A HISTORY OF MATHEMATICS

FOR HIGH SCHOOL STUDENTS

A THESIS

SUBMITTED TO THE DEPARTMENT OF

MATHEMATICS AND THE GRADUATE COUNCIL OF THE KANSAS STATE

TEACHERS COLLEGE OF EMPORIA IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

BY

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JULY 1934

EMPORIA KANSAS

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ACKNOWLEDGEMENT

Acknowledgement is made to the following persons who have assisted in making this thesis:

To Dr. Oscar J. Peterson, head of the Department of Mathematics of the Kansas State Teachers College of Emporia, under whose guidance this thesis has been written. His friendly criticisms and helpful suggestions have been very valuable.

To Dr. Edwin J. Brown, Director of the Graduate Division of the same institution, whose suggestions in the procedure have been helpful.

To Mrs. Margaret Gardner Thomas who has assisted in reading and correcting the manuscript.

To Miss Carol Gardner who has typed the thesis.

Erma S. Gardner

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CHAPTER I

INTRODUCTION

1. MATHEMATICS AND THE HIGH SCHOOL STUDENT

Lives of great men all remind us
  We can make our lives sublime,
And, departing, leave behind us
  Footprints on the sand of time;
Footprints, that perhaps another,
  Sailing o'er life's solemn main,
A forlorn and shipwrecked brother,
  Seeing, shall take heart again.
Let us, then, be up and doing,
  With a heart for every fate;
Still achieving, still pursuing,
  Learn to labor and to wait.
  --Henry Wadsworth Longfellow

The lives of great men have always held a powerful interest for those who have had an opportunity to view them. Very few high school students who are studying mathematics know anything about the history of the subject which they are studying. They do not realize that the mathematical forms which they see every day may have prompted some genius to make an important discovery. Too many times the high school student seems to think that mathematics "just happened." Mathematics needs to be made more real to the high school student. Before this can be done he needs to know something of the background of the subject, the places and conditions which prompted the different stages of its development, and something of the men whose entire lives were spent in the mathematical field.

2. PURPOSE OF THIS THESIS

It is the purpose of this thesis to present some of the events and persons associated with the history of mathematics which, it is hoped, will increase the interest of a high school student for the subject. It is hoped that this presentation will give the student a correct perspective of the way in which our modern mathematics has developed. Although many men have been called mathematical geniuses, not infrequently their achievements have been the result of long exacting hours or work. They kept on trying, undismayed by difficulties, until finally success rewarded their faithful endeavors. This has been expressed aptly in a verse written by an American poet.

> The heights by great men reached and kept,
> Were not attained by sudden flight;
> But they, while their companions slept,
> Were toiling upward in the night.

3. METHOD OF PROCEDURE

In compiling the data for this Thesis, effort has been made to select data which is as reliable as a study of this nature can secure. In order to make this history more suggestive it has been broken up into what seems to be rather natural divisions as to the period of time. In making the various maps, only those places have been indicated which have been mentioned in the section following the map.

4. SOURCES

The sources of the data have been many. These sources have been secondary due to the fact that primary sources were not available. The following are the classifications of the sources which were used:

1. Books
2. Magazine Articles
3. Newspapers
4. Encyclopedias

CHAPTER II

GENERAL HISTORY OF MATHEMATICS

5. EARLY HISTORY

There is a saying, for which Plato is often given credit, that "God eternally geometrizes". How very true is this statement. The study of nature alone reveals the presence of mathematical forms of all kinds. If a number of snowflakes were kept below a freezing temperature and observed under a microscope, two definite facts would be discovered about them. In the first place, no two flakes would be found which were exactly alike. In the second place, every flake would have the form of a regular hexagon, that is, a symmetrical six sided figure.

Before the creation of mankind, nature was busy building up parallel strata of rocks. Single rocks were formed in geometric shapes. A crystal is in the form of some geometric solid. An ordinary crystal of common salt is a cube. The diamond crystal is a regular octrahedron. It has eight faces, all of which are regular equilateral triangles. Its twelve edges are equal in length, and the six polyhedral angles of the diamond are made up of four equal face angles each. Nature has many such examples of geometric figures. Plant seeds, the rings found in the trunk of a tree which show its age, the petals of flowers, and the veins in the leaves of trees are just a few of the many examples.

Thousands of years before the postulate, "a straight line is the shortest distance between two points" was explicitly stated, animals and humans were using the same idea when "cutting corners" to shorten distances. The spider seems to recognize regular polygons when weaving a web. Many years before the Greeks discovered and proved with mathematical rigor that regular hexagons may be made to fill a plane space without overlapping or leaving a space between them, the busy bees were making honey cells shaped like regular hexagons.

Along with the development of the human race came a need for what is now called mathematics. It appeared in art, religious mysticism, in war, in needs of pastoral life, in personal ornamentation such as rings, bracelets, and ear rings, and in architecture. Indeed mathematics, through its own development, has made possible and to some extent has guided the trends of modern civilization.

As far back as there is any record, there seems to be no uncivilized state in Egypt. All Greek writers have given to the Egyptians, without any jealousy, the credit for the invention of the mathematical sciences. Menes, the first king of Egypt, changed the course of the Nile River, made a large reservoir in which to store water for the dry seasons, and built one of the first pyramids. Even to-day it is marveled that the early pyramids of Egypt are built with such mathematical accuracy.[1] Sesostries, one of the first kings of Egypt, divided the land among the people so that each person would have a piece of land the same size. The king collected a revenue from each person and, since every one owned the same amount of land, the same amount of tax was paid by each.[2]

The king, in dividing the land and in making the tax law, forgot that the Nile River overflowed once a year and washed away all the landmarks. The people who lived along the river soon began to object to the paying of the same tax as the others were paying when part of their land was washed away. The people complained to the king who sent overseers and "rope stretchers" to measure the land which was left. The taxes were then levied in proportion to the entire tax imposed.

The "rope stretchers" were the surveyors of Egypt. It is surmised that the name "rope stretchers" came from the fact that they used a rope knotted in segments whose ratios were 3:4:5. The rope thus knotted was used in the construction of right angles. If the rope AD were divided so that AB would be the segment of length 3, BC of length 4, and CD

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[1] James Henry Breasted, A History of Egypt (New York: Charles Scribner's Sons, 1919), pp. 36-37.

[2] David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1923), Vol. I, p. 51.

of length 5, (A____B______C_________D), the right angle B would be constructed in the following manner. Stakes were placed at A and B. The end of the rope at D was placed on the stake at A. A stake was then placed in the knot at C and was moved until the lines BC and CD were both taut. The right angle thus formed was at the point B.[3]

The Egyptians were very particular about the orientation of their temples. In order to do this it was necessary to obtain with accuracy the north and south, the east and west lines. To do this a point on the horizon where a certain star rose and set was located. In this manner the north and south line was located. To find the east and west line the professional "rope stretchers" were employed. The line was located by the use of the 3:4:5 triangle.[4]

The geometry used by the Egyptians was therefore a practical geometry with no thought of logic. They used many rules but no attempt was made to prove that they were true. About the seventh century B. C. commercial trade began between Greece and Egypt. Naturally they exchanged ideas as well as merchandise. The Greeks were anxious for knowledge and sought out the Egyptian priests for instruction. The Egyptian ideas were taken back to Greece where they gave the

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[3] W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1924), pp. 6-7.

[4] Ibid., p. 6

Greeks new ideas and a new basis upon which to work. Plato said, "Whatever we Greeks receive we improve and perfect." From this time on the mathematical science developed rapidly.

There is no reason to believe that the Phoenicians or the other neighbors of Egypt paid much attention to mathematics. In I Kings 7:23 and in II Chronicles 4:2 of the Bible these words are found: "And he made a molten sea, ten cubits from one brim to the other: it was round all about, and his height was five cubits: and a line of thirty did compass it around about." According to this the Jews had not paid much attention to geometry. If this rule were true the circumference of the earth would be found by multiplying the diameter by three. The Babylonians used pi equal to three.

6. ORIGIN OF TERMS GEOMETRY AND ALGEBRA

It is natural that the Greek name would be the one to designate the science of geometry. It is derived from the Greek words for earth and to measure, Γη (ge) earth, and μετρεῖν (metrein) to measure. The term, geometry, therefore was originally, as it is in some languages today, synonymous with the English word surveying. Since the science was well started in Egypt before the University of Alexandria was founded by the Greeks, the word is probably the translation of an Egyptian word. It was being used at the time that

Plato and Aristotle were doing their work and no doubt goes back to the time of Thales. It might be noted that Euclid did not call his treatise a geometry since the word probably still referred to land measure. There have been a variety of names for geometry. During the sixteenth century this was true of all branches of learning. Among the best known titles was the one used by Robert Recorde, The Pathway to Knowledge.[5]

The term algebra came into use many years after the term geometry. When Ahmes copied the Papyrus, now known as the Ahmes Papyrus, in 1650 B. C., he had no word to use for the term algebra. He combined both it and arithmetic under the same heading, that of arithmetic. However, he gave as a definition of the subject, "the nature and for knowing all that exists, every mystery, every secret."[6]

It was from Al-Khowârizmî's title, ʻilm al-jabr waʻl muqabalah, that the modern word was derived. The Latin translation, Ludus algebrae almucgragalaeque and Gleba Mutabilia, was the first translation of the Arabic title that was made. The English translated the word into algrebar and almachabel in the sixteenth century. The word was then shortened into the word algebra. A good definition of the title, ʻilm al-jabr waʻl muqabalah, is "restoration and

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5 David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), Vol. II, p. 273

6 Ibid., p. 386

oppositions and the clearest explanation of their use."[7]

The following Table shows a list of the many names used for the word algebra, by whom it was used, and where possible the meaning of the word used as well as the date is given.

TABLE I

NAMES USED FOR THE WORD ALGEBRA

Name	Used By	Date	Nation-ality	Meaning of the Term
Āryabhatīyam[8]	Āryabhata	510	Hindu	
Kutakhādyaka[9]	Brahmagupta	628	Hindu	The pulverizer (chapter on indeterminate equations)
Ganita-Sāra-Sangraha[10]	Mahāvīra	850	Hindu	A brief exposition of the compenduim of calculation
Avyakta-ganitā[11]	Bhāskara	1150	Hindu	Calculation with knowns
Avyakta-kriyā[12]	Bhāskara	1150	Hindu	Calculation with unknowns

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[7] *Ibid.*, p. 388

[8] *Ibid.*, p. 387

[9] *Ibid.*, p. 387

[10] *Ibid.*, p. 387

[11] *Ibid.*, p. 387

[12] *Ibid.*, p. 387

TABLE I (Continued)

Name	Used By	Date	Nationality	Meaning of the Term
ʻilm-jabr wál-muqabalah[13]	Al-Khowârizmî	825	Arab	Restoration and opposition
Fakhrî[14]	Al-Karkhî	1020	Arab	
Gleba Mutabilia[15]	Guglielmo de Lunis	1250	Italian	
Algebra-et almochabala[16]	Roger Bacon	1250	English	
Ars Magna[17]	Cardan	1545	Italian	Great Art
Analysis[18]	Vieta	1590	French	
l'arte mayor[19]	Juan Diez	1556	Mexican	Major Art
Kigin Seiho[20]	Seki	1680	Japanese	Method of reverting the true and buried origin of things
Great Art[21]		15th & 16th Cen.	Italians	To distinguish arithmetic and algebra.

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13 Ibid., p. 387

14 Ibid., p. 387

15 Ibid., p. 390

16 Ibid., p. 390

17 Ibid., p. 392

18 Ibid., p. 392

19 Ibid., p. 392

20 Ibid., p. 386

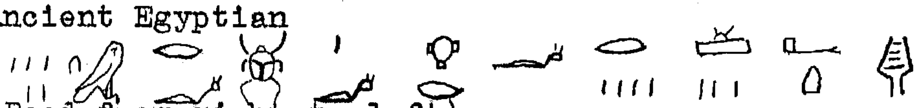
21 Ibid., p. 392

The term, algebra, was first adopted by the European schools. The Moors took the word to Spain. The Spaniards had a similar word, algebrista, which meant a restorer, or a person who sets bones. It was not unusual to see over the entrance of a barber shop these words, "Algebrista y Sungrador" which meant that a bonesetter and a bloodletter might be found in that particular shop. The striped pole which is used in America as a sign for a barber shop and a metal wash basin which is used in Europe for the same purpose are relics of the latter phrase, bloodletting, of the haircutters work.[22]

The word went to Italy from Spain. From there it spread rapidly over the world. Thus it took seven or eight centuries for the word algebra to be commonly used. This is an example of many of our common words. Although they may have been used centuries ago, they did not become common words until there arose a great need for them.

The different forms through which the algebraic equation has passed may be made the basis for a very interesting comparison. The following list shows the various forms assumed by equations as written in the many countries, from the early times to the present. All the forms are for the one equation, $X + X/4 = 15$.

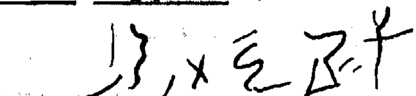
Ancient Egyptian



(Read from right to left)

22 Ibid., p. 382

<u>Ahmes Papyrus</u> (1650 B. C.)



(Read from right to left)

Vande Hoecke (1514)

1 Pri + 1/4 Pri dit is ghelijie 15

Cardan (1545)

1 rib^{9} p : 1/4 rib$^{\prime}$ aeqtis 15

Ghaligai (1521)

C^{o} ◻ 1/4 C^{o} — 15 numeri

Buteo (1559)

1 p P 1/4 p [15

Bombelli (1572)

$\overset{\smile}{1}$ · P · $\overset{\smile}{1/4}$ · equals A′15

Gosselin (1577)

1 L P 1/4 L aequalia 15

Ramus and Schoner (1586)

1 —+— 1/4 1 aequatus sit 15

Vieta (1590)

N + 1/4 n aequatur 15

Girard (1629)

1 (1) + 1/4 (1) = 15

Oughtred (1631)

X + 1/4 X = 15

Harriot (1631)

X + 1/4 · X = + 15

Herigone (1634)

1 X ⊥ 1/4 X 2/2 15

Descartes (1637)

X + 1/4 X ∝ 15

Wallis (1693)

The present modern form: $X + X/4 = 15$

7. NUMERICAL SYSTEMS

From the very earliest times the primitive man has had some use for numbers. Although his needs were very simple, he needed some form to designate the size of his family, the number of enemies he had, and the number of wild animals he had killed. The herdsman in the early days could tell that one sheep was missing out of his flock before he was able to count the number of sheep in the flock.[23]

Nearly all number systems, both ancient and modern, are based on the scales of five, ten, or twenty. It is not difficult to see the reason for this. When a child is learning to count, he uses his fingers and sometimes his toes. In the same way the primitive people counted on their fingers and toes. One of the best proofs of this is the various number systems of the savage tribes which are still in existence today.

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[23] David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), p. 6, Vol. I.

Among the South American Indians are certain tribes which have been observed to be counting by their hands. They might say one, two, three, four, hand, hand and one, and so on until they would have ten for both hands, or hands finished. Eleven would be one on the foot. This could be continued until twenty which would be man. Similar tribes have been found in Africa. When a Zulu wishes to say six he says "taking the thumb" which means that he has counted all the fingers on the left hand and has started with the thumb on the right. In some of the South Sea Islands when the natives are counting they use nuts. After they have reached five they lay aside one nut and say "heap". Another tribe counts to ten and then lays down a small stalk of cane for the first ten. After he has counted to ten the second time he lays down the second stalk. This is continued until one hundred is reached and then a large stalk is placed on the ground and all the smaller ones removed.[24]

Thus it can easily be seen that there have been ways invented by the various tribes to take care of their needs. However, there have been tribes found which seem to have no conception of numbers. The Chiquites of South America have

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[24] Levi L. Conant, "Primitive Number Systems." Smithsonian Report, 1892, pp. 584-85.

no word which might be accepted as a substitute for the word one. Their knowledge of numbers is so slight that the statement, their language contains no numbers, has been made. Some of the lower class of natives of Australia seem to have no system of counting above four. They count by saying, "one, two, two and one, two twos, much". The term "much" covers all numbers larger than four.[25]

In general it might be said that mathematics met the needs of the primitive life, with respect to number, as the needs for the numbers arose. Until some method of communication was found, each group of people had a system all their own. The Hindu-Arabic numbers that eventually replaced the ancient number systems of Europe originated in India, it may be as early as the third century B. C. The origin of the symbol for zero is uncertain. With the addition of a symbol for zero, however, a place value system was obtained which has proved far superior to other systems of calculation. The zero was not in general use until the ninth century. [26]

Before the invention of the printing press, even though two people might be using the same system of numbers, they could hardly be recognized as the same after they were written.

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25 Ibid., p. 586.

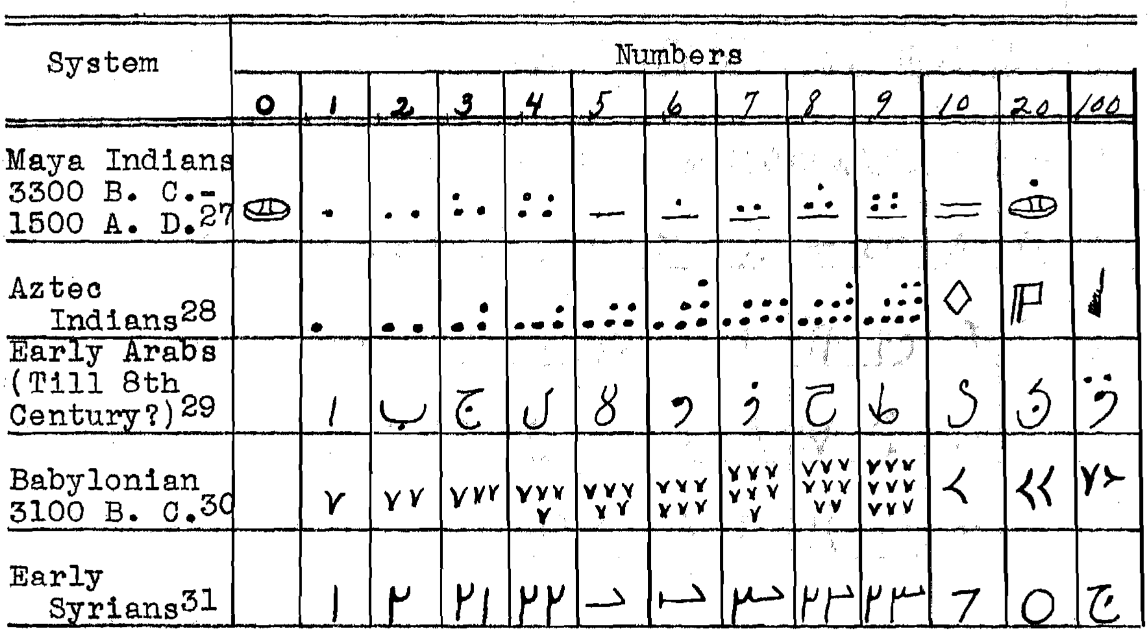
26 David Eugene Smith, op. cit., p. 69

This was because the hand-writing of people varied so much. After the printing press was invented the numerals became more and more uniform.

Table II shows some of the numbers used by different countries.

TABLE II

DIFFERENT TYPES OF NUMBER SYSTEMS



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[27] Florian Cajori, The Early Mathematical Sciences in North and South America (Boston: Gorham Press, 1928) p. 12

[28] Florian Cajori, A History of Mathematical Notations (Chicago: Open Court Publishing Company, 1928), Vol. I, P. 41

[29] Ibid., p. 29

[30] David Eugene Smith, op. cit., Vol. II, p. 37

[31] Florian Cajori, op. cit., p. 19

TABLE II (Continued)

System	Numbers											
	0	1	2	3	4	5	6	7	8	9	10	20	100
Roman[32]		I	II	III	IV	V	VI	VII	VIII	IX	X	XX	C
Greek[33]		A	B	Γ	Δ	E	F	Z	H	Θ	I		
Greek Alphabetic[34]		α	β	γ	δ	ε	ϛ	ζ	η	θ	ι	κ	ρ
Greek Herodian[35]		I	II	III	IIII	Γ	ΓI	ΓII	ΓIII	ΓIIII	Δ	ΔΔ	H
Egyptian Hieroglyphic 3300 B. C.[36]		I	II	III	II II	III II	III III	IIII III	IIII IIII	IIIII IIII	∩	∩∩	ϱ
Egyptian Hieratic 3300 B. C.[37]		I	ιι	ιιι	—	[illegible]	[illegible]	2	=	U/L	ʌ	π	ϱ
Hebrew[38]		א	ב	ג	ד	ה	ו	ז	ח	ט	י	כ	ק
Modern Arabic[39]		١	٢	٣	٤	٥	٦	٨	٧	٩	٠		
Modern Chinese[40]		一	二	三	四	五	六	七	八	九	十		

32 Ibid., p. 32

33 Ibid., p. 25

34 Ibid., p. 25

35 David Eugene Smith, op. cit., p. 50

36 Ibid., p. 46

37 Ibid., p. 47

38 Ibid., p. 53

39 Ibid., p. 39

40 Ibid., p. 70

TABLE II (Continued)

System	Numbers												
	0	1	2	3	4	5	6	7	8	9	10	20	100
Chinese Merchants[41]		〡	〢	〣	〤	〥	〦	〧	〨	〩	十		
Chinese Rod (Until 19th Century)[42]		𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨	𝍩	𝍪	
Modern Sanskrit[43]		१	२	३	४	५	६	७	८	९	०		
Modern Siamese[44]	๐	๑	๒	๓	๔	๕	๖	๗	๘	๙	๑๐		
Modern Burmese[45]	၀	၁	၂	၃	၄	၅	၆	၇	၈	၉	၁၀		
Modern Thibetan[46]	༠	༡	༢	༣	༤	༥	༦	༧	༨	༩	༡༠		
Modern Ceylonese[47]		𑇡	𑇢	𑇣	𑇤	𑇥	𑇦	𑇧	𑇨	𑇩	𑇪		
Modern Maylaan[48]		൧	൨	൩	൪	൫	൬	൭	൮	൯	൰		

41 *Ibid.*, P. 40

42 *Ibid.*, p. 42

43 *Ibid.*, p. 70

44 *Ibid.*, p. 43

45 *Loc. cit.*

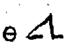
46 *Loc. cit.*

47 *Loc. cit.*

48 *Loc. cit.*

8. SYMBOLS

The history of symbolism covers a period of a great many years, yes, centuries. Before the invention of the printing press each author, if he wanted to use symbols of any kind, used his own symbols regardless whether or not anyone else had used the same ones. Consequently, the variations of symbols were about as numerous as the authors who used them. With the development of the printing press, the symbols became standardized.

In the Egyptian hieratic form of writing a sign was used for addition and subtraction. While these were not symbols in the sense that symbols are thought of now, they served the same purpose. The symbol  (a man walking forward) was used to designate addition while  (a man walking backwards or retreating) was used for subtraction. Ahmes varied these two symbols slightly in the Ahmes Papyrus when he wrote  for addition and  for subtraction.[49] Diophantus used  as a symbol for subtraction.[50] Widman was the first to use our present signs in a book. They may not have been original with him.[51] It was not until nearly fifty years after their first publication that they were used as

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49 Ibid, p. 396

50 Loc. cit.

51 Ibid, p. 399

the symbols for addition and subtraction in arithmetic.[52] Before that they were used to denote excess or deficiency. The + and - were not used in Italy for nearly a century after their introduction. Recorde introduced them in England about the middle of the sixteenth century.[53]

The use of the sign x for multiplication is probably due to Oughtred. Leibniz objected to the symbol saying that it could not be told from the letter x when used as an unknown term. Leibniz experimented with six signs. They were x, ·, ⌒, ', ;, and *. Out of the six, only two are used very extensively to-day. These are the x and the · .[54] The present symbol for division is a comparative new one, having been in use since the middle of the seventeenth century.[55] The early Egyptians wrote a ⬭ above a number which signified that 1 was to be divided by the number.[56] Brahmagupta was the first to write one number above another to signify division. The Arabs, to improve this practice, placed a bar between the two numbers. Leibniz, because of the difficulty in printing fractions with one number above the other, preferred the use of the colon to signify division. From these

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[52] Vera Sanford, *A Short History of Mathematics* (Boston: Houghton-Mifflin Company, 1930), p. 152.

[53] *Loc. cit.*

[54] *Loc. cit.*

[55] *Loc. cit.*

[56] Florian Cajori, *History of Elementary Mathematics* (Chicago: Open Court Publishing Company, 1928), p. 23

two symbols has come the modern symbol.[57]

The equality sign was the invention of Robert Recorde. One time when speaking of the symbol he said, "I will sette as I doe often in woorke vse, a pair of paralleles, or Gemowe (i. e., twin) lines of one lengthe, thus $=\!=$, bicause noe. 2. thynges can be moare equalle." The survival of the sign $=$ may be due partly to its simplicity and partly to the fact that Recorde's books were so popular.[58]

Oughtred used the symbols ⊏ and ⊐ with the meaning "Is greater than" and "is less than". The use of these two signs remained in use until about a hundred years after they were first used. Before the publication of the work of Oughtred, Harriot was using $>$ and $<$. These two seemed to be so much simpler than they have remained in use to the present time.[59]

The following Table shows some of the signs which have been used in addition and subtraction:[60]

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[57] Vera Sanford, _op. cit._, p. 152

[58] _Ibid._, pp. 152-4

[59] _Ibid._, p. 154

[60] _Ibid._, p. 151

TABLE III

METHODS OF INDICATING PLUS AND MINUS

Year	Name of User	Nationality	Addition	Subt.
1202	Fibonacci	Italian	plus	minus
1494	Pacioli	Italian	p̄	m̄
1549	Peletier	French	Plus	moins
1556	Tartaglia	Italian	p.	m.
1583	Clavius	German	P.	M.
1489	Widman	German	+	-
1522	Riese	German	+	÷
1542	Recorde	English	+	-
1568	Baker	English	x	-
1590	Vieta	French	•	=
1608	Clavius	German	+	-

The following table shows some of the symbols that have been used for the equality sign:[61]

TABLE IV

METHODS OF REPRESENTING EQUALITY

Date	Name	Symbol
1557	Recorde	=
1559	Buteo	[
1575	Xylander	//
1634	Herigone	2|2 or ⊔
1637	Descartes	∝
1680	Leibniz	⊓ and =

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61 *Ibid.*, p. 153

9. WEIGHTS AND MEASURES

From the very earliest times it has been necessary to have some standards of weights and measures. Before the invention of money, exchange of the necessities of life was carried on by barter; that is, the exchange of something one had and did not need for something he did need. At first there were no standards of weights and measures. Two adjoining countries or states might have had entirely different measures with no standard by which a comparison could have been made between their measures. It was soon discovered that this was not satisfactory, especially as the development of transportation made it possible and easier for one country to trade with another.

