540 '
2550 RETURN
2560 LOCATE 22,10:INPUT "Do you want to try another EPSILON ";A$
2570 '***********   SUBROUTINE TO DRAW THE GRAPH  ************
2580 '
2590 CLS
2600 LINE (1,150) - (630,150)            'FIRST DRAW THE AXES AND LABEL THEM
2610 LINE (YAXE,1) - (YAXE,165)
2620 LINE (YAXE+1,1) - (YAXE+1,165)
2630 PX3 = .12 * YAXE :LOCATE 2,PX3:PRINT "Y"
2640 LOCATE 20,77:PRINT "X"
2650 '
2660 '      Find the first set of pixels and draw that boundary
2670 X = A    : GOSUB 2920
2680 PX1 = YAXE + SCALE * X  :  PY1 = 150 - SCALE/2 * Y
2690 LINE (PX1,PY1) - (PX1,150)
2700 LINE (PX1+1,PY1) - (PX1+1,150)
2710 PX3 = .12*PX1 + 2
2720 LOCATE 20,PX3: PRINT A
2730 '
2740 '        Graph the function
2750 FOR I = A TO B    STEP .15
2760    X = I : IF X = 0 THEN 2840
2770    GOSUB 2920
2780    PX2 = YAXE + SCALE * X   :  PY2 = 150 - SCALE/2 * Y
2790    IF PY2 < 0 OR PY2 > 199 THEN 2830
```

```
28ØØ    IF PY1 < Ø OR PY1 > 199 THEN 283Ø
281Ø    LINE (PX1,PY1) - (PX2,PY2)
282Ø    LINE (PX1+1,PY1) - (PX2+1,PY2)
283Ø    PX1 = PX2  :  PY1 = PY2
284Ø NEXT I
285Ø '
286Ø LINE (PX1,PY1) - (PX1,15Ø)             'draw the last boundary and label
287Ø LINE (PX1+1,PY1)-(PX1+1,15Ø)
288Ø PX3 = .12*PX1 + 1 : LOCATE 2Ø,PX3:PRINT B
289Ø '
29ØØ RETURN
291Ø '
292Ø '**********   FUNCTION SUBROUTINE   ***********
293Ø Y =  SQR( X ) + 1
294Ø RETURN
```

```
10 '**********  DERIVATIVES           MIKE MOSIER  ***********
20 '
30 '                     STORED UNDER:   DERIV
40 '                        JUNE 15, 1985
50 '
60 CLEAR,,,32768! : SCREEN 6 : COLOR 3,4
70 'SCREEN 2
80 CLS
90 '
100 CLS
110 LOCATE 3,13:PRINT "PROGRAM DERIVATV
120 LOCATE 5,15:PRINT "COMMAND MENU
130 LOCATE 8,10:PRINT "1. Derivative Demonstration
140 LOCATE 10,10:PRINT "2. Graph of Function with its Derivative
150 LOCATE 12,10:PRINT "3. Table of Function and Derivative Values
160 LOCATE 14,10:PRINT "4. Terminate Program
170 '
180 LOCATE 20,10:INPUT "ENTER YOUR CHOICE ", CMD
190        ' this is an error trap for their choice!
200 WHILE CMD <> FIX(CMD) OR CMD < 1 OR CMD > 4
210    LOCATE 24,10:PRINT "Please enter a 1, 2, 3, or 4"
220    GOTO 180
230 WEND
240 '
250 WHILE CMD <> 4
260    ON CMD GOSUB 330, 880,1470
270    GOTO 100
280 WEND
290 '
300 SCREEN 0
310 END
320 '
330 '**************  SUBROUTINE TO GIVE EXPLANATION   ***********
340 CLS : YAXE = 100 : XAXE = 150
350 LOCATE 10,10 :  PRINT "The graph of f(X) = -1/6 ( X - 4 )^2 + 4 will be draw
n, and we"
360 LOCATE 12,10 : PRINT "will construct the tangent line to the curve at X = 3"

370 LOCATE 14,10 : PRINT "by first drawing a secant line to another point on the
"

380 LOCATE 16,10 : PRINT "curve, then bring that point over to meet X = 3."
390 LOCATE 18,10 : PRINT "The slope of any of these secant lines is given by"
400 LOCATE 20,10 : PRINT "the value of  ( f( 3+h ) - f(3) ) / ( 3+h - 3 ), where
"

410 LOCATE 22,10 : PRINT "h is how far the other end of the secant is from 3."
420 LOCATE 24,20 : INPUT "Press enter to continue.",A$
430 '
440 CLS : GOSUB 1770
450 BEGINX = 0 : STPX = 8 : GOSUB 790 ' to graph the curve
460  '
470 X = 3   : FX = -1/6 * (X - 4)^2+4
480 PNT = 7 : FPNT = -1/6 * ( PNT - 4 )^2 + 4
490 SLOPE = (FPNT - FX) / (PNT - 3)
500 LOCATE 5,60 : PRINT "SLOPE = ";:PRINT USING "##.###";SLOPE
510 REND = SLOPE*(7) + FX : RPX = 500 : RPY = 150 - 20*REND
520 LEND = SLOPE*(-4) + FX : LPX = 60 : LPY = 150 - 20*LEND
530 LINE (RPX,RPY)-(LPX,LPY)
540 LOCATE 22,1 : INPUT "Press enter to move secant.",A$
550 LOCATE 22,1 : PRINT "                                  "
```

```
560 LINE (220,155)-(220,151) : LOCATE 21,28 : PRINT "3"
570 '
580  '      following loop moves the secant line toward a tangent
590  FOR PNT = 6.7505 TO 3.0005 STEP -.25
600    PPNT = 100 + 40*PNT : LINE (PPNT,155)-(PPNT,151)
610    LINE (RPX,RPY)-(LPX,LPY),0
620    FPNT = -1/6 * ( PNT - 4 )^2 + 4
630    SLOPE = (FPNT - FX) / (PNT - 3)
640    LOCATE 5,60 : PRINT "SLOPE = ";:PRINT USING "##.###";SLOPE
650    R = 10 : L = -1
660    REND = SLOPE*(R-3) + FX : RPX = 100+40*R : RPY = 150 - 20*REND
670    LEND = SLOPE*(L-3) + FX : LPX = 100+40*L : LPY = 150 - 20*LEND
680    IF RPY < 0 THEN R = R - 1 : GOTO 660
690    LINE (RPX,RPY)-(LPX,LPY)
700    BEGINX = PNT-.5 : STPX = PNT + .5
710    GOSUB 790 'to redraw anything erased from the curve
720    IF PNT > 3.005 THEN LINE (PPNT,155)-(PPNT,151),0
730  NEXT PNT
740  '
750 LOCATE 21,1 : PRINT "The limit of ( f(3+h) - f(3)) / ((3+h) - 3), as h
760 LOCATE 22,1 : PRINT "approaches 0, is exactly the slope of the tangent line.