At first thought it seems as though it would have been easy for a group of men to have fixed a definite measure. They might have fixed the length of a yard then have cut a piece of board the right length. However, this would not have been as simple as it might appear. The piece of wood might have been cut the correct length, but the ends of it worn away with use would have destroyed the true measure. The same thing would have been true of metal measures. The metal would have corroded and even varied in size with the temperature.[62]

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62 Chambers Encyclopedia, "Weights and Measures", Vol. X, p. 125.

The preservation of standards of units of measures, since they could not be determined with accuracy, had to be safe gmarded against alteration. In the ancient times the priests and magistrates had charge of them. The Roman standards were kept in the Temple of Hercules. The Roman length of measure was chisled into the base of a statue located in the Roman Forum. It wasn't unusual in medieval times to find a standard length of measure for a particular locality to be designated by two iron pegsset in the walls of the city.[63]

In 1824 the English Parliament passed an act defining the standard weights and measures. Then in case that the standard weights and measures were lost, they might be replaced.[64]

Tradition tells many interesting incidents and ways in which some of the measures were determined. The yard measure was established by Henry I, a king of England, from 1068 to 1135. The length of the yard was to be the length of the distance between his nose and the end of the thumb when his arm was stretched out straight.[65]

Among the Greeks and Romans the pace was the measure used to correspond to the English yard. The pace was the

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[63] Vera Sanford, op. cit., p. 356

[64] Chambers Engyclopedia, op. cit., p. 127

[65] Editorial in the Kansas City Star (Kansas City,Missouri), "Gave Us Our Measurements", September 16, 1933.

length of a step. In the time of Alexander the Great official pacers were hired. They traveled from one end of the country to the other, giving the distances from one village to another, the dimensions of a piece of land, and providing scientists with whatever information they needed. In a Roman road book the distances were given in paces. A Roman soldier by looking at his road book could tell how long it would take to march from one place to another.[66]

In 1514 Köbel gave the following rule for the finding of the length of a rod:

> To find the length of a rod in the right and lawful way, and according to scientific usage, you should do as follows: Stand at the door of the church on Sunday, and bid sixteen men to stop, tall ones and short ones, as they happen to pass out when the service is finished: then make them put their left foot one behind the other and the length thus obtained shall be the right and lawful rod to measure and survey land with, and a sixteenth part of it shall be a right and lawful foot.[67]

In 1221 three barleycorns were established as one inch, twelve inches as one foot, and three feet as one ell.[68] The length of an inch was also determined by the length of the terminal joint of the thumb.[69]

A fathom was the distance from fingertip to fingertip when both arms were extended. The Hebrew cubit was the length

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[66] Vera Sanford, op. cit., p. 359

[67] Ibid., p. 354

[68] Kansas City Star, op. cit.,

[69] Vera Sanford, op. cit., p. 353

of the forearm from the elbow to the end of the middle finger. The Egyptian cubit was the same as the Hebrew cubit, but it was divided into digits which were finger-breadths. Each finger-breadth was regarded as four grains of barley-corn placed breadth-wise.[70]

The word acre is identical with the Latin word "ager" and the Greek word "agros," both words meaning "field," and the German word "acker" which means both field and a measure of land. Most of the different countries of the world have measures which correspond to the English acre. Perhaps the measure orginated in the amount of land that could be plowed with one plow in one day's time.[71]

Because the systems of measure differed in all countries, it has seemed necessary to find some kind of a system which would be uniform. Before the metric system was adopted, in the northern part of France alone there were eighteen different "aunes" (a measure for cloth) and in the entire country there were nearly four hundred different ways in which the area of land was expressed. This condition did not exist only in France but was found in all the European countries. Before the days of easy transportation and communication these different measurements were not troublesome.

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[70] Chambers Encyclopedia, "Cubit", Vol. VII, p. 350

[71] Chambers Encyclopedia, "Acre", Vol. I, p. 32

However, by the end of the eighteenth century, the people realized that some uniform system must be established.[72]

In 1789 the French Académie des Sciences appointed a committee to work on a new system of measures. It was finally decided to take one forty millionth of a meridian as a basal unit. A careful survey was made of the length of the meridian from Barcelona to Dunkirk. There was a revolution in Spain at that time which delayed the work. The original idea of the unit was not carried out. A standard meter was fixed and copies of it were made and sent to all the countries of the world.[73]

The metric system became obligatory in France in 1840. The measures were legalized in the United States in 1866. They are generally used in scientific laboratories. In 1933 they were adopted by the American Athletic Union to be used in measuring distances in all athletic events.[74]

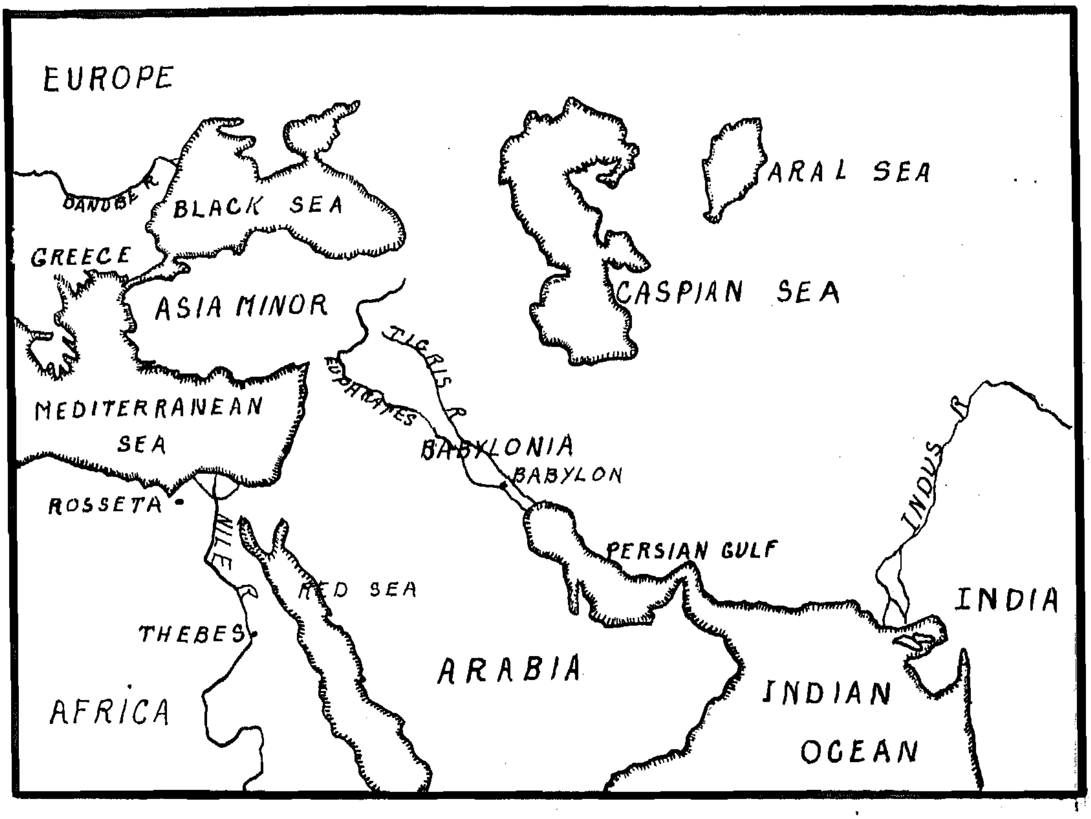
CHAPTER III

ANCIENT MATHEMATICS

INTRODUCTION

There is no definite knowledge as to the country in which mathematics first developed into anything like a science. There has been historical mathematical material left to us, principally from four countries; namely, Egypt, Babylonia, China, and India. Each of these countries claim to be the pioneer in mathematics. These countries all flourished along one or more rivers which furnished water for irrigation purposes. This gave an opportunity for the development of mathematics in problems concerning the irrigation projects.

Of the four countries mentioned in the preceeding paragraphs, only two, Babylonia and Egypt, will be considered in this section. It is through the Babylonian tablets that the knowledge of mathematics in Babylonia has been preserved. These records show that the Babylonian merchants were familiar with bills, receipts, notes, accounts, and systems of measures 3000 B. C. The Egyptians left records in the form of papyri and wall reliefs. The Ahmes Papyrus is the first mathematical handbook which has been found up to the present time.



MAP 1

PROFILE MAP FOR ANCIENT MATHEMATICS

The map shows the location of the cities and countries which are mentioned in the next section.

10. AHMES PAPYRUS

The Ahmes Papyrus, the oldest mathematical record in existence, was found at Thebes in the ruins of a small building near the Ramesseum. The Papyrus was copied by Ahmes in 1650 B. C. from mathematical records which were perhaps a thousand years old at that time. The Ahmes Papyrus was purchased soon after its discovery in 1858 by A. Henry Rhind. After his death the Papyrus came into the possession of the British Museum. About twenty years ago a group of students of Egyptology were allowed to study the roll.

Arnold Buffum Chase, Chancellor of Brown University, had long been interested in the literature and monuments of Egypt. In 1910 he and his wife made a trip to Egypt. The result of the trip was an increased interest in old Egyptian records. The Ahmes Papyrus came into the possession of the British Museum about two years later. Chancellor Chase gained permission to make an intensive study of it. Realizing its value in the study of mathematics, he obtained permission to prepare the Papyrus for publication. When they began the translation several pieces of the roll were found missing. In 1922 Professor Newberry, the English Egyptologist, found some fragments of a papyrus in The New York Art Museum. Suspecting that they might be the missing pieces of the Ahmes Papyrus, the pieces were taken to England. Professor T. E. Peet, with much skill and accuracy, arranged most of the pieces in their original position in the Papyrus. There are a few small pieces whose positions have not as yet been determined. When the translation was nearly complete, The American Mathematical Association came forward to help in the introduction of the Ahmes Papyrus in its translated form.[75]

The translation of the Ahmes Papyrus no doubt was a difficult task. If it had not been for the discovery several years before, it is doubtful it the translation could have been made. In August 1799, a French artillery officer was digging at Rosetta, near the mouth of the Nile River, for the purpose of throwing up fortifications. He found a large stone with strange markings on it several feet under the ground. After the stone was cleaned, it was found to contain three different inscriptions. The one at the top of the stone was in Hieroglyphics, the printing of the Egyptians. The second inscription was in Hieratic, which corresponds to the handwriting of the Egyptians. The third and last inscription was written in the Greek language. The Rosetta Stone may now be seen in The British Museum. The European scholars could easily read the Greek inscription. Here at last was a key to the many treasures of Egyptian History and to the many accounts of the life of the Egyptian people which are found on the monuments, tombs, and

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[75] Arnold Buffum Chace, "The Rhind Mathematical Papyrus" Mathematical Association of America (Oberlin, Ohio, 1927), Vol. I, pp. 1-2

papyrus rolls. Before the finding of the Rosetta Stone all efforts of scholars and archaeologists to translate the early Egyptian writings had been in vain.[76] [77]

In order that the *Ahmes Papyrus* might be more easily studied, thirty-one photoplates were taken from the original copy. From the photoplates one hundred nine fac-simile plates, showing small portions of the Papyrus magnified so as to bring out clearly just what the original writing looked like, were made. The one hundred nine fac-simile plates showed the entire Papyrus and by close inspection, the places where the small fragments were missing and had been replaced could be noticed.[78]

The *Ahmes Papyrus* was originally a roll eighteen and one-half feet long and thirteen inches wide. It was written in Hieratic, the handwriting of the Egyptian people.[79] The writing was done in two colors. Red was used for the first wrods of each problem, for headings, and for emphasis of any kind. The rest of the Papyrus was written in black ink.

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[76] Willis Mason West, *The Ancient World* (New York: Allyn and Bacon, 1913), pp. 11-12.

[77] Hutton Webster, *Ancient History* (New York: D. C. Heath and Company, 1913), pp. 71-72.

[78] Arnold Buffum Chace, *op. cit.*, p. 1

[79] Arnold Buffum Chace, "The Rhind Mathematical Papyrus" *Mathematical Association of America* (Oberlin, Ohio, 1927), Vol. II, p. 6.

The first part of the Ahmes Papyrus might be called the title page. Although a small piece seems to be missing, it has been translated as:

> Accurate reckoning. The entrance into the knowledge of all existing things and all obscure secrets. This book was copied in the year 33, in the fourth month of inundation season, under the majesty of the king of Upper and Lower Egypt, A-user-Re', endowed with life, in likeness to writings of old made in the time of the king of Upper and Lower Egypt, Ne-ma'et-Re'. It is the scribe A'h-mose' who copies this writing.[80]

If the Ahmes Papyrus had a Table of Contents, the table would probably read something like this:

Chapter I--Egyptian Arithmetic

Section I	Table of Division of 2 by Odd Numbers from 3 to 101.
Section II	Problems 1-6. Table of Division of the Numbers 1-9 by 10.
Section III	Problems 7-20. Multiplication by certain Fractional Expressions.
Section IV	Problems 21-23. Problems in Completion.
Section V	Problems 24-29. "AHA" or Quantity Problems.
Section VI	Problems 30-34. Division by a Fractional Expression.
Section VII	Problems 35-38. Division of a Hekat.
Section VIII	Problems 39-40. Division of Loaves. Arithmetic Progression.

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80 Arnold Buffum Chace, "The Rhind Mathematical Papyrus" Mathematical Association of America (Oberlin, Ohio, 1927), Vol. I, p. 49.

Chapter II--Geometry

Section I Problems 41-46. Problems of Volume.

Section II Problem 47. Division of 1000 Hekat.

Section III Problems 56-60. Pyramids; the Relation of the Lengths of Two Sides of a Triangle.

Chapter III--Problems 61-87. Miscellaneous Problems.

Because the Egyptian fractions were limited to 2/3 and to the reciprocals of whole numbers such as 1/2, 1/3, 1/5, etc., and because the Egyptian arithmetic was chiefly concerned with fractions, Ahmes worked with fractions in the larger part of his Papyrus. The Egyptian method of writing fractions differed from the modern method. Instead of writing 2 7/12, as it is written now, Ahmes would have written it as 2 1/3 1/4. 3 37/42 would have been written 3 1/2 1/6 1/7 1/21 1/42.[81]

If a student of mathematics to-day wanted to add two groups of fractions, say 1/3 1/5 and 1/5 1/7, the first step would be to reduce all the fractions to the common denominator or 105. When the fractions had been reduced to a common denominator they would have 35/105 21/105 and 21/105 15/105. Adding the numerators the sum of the fractions would be found to be 82/105.

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[81] David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), Vol. II, pp. 210-13.

Suppose the Egyptians had had the same problem. To add the fractions, they would group a number of things of the same kind. The Egyptian might take for his number 105 loaves. He would say that 1/3 of 105 loaves is 35 loaves and 1/5 of 105 loaves is 21 loaves, making 56 loaves in the first group. In the second group 1/5 of 105 loaves is 21 loaves and 1/7 of 105 loaves is 15 loaves, making 36 loaves in the second group. In all, then, h is share of the 105 loaves would be 82 loaves. The Egyptian could not say, however, that he would receive 82/105 of all the loaves.

To find what part of 105 loaves would make 82 loaves, Ahmes would take fractional multipliers and try to multiply 105 by them so as to get 82. He might say:

	1.	105
\	2/3. . . .	70
	1/3. . . .	35
	1/30 . . .	3 1/2
\	1/15 . . .	7
	1/10 . . .	10 1/2
	1/5. . . .	21
\	1/21 . . .	5
	Total 2/3 1/15 1/21	82

The geometric part of Ahmes Papyrus reveals considerable

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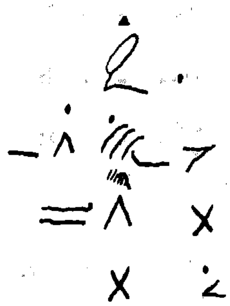
82 Arnold Chace, "The Rhind Mathematical Papyrus", Mathematical Association of America (Oberlin, Ohio, 1927), Vol. I, p. 8.

knowledge of geometric facts useful in the determination of volumes, areas, and line relationships. The section of miscellaneous problems is interesting because it gives some knowledge of Egyptian customs, their method of trading, of raising taxes, of feeding animals, and of fixing comparative values of different foods and drinks by the amount that can be made from a unit of material.[83]

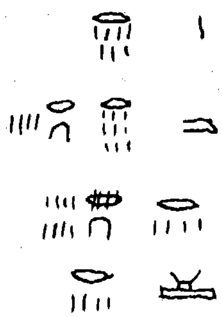
The following problems are problems which have been taken from the Rhind Papyrus. First is a copy of the problem as found in the Papyrus (in hieratic), second the problem written in hieroglypics, third the problem written in English, and fourth the method used by Ahmes in solving the problem. In reading the problem as it is written the first two times, it must be remembered that the Egyptians wrote from the right side of the page to the left. Since Ahmes was left handed, it might have been easier for him to write from the right side to the left.

(1)



(Ahmes hieratic)

(2)



(Egyptian hieroglyphics)

(3) Multiply 1/7 by 1 1/2 1/4

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83 Ibid., p. 2

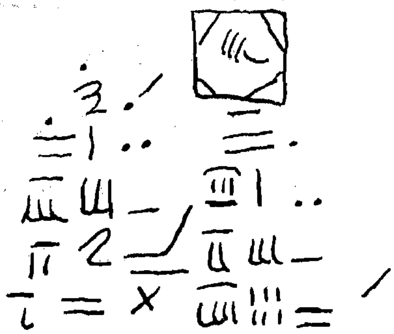
(4) 1 . . . . 1/7

1/2 . . . . 1/14

1/4 . . . . 1/28

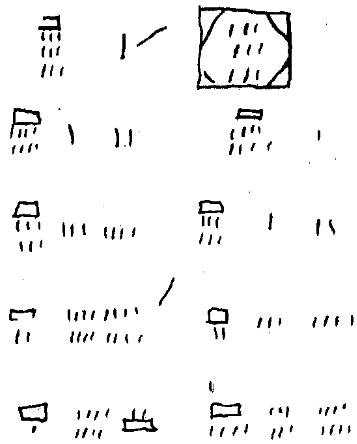
Total . . . . 1/4 [84]

(1)



(Ahmes hieratic)

(2)



(Egyptian hieroglyplics)

(3) Compare the area of a circle and its circumscribing square.

(4) The circle of diameter nine

1 . . .	8 setat
2 . . .	16 setat
4 . . .	32 setat
8 . . .	64 setat

The square of sides nine

1 . . .	9 setat
2 . . .	18 setat
4 . . .	36 setat
8 . . .	72 setat
Total	81 setat[85]

The Ahmes Papyrus was probably used both as a textbook and as a reference book. Although this one copy was almost

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84 Arnold Buffum Chace, "The Rhind Mathematical Papyrus" _Mathematical Association of America_ (Oberlin, Ohio, 1927), Vol. II, Plate 41, Problem 11.

85 _Ibid._, Plate 70, Rroblem 48

perfect when it was found in 1858, nearly 3500 years after it was written, more than five hundred pieces and copies of the same papyrus have been found in the various parts of Egypt.

Much has been written about the mistakes found in the Ahmes Papyrus. Because of the other possibilities for mistakes, it is not likely that Ahmes made all of them in copying the papyrus. There are but a few mistakes in the Papyrus and most of them are in the numbers. It may be true that Ahmes was an ignorant scribe, but he understood something at least of the problems that he copied and may have added details of his own. It may be that the manuscript which he copied had mistakes in it. Corrections in the copy that is now in the British Museum may have been made by a later hand than Ahmes.[86]

The papyrus roll went through a very interesting process in its manufacture. Four thousand years ago the people in Egypt did not know how to make paper. The Egyptians, however, were very similar to people to-day. If they needed something which they did not have and which they could not buy they tried to find something which they could use in the place of it. Since they had no paper, the Egyptians experimented until someone discovered the papyrus.

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[86] Arnold Buffum Chace, "The Rhind Mathematical Papyrus" Mathematical Association of America (Oberlin, Ohio, 1927), Vol. I, pp. 40-1.

An unusual water plant grew in Egypt along the Nile River. The same plant was found in Sicily and in Babylonia but the natives of those countries made no use of it. In many ways this plant was like some of the rushes which grew in the swamps of the United States except that it was larger. The stem of the papyrus reed grew from six to sixteen feet tall and was as thick as a man's arm at the lower part of the plant. The stem of the plant was smooth except near the bottom where the roots branched out and at the top where there was a bunch of leaves formed very much like an umbrella. The stem was triangular in shape. The plant is very rare in Egypt now but some of them may be found in Southern Italy and in Central Africa.

To make papyrus from the plant, the stem of the papyrus reed was cut in two equal parts. The bark of the plant was removed. The pith which remained was cut into thin strips by the use of some sharp instrument. The strips were then placed side by side on a smooth surface. At right angles with the first strips was placed a second layer of strips. A weight, probably a stone or some other heavy substance, was placed on top of the strips of papyrus and the upper surface was heated. The heat was probably given off by fire which was built on top of the surface. Some people have said that the papyrus was made where the sun could shine on it. The sun being very hot heated the upper surface or stone as a fire might heat it.

The sap of the papyrus reed was very much like our glue. When heat was applied the sap became warm and filled in all the space which might be left between the strips. When the sap became cold, the stone which had been placed on top of the strips was removed. What at first looked like several strips of the stem of a reed now resembled a sheet of course brown paper. The papyrus was very tough when it was new, but as it became older it became more brittle and was more easily torn. It was from the word "papyrus" that the English word "paper" orginated.

The papyrus was made in long strips. It was on such strips that the Egyptians wrote their books. They used a pen which was made from a small papyrus reed and an ink which was made from the sap of different kinds of vegetation. When the writing was finished the strips were rolled, and for this reason they were called papyrus roll.[87] [88]

Ahmes, no doubt, was an ordinary Egyptian lad and probably received the same education that other boys who were to become scribes received.

An Egyptian boy remained with his mother until he was four years old. At that time his education was turned over to his father. In a book which was written about 5000 years

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[87] Edward Maunde Thompson, "Papyrus". Encyclopedia Britannica, 14th edition, Vol. XVII, pp. 246-49.

[88] Chamber's Encyclopedia, "Papyrus", Vol. VII, pp. 253-254.

ago the following items were given which were to be used in the instruction of a boy:

> Be proud of thy learning, but be willing to learn from all. Treat a venerable wise man with respect but correct thy equal when he maintains a wrong opinion. Be not proud of thy earthly riches for they come to thee from God without thy help. Calumnies should never be repeated. Keep a contented countence, and behave to thy superiors with proper respect, then shalt thou receive that which is the highest reward to a wise man; the princes who hear thee shall say: "How beautiful are the words which proceed out of his mouth".[89]

The instructions given in the home related mainly to manners and morals. To illustrate the uniform character of the Egyptian ideas from age to age, the following quotation is taken from a book that was written something like 1500 years later than the one just mentioned:

> Be industrious; let your eyes be open lest you become a beggar; for the man that is idle cometh not to honor. Do not go into the house of another unless invited; if he bids you enter you are honored. When you go into a neighbor's house do not look around. If you see anything be silent about it, for to tell others about it is a crime worthy of death. Do not talk too much for a man of many words is not respected. Above all things, guard your speech, for a man's ruin lies in his tongue. Man's body is a storehouse full of all kinds of answers; choose the r ight one and express it well, and let the wrong answer stay imprisoned in your body. Behave well at meals and be not greedy. Do not eat bread while another stands near, unless you share with him. One is poor, one is rich, but the generous shall always have bread. He that was rich last year may be a vagrant next year. Do not sit while an older person or one holding a higher office

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[89] Charles H. Sylvester, editor, Progress of Nations (Chicago: National Progress League, 1912) 9 vols., Vol. I, pp. 52-53.

stands.[90]

The Egyptians, like the modern people of today, valued education mainly because of the superiority which an educated man may possess over a man who does not have an education. The Egyptian boy who was diligent in his studies might become a scribe. A scribe was exempt from all military service and, if they were very faithful to their work and showed great ability, they were apt to be given a higher office.

A boy intended for the career of a scribe wassent to the school of the priests when quite young. Here, even though he might be of low rank, he was classified with the children of the nobles. The children were not passed from one teacher to another but were taught by the same one until they had completed their work. The discipline in the schools was very strict. At an early hour a teacher came to the boy's bed and shook him very hard and rough until the boy was awake. The boy would arise and dress immediately and then begin his studies. The child was in class all forenoon. When the noon hour arrived, the air was filled with the shouts of the boys who were happy because of their half holiday. The food for the children was brought to the school by their mothers. A day's ration consisted of three rolls of bread and some beer. Severe punishments were given for

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[90] Ibid., pp. 53-54

disobeying any of the rules of the school. At times the boys were whipped until they were hardly able to stand.[91]

It was such an education that Ahmes received. Although it has been said that the scribe who copied the papyrus was illiterate, it may be that Ahmes had the best education that a young man could have had at that time.

11. BABYLONIAN TABLETS

Many records of the olden days are those which come from Babylonia. In certain parts of the country of Babylonia a soft clay was found. The clay was molded into tablets or cylinders. Figures were cut out or pressed in the clay by a sharp instrument made of metal or stone. After the tablet had been written on, it was baked so that it might be preserved. When the tablet was baked, the figures shrivelled and flattened into wedge shaped symbols. The scholars called this form of writing cuneiform, from the Latin word "cumeus" meaning wedge.[92]

Recently a Babylonian schoolhouse was excavated. On the floor of the building were found many slates which had been soft clay at the time they were used. When these tablets were filled with writing, the writing was cleaned off

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[91] Ibid., pp. 54-56.

[92] Willis Mason West, The Ancient World (New York, Allyn and Bacon, 1913), pp. 61-62.

by scraping smooth with straight-edged scrapers.[93]

Many times the Babylonians made what is now called duplicate tablets. After the first tablet had been written on and baked, another coat of clay was placed over the first. The writing which had been placed on the first time was then duplicated on the second. In case that there was ever any doubt about the record on the outside of the tablet or in case that part of it was broken off, by removing the outside layer the same record might be found on the inner layers.[94]

Each large town had a library building. The library contained a collection of clay tablets, cylinders, and bricks which were covered with cuneiform writing. The cuneiform writing averaged six lines to an inch. A library recently discovered contained over 30,000 tablets of the date 2700 B.C. all neatly arranged in order. Each tablet corresponds to a chapter. Each tablet had a library number stamped on it and was catalogued. The tablets were copied by scribes. Some of them contain a blank space once in a while. When the scribe could not make out a word when he was copying a tablet, he would leave out the word entirely and leave a blank in its place. For this reason many of the tablets are incomplete.[95]

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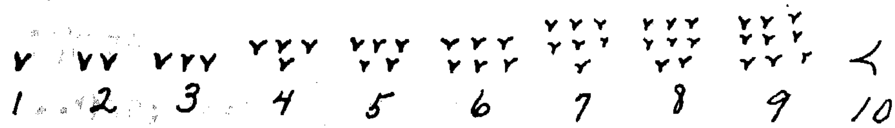
[93] Willis Mason West, The Story of the World's Progress (New York: Allyn and Bacon, 1928), p. 36.