770 LOCATE 23,10 : INPUT "Press ENTER to return to the command menu. ",A$
780 RETURN
790 PX1 = 100 + 40*BEGINX : BEGINY = -1/6*(BEGINX-4)^2 +4   'sub to redraw
800 PY1 = 150 - 20*BEGINY                                  'anything erased from
810 FOR X = BEGINX TO STPX STEP .2                         'the curve
820    Y = -1/6 * ( X - 4 )^2 + 4
830    PX2 = 40 * X + 100   :   PY2 = 150 - 20 * Y
840    LINE (PX1,PY1)-(PX2,PY2)
850    PX1 = PX2 : PY1 = PY2
860 NEXT X
870 RETURN
880 '**********  SUBROUTINE TO GRAPH THE DERIVATIVE FUNCTION  *******
890 '
900 CLS : LOCATE 10,20 : PRINT "Type in any function
910 LOCATE 12,20 : PRINT "after the Y# =    seen below.
920 LOCATE 14,20 : PRINT "Then, press enter twice."
930 LOCATE 18,1 : PRINT "goto 950"
940 LOCATE 17,1 : EDIT 1430
950 CLS : LOCATE 12,20 : PRINT "Enter the left and right endpoints of the "
960 LOCATE 13,20 : INPUT "the interval you want to graph over, as A,B ";A,B
970 X = A : GOSUB 1420 'to compute Y
980  FX# = Y#                             'SCALE is number of pixels per unit
990 SCALE = 600 / ( B - A ) : YAXE = 620 - SCALE * B : XAXE = 100
1000 CLS : GOSUB 1770 'to draw axes
1010 PX1 = YAXE + X*SCALE : PY1 = 100 - Y#*SCALE/2
1020 LCY = .12*PY1 : IF LCY < 1 THEN LCY = 1  'LCY is for locate command below
1030 IF LCY > 23 THEN LCY = 23
1040 LOCATE LCY,2  : PRINT "f(X)"
1050 H = .00005   ' H is given a value very near 0
1060 X = X + H : GOSUB 1420 ' to compute f(X+h)
1070 FXH# = Y#
1080 DY# = (FXH# - FX#) / H
1090 '
1100 PDY1 = 100 - DY#*SCALE/2
1110 LCDY = .12*PDY1 : IF LCDY < 1 THEN LCDY = 1
```

```
1120 IF LCDY > 23 THEN LCDY = 23
1130 LOCATE LCDY,2  : PRINT "f'(X)"
1140 FOR X = A TO B  STEP .1
1150    GOSUB 1420
1160    FX# = Y#
1170    PX2 = YAXE + X*SCALE: PY2 = 100 - Y#*SCALE/2
1180    X = X + H : GOSUB 1420
1190    FXH# = Y#
1200    DY# = (FXH# - FX#) / H
1210    PDY2 = 100 - SCALE/2 * DY#
1220    IF ABS(PDY2-PDY1) > 150 THEN PDY1 = PDY2 : GOTO 1240
1230    IF PDY2 > 0 AND PDY2 < 199 THEN LINE (PX1,PDY1)-(PX2,PDY2)
1240    PX1 = PX2 : PDY1 = PDY2
1250 NEXT X
1260 LOCATE 23,1 : INPUT "Press enter for f(X)",A$
1270 LOCATE 23,1 : PRINT "                    "
1280 X = A : GOSUB 1420
1290 PX1 = YAXE + A*SCALE : PY1 = 100 - Y#*SCALE/2
1300 FOR X = A TO B  STEP .1
1310    GOSUB 1420
1320    FX = Y#
1330    PX2 = YAXE + X*SCALE: PY2 = 100 - Y#*SCALE/2
1340    IF PY2 > 0 AND PY2 < 199 THEN LINE (PX1,PY1)-(PX2,PY2)
1350    PX1 = PX2 : PY1 = PY2
1360 NEXT X
1370 '
1380 LOCATE 23,1 : INPUT "Another ";A$
1390 IF A$ = "Y" OR A$ = "y" THEN 880
1400 GOTO 270
1410 '
1420 ' SUBROUTINE TO CONTAIN THE FUNCTION
1430 Y# = .1 * (X + 3)*(X - 1)*(X - 4)
1440 RETURN
1450 '
1460 '
1470 '********* SUBROUTINE TO GIVE TABLE OF VALUES ***************
1480 '
1490 CLS : LOCATE 10,20 : INPUT "Do you want to change the function ";A$
1500 IF A$ = "N" OR A$ = "n" THEN 1550
1510 CLS : LOCATE 10,20 : PRINT "Make desired changes, then press enter twice.
1520 LOCATE 13,1 : PRINT "GOTO 1540"
1530 LOCATE 12,1 : EDIT 1430
1540 CLS
1550 LOCATE 10,50 : PRINT "               "
1560 LOCATE 10,10 : INPUT "Over what interval do you the table: a,b ";A,B
1570 IF (B-A) < .0199 THEN LOCATE 12,15 : PRINT "Interval must be at least .02 i
n length." : GOTO 1550
1580 CLS : INC = (B - A) / 10
1590 '  loop to increment X over the interval, and print results
1600    LOCATE 2,2 : PRINT "X                         f(X)                    1st DER at X

1610 PRINT
1620 H# = .00005
1630 FOR I = A TO B STEP INC
1640    X = I : GOSUB 1420 : FX# = Y#
1650    X = I + H#/2 : GOSUB 1420 : FXH1# = Y#
1660    X = I - H#/2 : GOSUB 1420 : FXH2# = Y#
1670    DY# = (FXH1# - FXH2#) / H#
```

```
1680    PRINT USING "###.###"+SPACE$(10)+"###.########"+SPACE$(10)+"###.##########
";X,FX#,DY#
1690 NEXT I
1700 '
1710 LOCATE 23,10 : INPUT "Another interval ";A$
1720 IF A$ = "Y" OR A$ = "y" THEN 1540
1730 LOCATE 23,28 : PRINT "  "
1740 LOCATE 23,10 : INPUT "Another function ";A$
1750 IF A$ = "Y" OR A$ = "y" THEN 1510 ELSE 270
1760 '
1770 '***************  SUBROUTINE TO DRAW GRAPH  *****************
1780 '
1790 '           first draw the axes
1800 LINE (0,XAXE)-(640,XAXE)     :     LINE (0,XAXE+1)-(640,XAXE+1)
1810 LINE (YAXE,0)-(YAXE,200)     :     LINE (YAXE+1,0)-(YAXE+1,200)
1820 'LOCATE 20,77 : PRINT "X"    :     LOCATE 2,12 : PRINT "Y"
1830 '
1840 RETURN
```

```
10 '************ INTEGRATION          MIKE MOSIER **********
20 '
30 '                     STORED UNDER:   INTEGRAL
40 '                         JULY 30, 1984
50 '
60 CLS
70 CLEAR,,,32768! : SCREEN 6 : COLOR 3,4
80 'SCREEN 2
90 CLS
100 LOCATE 2,13:PRINT "PROGRAM INTEGRAL
110 LOCATE 4,15:PRINT "COMMAND MENU  (Note that choice 2. must be made before 3
- 6)
120 LOCATE 7,10:PRINT "1. Explanation of Riemann Sums and Integrals
130 LOCATE 9,10:PRINT "2. Input a Function and Define an Interval
140 LOCATE 11,10:PRINT "3. Draw and Figure an Upper Riemann Sum
150 LOCATE 13,10:PRINT "4. Draw and Figure a  Lower Riemann Sum
160 LOCATE 15,10:PRINT "5. Draw and Use the Trapezoidal rule
170 LOCATE 17,10:PRINT "6. List a Table of Values Using Each
180 LOCATE 19,10:PRINT "7. Terminate Program
190 '
200 LOCATE 21,10:INPUT "ENTER YOUR CHOICE ", CMD
210 WHILE CMD > 2 AND CMD < 7
220    IF TRAP = 2 THEN 260
230    LOCATE 23,10:PRINT "You must choose 2. before 3 - 6"
240    GOTO 200
250 WEND
260 WHILE CMD <> FIX(CMD) OR CMD < 1 OR CMD > 7
270    LOCATE 23,10:PRINT "Please enter a 1, 2, 3, 4, 5, 6, or 7"
280    GOTO 200
290 WEND
300 '
310 WHILE CMD <> 7
320    ON CMD GOSUB 400, 1490, 1740, 2140, 2510, 2790
330    LOCATE 23,1 :INPUT "Press enter to return to the command menu. ",B$
340    GOTO 90
350 WEND
360 SCREEN 0
370 END
380 '
390 '
400 '************** SUBROUTINE TO GIVE EXPLANATION  *************
410 '
420 CLS
430 PRINT TAB(20)"Riemann Sums are a method of approximating the area "
440 PRINT TAB(15)"between a function and the X-axis.  Suppose we have the func-
450 PRINT TAB(15)"tion defined f(x) = -.25* (X - 4)^2  +  4  , which is a
460 PRINT TAB(15)"parabola opening down, with its vertex at (4,4), and X-int-
470 PRINT TAB(15)"ercepts at (0,0) and (0,8).  If we want to find the area
480 PRINT TAB(15)"under the parabola for the interval [0,8], then we're look-
490 PRINT TAB(15)"ing for the area of the region enclosed by the curve and
500  PRINT TAB(15)"the X-axis.
510 PRINT
520 PRINT TAB(20)"To get some idea of what the area would be, let's
530 PRINT TAB(15)"look at the rectangle with a base on the X-axis, and sides
540 PRINT TAB(15)"at X=0 and X=8 . The top of the rectangle will be through
550 PRINT TAB(15)"the vertex, so its height is the same as the maximum height
```