[94] Ibid., p. 37

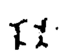
[95] Willis Mason West, The Ancient World (New York: Allyn and Bacon, 1913), p. 62.

The Babylonia numbers as found on the tablets are very interesting. The following is a list of the f orms used by the Babylonians for the numbers from one to ten:



In writing their numerals the Babylonians made a slight use of the subtraction method such as it is found in the Roman numerals. When the Romans wrote XIX it was the same as writing XX-I. The same principle was used by the Babylonians about two thousand years before. They wrote 19 , the symbol  meaning minus. Many people have wondered why it was written as 20 - 1 rather than 10+ 9. It was much easier to think of ninteen as being one less than twenty than as being nine more than ten. Ninteen was considered as an unlucky number by the Babylonians and they never used it in any of their numbers.[96]

Very few of the Babylonians could read and write. There was no attempt made to standardize their system of numbers for that reason. A few of their numbers will show some of their general characteristics. The illustrations are from numbers which were used about 2400 B. C.

 3600

 36,000, that is, 3600 x 10

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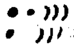
[96] David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1923), Vol. II, p. 37.

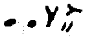
 72,000; that is, 3600 x (10 10)

2400

 10

 36

19; that is, 20 - 1

18; that is, 20 - 2

30 1/2; that is, 2 x 60 x 10 2/4 [97]

The Babylonians did some work in geometry. Just how much is not known. Among the figures that they used are parallel lines, a square, a figure with a re-entrant angle, and an incomplete figure, believed to represent three concentric triangles with their sides respectively parallel. The word "tim" was used by the Babylonians. The word means "line" but originally meant "rope". Since the Babylonians used the word, the conclusion might be that they, like the Egyptians, used a rope in measuring distances and in the measuring of angles. It is quite evident that the Babylonians knew how to divide the circumference of a circle into six equal parts by the use of the radius. That this division was knows in Babylonia follows from the inspection of the six spokes in the wheel of a royal carriage represented in a drawing found in the remains at Ninevah.[98] The Babylonians knew how to divide

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[97] Ibid., p. 38

[98] Florian Cajori, A History of Elementary Mathematics (New York: Macmillan Company, 1929), pp. 43-44.

the circumference of a circle into 360 degrees.[99] They used pi equal to three which is shown by the problem which gave the circumference of a circle as sixty and the diameter of the circle as twenty.[100]

It is interesting to note that the mathematical literature of Babylonia seemed to be especially rich in algebra. They seem to have had a formula which was equivalent to the modern formula for the solution of a quadratic equation.[101] It is not known just what the form of their formula was. The Babylonians considered the two roots of a quadratic equation when both of them were positive roots.[102]

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99 G. A. Miller, "Babylonian Mathematics". Science, 72: 601-2 (December 12, 1930)

100 R. C. Archibald, "Babylonian Mathematics." Science, 70: 66-67 (July 19, 1929)

101 Loc. cit.

102 G. A. Miller, "Oldest Extant Mathematics." School and Society, 35: 833-34 (June 18, 1932)

CHAPTER IV

THE GOLDEN PERIOD OF GREEK MATHEMATICS

INTRODUCTION

Mathematics, like all the finer products of the mind, develops best in peaceful surroundings. Greece, the Grecian towns on the Italian peninsula and in Asia Minor, and the small islands of the Aegean Sea were all ideal spots for such developments from the sixth century B. C. until the sixth century A. D. In all these places invasion was difficult. Even in case of invasion the rewards to the invader were few.[103]

About the seventh century B. C. a lively commercial trade arose between Greece, Babylonia, Egypt, and the other countries surrounding the Mediterranean. An intellectual interest followed the commercial trade. The first Greeks to show interest in mathematics were the Ionian Greeks who lived in the sheltered islands of the Aegean Sea and in the Greek cities of Asia Minor. To this group belonged Thales of Miletus. The Ionian Greeks possessed a lively curiosity. They visited the land of the pyramids just as American scholars in our time go to European schools to study. Therefore,

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[103] David Eugene Smith, A History of Mathematics (Boston: Ginn and Company, 1925), Vol. I, p. 54.

even though the Greek culture was not primitive, it commands our admiration. It was under the Ionian school that geometry became a logical and abstract science instead of a practical science as it was in Egypt. It was under their influence that the first map of the world was made.[104]

The Pythagorean school followed the Ionian school. Pythagoras, the founder of this school, was a scholar of Thales who influences him in his study of mathematics. This school was founded in the southern part of Italy. The school grew rapidly and had considerable political power for a number of years. This school was not only for the study of mathematics, natural science, and philosophy, but also a brotherhood which in some ways might approach the Masonic lodge. This school was the first to believe that the world was round instead of a flat disk.[105]

The Eleans were a group of philosophers who revolted from the Pythagorean school. The home of this school was Elea, a Greek city in Italy. The outstanding man in the Elean school was Zeno who is remembered for his paradoxes. Other members of the school were interested in such problems as trisecting an angle and the construction of a cube whose volume is twice as large as a given cube.[106]

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104 Vera Sanford, A Short History of Mathematics (Boston: Houghton Miffin Company, 1930), p. 5.

105 Florian Cajori, A History of Mathematics (New York: Macmillan Company, 1924), pp. 17-20.

106 Vera Sanford, op. cit., p. 8

During the Peloponnesian War (431-404 B. C.) the progress of geometry was checked. All the leading mathematicians from the Aegean Islands and Asia Minor emigrated to Athems. Although Athens lost some of its political power it sprang forward as a leader in philosophy and science. The foremost man in the Athenian school was Plato. Plato was not primarily a mathematician, but he did much for geometry. Another man of importance in the Athenean School was Aristotle. He was a tutor to Alexander the Great. Because of his position he was able to collect great quantities of scientific information from the countries which were conquered by Alexander. Although his chief interest was in the application of mathematics, he advocated the separation of arithmetic and geometry.[107]

After the death of Alexander the seat of learning moved to Alexandria, a seaport at the mouth of the Nile River founded by Alexander. Ptolemy, who was interested in mathematics and science, founded the Alexandrian Library, and built laboratories and museums. The government not only gave funds for research work but excused citizens from duties to the state in order that they might aid in educational work. Because of the cheapness of papyrus, Alexandria was the center of the book copying. The men whose works were outstanding in the first Alexandrian School were Euclid, who summarized

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[107] Florian Cajori, op. cit., pp. 25-6.

all the mathematical knowledge before his time, Eratosthenes, a librarian at the University of Alexandria, and Archimedes, who was the first to apply mathematics to mechanics.[108]

The second Alexandrian School may be said to commence with the Christian Era. Egypt had been absorbed into the Roman Empire. Alexandria was still the center of commercial trade and intellectual learning. Scholars from all over the world still met at the Alexandrian University. Even though geometry was still one of the most importatn studies, the theory of numbers had become the favorite one. The Romans were not interested in Greek mathematics. Cicero on one occasion congratulated the Romans because they were concerned only in the mathematics which they needed in measuring and reckoning. The Romans knew so little mathematics that they were required to employ mathematicians from Alexandria to do their surveying. Diophantus was the last great mathematician of the Second Alexandrian School. Boethius, a Roman, lived just following the fall of the Roman Empire. To him goes the credit for transmitting the knowledge of the Greek mathematics from the Romans to the Middle Ages.[109]

The Neo-Pythagorean School, a division of the second Alexandrian School, was made up of philosophers who tried to revive the teachings of Pythagoras. They flourished in

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108 Ibid., p. 29

109 Ibid., p. 45

Alexandria. Nichomaches was in this group.[110]

Although the works of many of the early Greek mathematicians have been lost, their influence upon present mathematics is very great. If it had not been for such men as Thales, Pythagoras, and Euclid it is hard to tell just what our present day mathematics might contain.

12. THALES

The first name which is of importance in the history of Greek mathematics is that of Thales of Miletus. Thales was born in Miletus in 640 B. C. and died in 546 B. C., ninety-four yers later. Thales studied science as well as mathematics. He spent a great deal of time trying to find an explanation of the universe. After many years of study, he decided that the earth was a large flat disk floating in a large body of water. He also believed that water was the first thing in the universe and from it all other things were made.[111]

During his early life Thales was actively engaged in commerce and also devoted some time to public affairs. Thales was regarded as one of the wisest men of his time. A great many stories are told of his shrewdness and resourcefulness.

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110 Ibid., p. 58

111 Henry Jackson, "Thales of Militus." Encyclopedia Britannica, 14th edition, 1929. Vol. 22, p. 12.

MAP 2

PROFILE MAP FOR THE GOLDEN PERIOD OF GREEK MATHEMATICS

The map shows the location of the cities which are mentioned in the next section. The following is a list of the mathematicians, their chronological age, and the cities with which they were associated:

Thales	600 B. C.	Miletus
Pythagoras	540 B. C.	Crotona
Pythagoras	540 B. C.	Samos
Plato	380 B. C.	Athens
Euclid	300 B. C.	Alexandria
Archimedes	225 B. C.	Syracuse
Nicomachus	1 00 A. D.	Gerasa
Diophantus	275 A. D.	Alexandria
Boethius	510 A. D.	Rome

It seems that Thales owned a large grove of olive trees. Olive oil at that time was used in the place of soap, for fuel in lamps, and in the place of animal fat in cooking. One year olives were very plentiful and Thales was afraid he would not be able to sell his crop for any great amount of money. The oil was extracted from the olives by means of presses. Thales realized that the only way to raise the price of olive oil was to control the amount of oil put on the market for sale. It is not known whether he bought or rented the presses but by some manner he obtained control over all of them in the country which surrounded Miletus. Thales was then able to control the price of olives, the amount of oil to be placed on the market at one time, and the price of the oil.[112]

Among the many stories, which have been built around the legendary wisdom of Thales, is the salt and donkey story. The salt which was used in Asia Minor around Miletus was brought in from other parts of the country. At one time Thales had charge of the pack train which transported the salt. The pack train was made up of mules, one of which seemed to be very stubborn. Once when the pack train was fording a stream of water, the stubborn mule decided to lie down and roll over. Of course the salt on the mule's back

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[112] W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 14

became wet and part of it dissolved. The mule realized that the load was much lighter and after that, whenever fording a stream, he would lie down and roll over. Thales decided to break the mule of the habit. One day when he was placing the packs on the backs of the mules, instead of putting salt in the pack of this one mule, he filled it with rags and sponges. The animal discovered when his pack became wet that it was heavier than before. Thales left the same pack on the mule for some time and finally cured him of the undesirable habit.[113]

Thales must have had one quality of a good scholar--that of being able to concentrate on different subjects. Tradition tells that one evening while walking down the street gazing at the stars and studying them, he was paying no attention to where he was going and suddenly found himself falling into a well. Afterward he was taunted about being so eager to know what was occuring in the heavens that he did not know what lay at his feet.[114]

This anecdote has been told in a little different way. One evening Thales stumbled into a ditch while studying the stars. An old woman passed by before he had had time to get up and said, "How can you tell what is going on in the sky

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[113] Loc. cit.

[114] Vera Sanford, A Short History of Mathematics (Boston: Houghton Mifflin Company, 1930), p. 6.

if you can't see what is lying at your own feet." This anecdote is often quoted to illustrate one of the impractical characteristics of philosophers and the absent mindedness of teachers.[115]

Egypt was a very important country at that time and was the center of trade for all the surrounding countries. Thales probably went to Egypt as a merchant and while there studied geometry and astronomy in his leisure time. He was middle aged when he returned to Miletus and, having become quite wealthy, he abondoned public and business life. He devoted the rest of his life to the study of philosophy and science. He lived in Miletus until his death.[116]

The value of the work of Thales cannot be measured by the number of theorems that have been attributed to him. The value of his work rests on the supposition that he used logical reasoning in the proofs of the theorems rather than experimentation. Thales made geometry abstract rather than limiting himself to practical applications such as the Egyptians had been doing before this time.[117] The credit for the following list of theorems has been given to Thales:

1. Any circle is bisected by its diameter. (This was

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[115] Florian Cajori, A History of Elementary Mathematics (New York: Macmillan Company, 1924), p. 48.

[116] W. W. Rouse Ball, op. cit., p. 15

[117] George Johnson Allman, Greek Geometry from Thales to Euclid (Dublin: University Press, 1889), p. 7.

known before probably but had never been stated as such.)

2. The base angles of an isosceles triangle are equal.
3. When two lines intersect, the vertical angles are equal.
4. An angle inscribed in a semicircle is a right angle.
5. The sides of similiar triangles are proportional.
6. Two triangles are congruent if they have two angles and a side respectively equal.[118]

It is interesting to note the kind of symbols used for numbers by Thales. These numerals are the same as the characters of the Greek (Herodian) alphabet of that time. Some of these were:

M	X	H	Δ	Π or Γ
10,000	1,000	100	10	5

He used four straight lines //// to represent four and ΔΔΔΔ to represent forty. The symbol for five, Γ , was combined with symbol for ten, Δ , to give the symbol for fifty,  or .[119]

In the Metropolitan Museum in New York at the present time is a vase of the period in which Thales lived. This Cypriote vase bears the inscription of the name of the owner.

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118 Vera Sanford, "Thales, The First of the Seven Wise Men of Greece", Mathematics Teachers, 23:85-86 (February, 1930)

119 Louis Charles Karpenski, The History of Arithmetic (Chicago: Rand, McNeely and Company, 1925), p. 12.

TA LE SE

Thales

It is not at all impossible that the vase belonged to Thales of Miletus. However, since there were many men by the same name, there is a chance that the vase might have belonged to some one else.[120]

13. PYTHAGORAS

Pythagoras probably ranks first among the interesting figures in the history of ancient mathematics. No doubt this is due to four things, namely:

1. The mystery surrounding his life.
2. His mysticism.
3. The brotherhood he established.
4. His mathematical ability.

The exact time and place of the birth of Pythagoras is not known. The majority of authors, however, have used the date 580 B. C. although some have used 568 B. C. Most authorities have given the Island of Samos as the birthplace of Pythagoras. Some say that he was born in Italy and migrated to Samos with his father when but a child. In the British Museum in London are a number of coins which have been found

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[120] David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), p. 65, Vol. I.

on the Samian Island. These coins, which were made several centuries after the birth of Pythagoras, bear his name and figure. This would not likely have been the case had he been born elsewhere. His father was probably a merchant by trade and an engraver of seals as a side occupation.[121]

Regardless of his birthplace, time, and parentage, Pythagoras lived in stirring times and was one of the great makers of civilization of his period. At this time Samos was the center of Greek art and culture, while Confucius was teaching in China, and Buddha was spreading his doctrine in India.

Little is known about the life of Pythagoras. It is known that he was a pupil of Thales for several years. Thales persuaded him to visit Egypt where he studied Egyptian mathematics and religion. Nothing is definitely known about his life after this for several years but it is thought on his return from Egypt, he was captured by the Persians and studied with the Magi, the wise men of Persia. Probably he spent some time in traveling there and studying the religious doctrines of that country. He may have traveled in Babylon, India, and China before his return to Greece.[122]

After returning from his travels, Pythagoras returned to

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121 Ibid., p. 70

122 Ibid., p. 71

Samos where he started giving lectures to the young people of the Island. They seemed to have taken his lectures as a joke. The government was opposed to his teachings, so he soon went to Sicily with his mother and finally located in Crotona, a town on the southeastern coast of Italy. Crotona was a wealthy seaport. This part of the country had a democratic form of government. The aristocrats were looking for a leader. Pythagoras soon took this place and before long had gathered around him three hundred of the noble and wealthy men. They established a brotherhood which has been an example for all secret organizations since that time. His school was crowded with enthusiastic audiences. Women at that time were forbidden by law to attend any public meetings; nevertheless, they broke the law and flocked with the men to hear Pythagoras.[123]

Among the most attentive listeners at the Pythagorean school was Theano, the young and beautiful daughter of Milo, at whose home Pythagoras stayed. In spite of a probable forty years difference in their ages, they were married. Theano wrote a biography of Pythagoras but unfortunely it has been lost.[124]

The Pythagorean school was divided into two parts, the

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[123] W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 19.

[124] Ibid., pp. 19-20

probationers (or listeners) and the Pythagoreans (or mathematicians), the latter forming the brotherhood.[125] The members of the brotherhood had all things in common; that is, they had the same philosophical and political beliefs, bound by oath to the secrets of the brotherhood, ate simple food, had strict discipline, and their method of living encouraged self-command, temperance, purity, and obedience.[126] The Pythagorean brotherhood was a mystic circle and each member was purified from some imaginary guilt. They were superstitious as is shown by the following rules which they believed would bring great harm to them if disobeyed:

1. Not to sit on a quart measure.
2. Not to eat beans or the hearts of animals.
3. Not to stir fire with an iron rod.
4. Not to look in a mirror which was near a light.
5. Not to step across the beam of a balance.[127]

The belief of the transmigration of souls which is prevalent in India to-day is practically the same belief which Pythagoras held twenty-five hundred years ago.[128] Pythagoras

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125 Vera Sanford, "Pythagoras." Matheamtics Te acher, 23: 186, (March, 1930)

126 W. W. Rouse Ball, Primer of the History of Mathematics (London: Macmillan and Company, 1927), p. 21.

127 New International Encyclopedia, 2nd edition, "Pythagoras", Vol. XIX, p. 408.

128 Thomas Little Heath, "Pythagoras." Encyclopedia Britannica, 14th edition, Vol. XVII, p. 803.

led his brotherhood to believe that he had been on this earth before but in a different form. One day, when walking down the road, he saw a man whipping a dog. The dog was howling. Pythagoras asked the man who was whipping the dog to stop, saying, "It is the soul of a friend of mine, whom I recognize by his voice."[129] The same idea was held by the entire brotherhood. The reason that they would eat no meat was that they were afraid that they might eat the flesh of some blood relative. They were against the sacrificing of animals. They considered it murder.[130]

Shakespeare refers to the acceptance of the beliefs of Pythagoras and his school of the transmigration of souls in the Merchant of Venice.

> Thou almost mak'st me waver in my faith,
> To hold opinion with Pythagoras,
> That souls of animals infuse themselves
> Into the trunkes of men.[131]

The symbol of the brotherhood was five pointed star, the pentagram, which was probably discovered by Pythagoras. It was a symbol of health and gymnastics, both of which were taught in the school.[132]

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129 Chamber's Encyclopedia, "Pythagoras.", Vol. VIII, p. 38.

130 Thomas Little Heath, op. cit., p. 803.

131 William Shakespeare, Merchant of Venice (Chicago: Scott, Foresman and Company, 1919), edited by Robert Morss Lovett, p. 128

132 Florian Cajori, History of Mathematics (New York: Macmillan Company , 1924), p. 19.

All members of the brotherhood were bound by oath to secrecy. The only secrets which were revealed were told by Pythagoras, himself. It has been told that one member became disgusted with the brotherhood and left the school. After he had gone he told many things concerning the school. The man was instantly captured by the Pythagoreans and was thrown into the sea where he was drowned.[133]

Pythagoras would allow none on his teachings to be put into writing. This may have been because the papyrus which was used in Egypt was not readily available in Crotona, parchment had not been invented as yet, and the wax and clay tablets, although Pythagoras may have learned about them in his travels, were hardly usable for such things. The teachings of the brotherhood were, therefore, handed down from mouth to mouth. The first book containing any of the teachings was written by Philolous in 385 B. C. As far as it is known this book is not in existence at the present time. The library in Alexandria which was burned in 47 B. C. contained many ofthe books written by the early writers and it is possible that the book written by Philolous was destroyed at that time.[134]

Many things about mathematics which were not known

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[133] W. W. Rouse Ball, _A Short Account of the History of Mathematics_ (London: Macmillan and Company, 1915), p. 20.

[134] _Loc. cit._

prior to this time were added by Pythagoras. Among these were:

1. The Pythagorean Theorem which states, "The square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides." was named for Pythagoras. There is no doubt but what Pythagoras was the first to discover the proof for this theorem even though it is commonly believed that he received the idea while traveling in Egypt.[135]
2. The theorem "The sum of the angles of a triangle is equal to two right angles or 180 degrees."[136]
3. The five regular solids--the cube, tetrahedron, octahedron, icosahedron, and dodecahedron.[137]
4. The construction of a rectilinear figure equal and similar to another figure.[138]
5. The definitions of many geometric terms. Some of the definitions used by Pythagoras are still in use

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[135] Jasper O. Hassler--Roland R. Smith, The Teaching of Secondary Mathematics (New York: Macmillan Company, 1930), p. 68.

[136] Thomas Little Heath, "Pythagoras." Encyclopedia Britannica, 14th edition, Vol. XVIII, p. 803.

[137] David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), p. 75, Vol. I.

[138] Thomas Little Heath, op. cit., p. 804.

to-day.[139]

6. The division of integers into such groups as odd and even, prime and composites, etc.[140]

Either Pythagoras or his school knew that a plane space about a point may be filled by six equilateral triangles, four squares, or three regular hexagons. This fact doubtless was inferred as a result of the observation of mosaic pavements long before this time. Pythagoras, no doubt, was the first to be able to prove that this was true.[141]

Pythagoras was the first man of whom any record has been found who believed the world to be round. It is not known why he came to this conclusion but no doubt there were few people who accepted the belief at that time.[142]

There are several stories about the death of Pythagoras. About 500 B. C. his brotherhood became more powerful and the government of the country more democratic. The leaders of the government decided to do away with the brotherhood. Meeting houses of the brotherhood were sacked and burned. The house of Milo was among those burned. Fifty followers of Pythagoras were killed or burned to death at the same

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139 Ibid., p. 803.

140 Loc. cit.

141 Florian Cajori, A History of Mathematics (New York: Macmillan Company, 1924), p. 19.

142 Thomas Little Heath, op. cit, p. 804.

time. Some authorities say that Pythagoras was among those killed.[143]

Other authorities say that Theano, the wife of Pythagoras, was killed in the sacking and that Pythagoras spent the rest of his life traveling. One day he became ill and, although nursed by a kind hearted innkeeper, did not recover. Before his death he inscribed the pentagram of his brotherhood and the motto "A figure and a step forward; not a figure to gain through oboli" upon the board. Oboli were Greek coins worth about three cents each in United States money. The innkeeper hung the board on the outside of his home where it was discovered several years afterwards by a member of the school of Pythagoras.[144]

Two centuries later, 343 B. C., the Senate erected the statue of Pythagoras in Rome. On the statue were carved the following words: "To the wisest and bravest of all Greeks". The statue was still to be seen in Rome at the beginning of the Christian era.[145]

14. PLATO

Never in all the early history of Greece was there more

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143 _Ibid._, p. 802

144 Vera Sanford, "Pythagoras." _Mathematics Teacher_, 23: 186 (March, 1930)

145 David Eugene Smith, _op. cit._, p. 76.

need of a leader in education than during the time in which Plato lived. Plato was born in Athens in 430 B. C. Both his father and mother were descendents of the most distinguished families of Athens.[146] Little is known about the early life of Plato. This is due to the fact that so many of the early records have been lost or destroyed.

Plato's real name was Aristocles. He was surnamed Plato because of his broad shoulders.[147] It is possible that he made his home with a wealthy uncle, his father having died when Plato was but a lad. He was well educated for a youth of that period. From his writings a person might judge that music and gymnastics were the two branches which were his specialities. No doubt he was well versed in poetry, art, and all the culture of his age.[148] Tradition tells us that at the age of twenty Plato burned all the poems which he had written.[149]

For a youth of Plato's birth and endowments, it seems as though politics would have been the natural career. Plato, however, is said to have witnessed the murder of his teacher, Socrates, by some government officials which turned

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146 Ibid., p. 87.

147 William A. Hammond, "Plato." New Americana Encyclopedia, 2nd edition, Vol. XXII, P. 233.

148 Alfred Edward Taylor, "Plato." Encyclopedia Britannica, 14th edition, Vol. XVIII, p. 48

149 Chamber's Encyclopedia, "Plato." Vol. VII, p. 587.

him against leading the life of a politician. Plato left Athens soon after the death of Socrates and traveled extensively in Greece, Southern Italy, Sicily, and even in Egypt and northern Africa. Although one of the purposes of the visits to these countries may have been commercial policies, probably the chief reason was that he might obtain that knowledge which was available in other countries and expecially the learning of the Egyptian priests. Plato no doubt studied with some of the followers of Pythagoras in Italy, and it was from them that he learned to appreciate the value of geometry. Plato returned to Athens when he was about forty years old.[150]

According to one story, Plato visited the king of Syracuse during his travels. This ruler, Dionysius, was a very cruel man and by many considered a tyrant. Dionysius, while conversing with Plato, became very much offended by the freedom of Plato's speech. Not wanting to murder him because of the disasterous results which might come to him, Dionysius planned to have Plato captured while on his homeward journey. Plato was then sold into slavery; his friends in Athens soon heard of the plight of Plato and sent ransom money to have him released.[151]

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[150] New International Encyclopedia, "Plato.", 2nd edition Vol. XVIII, p. 713.

[151] William A. Hammond, "Plato". Americana Encyclopedia, 2nd edition, Vol. XXII, p. 233.

Soon after his return to Athens, Plato established a school in a gymnasium about a mile from Athens. The school was called the "Academy". The name was derived from the Greek hero, Academus, whose home was surrounded by beautiful gardens and trees.[152] This place adjoined the small estate which Plato dedicated to the use of the school. When Plato died he left his gardens and other property including the school to a group of his followers, organized into a club. The Athenian law did not recognize the right of a club to own property. The club, not wishing to see the work of Plato and his school cease, claimed to be organized for the worship of the Muses, the patrons of literature and learning. The name Museum was given to the academy.[153] This was the first endowed academy or university. This university was in existence for eight centuries or until about fourteen hundred years ago.[154]

The Academy was the first school to have an entrance requirement. This requirement was carved in a slab of stone which was above the main entrance of the school. The requirement was "Let no one ignorant of geometry enter my doors."[155]

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152 New International Encyclopedia, op. cit., p. 713.

153 Willis Mason West, The Ancient World (New York: Allyn and Bacon, 1913) pp. 291-92.

154 New International Encyclopedia, op. cit., p. 713.

155 Vera Sanford, "Plato, One of the Three Greatest Athenian Names." Mathematics Teacher, 23: 268 (April, 1930)

Although Plato was not primarily a mathematician, he added a great deal to the study of mathematics. His theory seemed to be that everyone should learn the fundamental parts of mathematics the same as they should learn the alphabet. Plato insisted upon very accurate definitions in geometry.[156]

Plato died in 349 B. C., while at the marriage feast of a friend, on his eightieth birthday.[157] His name has gone down in history as one of the most noted scholars of his day and one of the most influential men of all times.

15. EUCLID

The most successful textbook writer the world has ever known was Euclid. This is shown by the fact that over one thousand editions of his geometry have been printed since 1482 and his work dominated the teaching of the subject for eighteen hundred years before that time.[158] The geometry which is used as a textbook in the state of Kansas at the present time is only one of a great many geometries which are in use, all of them based upon Euclid's work. It is interesting to compare one of these late geometry textbooks with

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[156] Vera Sanford, A Short History of Mathematics, (Boston: Houghton Mifflin Company, 1930) p.9.

[157] William A. Hammand, "Plato." Americana Encyclopedia 2nd edition, Vol. XXII, p. 233.

[158] David Eugene Smith, History of Mathematics, (Boston: Ginn and Company, 1925), pp. 103-4.

an English translation of Euclid, such as the Tedhunder edition.

Euclid, sometimes called the father of mathematics, was born about 300 B. C. Almost nothing is known about the life of Euclid--not even his birthplace, the date of his birth, the date of his death, or even his nationality. He was either a Greek or an Egyptian who studied at the University of Alexandria and later taught there.[159]

Euclid's teaching was distinguished by fairness, kindness, gentleness, and modesty. One incident in his teaching experience is frequently related. In one of his classes Euclid had a lad who had just begun the study of geometry. After he had studied the first theorem he asked Euclid what he would gain by learning all that stuff. Euclid insisted that the knowledge was worth acquiring for its own sake but the youth was not satisfied. Finally Euclid called his slave and had the slave give the boy some coppers since the lad could not learn unless he made aprofit from it.[160] One day a student asked Euclid if there was not an easier way to learn geometry and Euclid replied, "There is no royal road to geometry."[161] He went on in the explanation that the only

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[159] Ibid., p. 105.

[160] R. R. Vivian, "Mathematics: A Great Inheritance." Educational Review, 53: 34 (July, 1917)

[161] Thomas Little Heath, "Euclid." Encyclopedia Britannica, 14th edition, Vol. VIII, p. 802.

way to master the subject was by hard work.

All treatises at the time of Euclid were written on long strips of parchment or papyrus. Each long strip or part of the treatise was rolled separately and called a "volume", a word which came from the Latin word meaning "to roll". It was difficult to handle these long strips even when rolled. Soon they were divided into parts or smaller rolls known as "biblia", derived from "biblos", a Greek word meaning "books", originally meaning "papyrus". Because of this we have books of the Bible, the books of geometry, the books of Homer, and so on.[162]

Although he is the author of several books on mathematics and physics, the fame of Euclid at all times has rested mainly upon his books on geometry, called the _Elements_. The _Elements_ were made up of thirteen books, but not all of them dealt with geometry. Some of the books dealt with the theory of numbers, and some with algebra. In fact, the _Elements_ might be called a "General Mathematics". The books relating to geometry were a collection of all the material relating to the subject before the time of Euclid. Many of the propositions and theorems were original with Euclid, but it is not known how many. In a report of a Committee of the British Association for the Advancement of Science (comprising some of England's ablest mathematicians) in 1887, a

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[162] David Eugene Smith, _op. cit._, p. 105

statement was given which seems to evaluate the work of Euclid. It was as follows: "No textbook that has yet been produced is fit to succeed Euclid in the position of authority."[163] It may be well to admit that Euclid's Elements were written in the first place for mature philosophers. It was soon afterwards used by boys of fourteen. But in those days it took a mature philosopher to work what some of the modern high school students would consider easy problems.[164]

In the Elements were found assumptions with some explanation of each, three postulates, and twelve axioms. Although the word "axiom" had been used by Procius before the time of Euclid, Euclid did not use it. He used instead "common-notions"--common either to all men or to all sciences. The proofs for the theorems found in the Elements were very systematically arranged. These proofs have been the standard formation for all geometric proofs to this time. The fact that until recent years the English translation of the Elements was used as a textbook, show how complete the works of Euclid were.[165]

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163 Florian Cajori, A History of Elementary Mathematics (New York: Macmillan and Company, 1924), pp. 67-9.

164 George W. Evans, "Heresy and Arthodoxy in Geometry." Mathematics Teacher, 19: 195, (April, 1926)

165 W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 54.

16. ERATOSTHENES

Eratosthenes was born in 275 B. C. and died in 194 B. C. He was the librarian at the University of Alexandria. It was Eratosthenes who first suggested the calendar which is now known as the Julian calendar and the one which is in use today. He suggested that every fourth year should contain three hundred sixty-six days. Eratosthenes was also interested in geography. He was the first to measure the circumference of the earth. He measured one degree as nearly seventy-nine miles on the earth's surface. Using his measurements the circumference of the earth would be about 3925 miles.[166]

Eratosthenes was especially interested in prime numbers. He constructed a table, called a "sieve," for the construction of prime numbers. The rule was:

> Write down all the numbers from 1 upward; then every second number from 2 is a multiple of 2, and may be cancelled; every third number from 3 is a multiple of 3 and may be cancelled; every fifth number is a multiple of five, and may be cancelled; and so on. It has been estimated that it would involve working for about 300 hours to find all the primes between 1 and 1,000,000 by this method.[167]

Upon catching an eye disease which was very common along the Nile River, Eratosthenes committed suicide rather

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166 W. W. Rouse Ball, Primer of the History of Mathematics (London: Macmillan and Company, 1927), p. 25.

167 Ibid., p. 26.

than live a sightless life.[168]

17. ARCHIMEDES

Archimedes, who was the first great mathematician to apply mathematics to mechanics, was born in Syracuse, Sicily, in 278 B. C. and died in 212 B. C. Some biographies of Archimedes say that he was a relative, or certainly a friend, of King Hiero. If he held a public office, which no doubt would have been true had he been a relative of the king, it is not known to-day.[169] Cicero, writing about Archimedes, said that he was of low birth.[170] Archimedes studied at the University of Alexandria. After his return to Sicily he became a close friend of King Hiero who was his great admirer.[171]

There is no doubt but that Archimedes did some work in the Library of Alexandria, and he may have helped collect the rolls for the library. This library, which contained an unusual collection of rolls and was the largest library of the ancient world, was founded by Ptolemy I about

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168 Vera Sanford, A Short History of Mathematics, (Boston: Houghton Mifflin Company, 1930), p.11.

169 Encyclopedia Americana, 2nd edition, Archimedes." Vol. II, p. 168

170 Florian Cajori, A History of Mathematics (New York: Macmillan Company , 1924), p. 34.

171 W. E. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 64.

300 B. C. It grew quickly, and it is thought to have contained from 490,000 to 700,000 volumes. The greater part of the library, which contained collections from Greece, India, Rome, and Egypt, was located in a museum. However, part of the library was in the temple of Serapis. The library was at the disposal of all the scholars of the world. The material in the library was catalogued according to the principal theme, such as history, literature, mathematics, etc., with a biographical sketch of each author.

When Julius Caesar laid seige to Alexandria in 47 B. C., the part of the library located in the museum was destroyed by fire which spread from a burning fleet of ships. The library was replaced by the part of the original library in the Temple of Serapis and by the collection of rolls given to Cleopatra by Mark Antony. The new library was placed in the Temple of Serapis. The fate of this part of the library is not known. Some say that when the heathen temples in the Roman Empire were destroyed in 391 A. D. the Temple of Serapis was not spared in spite of its great value. Others say that it was destroyed by the Arabs when they captured and burned a part of the city of Alexandria. This later story has been discredited by most authorities who believe the library was not in existence at that time.[172]

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[172] Chamber's Encyclopedia, "Alexandria, Library of", Vol. I, p. 131.

If the Alexandrian Library were in existence today, so many things which are uncertainities could easily be settled. The translations of the rolls and tablets would extend our knowledge of ancient works in literature, mathematics, science, and other subjects. No doubt some of the lost arts of the older days might be r ecovered.

Archimedes discovered the principle, "that a body immersed in a fluid loses as much in weight as the weight of an equal volume of fluid," which is one of the fundamental principles in physics to-day. King Hiero had had a crown of gold made. The crown was to be used as a sacrifical offering to the Gods. The king had ordered that the crown be made of pure gold. When it was brought to him, the king suspected that perhaps the goldsmith had fraudulently added too much alloy or cheaper metal to the gold. The king asked Archimedes to find out whether or not his suspicions were true. Archimedes was somewhat troubled about it. It was an entirely new problem about which to think and naturally he was anxious to please the king by solving it. One day while bathing in one of the public baths in Syracuse, he suddenly jumped from the pool and hastened home, still undressed. As he ran through the streets he cried out "Eureka! Eureka! I have found it! I have found it!"[173] No doubt it gave

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[173] R. R. Vivian, "Mathematics: A Great Inheritance." Educational Review, 53: 34 (July, 1917).

Archimedes great joy to solve such problems.

King Hiero had a ship built for commercial purposes. After it was finished they discovered it was so large that they could not launch it in the sea from the frame in which it was built. Archimedes was called to the king and asked if he could contrive some means by which the ship could be launched. Archimedes built a machine which was used in the launching of the ship but no description of the machine is available to-day. When being congratulated upon his invention, Archimedes made the well known remark, "given the place to stand, he could move the earth".[174] This is sometimes quoted as "give me a fulcrum and I will move the world".[175]

While Archimedes was studying in Egypt he spent some of his leisure time along the Nile River watching the Egyptian slaves trying to drain some of the low land. In other sections the slaves were carrying water from the river to irrigate some of the fertile land which needed only water to make it produce large crops. All the draining and irrigating was done with such difficulty that Archimedes started to work on something which would be of benefit to them. The result of his labors was the Archimedes' Screw. It is still used to-day, especially in Holland, for the draining and irrigating

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[174] Thomas Little Heath, "Archimedes." Encyclopedia Britannica, 14th edition, Vol. II, p. 269.

[175] E. W. Brown, "History of Mathematics." Scientific Monthly, 12: 392 (May, 1921).

of land.[176]

Many legends are told about the different inventions of Archimedes, but the validity of some of them has never been established. In fact some of them are almost fantastic. Among these is a legend which tells that a fleet of war vessels was in the harbor of Syracuse with the plan to capture the city. Archimedes with the use of mirrors and sun glasses set fire to the ships.[177] Another story tells that Archimedes invented a war engine which was used during the time the Romans were trying to capture Syracuse. The engine could hurl rocks and other sharp missiles a very great distance and to any location desirable terrifying the Romans to such an extent that they soon left the harbor of Syracuse.[178] This is of course very possible and it is known that the Romans used a similar machine later in their warfare.

There are three stories about the death of Archimedes. All of them are similar and no doubt started from one story. One of the three stories relates that Archimedes was on the seashore working with some of his diagrams. He was so intent on his work that he didnot notice that the Romans had entered the city. A soldier approached Archimedes and asked

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[176] New International Encyclopedia, "Archimedes' Screw." 2nd edition, Vol. II, p. 56.

[177] W . W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 65.

[178] Vera Sanford, A Short History of Mathematics (Boston: Houghton Mifflin Company, 1930), p. 12.

him to report to Marcellus, the Roman general who had taken charge of Syracuse. Archimedes refused to do so until he had finished the problem on which he was working. The soldier then drew his sword and killed Archimedes.[179]

A second story tells that as Archimedes was sitting by the seashore drawing pictures in the sand, a soldier came and stood by him for a time unnoticed by Archimedes. After some time the soldier, probably because he was attracting no attention, began to destroy the figures with his foot. Archimedes remonstrated and asked the soldier not to destroy his work. The soldier was an independent fellow and, when asked not to destroy the figures, became very angry and killed Archimedes.[180]

According to the third version of the story, the soldier, after watching Archimedes work with some of his instruments, asked him to take them to Marcellus who was much interested in such things. Archimedes loaded his arms with spheres, angles, dials, and other instruments and started down the road toward the Roman encampment where Marcellus had his headquarters. Archimedes was proud of his instruments and always kept them polished. Some of the soldiers, who were on guard, saw Archimedes approaching. As the sun

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[179] T. L. Heath, <u>The Works of Archimedes</u> (Cambridge: University Press, 1897), p. xviii.

[180] <u>Encyclopedia Americana</u>, 2nd edition, "Archimedes." Vol. II, p. 169.

shone on the instruments which he was carrying, they appeared as if they were made of gold. The soldiers, tempted by the opportunity to obtain so much gold, killed Archimedes. As the soldiers started to leave, they were caught and taken to Marcellus.[181]

Whether or not any of these legends are true, it is known that Marcellus was greatly grieved when he learned of the death of Archimedes and had a tombstone erected to his memory. On the tombstone was placed a cylinder with a sphere inscribed in it. This was symbolic of the relationship Archimedes had found to exist between the two figures. The people of Sicily soon forgot that such a man as Archimedes ever existed. Soon brush and weeds were allowed to grow around the tombstone, which was eventually hidden from sight. About one hundred fifty years later Cicero, while ruler of Sicily, wandering about the city one day, noticed the thicket and wondered what it might hide. Thus he found the tomb of Archimedes and he had it restored to good condition.[182]

Archimedes wrote several books. The most important among these were three on plane geometry, three on solid geometry, and one on arithmetic, and three on mechanics.[183] In

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181 T. L. Heath, Op. cit., p. xviii

182 Thomas Little Heath, "Archimedes." Encyclopedia Britannica, 14th edition, Vol. II, p. 269.

183 Florian Cajori, A History of Mathematics (New York: Macmillan Company, 1924), p. 34.

1906 Professor Heiberg found in a museum in Constantinople a manuscript written by Archimedes on some geometric solutions. Upon examining the work and having it translated, it was understood why a problem with its solution was too deep for the ordinary mind to solve.[184] One of the important rules which Archimedes used in the solution of his problems was that pi was less that 3 1/7 and greater than 3 10/71.[185] In his arithmetic known by its Latin name, Arenarius (sand-reckoner), Archimedes tried to express the amount of sand which it would take to fill the universe. This work has given rise to the idea that Archimedes invented a new and powerful system of notation, all knowledge of which perished with the work itself.[186]

18. NICOMACHUS

Nicomachus, a Jew, was born at Gerasa about 100 A. D. Gerasa was probably located where now is found the modern town Jerash, fifty-six miles north east of Jerusalem. Nicomachus was a member of the Neopythagoreans, an organization of prominent men who lived in Alexandria and who were trying to revive the teachings of Pythagoras. He probably lived in

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[184] Ibid, p. 35

[185] W. W. Rouse Ball, Primer of History of Mathematics (London: Macmillan and Company, 1915), p. 22

[186] Florian Cajori, op. cit., p. 29

Alexandria for some time working with the organization and studying their beliefs.[187]

The period in which Nicomachus lived was a period of decay for the intellectual world. If it had not been for the fact that Nicomachus did something which had not been done before, his name would no doubt have been forgotten. Several men before this time had done much work in arithmetic, but none of them had paid much attention to what had been worked out before them. Nicomachus wrote a book on arithmetic in which he summarized all the existing knowledge of the subject. He must have spent a long time in collecting it and organizing it in a systematic order. Because it was the best book in arithmetic at that time, it was adopted as a textbook in a few remaining schools. Boethius translated the book from the Greek, in which it was written, into Latin. The same book was used in the European schools, especially the church schools, until near the close of the seventeenth century and was the only book on arithmetic until the close of the twelfth century.[188]

One of the things of interest which Nicomachus had in his arithmetic book was his method for finding cubes or numbers. According to his method the cube of any number is equal to the sum of successive odd numbers. For example the

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[187] David Eugene Smith, A History of Mathematics (Boston: Ginn and Company, 1925), Vol. I, pp. 127-28.

[188] Ibid., p. 128-29.

following cubes might be given:

$$8 = 2^3 = 3 + 5$$

$$27 = 3^3 = 7 + 9 + 11$$

$$64 = 4^3 = 13 + 15 + 17 + 19$$

$$125 = 5^3 = 21 + 23 + 25 + 27 + 29$$ [189]

If the students in arithmetic today were to study the book written by Nicomachus, there would be little doubt that they would find it very difficult to read. This may be judged by the following quotation:

> It comes about that even as fair and excellent things are few and easily enumerated, while ugly and evil ones are widespread, so also the superabundant and deficient numbers are found in great multitude and irregularly placed-------but the perfect numbers are easily enumerated and arranged in suitable order; for only one is found among the units, 6, only one among the tens, 28, and a third in the ranks of the hundreds, 496, alone, and a fourth within the limits of the thousands, that is below ten thousand, 8128.[190]

The above quotation is a bout one-tenth of the discussion on perfect numbers. By reading and summarizing the quotation, the definition of a perfect number is one which is equal to the sum of its aliquot parts. The first perfect number is six because it is the sum of its aliquot parts; namely, one, two, and three. Numbers, whose aliquot parts when added exceeded the number itself, were called abundant

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189 Florian Cajori, op. cit., p. 32

190 Louis Charles Karpenski, The History of Arithmetic (Chicago: Rand McNally and Company, 1925), pp. 16-17.

or excessive numbers. Take for example the number twelve. The aliquot parts are one, three, two, four, and six. These aliquot parts added together make sixteen which is larger than the original number twelve.[191]

19. DIOPHANTUS

In spite of the fact that Diophantus was one of the greatest of all Greek mathematicians, the only things which are known about his life are found in a curious problem found in the Greek Anthology which dates back to the fourth century. According to this problem Diophantus' boyhood lasted one-sixth of his life, his beard grew after one-twelfth more, after one-seventh more he married, five years later his son was born, the son lived to be half of his father's age, and the father died four years after his son.[192] If the problem is authentic, the algebra student can easily find that Diophantus lived to be eighty-four years old. It is not known where he went to school. If he went at all, it is not known who his teachers were, what interested him in mathematics in the first place, or what books he had a chance to read.

Although Diophantus wrote three different works, the

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[191] G. A. Miller, "Widespread Error Relating to Greek Mathematics." School and Society 18: 621 (November 24, 1923)

[192] Thomas L. Heath, Diophantus of Alexandria (Cambridge: University Press, 1910), p. 3.

most important one was Arithmetica. The Arithmetica consisted of thirteen books. All of these have been lost with the exception of six.[193] In these books Diophantus brought together all of the knowledge which the Greeks had of algebra.[194] Although he had many sources from which to draw his material, there is no doubt but that Diophantus made many contributions himself. The people who had written about algebra before had used words instead of symbols. Diophantus could see no value in this practice so in his books he used the first symbols of which there is any record.[195] ⋔ was the symbol he used for subtraction[196] and ι the symbol he used for "equals".[197]

Diophantus was the first to use coefficients and exponents in an abbreviated form. He wrote X^2 as Δ^{υ}, X^3 as K^{υ}, X^4 as $\Delta\Delta^{\upsilon}$, X^5 as ΔK^{υ}, and X^6 as KK^{υ}.[198]

The following paragraphs are the paragraphs used by Diophantus in the front of his Arithmetica for the dedication

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[193] Thomas Little Heath, "Diophantus." Encyclopedia Britannica, 14th edition, Vol. VII, p. 400.

[194] W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 104.

[195] Vera Sanford, A Short History of Mathematics (Boston: Houghton Mifflin Company, 1930), p. 143.

[196] Ibid., p. 149

[197] Ibid., p. 153

[198] Karl Fink, A Brief History of Mathematics (Chicago: Open Court Publishing Company, 1903), p. 65.

of the book:

> Knowing my esteemed friend, Diophantus, that you are anxious to learn how to investigate problems in numbers, I have tried, beginning from the foundations on which the science is built up, to set forth to you the nature and power subsisting in numbers.
>
> Perhaps the subject will appear rather difficult, inasmuch as it is not yet familiar (beginners are, as a rule, too ready to despair of success); but you, with the impulse of your enthusiasm and the benefits of my teaching, will find it easy to master; for eagerness to learn, when seconded by instruction, insures rapid progress.[199]

20. BOETHIUS

Anicius Manlius Severinus Boethius was born in Rome about 475 and died in 526. Boethius belonged to a family which for the two preceding generations had been esteemed one of the most illustrious in Rome. He may have been educated in Athens as he seemed to be familiar with Greek literature and science.[200]

Although Boethius liked to write, he realized that if he followed the career of a writer he probably would not become prominent in the affairs of Rome. He became a politician and was soon a favorite of King Theodoric. Boethius took advantage of his position to reform the coinage system

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[199] Thomas Little Heath, Diophantus of Alexandria (Cambridge: University Press, 1910), p. 129.

[200] Chamber's Encyclopedia, "Boethius." Vol. II, p. 185.

of Rome and he introduced sun dials and water clocks for public use.[201]

During the time in which Boethius lived, the public officials of Rome were in the habit of plundering the homes and land of the people for whatever they might want. Boethius was very much opposed to this custom and fought against it. This brought the hatred of the court upon him, and one time when he was absent from Rome, he was sentenced to death. He was found in a church in Ticinum and was captured. His captors placed him in the baptistry of the church and tortured him by drawing a rope around his head until his eyes were forced out of their sockets and then they beat him to death with clubs.[202]

After his death the church said that Boethius was a martyr and consequently his works were considered authentic. The public recognized his worth and later the state erected statues in his honor.[203]

The Romans generally, unlike the Greeks, were not interested in abstract mathematics. They were matter-of-fact people, interested only in enough mathematics to satisfy their few immediate needs. In philosophy, poetry, and art,

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[201] W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 32.

[202] Ibid., p. 132

[203] Vera Sanford, A Short History of Mathematics (Boston: Houghton Mifflin Company, 1930), p. 13.

the Romans were imitators. But in mathematics they did not even rise to the desire of imitation. As a consequence, not only the geometries of Archimedes and Apollonius, but even the Elements of Euclid were neglected. What little mathematics the Romans possessed did not come altogether from the Greeks, but came in part from more ancient sources. Exactly where and how some of it originated is a matter of doubt. It seems as though the "Roman notation" and the early practical geometry of the Romans may have come from the old Etruscans, who inhabited the district between the Arno and Tiber Rivers. It has been said that the Etruscans kept track of the years by driving a nail into a statue of Minerva whenever a year elapsed and that the Romans continued this practice.[204]

At one time during his reign, Julius Caesar desired to have the Roman Empire surveyed in order to secure a better division of the taxes. Their own people in Rome could not make the survey. Mathematicians were called from Alexandria to do the work. Caesar reformed the Roman calendar, but, to do so, he secured the services of the Alexandrian astronomer, Sasigenes.[205]

Boethius was almost alone in experiencing any great interest in mathematics in Rome. He must have studied the

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204 Florian Cajori, A History of Mathematics (New York: Macmillan Company, 1924), p. 63.

205 Ibid., pp. 90-91.

works of the Greek mathematicians quite thoroughly. The greatest work of Boethius, Consolation of Philosophy, was written while he was in prison. He wrote Institutis Arithmetic, which was essentially a Roman translation of the work of Nicomachus. Boethius also wrote a Geometry, which was made up of several books. The first book, being based on Euclid's Elements contained definitions, postulates, and axioms as well as the theorems of the first three books of Euclid. The proofs for the theorems were not given. The second book taught by numerical examples the mensuration of plane figures.[206]

A part of the Geometry of Boethius was given to the abacus, which he attributed to the Pythagoreans. Boethius introduced a great improvement to the old abacus. He discarded pebbles and "apicis" (probably small cones) were used. Boethius did not mention the symbol, 0, in his works.[207]

Another of Boethius' works was a book on music. Music at that time was ranked as a part of mathematics. This book was used as a textbook by Oxford until the present century. The other books of Boethius remained standard textbooks for several centuries.[208]

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206 David Eugene Smith, A History of Mathematics (Boston: Ginn and Company, 1925), pp. 178-9.

207 Florian Cajori, op. cit., pp. 67-8.

208 Encyclopedia Britannica, "Boethius.", 14th edition, Vol. III, p. 777.

CHAPTER V

HINDU ARABIC MATHEMATICS

INTRODUCTION

Soon after the decadence of the Greek mathematical research, the Hindus began to display brilliant mathematical powers. In geometry the Hindus were even weaker than the Greeks had been in algebra and arithmetic. However, it was in the field of algebra and arithmetic that the Hindus seemed to do their greatest work. It is certain that part of the Hindu mathematics was influenced by the Greek mathematics, but just how much is hard to say. The Chinese influence may have been felt at a later period. India had several great mathematicians. Among the most important ones are Āryabhata, Mahāvīra, Brahmagupta, and Bhāskara. These Hindu writers created a rhetorical algebra and applied it to the solution of equations. They made trigonometry an adjunct to astronomy.[209] They seem to have invented numerals which developed into the Hindu-Arabic system and the Zero which made the place value possible. Like other things, the Hindus probably did not realize the value of their number system. The symbol for zero, itself, was not invented until after the idea of a zero had become common property. The

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209 W. W. Rouse Ball, Primer to the History of Mathematics (London: Macmillan and Company, 1927), p. 41.

names of the men who first started using symbols are not known. It wasn't until after the numerals had been in use several centuries that the history of mathematics in India reached such a stage that the names of the most important contributors to the field of mathematics were found.[210]

The Arabs carried on a commercial trade with India. The introduction of the Hindu science into Arabia probably took place near the close of the eighth century. They adopted the Hindu decimal system of numeration. The scientific works of the Greeks were accessible to the Arabs and before the end of the ninth century the works of Euclid, Archimedes, and other Alexandrian writers were being translated under the authority of the caliphs. Al Khowarizmi was one of the Arabs whose works has come down to us at the present time.[211]

21. ĀRYBAHATA

The first of the great Hindu mathematicians, Āryabhata, was born at Kusumapura about 475 and died about 550. Kusumapura is not far from the present town of Patna on the upper Ganges River.

It is quite evident that Aryabhata did his writing near

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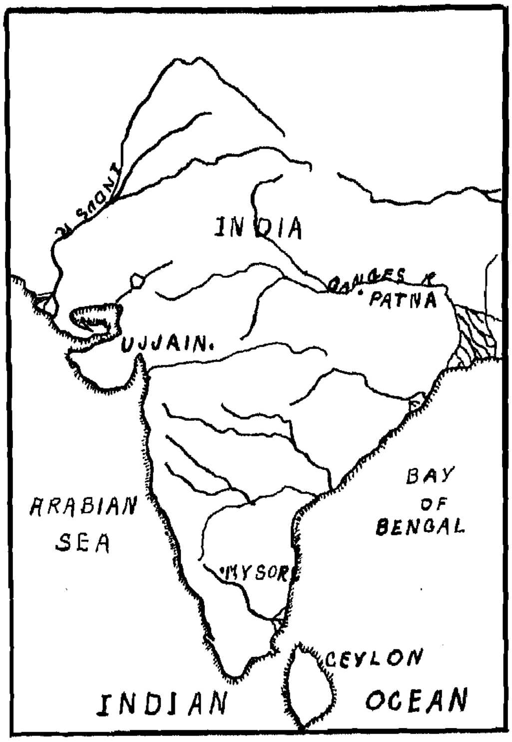
[210] Vera Sanford, A Short History of Mathematics (Boston: Houghton Mifflin Company, 1930), p. 14.