```
560 PRINT TAB(15)"of the function.  Since our parabola is completly inside
570 PRINT TAB(15)"this rectangle, we know the actual area is less than the
580 PRINT TAB(15)"area of the rectangle,  which is 8x4 = 32.
590 PRINT
600  LOCATE 20,15:INPUT "Press enter and I'll draw this for you. ",A$
610 '
620 N = 1 : ILNTH = 8 : YAXE = 30 : UNIT = 30 : UNIT2 = 60 : FROM1 = 1
630 RE = 8 : LE = 0 :  GOSUB 1830
640 '
650 CLS
660 PRINT TAB(20)"Well, now we know the area under our parabola is less
670 PRINT TAB(15)"than 32.
680 PRINT TAB(20)"Let's take our interval [0,8], and partition it into 3 equal
690 PRINT TAB(15)"subintervals, [0,2.6] , [2.6,5.2] and [5.2,8].
700 PRINT TAB(20)"Now, just like before, we'll draw rectangles, only three
710 PRINT TAB(15)"this time.  Each will have a base equal to the length of
720 PRINT TAB(15)"the subintervals, which is 8/3 = 2.66 .  For the height of
730 PRINT TAB(15)"each rectangle, we'll again use the maximum height of the
740 PRINT TAB(15)"function, only this time the height in each subinterval.
750 LOCATE 18,15:INPUT "Press enter to continue. ",A$   : CLS
760 PRINT TAB(20)"Remember, we've divided the interval [0,8] into three parts,
770 PRINT TAB(15)"and we'll draw a rectangle for each of the subintervals.
780 PRINT
790 PRINT TAB(20)"To find the area of the first one, find the maximum height
800 PRINT TAB(15)"of the first subinterval ( which is f(2.66), or 3.55 ), then
810 PRINT TAB(15)"multiply by the interval length, or 2.66. All this gives
820 PRINT TAB(15)"an area of about 9.45.
830 PRINT TAB(20)"The middle rectangle has a maximum height of 4, times the
840 PRINT TAB(15)"length of the subintervals (2.66), gives about 10.64 .
850 PRINT TAB(15)"Notice that the area of the third rectangle will be the
860 PRINT TAB(15)"same as the area of the first.  Add the three up and
870 PRINT TAB(15)"we have 29.54 , which is closer to the real thing.
880 LOCATE 24,15:INPUT "Press enter and I'll draw this for you. ", A$
890 N = 3 : GOSUB 1830
900  '
910 CLS
920 PRINT TAB(20)"AS you can see, by dividing the interval into three rectan-
930 PRINT TAB(15)"gles, we've cut off some of the excess area.  At this point
940 PRINT TAB(15)"you might guess that if we divided it into more subintervals
950 PRINT TAB(15)"yet, we'd cut off more off the excess.
960 PRINT
970 PRINT TAB(20)"Press enter, and I'll divide the interval into 10
980 LOCATE 7,15:INPUT "subintervals (giving 10 rectangles to add up).",A$
990 N = 10: GOSUB 1830
1000 '
1010 CLS
1020 PRINT TAB(20)"The total area is going down, as the size of the partition
1030 PRINT TAB(15)"goes up. This is because with more and more rectangles, there