[211] W. W. Rouse Ball, op. cit., p. 41.



MAP 3

PROFILE MAP FOR HINDU ARABIC PERIOD OF MATHEMATICS

The map shows the location of the cities which are mentioned in the next section. The following is a list of the mathematicians mentioned, their chronological age, and the cities with which they were associated:

Āryabhata	510	Patna
Brahmagupta	628	Ujjain
Mahāvīra	850	Mysore
Bhāskara	1150	Ujjain

the city of Patna, for at the beginning of one of his works these words are found,

> Having paid homage to Brahma, to Earth, to the moon, to Mercury, to Venus, to the Sun, to Mars, to Jupiter, to Saturn, and to the constellations, Aryabhata, in the City of Flowers, sets for the science venerable.

The City of Flowers is one name which has been given to the town, Patna.[212]

The chief work of Aryabhata is Āryabhatīya which is written entirely in verse. The book has never been entirely translated. The language used in it is not easily understood, and it is so different from the Hindu language used to-day that no one has been able to translate it. The book is divided into four parts, the first three being devoted to astronomy and trigonometry and the last to algebra, geometry, and arithmetic.[213]

Āryabhata's work shows that he understood quadratic equations.[214] In the part given to arithmetic, when one number was to be added to another, the first number was placed after the second without any particular sign. The same thing was true of subtraction except that the number

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212 David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), Vol. I, pp. 154-55. (Citing, "Lecons de Cabcul d'Aryabhata." Journal Asiatique, 13: 396)

213 W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 147.

214 Loc. cit.

to be subtracted had a dot placed above it. The powers of numbers also received special attention.[215]

second power is varga or va

third power is ghana or gha

fourth power is va-va

fifth power if va gha ghata

sixth power is va gha

seventh power is va va gha ghata

(ghata meant addition)[216]

Several geometric rules were given by Āryabhata, most of which were imperfect as far as statement was concerned. The rule which he gave for finding the area of an isosceles triangle was correct. His rule was "The area produced by a trilateral is the product of the perpendicular that bisects the base and half the base."[217] The rule which Āryabhata used for finding the value of pi gives the value correct to a number of places. This may have been accidental, however, as in other parts of his works he used the value of pi as three or the square root of ten. To find the value of pi, Aryabhata added four to one hundred, multiplied the result

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215 Florian Cajori, A History of Mathematics (New York: Macmillan and Company, 1924), p. 93.

216 Karl Fink, A Brief History of Mathematics (Chicago: Open Court Publishing Company, 1903), p. 72

217 David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), Vol. I, p. 156.

by eight, and added 62,000 to this product, giving as a result the approximate value of the circumference of a circle whose diameter is 20,000.[218]

22. BRAHMAGUPTA

Brahmagupta, who was born in 598 and died in 660, lived at Ujjain, the site of the great astronomical observatory in Central India. At the age of thirty he wrote a book on astronomy, Brahmasidhānta, (meaning Brahma Correct System), which contained twenty-one chapters. The part of it devoted to algebra was called Kutākhadyaka, which means pulverizer.[219]

Some of the more important things found in his works are:

1. x in the equation $x^2 + px - q = 0$ is found by the formula $x = \frac{\sqrt{p^2 + 4q} - p}{2}$ [220]
2. The area of a quadrilateral is found by the formula $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$ where the sides are a, b, c, and d and the perimeter of the quadrilateral divided by 2 is s. This formula holds true only if the quadrilateral can be inscribed in a circle.[221]
3. He found the value of pi correct to several places

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218 Loc. cit.

219 Ibid., p. 158.

220 Ibid., p. 159.

221 Karl Fink, A Brief History of Mathematics (Chicago: Open Court Publishing Company, 1903), p. 216.

but never used it.[222]

The Hindu mathematicians were noted for their impractical, fanciful problems stated in the most flowery phrases. In this respect Brahmagupta was like the other Hindu mathematicians. The following are two of the problems which have been attributed to him:

> On top of a certain hill live two ascetics. One of them being a wizard, travels through the air. Springing from the summit of the mountain he ascends to a certain elevation and proceeds by an oblique descent diagonally to a neighboring town. Their journeys are equal. I desire to know the distance of the town from the hill, and how high the wizard rose.[223]

> A cat sitting on a wall 4 cubits high, saw a rat prowling 8 cubits from the foot of the wall. The rat too perceived the puss and hastened toward its abode at the foot of the wall: but it was caught by the cat proceeding diagonally an equal distance. In what point within the 8 cubits was the rat caught?[224]

23. AL-KHOWARIZMI

Al-Khowârizmî, whose full name was Abû Abdallâh Mohammed ibn Mûsâ Al-Khowârizmî (meaning Mohammed, the son of Musa--of Moses--from Khowarizm, a locality south of the Black Sea) was the first notable author of mathematical books.

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222 W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 149

223 David Eugene Smith, op. cit., p. 159

224 Vera Sanford, History and Significance of Problems in Algebra (Boston: Houghton Mifflin Company, 1925), p. 76. (Citing: "Brahmagupta." Colebrooks translation, p. 310)

Very little is known of the life of Al-Khowârizmî. He was a native of Khwarezm, the country in which is now located the city of Khiva. He was also engaged as librarian by the caliph, Al-Mamun. Part of his duties were to revise the tables of Ptolemy, to take observations at Bagdad and Damascus, and to work on the measuring of a degree on a meridian of the earth. Al-Khowârizmî accompanied a mission to Afghanistan and possibly returned to his native country by the way of India.[225]

Although an astronomer and the author of several astronomical tables, and works on sun dials and chronology, Al-Khowarizmi is best known for having written the first book bearing the name "algebra", a treatise based upon Greek models, and in which the Hindu numerals were used.[226] The title of the algebra was '<u>ilm</u> <u>al-jabr</u> <u>wa'l</u> <u>muqabalah</u>, meaning the "science of reduction and cancellation" or referring to the fact that the same magnitude may be added or subtracted from both sides of an equation without changing its value. In the book of Al-Khowârizmî, the unknown quantity is termed either as "the thing" or "the root of a plant". It was from the latter term that our use of the word root, as applied to the solution of an equation, is used.[227]

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225 W. W. Rouse Ball, <u>op</u>. <u>cit</u>., pp. 155-56.

226 Charles Pomery Sherman, "The Origin of Our Numerals." <u>Mathematics</u> <u>Teacher</u>, 16: 398-401 (November, 1923)

227 W. W. Rouse Ball, <u>op</u>. <u>cit</u>., p. 157.

The work of Al-Khowârizmî is divided into five parts. In the first part he gives rules for the solution of the quadratic equation. Although Al-Khowârizmî considered only real and positive roots, he recognized the existence of two roots, which as far as is known was never done by the Greeks. In one part of his book Al-Khowârizmî solved some problems, such, for example, as to find two numbers whose sum is ten and the difference of whose squares is forty.[228]

One of the interesting things given by Al-Kworârizmî was the geometric proof for an equation such as $x^2 + 10x = 39$, or any equation of the form $x^2 + px = q$. To do this he let AB represent the value of x and constructed the square ABCD on it. (See the figure below). Then he extended the line DC to F and the line AD to H, making AH = CF = 5 or 1/2 p. The figure was then completed as drawn below. Then the areas of ABCD, AHKB, and CBGF represent respectively the magnitudes x^2, 5x, and 5x. Thus the left side of the equation is represented by the sum of the areas of ABCD, HKBA, and BGFC. To both sides of the equation the area of the square KLGB, the area of which is 25 or $1/4\ p^2$ was added. The new square has the area of 39 + 25; that is, to 64 or $q + 1/4\ p^2$. A side of the new square is therefore 8. The side of the square DH will exceed AH, which is equal to 5, by the value of the unknown required, which is therefore 3.[229]

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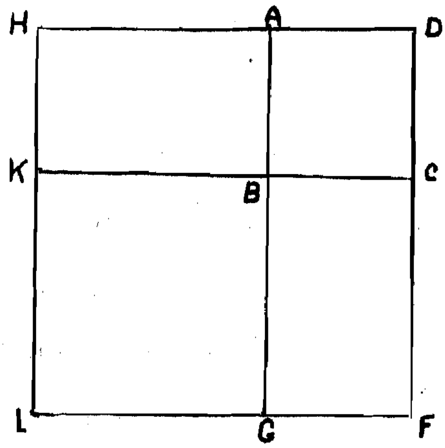
228 Ibid., pp. 151-158.

229 W. W. Rouse Ball, A Primer of the History of Mathematics (London: Macmillan and Company, 1927), pp. 42-43.



24. MAHĀVĪRA

Mahāvīrācārya, Mahāvīra the Learned, was the third of the great Hindu writers. He probably lived in the court of the monarchs or kings who ruled over what is now the native state of Mysore. The king's name is given as Amoghavarsha Nirapunga. It may be that Mahāvīra knew of the works of the other two Hindu writers, Brahmagupta and Āryabhata, and tried to improve upon their works.

The work of Mahavira, Ganita-Sāra-Sangraha, begins with a salutation of a religious nature. This was not unusual with Oriental treatises. In this case the words are addresses to the authors patron saint, the founder of the religious sect of Jainis (Jinas), a contemporary of Buddha. Thus:

> Salutation of Mahāvīra, the Lord of the Jinas, the protector (of the faithful), whose four infinite attributes, worthy to be esteemed in (all) the three worlds are unsurpassable (in excellence).
>
> I bow to that highly glorious Lord of Jinas, by whom, as forming the shining lamp of knowledge

of numbers, the whole of the universe has been made to shine.[230]

The work consists of nine chapters. Some of the interesting features of his book are:

1. The law relating to zero--"A number multiplied by zero, and that (number) remains unchanged when it is divided by, combined with, (or) diminished by zero."[231]
2. The rule for the inverting of the divisor in division of fractions--

   > After making the denominator of the fraction its numerator (and visa versa), the operation to be conducted then is as in the multiplication (of fractions).[232]

3. The use of 10 for the value of pi.[233]

Some of the problems taken from Mahāvīra's work are very fanciful and interesting. The following are some of the problems which have been found and attributed to him:

> A powerful unvanquished excellent black snake which is 32 hastas in length, enters into a hole (at the rate of) 7 1/2 angulas in 5/14 of a day; and in the course of 1/4 of a day, it's tail grows by 2 3/4 of an angula. O ornament of arithemticians, tell me by what time the serpent enters

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230 David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), p. 161. (Citing: Rangācārya, "The Ganita-Sāra-Sangraka of Mahāvīrācārya")

232 Loc. cit.

233 Loc. cit.

fully into the hole.[234]

One fourth of a herd of camels was seen in the forest; twice the square root (of that herd) had gone on to the mountain slopes; and three times five camels (were) however, (found) to remain on the bank of the river. What is the (numerical) measure of that herd of camels?[235]

Of a collection of mango fruits, the king (took) 1/6; the queen 1/5 of the remainder, and the three chief princes took 1/4, 1/3, and 1/2 (of that same remainder); and the youngest child took the remaining three mangoes. O you who are clever in miscellaneous problems of fractions, give out the measure of that (collection of mangoes).[236]

In the course of 3/7 of a day, a ship goes over 1/5 of a kros'a in the ocean; being opposed by the wind she goes back (during the same time) 1/9 of a kros'a. Give out, O you who have powerful arms in crossing over the ocean of numbers well, in what time that ship will have gone over yojanas.[237]

In to the bright and refreshing outskirts of a forest, which were full of numerous trees with their branches bent down with the weight of flowers and fruit, trees such as jambu trees, lime trees, plantains, areca palms, jack trees, date palms, hintala trees palmyras, punnage trees, and mango trees (into the outskirts) the various quarters whereof were filled with many sounds of crowds of parrots and cuckoos found near springs containing lotuses with bees roaming about them (into such forest outskirts) a number of weary travelers entered with joy. (There were) sixty-three (numerically equal) heaps of plantain fruits put together and combined with seven (more) of these

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234 Vera Sanford, _A Short History of Mathematics_ (New York: Houghton Mifflin Company, 1930), p. 17. (Citing: Raṅgācārya, translation _The Ganita-Sāra-Sangraka of Mahāvīrācārya_, p. 89)

235 David Eugene Smith, _op. cit._, p. 162

236 Vera Sanford, _op. cit._, p. 211

237 _Ibid._, p. 211

> same fruits, and these were equally distributed among twenty-three travelers so as to have no remainder. You tell me now the numerical neasure of the heap of plantains.[238]

If the students of algebra today were to find a few problems such as those in the preceeding paragraphs in their textbooks they might appreciate the simplicity which modern textnook writers strive to achieve.

There are a few problems to-day which are similar to the preceeding problems of Mahāvīra's. One of them is known as the "Sailor's Riddle". It was used to initiate a group of new sailors who were crossing the equator for the first time. Those who solved the riddle in an hour escaped any further hazing.

Three men, A, B, and C, and a monkey were stranded on a desert island. Their only food was coconuts. A, B, and C decided that they would gather all the coconuts and then give each man his share. By doing this no man would have more food than any other. They gathered the coconuts one day. They had just finished when evening came, so they piled them in one stack and went to bed, planning to divide them the next morning. During the night A awoke. He decided that he would go to the stack of coconuts and divide them while the others were asleep. By doing this he would be sure

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238 David Eugene Smith, History of Mathemtaics (Boston: Ginn and Company, 1923), p. 163. (Citing: Rangācārya, translation The Ganita-Sāra-Sangraha of Mahāvīrācārya)

to get his share. A divided the nuts into three stacks and had one left over. He put his pile behind some bushes near by and finally decided it wouldn't do to take all the nuts from the monkey, so gave him the one that was left. The other two piles he stacked together and then went back to bed. During the night B and C did the same thing that A had done. Each man had one coconut left for the monkey. The next morning each man realized that the pile of nuts was smaller than it had been the night before, but, thinking he was responsible for it, said nothing. They gathered around the pile and upon dividing it found that it was exactly divisable by three. How many coconuts were in the pile at the beginning?

The problem may be worked by the following method:

Let $X =$ number of coconuts

If $y =$ first man's share

$$3y + 1 = X$$

If $z =$ second man's share

$$3z + 1 = 2y$$

If $w =$ third man's share

$$3w + 1 = 2z$$

$2w/3 =$ each man's portion in the morning

$2w/3 = (8X - 38)/81 =$ (integer)

$$X = (81n + 38)/8$$

$$= (80N + 32)/8 + (N + 6)/8$$

$N = 2 + 8k$ when k may be any of the values of 0, 1, 2, 3, ...

$N = 2$ or 10, etc.

$X = 25$ or 106, etc.

Proof:

$25 \div 3 = 8$--1 left over

$16 \div 3 = 5$--1 left over

$10 \div 3 = 3$--1 left over

$6 \div 3 = 2$

25. BHĀSKARA

Bhâskâracārya, commonly known as Bhâskara the Learned, the last of the great Hindu writers lived near Ujjain about 400 years after Brahmagupta had worked in the same locality. He was born in 1114 and died in 1185. Bhāskara is especially noted for his treatment of the negative numbers which he regarded as debts and losses.[239]

Bhāskara had a daughter who was to be married. Being an astrologer as well as a mathematician, Bhāskara had discovered the proper day and right hour for the ceremony to take place. Any other time was said to bring misfortune to the girl. When the daughter, Lilāvati, was dressed for the ceremony, she placed on her head a beautiful crown which contained a number of pearls. There was a beautiful garden

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239 Vera Sanford, A Short History of Mathematics (Boston: Houghton Mifflin Company, 1930), p. 18.

surrounding the home of Bhāskara. There were several pools of water in the garden. When Lilāvati left her room, she took an hour cup and placed it on the water in one of the pools. The hour cup was made with a small hole in the bottom of it. Enough water would trinkle in the hole in an hour's time to sink the cup. Bhāskara had told his daughter that when the hour cup sank into the water the marriage ceremony was to take place. Lilāvati was an inquisitive girl and decided to watch the water run into the cup. As she was leaning over the pool watching the water rise into the cup, a pearl fell unnoticed from her crown into the cup. Not realizing that the hole in the cup had been stopped up, when the cup didn't sink, the girl took it to be an ill omen for her marriage.[240] For many days the father thought that the girl might lose her mind over the affair. To console her, Bhāskara wrote a book in her honor, saying:

> I will write a book of your name which shall remain to the latest times; for a good name is a second life and the groundwork of eternal existence.[241]

The *Lilāvati* began, as was the custom in the East, with an address to a Diety. The salutation that Bhāskara used was as follows:

> Salutation to the elephant headed Being who infuses joy into the minds of his worshipers, who

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240 *Loc. cit.*

241 David Eugene Smith, *History of Mathematics* (Boston: Ginn and Company, 1925), p. 277, Vol. I.

> delivers from every difficulty those who call on him, and whose feet are reverenced by the Gods.[242]

The whole book is written in verse. The arithmetic part included notations, eight operations with integers, operations with fractions and mixed numbers, systems of weights and measures, decimal numeration briefly described, and the rules relating to zero. His rules for the use of zero were: $a \pm 0 = 0$, powers of 0 are 0, any root of 0 is 0, and $a \cdot 0 = 0$. The statement that $a \div 0 = 0$ (corrected by his commentators) was evidently not clear to Bhāskara, for his statement is "a definite quantity devided by cipher is the submultiple of nought." For illustrations Bhāskara used $10 \div 0 = 10/0$ and $3 \div 0 = 3/0$. These examples were accompanied by the statement that "this fraction, of which the denominator is cipher, is termed an infinite quantity." For the value of pi, Bhaskara used 3.1255.[243]

The *Bija Ganita* is the name of the algebra book written by Bhāskara. He did not use imaginaries, saying that there was not a square root of a negative number for it is not a perfect square. Colors were used for unknowns when there were more than one. Bhāskara discussed simple and quadratic equations.[244] In speaking of the so-called negative numbers

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242 *Loc. cit.*

243 *Ibid.*, pp. 277-78

244 *Loc. cit.*

Bhaskara said, "Most people approved them not."[245]

It is not known if the work of Bhāskara was original or if it was a compilation of what was known in India at that time. He probably knew of the work of the Arabs, and they probably knew of his work, as soon as it was written, and it influenced their subsequent writing. However, they failed to make use or extend some of the discoveries which were contained in it. The works of Bhāskara, thus, became indirectly known in Europe before the end of the twelfth century. The text itself was not introduced into Europe until more recent time.[246]

The word used for addition by Bhāskara was yuta, or yu, the word for "equals" was phalam which was usually abbreviated to pha. The problem 5 + 7 = 12 would have been written pha 12 [5/1 7/1] yu by Bhāskara.[247]

The following are some of the problems found in the works of Bhāskara:

> The son of Prit'ha', exasperated in combat, shot a quiver of arrows to slay Carna'. With half his arrows he parried those of his antagonist; with four times the square root of the quiverful he killed his horse; with six arrows he slew Salya; with three he demolished the umbrella, standard, and bow; and with one he cut off the head of the

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245 R. R. Vivian, "Mathematics: A Great Inheritance." Educational Review, 53: 35 (July, 1917)

246 W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan Company, 1915), pp. 150-51.

247 Florian Cajori, A History of Mathematics (New York: Macmillan Company, 1924), p. 91.

foe. How many were the arrows which Arjuna let fly.[248]

A trader paying 10 coins upon entrance to a town doubled his remaining capital, consumed 10 (during the stay) and paid 10 on his departure. Thus in three towns (visited by him) his original capital was doubled. Say what was the amount?[249]

A person gave a medicant a couple of cowry shells first; and promised a twofold increase of the alms daily. How many nishcas does he give in a month?[250]

The square root of half the number of bees in a swarm has flown out upon a jessamine-bush, 8/9 of the whole swarm has remained behind; one female bee flies about a male that is buzzing within a lotus-flower into which he was allured in the night by its sweet odor, but is now imprisoned in it. Tell me the number of bees. (Answer 72)[251]

On an exposition to seize his enemies elephants, a king marched 2 yojanas the first day. Say intelligent calculator, with what increased rate of daily march did he proceed, since he reached his foe's city, a distance of eighty yojanas in a week.[252]

A snake's hole is at the foot of a pillar, and a peacock is perched on its summit. Seeing the snake,

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248 David Eugene Smith, A History of Mathematics (Boston: Ginn and Company, 1923), Vol. I, p. 280.

249 Vera Sanford, A Short History of Mathematics (Boston: Houghton Mifflin Company, 1930), p. 176. (Citing: H. T. Colebrooke, Bhāskara, p. 54)

250 David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), Vol. II, Topical Survey, p. 501. (Citing: H. T. Colebrook, Bhāskara, p. 55)

251 Florian Cajori, A History of Elementary Mathematics (London: Macmillan and Company, 1929), p. 100. (Citing: Herman Hankel, Die Entwickelung der Mathematik in den letzten Jahrkundertin, Tiibingen, 1884, p. 191)

252 Vera Sanford, The History and Significance of Certain Problems in Algebra (New York: Teachers College, Columbia University, Bureau of Publications, 1927), p.7/

at the distance of thrice the pillar, gliding towards his hole, he pounces obliquely upon him. Say quickly at how much cubits from the snake's hole do they both meet, both preceeding an equal distance?[253]

How many are the variants in the form of God Sambu (Siva) by the exchange of his ten attributes held reciprocally in his several hands; namely the rope, the elephants hook, the serphent, the tabor, the skull, the trident, the bedstead, the dagger, the arrow, the bow: as those of Hari by the exchange of the mace, the discus, the lotus, and conch.[254]

Five doves are to be had for three drammas; seven cranes for five drammas; nine geese for seven drammas; and three peacock for nine drammas; bring a hundred of these birds for a hundred drammas for the prince's gratification.[255]

Lovely and dear Lilâvati, whose eyes are like a fawn's, tell me what are the numbers resulting from 135 multiplied by 12. If thou be skilled in multiplication, whether by whole or by parts, whether by division or separation of digits, tell me, auspicious damsel, what is the quotient of the product when divided by the same multiplier.[256]

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253 Ibid., p. 77

254 Ibid., p. 90

255 Ibid., p. 92

256 W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1924), p. 152.

CHAPTER VI

PERIOD OF TRANSMISSION

INTRODUCTION

While the Hindus and Arabs were making progress in the field of mathematics, Europe was going through the dark ages. The barbaric nations from the swamps and forests of the North and from the Ural Mountains swept down into Europe and destroyed the Roman Empire in 455. These people were very slowly civilized. They had little regard for the intellectual treasures of the countries which they had captured.[257]

From the sixth century until the eighth century the only places of study in western Europe were the Benedictine monastaries. Some literature was taught in these schools. The science which was taught seemed to be limited to methods of keeping accounts and the rules by which the date of Easter might be determined. The monks had renounced the world and they apparently saw no necessity for learning more science than was required for the services of the church and monastaries.[258]

The tradition of the Greek and Alexandrian learning soon

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257 Florian Cajori, *A History of Elementary Mathematics* (New York: Macmillan Company , 1924), p. 111

258 W. W. Rouse Ball, *Primer of the History of Mathematics* (London: Macmillan and Company, 1927), p. 36.

died away. It is possible that in a few places the works of the Greek mathematicians were available, but not readily so. There were no students and because of this the earlier works had no a ttraction and soon became scarce.[259]

In the latter half of the eighth century, Charles the Great commanded that schools be opened in connection with the monastaries and cathedrals in his kingdom. The mathematics in these schools probably did not go beyond the geometry and arithmetic of Boethius. After the death of Charles the Great the schools neglected the teaching of science. The subjects taught were limited to Latin, music, and theology. The fact that the schools continued gave the opportunity, for those desiring it, to get enough mathematics to keep their accounts, enough geometry for land surveying, and astronomy sufficient to enable them to calculate the feasts and fasts of the church.[260]

At the end of the eleventh century or the beginning of the twelfth century a revival of learning took place. There was a tendency for teachers to settle in the vicinity of some school, and, with the consent of the authorities, give lectures on theology, logic, and civil laws. This custom led to the formation of guilds or trade unions, and was the first stage in the history of the medieval university. If the

259 Ibid., p. 37.

260 Ibid., pp. 37-8.

school was successful and its members wanted it to be perfect, they asked the state for legal privileges and they were usually given. The last step in the evolution was recognition by the pope or emperor.[261]

It was from Spain that a knowledge of eastern mathematics first came into western Europe. The Moors had captured Spain in 747 and by the beginning of the tenth or eleventh century had attained a high degree of civilization. Although the Moors were not friendly with the Arabian government, they gave great mathematicians a hearty welcome in their schools. In this way the Arabic translations of the works of the Greek writers as well as the works of the Arab algebraists were read in the Spanish schools of Granda, Cordova, and Seville.[262]

Frederick II did much or more than any other single man of the thirteenth century to introduce the Arab mathematics into western Europe. At that time Jewish physicans were tolerated in Spain because of their scientific knowledge and their medical skill. Frederick II made use of this fact and engaged a staff of educated Jews to obtain for him copies of Arab works and Greek editions which were in circulation in Arabia. These works were translated and placed in the library at the University of Naples. From this time on it might

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261 Ibid., p. 39.

262 Ibid., p. 45.

he said that the development of science in Europe was independent of the aid of Arab scholars.[263]

When the refugees escaped from Constantinople after the fall of the eastern empire in 1453, many of them came to Europe. With them they brought many copies of Greek books and editions. This gave an added stimulus for the study of mathematics.[264]

MAP 4

PROFILE MAP FOR TRANSMISSION PERIOD

The map shows the mathematical routes during the transmission period as numbered and listed below, also the location of cities mentioned in the following section:

(1) Hindu Astronomers to Bagdad--773

(2) Al-Khowârizmî-Hindu Numerals taught to Arab Mathematicians--825

(3) Arabic learning follows conquests into Northern Africa

(4) Arab learning introduced into Spanish Universities during tenth century

(5) Adelard of Bath studies in Syria and takes manuscripts back to England with him--twelfth century

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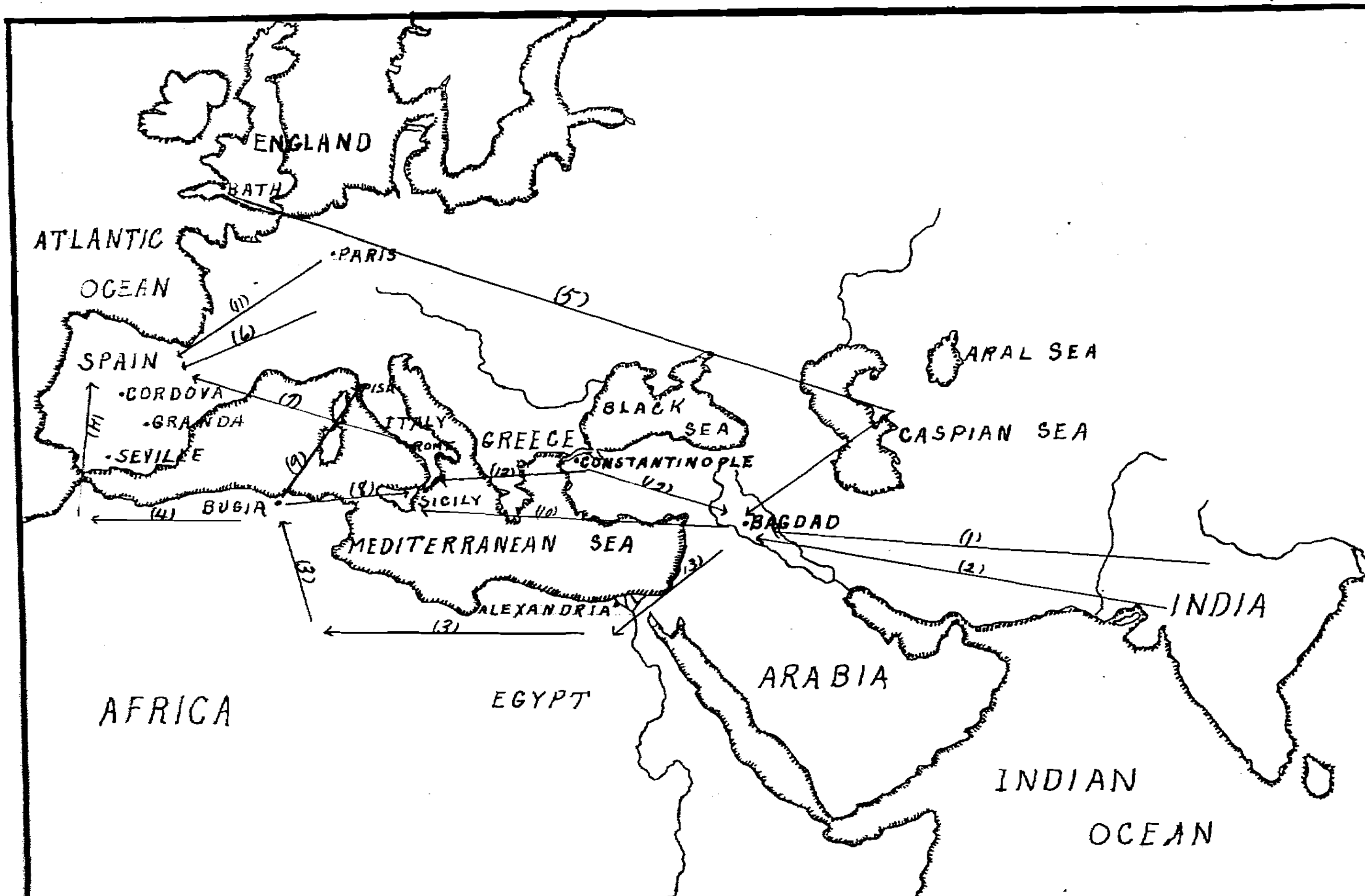
263 Ibid., p. 47

264 W. E. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 120.



MAP 4

PROFILE MAP FOR TRANSMISSION PERIOD

(6) Christian Scholars study in Spain during the tenth, eleventh, and twelfth centuries.

(7) Gerbert studies in Spain during tenth century

(8) Arabic learning introduced into Sicily

(9) Fibonacci takes Arabic learning to Pisa--twelfth century

(10) Frederick returns manuscripts to Sicily and Rome--thirteenth century

(11) Nemorarius studies in Spain in thirteenth century

(12) With the fall of Constantinople manuscripts were sent to Bagdad and Italy

The following is a list of the mathematicians mentioned, their chronological age, and the cities which they are connected:

Gerbert	1000	Rome
Adelard of Bath	1120	Bath
Fibonacci	1200	Pisa
Nemorarius	1200	Paris

26. GERBERT

Gerbert, like many men who have become famous during their life, was born of very humble parents at Aurillac in Auvergne early in the tenth century. At an early age his parents as well as many other people in the village realized that he was naturally a very brilliant child. He was sent to a monastary of his native village where he began his education. In 967 he went to Barcelona, Spain, which was under the Christian rule at that time, to study mathematics. In 971 he went to Rome where his proficiency in music and astronomy excited much interest. His interests were not confined to these two subjects, however. In fact he mastered every course in the curriculum with the exception of logic. The emperor requested that he would also study this, so in 972 he went to Rheims where he also studied philosophy and helped make Rheims an educational center.[265] In 983 Gerbert was appointed abbot of the monastary of Bobbio, where he taught with much distinction and success.[266] After this he held various offices in the church, each being a step forward, until in 999 he was elected to the papacy. He was called Pope Sylvester II.

During the time he was lecturing, Gerbert, in his spare moments, made some interesting terrestail and celestial globes which he used to illustrate his lectures. These globes were admired by many people, several of whom offered to buy them. However, Gerbert, instead of selling them, offered to exchange them for copies of Latin classics which were already becoming scarce. He appointed agents in the chief towns of Europe to carry on the trade of globes for classics. It was through his efforts that several of the important Latin works were preserved.[267]

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265 Ibid., p. 137

266 Chamber's Encyclopedia, "Sylvester," Vol. IX, p.252

267 Loc. cit.

All of the things that Gerbert invented were not for the betterment of mathematics. Among his inventions was a large clock which was used and preserved for many years at Madgeburg. He made an organ which was worked by steam. This organ was used at Rheims for two centuries after the death of Gerbert.[268]

Many of the contemporaries of Gerbert said that he had sold himself to the devil when a young man. Their theory was that it was only through the help of the devil that Gerbert was able to make the clock and organ, to become Pope, and to write the mathematical works in which he gave the public many things for the advancement of the subject. As late as 1522 in a biography of Gerbert published in Venice, it was related that he obtained the papacy by black art, having given himself to the devil. Others tell of his effort to escape from his bargain when he was on his death bed.[269]

It was through the work of Gerbert that the Hindu-Arabic numerals were brought to Europe. His works do not show that he had a symbol which he used for zero. His other nine symbols looked like this and were called by these names.[270]

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[268] Ibid., p. 138.

[269] Loc. cit.

[270] David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), p. 75, Vol. II.

igin /	calatis Lo
andras C	zenis 7
ormis Ƹ	temenias 8
arbas 4	celentis 9
quimas X	

The Geometry written by Gerbert was based on a copy of the geometry written by Boethius. Although it did not contain much original material, it does indicate that Gerbert had a great ability. His own contributions were some applications of land surveying, and the determination of the height of inaccessible objects. For this he used the Pythagorean theorem.[271] The condition in which the geometry of that time found itself may be judged by the fact that Gerbert used the same inaccurate rule for the finding of the area of a trapezoid as was used by Ahmes. His expression for the area of an equilateral triangle of side a was $1/2\ a\ (a - 1/7\ a)$, which was equivalent to using 1.714 for the square root of three.[272]

Gerbert was able to solve a problem which was very troublesome in those days. The problem was to find the sides of a right triangle when the area and the hypotenuse were given, using h^2 as the area, his formulae were

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271 W. W. Rouse Ball, op. cit., p. 138

272 Vera Sanford, A Short History of Mathematics (Boston: Houghton Mifflin Company, 1930), p. 239.

$\frac{1}{2}\sqrt{(c^2 + 4h^2 + \sqrt{c^2 - 4h^2})}$ and $\frac{1}{2}\sqrt{(c^2 + 4h^2 - \sqrt{c^2 - 4h^2})}$[273]

Many pupils of France, Germany, and Italy gathered at Rheims and any other place where Gerbert was located to enjoy his instruction.[274]

27. ADELARD OF BATH

During the twelfth century most of the scholars of note studied in Spain. Some of them went to Syria. Adelard of Bath, an English monk, was a scholar who traveled very widely, visiting Egypt, Greece, Asia Minor, Syria, and Arabia as well as Spain and France. His education in Europe was received at monastaries which were located at Toledo, Tours, and Leon. This education gave him a good foundation for his work in the East.[275]

The Mohammedans in the East would not allow foreigners to attend their lectures. Adelard, after studying the language and customs of the Arabs for sometime, disguised himself as a Mohammedan student and attended some of their lectures at Cordova. While attending these lectures he obtained a copy of Euclid's Elements. This Adelard brought back to England with him together with other mathematical

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273 W. W. Rouse Ball, op. cit., p. 5.

274 Chamber's Encyclopedia, "Sylvester." Vol. IX, p. 252.

275 David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1926), p. 203.

works. He translated the _Elements_ into Latin and this translation became the foundation of all editions known in Europe till 1533 when the Greek text was recovered. The knowledge of Euclid's _Elements_ spread rapidly, and by the end of the thirteenth century Roger Bacon was familiar with it. By the close of the fourteenth century the first five books formed a part of the curriculum of many of the universities.[276]

After Adelard returned to England from his travels in the East, he was granted a pension by Henry I. He then spent most of his time writing. Some of his writings are still preserved.

28. FIBONACCI

Leonardo Pisano, or Fibonacci as he is generally called, was born at Pisa in 1170 and died in 1250. The name Fibonacci, means the son of Bonaccio. Bonaccio was a nickname meaning "a good stupid fellow". Later Fibonacci was called Leonardo Bigollone. This was a nickname which was no doubt given him by some of his fellow townsmen. This nickname meant a "stupid loiterer" or "blockhead" as it is usually translated.[277]

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[276] W. W. Rouse Ball, _A Short Account of the History of Mathematics_ (Boston: Ginn and Company, 1925), p. 203, Vol. I

[277] Moitiz Cantor, "Leonardo Pisano." _Encyclopedia Britannica_, 9th edition, Vol. XIX, p. 540.

Pisa ranked with Venice and Genoa as one of the greatest commercial centers of Italy. These towns had large warehouses where goods could be stored and where duty was paid on all goods imported from other sections of the Mediterranean. The secretary of such an establishment was a man of considerable importance. It was such a position that the father of Fibonacci held at Bugia on the north coast of Africa. It was at Bugia that Fibonacci received his early education from a Moorish school master. While yet a young man he visited Egypt, Syria, Greece, Sicily, and Southern France, meeting with scholars and becoming acquainted with the arithmetical systems used by the merchants of the various countries. Fibonacci was convinced that the Hindu system was superior to all others.[278]

Fibonacci's fame was spread over Europe very rapidly. At one time he was summoned to the court of Frederick II to engage in a mathematical duel with the court mathematician, John of Palermo. Frederick had heard about the skill of Fibonacci in solving mathematical problems and desired to find out if all the marvelous accounts were true. This was the first mathematical duel held. In the sixteenth and seventeenth century they became more common. The competitors were informed before the time of the contest the type of questions to be asked. None of Fibonacci's competitors were

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[278] David Eugene Smith, op. cit., p. 1925.

able to answer any of the questions asked. Fibonacci solved the problems with little difficulty. One of the problems was a cubic equation. Cubic equations had not been solved by the use of algebra before this time. Fibonacci discovered that the problem given him could not have rational roots. For his answer he gave a very close approximation.[279]

The chief work of Fibonacci was his book, Liber Abaci. The first part of the book contained a detailed account of the Hindu-Arabic numerals and their used. Fibonacci urged that the Hindu-Arabic numeral system be adopted. He attempted to show its superiority over the Roman system. Europe did not accept the new system very quickly. A few of the merchants, especially those trading with the countries in the East, made some use of it. Nearly a hundred years after Liber Abaci was written, the Florentine bankers were forbidden to use the Hindu-Arabic numerals. No doubt this was due to the fact that there was a large variety of forms being used and the lack of standardization of the digits. The Roman numerals were used in France and England in commercial and governmental accounts as late as the sixteenth century.[280]

Probably the only reason that the Roman numerals have

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279 Florian Cajori, A History of Mathematics (New York: Macmillan and Company, 1924), pp. 123-24.

280 Ibid., p. 123.

survived to this time is because they were in such popular use before the Hindu-Arabic system came into use. An interesting incident happened a few years ago. It was noticed that a store keeper in Pisa was using the Roman numerals to mark the prices on all his goods. When asked why he was still using that system when it was out of date, he replied, "People from all over the world can read them regardless of their own numerals, Turks, Italians, French, and even Americans."[281]

Fibonacci introduced new rules for multiplication and division. Such words as capital and percent were introduced by him. He was the first to use the "borrow and pay" method of subtraction. According to this method, when twenty-four is subtracted from fifty-two, four is taken from twelve and the one which was borrowed from the four is paid back to the two and then three is subtracted from five. He used 3.1418 for the value of pi.[282]

Fibonacci used the word "res" to designate unknown quantities. The English word "cipher" came from the Arabic word sifr or sifra which means empty. Fibonacci called the word zero, zepherum.[283]

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281 Vera Sanford, "Roman Numerals." _Mathematics Teacher_, 24: 23 (January, 1931)

282 Karl Fink, _A Brief History of Mathematics_ (Chicago: Open Court Publishing Company, 1903), p. 218.

283 Florian Cajori, _op. cit._, p. 121.

Among the other books written by Fibonacci were _Practica Geometriae_ and _Flos_. The first book presented the subject of geometry in a systematic form. It treated such topics as the finding of areas and volumes, square roots and cube roots, and dealt with a surveying instrument called the quadrans.[284] In the second book, _Flos_, Fibonacci discussed the cubic equation which had been given him by Frederick II in the mathematical duel. The cubic equation was $X^3 + 2X^2 + 10X = 20$.[285]

Many of the problems found in Fibonacci's books are similar to those used today.

> There is a lion in a well whose depth is 50 palms. He climbs 1/7 of a palm daily and slips back 1/9 of a palm. In how many days will he get out of the well?[286]

The problem might be worked by the following method: if the lion climbs 1 palm in seven days and falls back 1 palm in nine days, it would take him sixty-three days to climb 2 palm. If the lion worked constantly, it would take him sixty-three times fifty divided by two or 1575 days to climb from the well.

> A certain king sent 30 men into his orchard to plant trees. If they planted 1000 trees in 9 days,

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284 Vera Sanford, _A Short History of Mathematics_ (Boston: Houghton Mifflin Company, 1930), p. 25.

285 Florian Cajori, _op. cit._, p. 124.

286 Vera Sanford, _The History and Significance of Problems in Algebra_ (New York: Teachers College, Columbia University, Bureau of Publications, 1927), p. 63.

in how many days would 36 men plant 4400 trees? (Answer 33 days)[287]

A man went into an orchard in which there were seven gates, and there took a certain number of apples. When he left the orchard, he gave the first guard half the apples that he had and one more apple. To the second, he gave half the remaining apples and one apple more. He did the same in the case of each of the remaining five guards and left the orchard with one apple. How many apples did he gather in the orchard?[288]

A man who was approaching his end, called his eldest son to him and said,"divide my estate among yourselves in this way: You are to have one bezant and one seventh of the rest of my property." He said to the second son, "You are to have two bezants and a seventh of what then remains." To the next one he gave three bezants and one seventh of what was left. And thus he called all of his sons in order, giving to each one bezant more than the one before, and a seventh of his property and and the last one had all that was left. It happened, moreover, that each had an equal share in the estate according to these conditions. The question is how many sons there were, and how large was the estate.[289]

A certain man says that he can weigh any amount from 1 to 40 pounds using only 4 weights. What size must they be? (Answer 1, 3, 9, 27)[290]

29. JORDANUS NEMORARIUS

Jordanus Nemorarius was the first German mathematician, especially algebraist, of any note. His works have only

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287 Vera Sanford, The History and Significance of Problems in Algebra (New York: Teachers College, Columbia University, Bureau of Publications, 1927), p. 63. (Citing: Fibonacci, Liber Abaci, Bon Compagni edition, p. 134)

288 Ibid., p. 58.

289 Ibid., p. 61.

290 Ibid., p. 90.

recently been discovered. Nothing is known of his life except that he was a monk who studied in Paris. Later he became general of the Dominican order and died in 1236.[291]

The first book written by Jordanus was De Triangulus, a geometry written in four books which contained definitions, seventy-six theorems and propositions on triangles, all of which were based on Euclid's Elements, propositions on the ratio of straight lines, comparison of areas of triangles, arcs and chords of a circle, regular polygons, the duplication of a cube, and the trisection of an angle.[292]

Algorismus Demonatratus was the name of an arithmetic book written by Jordanus. In this book he gives the rules for the four fundamental processes used in arithmetic. In the writing of the book he used the Hindu-Arabic numerals.[293]

De Numeris Datis contained 115 problems in linear equations with their solutions. These problems were much like the ones which are used in the algebra books today. Jordanus used more than one unknown in some problems.[294]

The historical importance of Jordanus lies in the fact that he was the first to make regular use of letters to

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291 W. W. Ball Rouse, A Short Account in the History of Mathematics (Boston: Ginn and Company, 1925), p. 171.

292 Ibid., pp. 171-72.

293 Ibid., p. 172.

294 Loc. cit.

represent algebraic magnitudes.

CHAPTER VII

MATHEMATICAL PERIOD FOLLOWING RENAISSANCE

INTRODUCTION

The invention of the printing press marks the beginning of the modern era. Following this invention came an outburst of activity in all branches of learning. Before this time it was only a select few who had access to the copies of the books which had been written. After the introduction of the printing press, book stores were opened and the things which had been available to a few became the property of the public.

Along with the modern period came a rapid development in the mathematical sciences and the development of symbolism. During the end of the transmission period the universities had become firmly established and, with the aid of the printing presses, they became the centers of mathematical sciences. No longer was all the work in mathematics confined to Italy and Greece. Important discoveries were being made all over the continent of Europe.[295]

30. GIROLANIO CARDANO

Girolanio Cardano, a celebrated mathematician,

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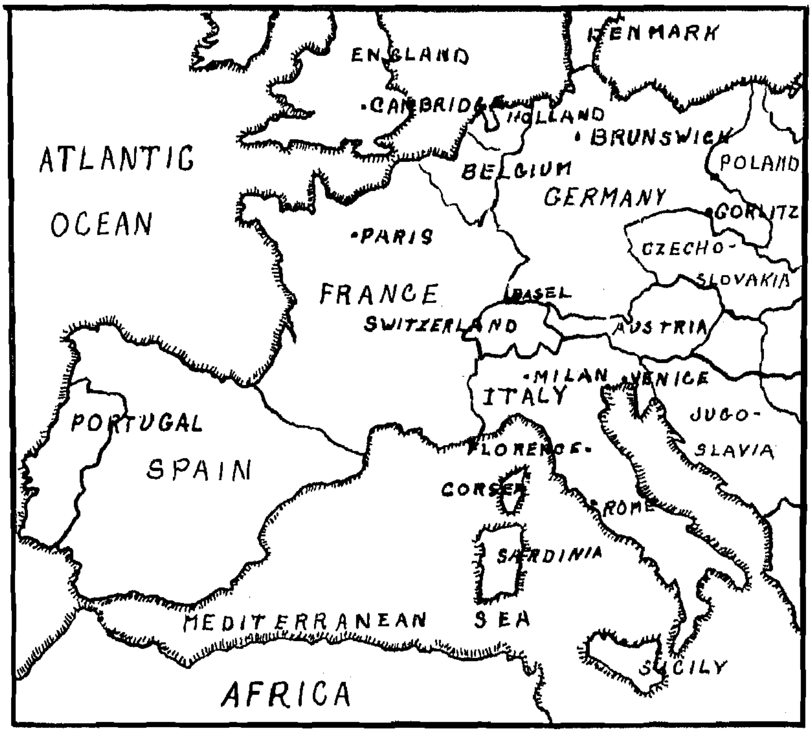
295 W. W. Rouse Ball, A Primer of the History of Mathematics (London: Macmillan and Company, 1927), pp. 57-8.



MAP 5

PROFILE MAP FOR MATHEMATICAL PERIOD FOLLOWING RENAISSANCE

The map shows the location of the cities which are mentioned in the next section. The following is a list of the mathematicians mentioned, their chronological age, and the cities with which they were associated:

Cardan	1545	Milan
Tartaglia	1545	Venice
Ferrari	1545	Milan
Galileo	1600	Florence
Descartes	1637	Paris
Pascal	1650	Paris
Newton	1680	Cambridge
Bernoullis	1690-1860	Basel
Euler	1750	Basel
Gauss	1799	Braunchweig (Brunswick)

naturalist, physician, and philosopher, was born at Pavia in 1501. Girolanio Cardano is commonly referred to as Cardan by English writers. However, his name also appears as Hieronymus Cardanus and Jerome Cardan.[296] Cardan was the illegitimate son of a physician and lawyer. The first part of his education Cardan received at home, later attending the universities at Pavia and Padua to complete his studies. He commenced life as a physician after he had received his degree. It was while Cardan was following this profession that he began the study of mathematics.[297] Cardan met his wife at Sarco soon after he started practicing medicine. She evidently was a woman of wealth as Cardan is said to have squandered her wealth in gambling.[298]

By 1546 Cardan's reputation as a doctor had spread over Europe. At this time the King of Denmark asked Cardan to accept a chair as professor of medicine at the University of Copenhagen, but Cardan turned down the invitation. He spent a year or so visiting France, Scotland, and England. While in Scotland he visited the primate of that country. The primate had been troubled years with asthma. All the

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[296] Vera Sanford, "Cardan." Mathematics Teacher, 23: 258 (November, 1930).

[297] W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 222

[298] The New International Encyclopedia, 2nd edition, "Cardan". Vol. IV, p. 535.

noted physicians of the country had tried to affect a cure, but one after another had failed. Cardan by chance cured the primate and was paid a large fee for his services. Cardan returned to Milan where he lived a short time before accepting a professorship of medicine at Pavia in 1558.[299]

Cardan had two sons. In 1560 the elder of the two sons poisoned his wife and was executed for the crime. A short time afterwards Cardan found that his younger son had committed some crime. In a fit of anger Cardan cut off the ears of this son with a large knife. The Pope Gregory XII granted him protection so that he would receive no punishment for the offense.[300]

In 1562 Cardan moved to Bologna. The story of his crime followed him and the University took steps to keep him from lecturing. The Pope interferred again and Cardan was allowed to continue his lectures. Cardan had never been able to live within his income. In 1570 he became so heavily in debt that he was thrown in prison. About the time he left prison he published a horoscope of Christ. Because this brought so much hatred on him, and because of his indebtedness, Cardan fled to Rome for protection. Here he was pensioned by the Pope and became astrologer at the papal court.

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299 Chamber's Encyclopedia, "Cardan." Vol. II, p. 607.

300 W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 222.

Cardan died in 1516, a few weeks after he had published his autobiography. Some writers assert, without sufficient authority, that Cardan had predicted that he would dir on September 2, 1516. When the day arrived and he was in perfect health yet at the end of the day, he committed suicide in order that his predictions would be fulfilled.[301]

Cardan was very much interested in a contest between Tartaglia and Fior. The contest was of a mathematical nature and was concerned with the solution of the cubic equation. Cardan had begun the writing of his monumental work on algebra, Ars Magna. When he found that Tartaglia had been able to solve a general cubic in the contest with Fior, Cardan asked Tartaglia to tell him the method he used in the solution in order that he might publish it in the Ars Magna. Cardan was very angry when Tartaglia refused. Shortly afterwards Cardan wrote to Tartaglia saying that an Italian nobleman of great fame had heard of Tartaglia's success in solving the cubic equations and was anxious to meet him. Cardan begged him to come to Milan at once. Tartaglia went to Milan but no nobleman was waiting for him. Cardan had used the fictitious nobleman as a decoy to bring Tartaglia to him. Cardan used all of his powers of persuasion and must have been quite plausible, for Tartaglia finally gave Cardan the formula under oath of secrecy.[302]

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301 Loc. cit.

302 Ibid., pp. 222-23.

When the Ars Magna was published in 1545, Tartaglia was one of the first people to study it. Imagine his amazement when he found his formula in the book. Naturally he was very angry and finally decided to challenge Cardan to a mathematical duel. In a mathematical duel each contestant made out a list of problems for the other contestant to work. The preliminaries were settled and the place where the duel was to take place was a certain church in Milan. When the day came Cardan failed to appear but in his place sent Ferrari who had been living in his home as an errand boy. Both sides claimed victory but most historians agree that Tartaglia deemed himself lucky in escaping alive.[303]

Ars Magna was far superior to any algebra published before this time. Before the algebra books had contained only equations in which the roots were positive. Cardan included negative and complex roots in his book. He discovered that complex roots were always found in pairs. Many authorities believe that part of the analysis of the cubic equation which Cardan published in his book might have been original with him.[304]

Many of the problems which are found in Cardan's algebra are very interesting. The following problems have been attributed to Cardan:

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303 Ibid., p. 223.

304 Ibid., pp. 223-24.

A dog is chasing a hare. The hare is 60 leaps ahead of the dog and 3 leaps of the dog equal 5 those of the hare. The dog takes three leaps in twenty-seconds and the hare 5 in twenty-one seconds, and three leaps of the dog are greater than 7 of the hare by 1/20 of a leap of the dog. When does the dog catch the hare?[305]

A certain slave fled from Milan to Naples going 1/10 of the whole journey in one day. At the beginning of the third day the master sent a slave after him and the slave sent 1/7 of the whole journey in one day. I do not know how far it is from Milan to Naples but I wish to know when the latter overtook him.[306]

If Saturn moves once around the earth in 30 years, and if Jupiter goes around it once in 12 years, how many years will there be in the interval between two conjunctions?[307]

Cardan at one time tried to figure on the number of days actually spent if a ship sailed westward on the Kalends of January 1517 and went three times around the earth returning on the seventh of May, 1526.[308]

Cardan was a man of remarkable contrasts. He was a heretic, yet frequently was befriended by the Pope. On the one hand he was a gambler, the father of a murderer, an inmate of an almshouse, and a failure as a citizen; on the

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305 Vera Sanford, The History and Significance of Problems in Algebra (New York: Teachers College, Columbia University, Bureau of Publications, 1927), p. 72. (Citing: Cardan, Practica Arithmetica)

306 Ibid., p. 73.

307 Ibid., p. 75.

308 Vera Sanford, A Short History of Mathematics (Boston: Houghton Mifflin Company, 1930), p. 214.

other hand he was a physicist, one of the finest algebraists of his time, a university professor, and a physician. He pretended to tell fortunes by the stars, was dishonest, deceptive, and unreliable in many of his affairs; yet he was a keen student of philosophy, and was wholly reliable in his scientific work.[309]

31. NICHOLAS TARTAGLIA

Nicolo Fontana, generally known as Tartaglia or Nicholas the stammerer, was born at Brescia in 1500. Nicholas was twelve years old when the French captured Brecia. When the people of the city learned that the French were coming a and would capture the town they took refuge in the cathedral, thinking this would be the one place where they would be safe. It puzzled the French, upon entering the town, to find the houses empty and searched until they located the people in the cathedral. The battered down the doors and massacred those in the church. Nicholas' father, who was a postal messenger in the town, was among those killed. Nicholas' skull, jaw, and palate were cut open by the sword of some soldier and he was left as dead when the soldiers vacated the cathedral.[310]

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309 David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), Vol. I., p. 296.