1040 PRINT TAB(15)"is less and less excess area above the function being added.
1050 LOCATE 10,15: INPUT "Now you tell me how many subintervals to use this time
.",N
1060 WHILE N <= 10
1070    PRINT TAB(15)"Please enter a number bigger than 10."
1080    GOTO 1050
1090 WEND
1100 GOSUB 1830
1110 '
```

```
1120 CLS
1130 PRINT TAB(20)"By now, you can guess that the more times we partition the
1140 PRINT TAB(15)"interval, the closer our area is to the real thing, so the
1150 PRINT TAB(15)"better our approximation.
1160 PRINT
1170 PRINT TAB(20)"The process of partitioning the interval into subintervals,
1180 PRINT TAB(15)"then finding rectangles that go up to some point on the
1190 PRINT TAB(15)"function inside each subinterval, is called a RIEMANN
1200 PRINT TAB(15)"SUM.  Any point inside would do, but so far we've always
1210 PRINT TAB(15)"used the maximum height in each subinterval for the height of

1220 PRINT TAB(15)"the correspoding rectangle.  This is called the UPPER RIEMANN

1230 PRINT TAB(15)"SUM.
1240 PRINT TAB(20)"If, instead, we used the minimum height in each subinterval,
1250 PRINT TAB(15)"they would be LOWER RIEMANN SUMS.
1260 PRINT
1270 PRINT TAB(20)"If we took the LIMIT as the number of subintervals goes to
1280 PRINT TAB(15)"infinity, of either the UPPER or the LOWER sums, we should
1290 PRINT TAB(15)"have the exact area under the curve.
1300 PRINT
1310 PRINT TAB(20)"That value is called the RIEMANN INTEGRAL of the function
1320 PRINT TAB(15)"over that interval.
1330 LOCATE 20,15:PRINT "Press enter and I'll draw it with a very large number
1340 LOCATE 21,15:INPUT "of subintervals, and thus get a good approximation.",A$

1350 N = 200 : GOSUB 1830
1360 CLS
1370 PRINT TAB(20)"So, to rehash this a little, the UPPER RIEMANN SUM for
1380 PRINT TAB(15)"N subintervals, over any interval [ a,b ] , is the sum of the

1390 PRINT TAB(15)"area of N rectangles, each with a base length of ( b - a )
1400 PRINT TAB(15)"divided by N.  The maximum height of the function in each
1410 PRINT TAB(15)"subinterval is used for the height of the corresponding recta
ngle.
1420 PRINT
1430 PRINT TAB(20)"The LOWER RIEMANN SUM is the same, except you use the minim-
1440 PRINT TAB(15)"um in each subinterval.
1450 PRINT
1460 PRINT TAB(20)"The LIMIT of these as N approaches infinity, is the RIEMANN
1470 PRINT TAB(10)"INTEGRAL over the interval.
1480 RETURN
1490 '************  SUBROUTINE TO INPUT FUNCTION AND DECIDE AXES  ******
1500 '
1510 CLS
1520 LOCATE 5,10:PRINT "Written on the screen below, is the number
1530 LOCATE 6,10:PRINT "9100, followed by Y = ( some function of X)
1540 LOCATE 8,10:PRINT "The cursor is at the start of that line, so if
1550 LOCATE 9,10:PRINT "that's not the function you want, move the cursor
1560 LOCATE 10,10:PRINT "to the right of the equals sign, and type your
1570 LOCATE 11,10:PRINT "function, then hit ENTER two times.
1580 LOCATE 14,1 : PRINT "GOTO 1600" : PRINT
1590 LOCATE 13,1 : EDIT 3620
1600 '
1610 CLS
1620 LOCATE 10,10:PRINT "You will now want to choose an interval to figure
1630 LOCATE 11,10:PRINT "the sums on.
1640 LOCATE 15,10:INPUT "Enter the Left and Right endpoints as A,B.  ", LE, RE
1650 '
1660 ILNTH = RE - LE      'ILNTH is the length of the interval
1670 UNIT2 = 600 / ILNTH  : UNIT = UNIT2/2                        'pixels in a unit
```