310 W. W. Rouse Ball, A. Short Account of the History of Mathematics (London, Macmillan and Company, 1915), p. 217.

Nicholas' mother had not been able to go to the cathedral with her husband and son. When she found that she had been left behind she hid in her home. After the French soldiers were gone she went to the church for what she supposed was the last look at her husband and son. Imagine her surprise when she found that Nicholas was not dead. In some way the mother managed to carry twelve year old son from the cathedral. They were very poor people and had no money with which to pay a doctor. Suddenly she remembered that when dogs were wounded they always licked the wounded place. Why couldn't she lick the wounds of her son and perhaps thereby bring relief to his suffering? Whether due to this simple treatment or in spite of it, the boy recovered.[311] The injury to his palate caused Nicholas to stammer for the rest of his life. It was from the stammering that he received the name "Tartaglia".[312]

Tartaglia's mother someway received enough money to send him to school for fifteen days. He took advantage of this opportunity and stole a copy book from which he taught himself to read and write. They were so poor that they could not afford to buy paper for Nicholas. He was obliged to use tomb stones as slates on which to work his problems. For

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311 _Loc. cit._

312 _Encyclopedia Britannica_, "Tartaglia." 14th edition, Vol. XXII, p. 823.

several years he worked on mathematics alone and soon became known for his skill along this line. He entered upon his public work by giving lectures at Verona but he was soon appointed to the chair of mathematics at Venice, which he occupied until his death.[313]

In 1535 Antonio Fiori discovered the solution for a certain type of a cubic equation. When Tartaglia heard of this he made the statement that he had a formula by which he could work any cubic equation. Fiori, who believed that Tartaglia must be an imposter, challenged him to a contest. According to the challenge, each contestant was to make a list of thrity problems. On the day that the contest was to begin, each was to take his problems and a certain amount of money to a notary. Each was to deposit the money with the notary and the one who worked the largest number of problems out of the thirty given him in thirty days time was to receive the money.

Tartaglia had an idea as to the kind of a cubic which Fiori could work. His problems were made so that they could not be worked by any method but by the use of the formula for working the general cubic equation. On the day of the contest Tartaglia worked the thirty problems given him in two hours while Fiori failed to work a single one of the problems given him by Tartaglia.[314]

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313 W. W. Rouse Ball, op. cit., p. 218.

314 Ibid., pp. 218-19.

Tartaglia had many problems which are interesting. The following are three taken from Tartaglia's works.

> A man went to a draper and bought a length of cloth 35 braccia long to make a suit of clothes. The draper told him when it was shrunk and clipped, every seven braccia would shrink in one braccia. The man took him at his word, but instead for every six braccia the cloth shrunk one. How much cloth did he lack?[315]

> A man had a certain amount of capital but fell to gambling and made as many denarii as he had to start with. He then spent 20 ducats on a horse, he rode away on the horse to an inn where he gambled with the inn keeper and redoubled his money. He spent 20 ducats on a beautiful robe. He then left the inn and went to the gate of the city where he found some people gambling. There he doubled what he had left, and bought a ring for 20 ducats and found that he had nothing left. How much money did he have when he started?[316]

> A man has 3 pheasants which he wishes to give to two fathers and two sons, giving each one pheasant. How can it be done?[317]

> A man gives a shepherd 720 sheep to pasture for five years on the agreement that at the end of this time they will divide the flock evenly. It happened that the shepherd died at the end of three years and eight months and his widow who had no trustworthy person to give the care of the sheep was forced to break the agreement though in truth she had a son by the shepherd, but he had not been brought up in that trade. There are then 1060 sheep. How shall they be divided?[318]

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315 Vera Sanford, History and Significance of Problems in Algebra (New York: Teachers College, Columbia University, Bureau of Publications, 1927), p. 13. (Citing: Tartaglia, Nicolo, General Trattato, Vol. I, fol. 169).

316 Ibid., p. 253

317 Ibid., p. 52

318 Ibid., p. 82

32. GALILEO GALILEI

It seems a coincidence that such a great man as Galileo should be born on the very day of Michelangelo's death and should die in the same year as Newton's birth. It seems as though he must have been born to fill the gap between the lives of the other two great leaders, Newton and Michelangelo.[319]

Galileo's father, who was a mathematician and musician of some renown, was an Italian nobleman living in Florence. At one time his ancestors had been quite wealthy, but before young Galileo was born the family fortune had dwindled to practically nothing.[320]

Galileo was born in Pisa on February 18, 1564. He died in 1642.[321] His tomb is in the church of Santa Crose, close to that of his fourteenth century ancestor named Galileo. This ancestor is said to have excelled in mathematics and science. When he was appointed to an important office in the city government, he changed his former surname to that of Galilei.[322]

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[319] David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), p. 363, Vol. I.

[320] J. G. Edgar, The Boyhood of Great Men (New York: Harper Brothers, 1853), pp. 147-48.

[321] W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 247.

[322] J. G. Edgar, op. cit., p. 145.

Galileo was the eldest child of the family. At the time of his birth his father was determined that he should become a cloth merchant in hopes that he might restore the family fortune.[323] When he was but a lad his chief pastime seemed to be collecting the broken toys of the children of the neighborhood and mending them. Many times to the delight of the owner he would add some mechanical device.[324] Although money was mighty scarce, the family managed to send Galileo to the convent at Vallombrosa. Here his literary ability and mechanical ingenuity attracted a great deal of attention. The monks of the monastary persuaded young Galileo that he should join their ranks and it was only by persuasion and almost force that his father removed him from the monastary.[325]

It was while in Pisa that Galileo attended the church services in Pisa's great cathedral. In the cathedral was a beautiful bronze lamp hanging from the ceiling. The lamp had just been lighted and returned to its place. Galileo happened to notice it swing to and fro. Finally he discovered that each oscillion was made in the same length of time. Soon after this he made known his laws of the pendulum. He

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323 David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1915), Vol. I, pp. 363-64.

324 J. G. Edgar, op. cit., p. 144.

325 W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), pp. 247-8.

had timed the swinging with the beating of his pulse. He conferred with some of the doctors of the university and suggested that the first thing that should be done in every medical diagnosis was the counting of the pulse beat to see whether or not it was normal.[326]

Galileo had never been allowed to study mathematics. One day, unknown to his father, he attained admittance to a lecture on geometry. He found it immensely interesting and soon began to study it. Galileo's father finally gave his consent to Galileo's plan of dropping the study of medicine and devoting his entire time to the study of mathematics. By 1589 his reputation as a mathematician was so great he was granted the professorship of mathematics at the University of Pisa. Mathematics was not considered a very necessary or important subject at that time. This is indicated by the fact that a professor of medicine at that time received the salary of $2150 a year while the professor of mathematics received a salary of only $65.[327] During the next three years Galileo carried on experiments in physics. Galileo's experiments brought him much criticism and he did not hesitate to ridicule people who made unwelcome remarks about them. Galileo's career in Pisa ended when he

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[326] New International Encyclopedia, "Galileo." Vol. IX p. 410.

[327] David Eugene Smith, History of Mathematics, (Boston: Ginn and Company, 1925), Vol. I, p. 365.

tactlessly condemned an invention of a relative of the Medici family who had been very influential in securing the position at Pisa for him.[328]

Because of his influence with the Venitian senate, Galileo was appointed professor of mathematics at Padua. Here he had sufficient time to carry on his experiments. It was here that he made the first thermometer and stated the law of falling bodies. This law states that two bodies of different weights will fall at the same speed through the air if they are dropped from a high place. He had carried on experiments testing this law by droping bodies of different weights from the top of the Leaning Tower of Pisa.[329]

In 1602 Galileo heard that a tube containing a lens which magnified objects had been made by Hans Lippershey. This gave Galileo an idea, and he immediately set to work on what became the first telescope. This is the reason that some of the telescopes bear the name of Galileo to-day. In a few months he had made instruments which would magnify objects thirty-two times. The invention brought honors as well as a small fortune to Galileo. He gave up his position at Padua.[330]

Galileo published a book by the name <u>Dialogo sopra dei</u>

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[328] W. W. Rouse Ball, <u>op. cit.</u>, p. 248.

[329] <u>Ibid</u>, pp. 248-49.

[330] <u>Ibid</u>, p. 249.

due massini sistemi del momdo Tolemaice e Copernicano. The book was written in the form of conversation between three men. The book was published in 1632.[331] Soon afterwards he was called to Rome to defend his book which had been condemned as being heretical. Galileo pleaded ill health but was obliged to make the journey. He was tried for heresy on the charge that he had said that the earth moved around the sun. Galileo was compelled to renounce, in the presence of a great assembly of cardinals, monks, and mathematicians, kneeling before them with his hand upon a gospel, the great truth he had maintained. Tradition tells us that he recanted to save his life, but under his breath he murmured, "But nevertheless it (the earth) does move just the same."[332]

Some people have told the pitiful story of how Galileo was taken to prison and there tortured for the rest of his life. However, it is generally agreed that he returned to Florence. Here he carried on his work in spite of the fact that he lost his eyesight. His blindness may have been due to the imperfections of the telescope which he worked with almost constantly. Some of his students helped him with his work until his death. Blindness, deafness, want of sleep, and pain in his limbs helped imbitter the last years of

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331 Vera Sanford, "Galileo." *Mathematics Teacher*, Vol XXIV, p. 119, (February, 1931)

332 *Encyclopedia Americana*, 2nd edition, Vol. XXII, p. 238.

the life of Galileo.[333] After Galileo had lost his eyesight he was visited by the poet Milton. Milton mentioned this visit in his great epic, "Paradise Lost".[334]

33. RENÉ DESCARTES

René Descartes was born near Toura, France on the thirty-first of March, 1596. His father was a man of wealth and a member of one of the best families of France. Half of the year, during which time the French Parliament was in session, he spent in Rennes. The rest of the time he spent on the family estate, Des Cartes, at La Haye.

René was the second child of a family of two sons and one daughter. At the age of eight he was sent to a Jesuit school at La Fleche. In his later years Descartes remembered and praised highly the fine education and discipline of the school. René was not strong, and because of his delicate health he was permitted to lie in his bed until late in the mornings. He followed this custom the rest of his life. While visiting Pascal during the year of 1647, Rene told him that the only way to do good work in mathematics and at the same time preserve one's health was to lie in bed in the mornings until he felt inclined to get up.[335]

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333 Ibid., p. 239.

334 Vera Sanford, op. cit., p. 120

335 W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 268-9.

When Descartes left school in 1612, he went to Paris to be introduced into the world of fashion. At this time the men of rank usually entered the army or the church. Descartes chose the former and joined the army of Prince Maurice of Orange.[336] One day whenw alking through the streets of Breda he saw a placard written in Dutch. The placard aroused his curiosity. Descartes stopped the first man who passed by and asked him to translate the works on the placard into either Latin or French. The stranger happened to be Isaac Beechman, the head of the Dutch college at Dort and one of the best known educators at that time. He told Descartes that on the placard was written a geometric problem which had been given as a challenge for anyone to solve. At that time no one had succeeded in obtaining the answer. Beechman told Descartes that he would translate the problem for him if he would solve it. Descartes worked on the problem only a few hours and then greatly surprised Beechman with its solution.[337]

The solving of the geometric problem made army life more distastful than ever to Descartes. However, because of the tradition and influence of his family, he remained in the army. All of his leisure time was now spent in the study of mathematics. Descartes' ideas for his analytic geometry were

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336 Ibid., p. 269.

337 Abraham Wolf, "Descartes." Encyclopedia Britannica, 14th edition, Vol. VII, p. 245.

received while yet a soldier. According to one story, he received his first ideas in a dream. Another story tells that his first idea came while watching a fly crawling along the ceiling of his room. The fly was near the corner and his first problem was to express the motion of the fly in terms of its distance from the walls. The third account states that he developed the subject while lazily lying abed on a cold day when the army was laying seige to a city.[338]

Descartes resigned from the army in 1621 and spent the next five years traveling. He settled in Paris in 1626,[339] and there he has been described as "a little well built figure, modestly clad in green taffety, and only wearing a sword and feather in token of his quality as a gentleman."[340]

In 1628 he moved to Holland where he spent the next twenty years, giving his time entirely to philosophy and mathematics. He disregarded all social functions and lived what might be called an unsocial life. Descartes was summoned to the court of Queen Christina of Sweden in 1649.[341] He had never been strong. The rigor of the northern winter

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338 Vera Sanford, <u>A Short History of Mathematics</u> (Boston: Houghton Mifflin Company, 1930), p. 43.

339 David Eugene Smith, <u>History of Mathematics</u> (Boston: Ginn and Company, 1925), Vol. I, p. 374.

340 W. W. Rouse Ball, <u>A Short Account of the History of Mathematics</u> (London: Macmillan and Company, 1915), p. 270.

341 Vera Sanford, <u>op. cit.</u>, p. 43.

combined with the long hours which he was expected to work caused his death from inflammation of the lungs a few months after he reached Stockholm. Sixteen years later his body was brought to Paris and buried in the church of St. Genevieve de Mont.

Descartes was a small man with an unusually large head, a projecting brow, prominent nose, and black hair coming down to his eyebrows. His voice was very feeble. His two chief characteristics were his selfishness and his coolness toward other people. He was not as widely read as might be expected of one of his genius. He despised both learning and art unless it could be used directly in one's work. Descartes never married so left no descendants although he had one illegitimate daughter who died quite young.[342]

The most important contribution of Descartes but by no means the only important one, was his invention of analytical geometry which is the basis of nearly all modern mathematics. This invention on geometry was given to the world in his book La Geometrie, which appeared in 1637.[343] In analytical geometry the position of a point is described by two numbers which represent the distances of the point from two perpendicular lines of reference. By this device it proves to be possible to solve geometric problems by the means of algebra.

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342 W. W. Rouse Ball, op. cit., p. 271.

343 Loc. cit.

This was a most significant step for it broke down barriers which had prevented geometry from making advancement, and was a stepping stone to the invention of calculus.

34. BLAISE PASCAL

Blaise Pascal was born in Clermont, France, June 19, 1623 and died in Paris August 19, 1662. He was the son of Etienne Pascal who was a judge and educator in Auvergne. Pascal's mother died when he was four years old. Soon afterwards his father, who was a skilled mathematician, resigned his position in Clermont and moved with his son to Paris where he personally supervised the training of his young son.[344] Pascal's father insisted that his education should include nothing but Latin and Greek. He hid all the mathematics books, as he did not want his young son to study or even see a book which related in any way to mathematics. One day while reading, Pascal ran across the word "mathematics". He asked his father what the word meant. He was told that it was a subject to be studied such as Greek and Latin. Pascal then asked what the subject treated. His father replied that it included the methods of making figures with exactness and the finding out what proportion they relatively had one to another. At the same time he forbade

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[344] Encyclopedia Britannica, "Pascal." Vol XVII, p. 350

young Pascal to talk or even to think about it.[345]

Pascal's genius could not thus be confined. He thought about the definition which his father had given him. Soon he began slipping from his room with a piece of charcoal and began drawing figures on the tile pavement which ran in front of the house in which they lived. He gave names of his own to all his figures, made his own set of axioms, and soon began making perfect demonstrations. In this way he arrived at the theorem, "The sum of the angles of a triangle equals two right angles." His father found him studying over the theorem one day and was so surprised at the genius of his son that he wept for joy.[346] When Pascal's father asked him what some of his figures were called, Pascal pointed to a circle and told him that it was a round. He also called a straight line a bar. Pascal explained to his father that it had been with great difficulty that he had discovered how to draw a perfect circle, equilateral triangles, and other figures which he had used in his work.[347]

Pascal's father was still afraid to let his young son go ahead with the study of mathematics. Among the elder Pascal's friends were several scientists of renown. He went

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345 W. W. Rouse Ball, op. cit., p. 282.

346 Florian Cajori, A History of Mathematics (New York: Macmillan Company, 1924), pp. 164-65

347 J. G. Edgar, The Boyhood of Great Men (New York: Harper Brothers, 1853), pp. 201-2.

to consult them at once and the scientists advised him to let young Pascal pursue the study of mathematics in whatever way he desired. Imagine young Pascal's joy when he was told that he could continue his studies and might even be allowed to have books to aid him in his work. No doubt he had been afraid that his father would deprive him of some of his privileges because of his disobedience.[348]

Many historians believe that it would have been impossible for a boy of twelve years old to have rediscovered so much of Euclid's Elements. However, Pascal's sister, Madame Périer, in her biography of Pascal, gave the above account. In the biography she also made the statement that the lad at the age of fourteen was admitted to the weekly meetings of the Geometry Circles. The Geometry Circles were composed of some of the best known mathematicians at that time. It was very unusual for a boy of fourteen to be admitted into the group.[349]

Pascal was still forced to spend most of his time in the study of languages. His sister said that geometry was his recreation and conics were his toys. At the ageof sixteen Pascal wrote a treatise on conics which was said to have been the best thing of its kind to be written since the time of Archimedes. When Descartes heard about the treatise he

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348 Ibid, pp. 202-3.

349 W. W. Rouse Ball, A Primer of the History of Mathematics (London: Macmillan and Company, 1927), p. 83.

refused to believe that it could have been written by anyone so young as Pascal and thought that surely it had been written by his father. Leibniz saw the treatise when he was in Paris. After he had reported on its contents, he suggested that it should be published. However it never was and is now lost.[350]

About the same time Pascal discovered the famous theorem known as "Pascal's Theorem". The theorem states "The three points determined by producing the opposite sides of a hexagon inscribed in a conic are collinear." He deduced more than four hundred corollaries from this theorem.[351]

Pascal's father returned to work in an importer's office when Pascal was eighteen. One day Pascal visited the office and watched his father labor for hours over long columns of figures. He made up his mind that it was foolish to spend so much time working over such columns of figures. He immediately set about to make a calculating machine which he finished at the age of nineteen. The king of France heard about the machine which could add such long columns of numbers and commanded Pascal to bring it to court. After exhibiting the machine Pascal was given the right to manufacture it. The modern adding and calculating machines are

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350 Florian Cajori, A History of Mathematics (New York: Macmillan Company, 1924), p. 165

351 W. W. Rouse Ball, op. cit., p. 84

all developments of this invention by Pascal.[352]

At the age of twenty-seven Pascal's health was very much impaired. He gave up the study of mathematics and went to a monastary at Boveu where he began the study of religion. Three years later his father died and Pascal went to Paris to take care of his father's estate. He temporarily returned to his study of mathematics and science.[353] In 1657 while riding one day in a carriage drawn by four horses, the horses ran away and dashed over the parapet of the bridge. The only thing that saved the life of Pascal was the fact that the traces broke. This event caused Pascal to believe that he hadn't been leading the right kind of a life. He moved to the monastary at Port Royal where his sister was a nun.[354] Although he did not become a member of the Jesuit order, he spent his time in prayer and practices of mortification. One author quoted from a letter of Pascal's in which he spoke of a sinful thought he had had and afterwards for some time wore an iron girdle studded with sharp pieces of iron with which he would pierce himself.[355] It is said that one night Pascal was suffering with a toothache. He was unable to sleep and some time during the night he discovered certain important properties of

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352 Vera Sanford, <u>A Short History of Mathematics</u> (Boston: Houghton Mifflin Company, 1930), p. 46.

353 W. W. Rouse Ball, <u>op. cit.</u>, p. 283.

354 <u>Loc. cit.</u>

355 <u>Ibid.</u>, p. 284.

the cycloid. This is the curve which is traced by a point on the rim of a wheel rolling on a straight line. This work with the cycloid helped to prepare the way for Newton's invention of the calculus.[356]

The following group of figures is known as the "Pascal's Triangle":

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1

Each number, excepting the border one, is the sum of the two nearest numbers in the row immediately above. The numbers in the Nth row are the coefficients in the expansion of $(a + b)^n$. Thus, $(a + b)^4, a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$.[357]

Pascal's utter disregard of the rules of healthful living hastened his death which occured at the early age of thirty-nine.

35. SIR ISAAC NEWTON

Leibniz gave a very generous tribute to Sir Isaac Newton

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356 Florian Cajori, A History of Mathematics (New York: Macmillan Company , 1924), p. 165.

357 New International Encyclopedia, 2nd edition, "Pascal Triangle." Vol. XVIII, pp. 136-37.

when he said that if one were to consider all the mathematicians from the beginning of the world to the time of Newton, that part of mathematics which was due to Newton would be by far the most valuable.[358]

Isaac Newton was born in Lincolnshire, near Grantham, England, on December 25, 1642. He was so weak and feeble at his birth that there was very little hope of his surviving more than a few hours at the most. Two women were sent for medicine to strengthen him and were greatly surprised to find the boy still alive on their return. Isaac's father died a short time before his birth. His mother later married the village rector.[359]

Isaac's mother sent him to a village school at Skillington when he was quite young. At the age of twelve he entered the public schools of Grantham. Newton was very inattentive to his schoolwork, was a general nuisance to his teachers because of his indifference, and always ranked the lowest in his classes. One day while Isaac was playing with some of his school mates, a boy who always ranked high in his class gave Isaac a kick in the stomach. The boy was larger and stronger than Isaac, and for several days Isaac could think of no way in which he could get even with the

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[358] David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), p. 404, Vol. I.

[359] J G. Edgar, The Boyhood of Great Men (New York: Harper Brothers, 1853), p. 171.

boy. One day while sitting in the class room he had a happy thought. There was one way in which he could get even with the boy who had kicked him. He would study hard and show the boy that someone could rank above him. From that time on Isaac continued to rise until he held the place of highest rank in his class.[360]

Although Isaac was considered a sober, silent, thinking lad who was somewhat fond of retirement, he was always very observant. One day he noticed that a windmill was being constructed near his home. Isaac went every day to watch the progress in its construction, and several men remarked that he seemed unusually interested in its design. Soon afterwards on the roof of the Newton home was found a miniature windmill modeled after the one whose erection had interested young Isaac. It was so perfect that many people praised him for his work. To their surprise he seemed to think nothing of the perfection of his model but was very happy to think that the wind would actually turn the wheel.[361]

Isaac introduced about this time the flying of paper kites. This past time had never been introduced in England before. Another of Isaac's inventions was the water clock. Long after his departure from Grantham, the clock was used by the surgeon with whom Isaac had made his home while in

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[360] Ibid., p. 172.

[361] Ibid., p. 173.

school. Another thing in which Newton seemed to find enjoyment was adding mechanical devices to the toys belonging to the neighbor children.[362]

Isaac's family had planned that he should be a farmer and carry on the work of his father. At the age of fifteen he was taken from school to assist his mother in the management of the farm. However, he was of no assistance. Instead of helping with the work he spent his time making experiments, solving problems, or working with some mechanical device. One time Isaac's mother sent him to town to market some pigs. Isaac stabled his horses at the inn and immediately went to the room which he had used when attending school. When a servant found him he was poring over a volume which had been covered with dust. His mother soon realized that it would be impossible to interest Isaac in farm work and he was allowed to return to school at Grantham.[363]

Isaac's uncle had received his education at Cambridge and it was through his recommendation that Newton was sent there to finish his education. He entered Cambridge in 1661 and for the first time in his life found himself among surroundings which were to develop his character and powers of genius.[364]

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362 Ibid., p. 173.

363 Ibid., p. 175.

364 W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 320.

Newton kept a diary in which he mentioned the fact that he had never studied mathematics before this time. It is possible that he might have read a book on logic by Sanderson which was taught in the elementary schools as a foundation for the study of mathematics. In October during his first year at Cambridge, Newton walked to Stouridge where a county fair was being held. Here he picked up a book on astrology and upon examining it found that he could not understand the book because of his lack of knowledge of geometry and trigonometry. Newton bought a copy of The Elements of Euclid and upon examining it decided that the theorems were too easy and trivial for him to waste his time studying. He then bought Descarte's Geometrie which he mastered with some difficulty. It is said that Newton made the statement later in his life that he had always been sorry that he had not mastered Euclid's geometry and made a thorough study of his other works before beginning the rest of his mathematical work.[365]

During the year of 1665 a great plague swept over England. The schools and all public buildings in the country were closed. Newton spent several months during this time at his home in Woolsthrope and here discovered the binomial theorem which is so important in algebra.[366]

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365 Ibid., pp. 320-21.

366 Ibid., p. 321.

Newton was given a Bachelor of Arts degree from Cambridge in 1665. By the time he received his Master of Arts degree in 1668 he was considered the greatest and most promising mathematician and physicist in England and one of the greatest if not the greatest in the world.[367]

In 1669 Newton was given the Lucasian professorship at Cambridge. His duties as a professor were not heavy. Once a week he would don his black robe and powdered wig and lecture to his class for a half hour. His lectures were given as fast as the students could take notes on them. The following week he would set aside four hours during which time the students who wished to come to his room to discuss the previous lecture might do so. Newton gave lectures dur- one term each year and never repeated a lecture. Each course consisted of nine or ten lectures and usually the lectures of one course began where the lectures of the previous one ended.[368]

There are several stories told about Newton during the time he was lecturing at Cambridge. One of them relates that Newton had placed his notes for the lectures he had given for several years on the table one evening, planning to start organizing them for a book he intended to write. While

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[367] David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), Vol. I, p. 400.

[368] W. W. Rouse Ball, op. cit., p. 324.

Newton was busy doing something else, his pet dog jumped upon the table upsetting a burning candle. The notes were ablaze and before the flames could be put out the notes were destroyed. Newton looked at the dog, Diamond, and with a sad note in his voice said, "Diamond, Diamond, you little know what damage you have wrought."[369]

Another story tells that Newton had a pet cat and a pet kitten. These two pets stayed in his room most of the time. Newton didn't like to leave his work long enough to open the door for them whenever they wanted to leave the house. After some thought and deliberation Newton cut two holes in the door of his room, a large one for the cat and a small one for the kitten.[370]

In spite of the persistence of such stories it is dubious if few or any of them are true. Newton had an aversion for pets or any kind and probably neither cat nor dog were ever in his possession.[371]

While sitting in the garden one day, so the story goes, an apple fell from a near-by tree to the ground. Newton had spent a great deal of time studying gravitation and why it was that people and objects here on the earth could remain

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369 Vera Sanford, A Short History of Mathematics (Boston: Houghton Mifflin Company, 1930), p. 50.