```
1680 YAXE = 620 - UNIT2 * RE
1690 '                                                    pixel the Y-Axis is at
1700 '
1710 TRAP = 2  'this is an error trap making sure the user picks 2 before 3-5
1720 '
1730 CLS : GOTO 330
1740 '**************  SUBROUTINE TO DRAW THE UPPER SUMS   ************
1750 '
1760 CLS
1770 LOCATE 10,30:PRINT "                         "   ' to clear the line first
1780 LOCATE 10,10:INPUT "How many subintervals do you want "; N
1790 WHILE N < 1 OR N <> FIX(N)   'error trap
1800    LOCATE 12,10:PRINT "POSITIVE WHOLE NUMBERS ONLY, PLEASE !!"
1810    GOTO 1770
1820 WEND
1830 GOSUB 3260 'to draw the graph
1840 '
1850 DELTA = ILNTH / N               'figures the width of each subinterval
1860 STP   = RE - DELTA + .0001      'stopping point for loop
1870 SUM = 0                         'SUM is the Riemann Sum
1880 FOR I = LE TO STP  STEP DELTA
1890    X = I : GOSUB 3610 'to figure the functional value for that X
1900    MAX = Y
1910    FOR J = I TO I+DELTA  STEP .05  'goes thru the Ith subinterval
1920       X = J
1930       GOSUB 3610
1940       IF Y >= MAX  THEN MAX = Y
1950    NEXT J
1960 '
1970    SUM = SUM + MAX * DELTA                        'figure the Riemann Sum
1980    PX1 = YAXE + UNIT2*I :  PY1 = 150 - UNIT * MAX 'now draw the rectangles
1990    LINE (PX1,PY1) - (PX1,150)
2000    PX2 = PX1 + UNIT2 * DELTA
2010    LINE (PX2,PY1) - (PX2,150)
2020    LINE (PX1,PY1) - (PX2,PY1)
2030 '
2040 NEXT I
2050         IF FROM1 <> 1 THEN 2090    'this tells it if we came from sub 1
2060         LOCATE 23,10:PRINT "The total area is ";SUM
2070         LOCATE 24,10:INPUT "Press enter to continue ",A$
2080         RETURN
2090 LOCATE 1,10:PRINT "The UPPER SUM for ";N ;"Subintervals is "; SUM
2100 LOCATE 2,10:INPUT "Do you want another Upper Sum "; A$
2110 IF MID$(A$,1,1) = "Y" OR MID$(A$,1,1) = "y"  THEN GOTO 1760
2120 '
2130 RETURN
2140 '**************  SUBROUTINE TO DRAW THE LOWER SUMS   ************
2150 '
2160 CLS
2170 LOCATE 10,30:PRINT "                         "   'to clear the line first
2180 LOCATE 10,10:INPUT "How many subintervals do you want "; N
2190 WHILE N < 1 OR N <> FIX(N)
2200    LOCATE 12,10:PRINT "POSITIVE WHOLE NUMBERS ONLY, PLEASE !!"
2210    GOTO 2170
2220 WEND
2230 GOSUB 3260 'to draw the graph
```

```
2800 CLS
2810 PRINT TAB(10)"SUBINTERVALS";TAB(27)"UPPER SUM";TAB(45)"LOWER SUM";TAB(63)"T
RAP. RULE
2820 N = 1            'N is number of subintervals, start with 1
2830 FOR K = 1 TO 3   'the following loops all increase N
2840    N = N*2
2850    GOSUB 3010
2860 NEXT K
2870 '
2880 FOR L = 20 TO 40 STEP 20
2890    N = L
2900    GOSUB 3010
2910 NEXT L
2920 '
2930 FOR M = 80 TO 200   STEP 60
2940      N = M
2950     GOSUB 3010
2960     IF DONE = 1  THEN 3000     ' upper = lower, so stop
2970 NEXT M
2980 '
2990 '
3000 RETURN
3010                 'Nested subroutine to figure sums for N
3020 '
3030 DELTA = ILNTH / N               'figures the width of each subinterval
3040 STP   = RE - DELTA + .0001    'stopping point for loop
3050 USUM = 0   :  LSUM = 0 : TRAP = 0
3060 FOR I = LE TO STP  STEP DELTA
3070    X = I : GOSUB 3610 : Y1 = Y
3080    MAX = Y : MIN = Y
3090    X = I+DELTA : GOSUB 3610 : Y2 = Y
3100    FOR J = I TO I+DELTA  STEP .1  'goes thru the Ith subinterval
3110       X = J
3120       GOSUB 3610
3130       IF Y >= MAX  THEN MAX = Y   'Maximum and minimum height in Ith
3140       IF Y <= MIN THEN MIN = Y
3150    NEXT J
3160    TRAP = TRAP + .5 * DELTA * ( Y1 + Y2 )
3170    USUM = USUM + MAX * DELTA       'Upper sum
3180    LSUM = LSUM + MIN * DELTA       'Lower sum
3190    IF USUM = LSUM THEN DONE = 1     'no need to continue
3200 NEXT I
3210 '
3220 PRINT
3230 PRINT USING SPACE$(10)+"####"+SPACE$(10)+"#######.##"+SPACE$(10)+"#######.##"
+SPACE$(10)+"######.##"; N, USUM, LSUM, TRAP
3240 '
3250 RETURN
3260 '***********   SUBROUTINE TO DRAW THE GRAPH  ************
3270 '
3280 CLS
3290 LINE (1,150) - (630,150)            'FIRST DRAW THE AXES AND LABEL THEM
3300 IF LE > 0 THEN 3340
3310 LINE (YAXE,1) - (YAXE,165)
3320 LINE (YAXE+1,1) - (YAXE+1,165)
3330 PX3 = .12 * YAXE :LOCATE 2,PX3:PRINT "Y"
3340 LOCATE 20,77:PRINT "X"
3350 '
```

```
3360 '       Find the first set of pixels and draw that boundary
3370 X = LE  : GOSUB 3610
3380 PX1 = YAXE + UNIT2 * X  :  PY1 = 150 - UNIT* Y
3390 LINE (PX1,PY1) - (PX1,150)
3400 LINE (PX1+1,PY1) - (PX1+1,150)
3410 PX3 = .12*PX1 + 2
3420 LOCATE 20,PX3: PRINT LE
3430 '
3440 '         Graph the function
3450 FOR I = LE TO RE   STEP .15
3460    X = I
3470    GOSUB 3610
3480    PX2 = YAXE + UNIT2 * X   :  PY2 = 150 - UNIT * Y
3490    IF PY2 < 0 OR PY2 > 199 THEN 3520
3500    LINE (PX1,PY1) - (PX2,PY2)
3510    LINE (PX1+1,PY1) - (PX2+1,PY2)
3520    PX1 = PX2   :   PY1 = PY2
3530 NEXT I
3540 '
3550 LINE (PX1,PY1) - (PX1,150)            'draw the last boundary and label
3560 LINE (PX1+1,PY1)-(PX1+1,150)
3570 PX3 = .12*PX1 + 2 : LOCATE 20,PX3:PRINT RE
3580 '
3590 RETURN
3600 '
3610 '**********   FUNCTION SUBROUTINE   ***********
3620 Y =  -.25*(X-4)^2 + 4
3630 RETURN
```