370 *Loc. cit.*

371 *Ibid.*, p. 51.

on the earth while it was rotating on its axis at such a rapid rate. The falling of the apple gave Newton an idea from which he was able to work out the law of gravitation. This law states that every particle of matter in the universe attracts every other particle with a force directly proportional to the mass of the attracting particle, and inversely to the square of the distance between them.[372]

In 1672 Newton was elected a member of the Royal Society, which was made up of men who had done important things in the field of science or mathematics. A year after his election he sent his resignation to the secretary. He lived too far from London to attend the meetings, was the reason he gave for his resignation. The secretary realized that the true reason that Newton desired to leave the society was because he could not afford to pay the dues which were one shilling a week. The secretary immediately corresponded with several members of the society and at the next meeting it was voted to excuse Newton from the payment of dues.[373]

Newton's position as a professor at Cambridge did not give him an income which was sufficient for his living expenses; therefore, in 1696 he resigned the Lucasian chair and accepted a position as Warden of the Mint of England.

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[372] J. G. Edgar, The Boyhood of Great Men (New York: Harper Brothers, 1853), p. 179.

[373] Vera Sanford, op. cit., p. 53.

Some authorities say that Newton resigned his chair because he realized the time during which he could do the most for science and mathematics was over. Others believe that he resigned because he wanted to experiment with alchemy, which is the professed art of transmitting the baser metals into gold.[374]

Because of his work in science Newton was elected president of the Royal Society in 1703. He held this office for twenty-five years or until his death. Two years later he was knighted by Queen Anne.[375]

One evening while giving a dinner for some of his friends, Newton left the table to get a bottle of wine. On the way to the cellar he became interested in something else and forgot about his guests. When Newton did not return his guests began to wonder what had happened to him. After searching for some time they found that Newton had gone to his room, put on his surplice, and had gone to the chapel for some religious service.[376]

One day when Newton was riding horseback he noticed something by the side of the road which was of interest to him and which he wished to inspect. He dismounted and, hold-

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374 Ibid., p. 54.

375 Ibid., p. 55.

376 David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), Vol. I, p. 404.

ing the reins of the bridle in one hand, inspected the object. Newton then started to walk up the road and when he reached the top of a near-by hill turned to remount his horse. Imagine his surprise when he found that his horse was at the bottom of the hill grazing by the roadside and in his hand was the horses' bridle which he had dragged up the hill behind him.[377]

Newton was of average height and was somewhat stout in his old age. He had long wavy hair which became a silvery white when he was about thirty. After his hair became white he was never known to wear his wig.[378] From the time he was an infant, when he was not occupied with his studies, he preferred the company of the women in the household to that of his thoughtless schoolmates. Among these was a young lady, clever and attractive, for whom Isaac formed a deep friendship. However, the friendship or affection faded because of Newton's poverty, and he never married. Newton's married niece was his housekeeper. He kept a carriage and six servants, three men and three women. At his home there was always hospitality and his friends were assured a hearty welcome.[379]

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[377] Loc. cit.

[378] W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 348.

[379] Philip Lenard, Great Men of Science (New York: Macmillan Company , 1933), p. 111.

Newton died in 1721 at the age of eighty-five and was buried in Westminister Abbey.[380] Many people today, when visiting that famous old church, pause before his tomb and pay homage to one of the greatest men this world has ever known. Voltaire was one of the many noted men who attended the funeral of Newton.

Later when he was an old man, Voltaire did much to make Newton's philosophy known in France. It has been said that the eyes of Voltaire would grow bright and that his cheeks would flush when he said that at one time he had lived in a land where a professor of mathematics, only because he was great in his vocation, had been buried like a king who had done good to his subject.[381]

36. THE BERNOULLIS

The Bernoulli family, one of the most unusual families in history, for several generations provided leaders in the field of mathematics. Belgium was the original home of the family but during a religious persecution they were forced to leave that country and they made their home in Switzerland. The following is a family tree of the Bernoulli family showing only those who were mathematical leaders.[382]

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380 David Eugene Smith, op. cit., p. 404.

381 Loc. cit.

382 Ibid., pp. 426-33.

- Nicolaus Bernoulli (The Father)
  - James I (1654-1705)
  - Nicolaus
    - Nicolaus I (1678-1759)
  - John I (1667-1748)
    - Nicolaus II (1695-1726)
    - Daniel I (1700-1762)
    - John II (1710-1790)
      - John III (1744-1804)
      - Daniel II (1751-1834)
        - Christoph (1782-1863)
          - Jean Gustave (1811-1863)
      - James II (1759-1789)

The first of this family to become known because of their mathematical work was James Bernoulli. James' family had intended that he should become a minister. In spite of his father's objections, he made an extensive study of mathematics. After traveling in France, Holland, Belgium, and and England, he returned to Switzerland in 1682. He immediately began the study of calculus and in 1687 he became professor of mathematics at the University of Basel. He was one of the first to make calculus a popular study and wrote the first book on the theory of probability.[383]

John Bernoulli was thirteen years younger than his brother James. His father was determined that John should become a merchant. John, however, thought that he would prefer the study of medicine or literature. He soon found

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383 Ibid., pp. 427-428.

these distasteful and began the study of mathematics. In 1695 John became professor of mathematics at Groningen University. Ten years later he was elected to fill the place left vacant by his brother's death at the University of Basel. John probably wrote on a larger range of subjects than his brother James. He wrote on calculus and was one of the most influential scholars in Europe in making the subject appreciated by university students.[384]

Daniel Bernoulli, the son of John Bernoulli spent several years in Russia as professor of mathematics in the Academy of Petrograd. The social life of Petrograd was too rough to please Daniel. When he became ill in 1733 he took his illness as an excuse to resign his professorship and returned to Basel. Here he became a professor of mathematics. A few of his works related to pure mathematics. These included the computing of trigonometric functions and continued fractions.[385]

Nicholas I was a nephew of James and John. His first position as a professor was at Padua in 1716. Later he returned to Basel where he was the fourth member of the Bernoulli family to be professor of mathematics at the University. He had studied to be a lawyer and his first book was on the theory of probability in legal matters. Daniel also

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[384] Ibid., pp. 428-29.

[385] Ibid, p. 431.

wrote on geometry and differential equations.[386]

Nicholas II studied law. He spent some time in traveling before becoming a professor of law at Bern. He was called to Petrograd to become professor of mathematics at that place. Here he died at the age of thirty-one.[387]

John II studied law but later in his life he spent several years as professor of mathematics in his native city.[388]

John III also studied law. He soon turned to mathematics and became a teacher of mathematics at the Academy of Sciences in Berlin. John III was very much interested in the history of astronomy. However, he wrote on factoring, indeterminate equations, and the doctrine of chance.[389]

Although the other Bernoullis were interested and were leaders in the field of mathematics, their work was not as important as those members of the Bernoulli family which have been mentioned.[390]

37. LEONARD EULER

Leonard Euler was born at Basel on April 15, 1707, and

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386 *Ibid.*, p. 432.

387 *Loc. cit.*

388 *Loc. cit.*

389 *Loc. cit.*

390 *Loc. cit.*

died in Petrograd on September 7, 1783. He was the son of a Lutheran minister who was well versed in mathematics. Leonhard received his first mathematical education from his father. Later he went to the University of Basel to study under John Bernoulli and it was here that he formed life-long friendship with both Daniel and Nicholas Bernoulli.[391]

When Nicholas and Daniel went to Petrograd to work in the University, they persuaded Catherine I, Queen of Russia, to invite Euler to come to the University where he taught mathematics and physics. In 1735 Euler solved an astronomical problem in three days which several eminent mathematicians had been working on for several months. To work the problem Euler used some improved methods of his own which had not been given to the public.

The climate had affected the eyesight of Euler. This and the strain of solving the problem caused Euler to lose the use of his right eye. When Anne I came to the throne of Russia, her despotism caused gentle Euler to devote all of his time to science and pay no attention to public affairs. Euler was called to Berlin in 1741. The Queen of Prussia, who received Euler kindly, couldn't understand why such a distinguished scholar should be so timid, reserved, and silent. When Euler was consulted about it, he replied, "Madam, it is because I come from a country where, when one speaks,

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391 W. W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Company, 1915), p. 393.

one is hanged."[392]

In 1766 Euler with difficulty received permission to leave Berlin and return to Russia upon the request of Catherine II. About three years later he became blind. In spite of this, and although his house burned in 1771, destroying most of his papers, Euler rewrote and improved his earlier works.[393]

When the French philosopher, Denis Diderot, paid a visit to the Russian court, so the story goes, he conversed very freely with many of the younger members. He displeased the Czarina with his anti-religious views and she asked Euler to help suppress him.

> Diderot was informed that a learned mathematician was in possession of an algebraical demonstration of the existence of God, and would like to give it to him before all the court, if he desired to hear it. Diderot gladly consented; though the name of the mathematician was not given, it was Euler. He advanced toward Diderot, and said gravely, and in a tone of perfect conviction:
>
> "Monsieur $(a \quad b^n)/n = x$, don Dieu existi, repondez." (Sir, $(a \quad b^n)/n = x$, therefore, God exists. Can you answer that?)
>
> Diderot, to whom algebra was Hebrew, was embarrassed and disconcerted; while peals of laughter rose on all sides. He asked permission to return to France at once, which was granted.[394]

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392 Florian Cajori, *A History of Mathematics* (New York: Macmillan and Company, 1924), p. 233.

393 W. W. Rouse Ball, *op. cit.*, p. 393.

394 Vera Sanford, *A Short History of Mathematics* (Boston: Houghton Mifflin Company, 1930), pp. 66-7. (Citing: David Eugene Smith, editor, *Budget of Paradoxis*, 2nd edition, Vol. II, p. 4, Chicago, 1915)

Euler's writings were so numerous in number that there has never been a complete edition of his works published. In 1909 the Swiss Natural Science Association voted to publish Euler's works in their original language. The mathematical organizations of France, Germany, America and other countries as well as individual donors are giving financial aid. When the work was started it was thought that 400,000 francs would cover the cost of the work. However, a mass of new manuscripts have been found in Petrograd. This new material will cause the cost of the publications to exceed the original estimate.[395]

Although Euler's work was mostly in higher mathematics, he gave some important things to the field of elementary mathematics. Foremost among these is the lettering of a triangle as it is lettered today; that is, if the angles of a triangle are designated by the letters A, B, C, the opposite sides are named a, b, c, respectively. Euler was the first mathematician to use i for the -1. Gauss used this same notation later.[396]

38. KARL FREDRICK GAUSS

Karl Frederick Gauss was born of poor parents in the town of Brunswick, April 23, 1777. His grandfather, having

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[395] Cajori, Florian, *op. cit.*, p. 233.

[396] *Ibid.*, p. 234.

tired of the farm and thinking there were other things he would rather do, had left the farm and moved to Brunswick. Here he earned a living for himself and his family by following the occupation of a gardener, which also became the principal occupation of Gauss' father.[397]

Although the Gauss family were very poor, they were taught to appreciate the better things of life. Young Karl gave indication of unusual gifts and talents. He learned to read by himself, asking the older people how certain letters were pronounced. His mastery of arithmetic, even before he had any training in the subject, greatly surprised every one. The marvelous aptitude of Karl for calculation brought him to the notice of Duke Karl Johann Ferdinand when he was fifteen years old. The Duke of Brunswick became and remained the protector and patron of Karl.[398]

It was to the Duke of Brunswick that Karl was indebted for a liberal education. His parents, who wished to profit by his wages as a laborer, did not encourage his education. In 1792 Guass was sent to Caroline College. At the end of three years the professors and pupils admitted that he had learned all that the professors could teach him. Gauss must have studied Newton's Principia because it was this that allowed him to enter Gottingen University in 1795 where he

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397 Philip Lenard, Great Men of Science (New York: Macmillan Company , 1933), p. 244.

398 Ibid., pp. 245-46.

studied for three years. Until the death of the Duke of Brunswick, Gauss received a pension, so that he was able to devote all of his time to his work in mathematics. In the year 1807 he went to the University of Gottingen as professor of mathematics, a post which he held until his death in 1855 at the age of seventy-eight.[399]

Gauss married twice but both of his wives died when they were quite young. The younger of his six children, a daughter, remained with her father until his death. Several of Gauss' descendants are now living in Missouri, near Kansas City.[400]

Gauss was the last of the great mathematicians whose interests were universal. While his interests may have been chiefly in the field of higher mathematics, many of his discoveries were in the lower branches. One of his many discoveries was the construction of a regular polygon of seventeen sides with a ruler and compasses. Along with this discovery Gauss gave a theorem by which it is possible to tell what regular polygons can be constructed. The formula is $2^m (2^n + 1)$ where m and n are integers and $2^n + 1$ is a prime. By the use of this formula it may be found that regular polygons of 3, 5, 17, 257, 65, 537, etc. or any multiple

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[399] W. W. Rouse Ball, A Short Account of the History of Mathematics (London, Macmillan and Company, 1915), p. 447.

[400] Philip Leonard, *op. cit.*, p. 245.

of these numbers may be constructed.[401]

Gauss is also noted for his work in theory of equations. The theory of numbers was his hobby. At one time Gauss made the statement "Mathematics is the queen of science and the theory of numbers is the queen of mathematics."[402] One of the many things which Gauss gave to the world as a result of his work in the theory of numbers is "the fundamental Theorem of Algebra". This theorem states that every algebraic equation with complex coefficients has a complex (real or imaginary) root.[403] Gauss was never known to rush the completion of a problem once he had it started. This was probably because he enjoyed the progression of his work and it has been said that the completion of a problem often was sorrowful to him and he was not satisfied until he had started work on another.[404]

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401 W. W. Rouse Ball, op. cit., p. 452.

402 David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1915), Vol. I, p. 504.

403 W. W. Rouse Ball, op. cit., p. 448.

404 Philip Leonard, op. cit., pp. 246-47.

CHAPTER VIII

RECENT MATHEMATICIANS AND THEIR CONTRIBUTIONS

INTRODUCTION

The history of mathematics does not close with Gauss. Before this time most of the mathematicians had covered the entire field. After the time of Gauss the field became so large that it was almost impossible for one person to study all of it as entirety. This has led to specializations in the various parts of the field. Many of the more recent discoveries have been in the field of higher mathematics. This does not mean that the elementary phase has been slighted. The work of the earliest mathematicians in elementary mathematics opened the field for discoveries of the many things which are more advanced. At the same time improvements have been made in the elementary field. As it would be impossible to mention all the recent mathematicians of note, only a few of the more important ones are considered.

39. GOTTFRIED WILHELM VON LEIBNIZ

Gottfried Wilhelm von Leibniz, a German, was born in 1646 and died in 1716. At an early age he showed great mathematical ability and read the most important mathematical works before he was twenty. Leibniz studied law before he entered the diplomatic service. He traveled quite extensively and met some of the leading mathematicians of Holland, England, and France. After Leibniz returned to Hannover he became librarian to the duke. The Leibniz home is now used as a museum in Hannover.

Leibniz's leisure time gave him sufficient opportunity to work on mathematics. His chief contributions to the field were differential and integral calculus. Leibniz also did some work on symbolism in algebra.[405]

40. JOSEPH LEWIS LAGRANGE

Joseph Lewis Lagrange, who was born in 1736 and died in 1813, was one of the greatest French mathematicians of the sixteenth century. When Fredrick the Great wrote to Lagrange that "the greatest king in Europe" wanted "the greatest mathematician in Europe" at his court, he did not exaggerate in the least his own reputation or that of Lagrange.

Lagrange showed no taste for mathematics until he was seventeen. At that time he commenced to study the subject without any help. In the next two years he began making discoveries of his own. Lagrange enriched algebra by his work on the solution of the algebraic equation. He also did extensive work in calculus, theory of equations, determinants, and differential equations.[406]

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405 David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1915), Vol. I, pp. 417-18.

406 Florian Cajori, A History of Mathematics (New York: Macmillan Company, 1924), pp. 250-59.

41. PIERRE-SIMON LAPLACE

Pierre-Simon Laplace, a French mathematician, was born in Normandy in 1749 and died in 1827. He was the son of a farm laborer and owed his education to some of the wealthy friends of the family. In later years, when Laplace had become distinguished for his mathematical works, he held himself aloof from both his parents and those who had helped him in his education.

Nathaniel Bowditch, an American astronomer, said that he never came across the phrase, "thus it plainly appears" which Laplace used, without feeling that he had hours of hard work ahead of him before he could indeed find out "just how plainly it appeared". Laplace is known chiefly through his work on celestial mechanics and the theory of probability.[407]

42. ADRIAN MARIE LEGENDRE

Adrian Marie Legendre was born in 1752 and died in 1833. He received his education in Paris. Legendre held a few minor government appointments during his lifetime but Laplace used his influence against Legendre. This kept Legendre from obtaining public recognition for his work. He was a very timid man and accepted his obscurity without any

[407] Vera Sanford, A Short History of Mathematics (Boston: Houghton Mifflin Company, 1930), pp. 67-68.

conflict.

Legendre is connected with elementary mathematics through his Elements de geometrie. This work was very valuable as a textbook. It was received in America very favorably and has been used as a model for the geometry texts in used in this country.[408]

43. NIELS HENRIK ABEL

Niels Henrik Abel was born in Norway in 1802. Although he died at the early age of twenty-six, he had accomplished much in the mathematical field. Abel was the first to prove that an algebraic solution of the general equation of the fifth degree was impossible. Abel is also known for his work on elliptic functions.[409]

44. AUGUSTIN LOUIS CAUCHY

Augustin Louis Cauchy, a French mathematician who lived at the same time as Napoleon, was born in 1789 and died in 1857. Cauchy's life seemed full of unrest, partly due to his own eccentricities and partly to the political situation in France at that time.

Cauchy's education was received in the technical and

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[408] Ibid., p. 68.

[409] David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1915), p. 527, Vol. I.

military schools which were established by Napoleon. He seemed to be especially interested in engineering. First his position was as a teacher of mechanics in the Ecole Polytechnique. Within the next eighteen years he held five different positions.

In spite of his unrest, Cauchy published about seven hundred works on mathematics. The most important of these were on methods of determining real and imaginary roots, on astronomy, and on work which helped bring determinants into general use.

Cauchy was considered a very conceited man, narrow in his views, a hard worker, and one who would argue over trifles.[410]

45. ÉVARISTE GALOIS

Évariste Galois, a staunch French republican, was born in 1811 and died in 1832. Because of his political beliefs he was thrown into prison twice. At the age of twenty he was killed in a duel over a love affair. Although Galois did most of his mathematical work in four years, to him is due one of the most important advances in the group theory. To him we also owe much of the modern theory of algebraic equations of higher algebra.[411]

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410 Ibid., pp. 496-97.

411 Ibid., p. 498.

46. CARL GUSTAV JACOB JACOBI

Carl Gustav Jacob Jacobi, who was born of Jewish parents at Potsdam in 1805, was educated at the University of Berlin. In 1827 he became professor of mathematics at the University of Konigsberg, where he remained until he was pensioned by the Prussian government in 1842. Jacobi then moved to Berlin where he remained until his death in 1851.

The chief works of Jacobi were in the fields of elliptic functions, determinants, theory of numbers, differential equations, and infinite series.[412]

47. JACOB STEINER

Jacob Steiner was born in 1796 in Switzerland. At the age of fourteen he was unable to read and write. His parents were poor and unable to send him to school. He went to school at the age of seventeen, being sent by Johann Heinrich Pestalozzi. When he was twenty-five years old he began giving private lessons in mathematics at Berlin. Later he became professor of mathematics at the University in Berlin. His later life was spent in Switzerland, where he lived until his death in 1827. Steiner was one of the greatest geometers of the modern time.[413]

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[412] W. W. Rouse Ball, _A Primer of the History of Mathemathics_ (London: Macmillan and Company, 1927), pp. 128-29.

[413] David Eugene Smith, _op. cit._, pp. 524-25.

48. GEORG FRIEDRICH BERNHARD RIEMANN

Georg Friedrich Bernhard Riemann was born in 1826 and died in 1866. Riemann received his education at the University of Berlin and the University of Gottingen. He became a Professor of Mathematics in the latter university in 1857.

Riemann's chief work was in the field of Non-Euclidean geometry. Non-Euclidean geometry is any system of geometry whose postulates contradict those of Euclid. The term is usually applied, however, to those geometries which deny Euclid's fifth postulate, known also as his parallel postulate.[414]

49. SIR WILLIAM ROWAN HAMILTON

William Rowan Hamilton, a mathematical product of Ireland, was born in 1805. Although a descendant of Scotch stock, Hamilton was always proud when he had a chance to proclaim himself an Irishman. Hamilton was one of the infant prodigies which one meets occasionally. At the age of three he could read English fluently and was somewhat advanced in arithmetic. By the time he was twelve, Hamilton had a working knowledge of twelve different languages. A year later he had written an algebra book which has never been published.

Hamilton was appointed Professor of Astronomy at Trinity

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[414] Vera Sanford, A Short History of Mathematics (Boston: Houghton Mifflin Company, 1930), pp. 275-76.

College of Dublin while he was an undergraduate in the school. He was knighted in 1835. Hamilton's first researches were on optics. Later he confirmed the conclusion Abel had made concerning the solution of a general equation of the fifth degree.[415] Hamilton's greatest work was in the field of quaternions.

50. JAMES JOSEPH SYLVESTER

Although James Joseph Sylvester, who was born in 1814 and died in 1897, was one of the most brilliant students in his class at Cambridge University, he was denied his degree because of his Jewish faith. He then went to the University of Dublin where he received his first degree. Later Cambridge awarded him both a bachelor's and master's degree.

Because he could see little opportunity for advancement in the teaching profession in England, he accepted an appointment as Professor of Mathematics in Virginia in 1841. He was a failure and due to some trouble with one of his students, he resigned in 1842. Sylvester then returned to England where he was Professor of Mathematics to Woolwick until 1869. In 1877 John Hopkins University asked him to accept a chair of mathematics. Sylvester did more than any other one man of his time to establish graduate work in America. To him is also due the founding of the American

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[415] David Eugene Smith, op. cit., p. 461.

Journal of Mathematics.[416]

51. KARL WEIRERSTRASS

Karl Weierestrass, one of the educational leaders who helped to make Germany a gathering place for scholars the last part of the nineteenth century, was born in 1815 and died in 1897. Weirerstrass first studied law and then began to study mathematics. He became Professor of Mathematics at Berlin in 1864. Weirerstrass was a leader in the theory of functions and in the theory of irrational numbers.[417]

52. ARTHUR CAYLEY

Arthur Cayley, a great friend of Sylvester, was born in England in 1821. He was the son of an English merchant in Petrograd, who hoped that Arthur would become his partner in business. When he was sent to King's College School in London at the a ge of fourteen, Arthur showed much mathematical ability. His father then sent him to Cambridge where he entered Trinity College at the age of seventeen. He graduated from this college with highest honors. About 1860 the Sadlerian professorship of pure mathematics was established at Cambridge and Cayley was the first to occupy the chair. America is indebted to Cayley for a series of le ctures given

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[416] *Ibid*., pp. 463-65.

[417] *Ibid*., p. 509.

by him at the John Hophins University on elliptic functions. Cayley's outstanding work was in the fields of analytic geometry and elliptic functions. He died in Cambridge in 1895.[418]

53. THE PEIRCE FAMILY

The Peirce Family was associated with mathematics at Harvard or elsewhere for nearly a century. The eldest member of this family, Benjamin Peirce, was considered one of the most prominent mathematicians of America until his death in 1880. He was a graduate of Harvard at the age of twenty and became a tutor there tow years later. He was soon awarded the professorship in mathematics and natural philosophy and later the professorship of mathematics and astronomy. His outstanding work in mathematics was the Linear Associative Algebra published in the American Journal of Mathematics in 1881.

James Mills Peirce and Charles Sanders Peirce, the two sons of Benjamin Peirce, were both students of Harvard. The first was professor of mathematics there from 1869 to 1906. The second son, Charles, for a number of years, worked on the United States Coast and Geodetic Survey and became well known for his researches in geodesy.

Another relative of Benjamin Peirce, a second cousin,

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418 Ibid., pp. 465-66.

once removed, was Benjamin Osgood Pierce. He wrote a number of valuable papers on mathematical physics.

One other member of the Pierce family was Leona May Pierce who worked at Yale University.[419]

CONCLUSION

In the preceding chapters many men, who have been famous for their works in mathematics, have been mentioned. Along with the biographies of these men have been given a little of the history of numbers, symbols, and weights and measures, as well as the subject matter included in the modern textbook of elementary mathematics.

However, the mathematicians mentioned in the preceding chapters are not the authors of textbooks used by the present day high school student. It is from the works of such mathematicians as Euclid, Pythagoras, Mahavira, and Boethius that the modern authors have received their ideas and subject matter, rewriting and reorganizing the material according to the form in which they think it should be presented.

It has been the tendency of the modern authors to simplify the problems given although they contain the same fundamental principles as the problems written several centuries ago. Upon comparing the two following problems taken

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[419] David Eugene Smith and Jekuthiel Ginsburg, A History of Mathematics In America before 1900 (Chicago: Open Court Publishing Company, 1934), pp. 118-24.

from an old English textbook, it is hoped that the student of mathematics will appreciate the simplicity of modern problems.

Question

A farmer with a plowman doth agree
That 30 days his servant he should be.
Each day he wrought the farmer is to pay
Him sixteen-pence; but when he was away
Five groates he is for each day to obate.
The time expired; they their accounts do state.
Whereby the master nothing is to give,
Nor has the servant any to receive.
How many days he wrought I do demand,
And how many he play'd I do understand.

Answer

That lazy drone who squandered away
13 days and one third in sleep and play,
In 30 days (for all he nothing got)
Deserved to have his bones broke, for an idle sot.

Question

I happened one evening with a tinker to sit
Whose tongue ran a great deal to fast for his wit.
He talked of his art with an abundance of mettle
I ask'd him to make me a flat bottom kettle.
That the top and the bottom diameters should be
In just such proportion as five is to three;
12 inches the deapth I would have and no more,
And to hold in ale-gallons seven less than a score.
He promised to do it, and to work he strait went;
But when he had done it, he found it to scant.
He altered it then, and to big he had made it,
And when it held right the diameters failed it;
So that making it so often too big or too little,
The tinker at last had quite spoiled the kettle.
Yet he vows that he will bring his said purpose to pass,
Or he'll utterly spoil every ounce of his brass.
To prevent him from ruin, I pray help him out,
The diameters length else he will never find out.

Answered by Mrs. Barbary Sidway

Well, bonny brave tinker, to save thee from ruin,
The kind british lasses are active and doing;
Because that thou art a brave fellow to mettle,
Take here the diameters both of they kettle;
One's 24 inches four tenths very near,
T'other fourteen, and 64 cent doth appear.

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