SENIOR MATHEMATICS IN KANSAS HIGH SCHOOLS

A Thesis

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Master of Science in Mathematics

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by

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TABLE OF CONTENTS

CHAPTER PAGE

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION AND THE PROBLEM

During the past twenty years, there have been tremendous changes in the secondary school mathematics curriculum. Many reasons have been given for these changes. One reason, of course, being the launching of Sputnik I by the Russians in 1957 and the realization by many Americans that we were lagging behind in science and mathematics. But the "new mathematics" movement was underway before 1957. Other factors contributing to the changes were the rapid growth of mathematics during the past one hundred fifty years and the revolutionary development of science and technology. Also, there was a growing concern about the neglect of the superior student. And finally, imaginative leadership in mathematics education began to emerge in various universities and professional organizations.[1]

I. NATIONAL GROUPS IN MATHEMATICS

College Entrance Examination Board. One of the groups most influential in mathematics change was the Commission on Mathematics of the College Entrance Examination Board (CEEB).

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[1]John J. Kinsella, Secondary School Mathematics (New York: The Center for Applied Research in Education, 1965), p. 15.

The Commission began its study in 1955 and published its nine-point program in 1959.[2]

Its program called for:

1. Preparation for college mathematics at the level of calculus and analytic geometry;
2. Understanding of deductive reasoning not only in geometry but in other fields, such as algebra;
3. Appreciation of mathematical structure illustrated by properties of sets of numbers;
4. Judicious use of unifying ideas, such as sets, variables, functions, and relations;
5. Treatment of inequalities along with equations;
6. Insertion of some coordinate geometry and solid geometry into the plane geometry course;
7. Introduction in Grade 11 of fundamental trigonometry centered on coordinates, vectors, and complex numbers;
8. Emphasis in Grade 12 on elementary functions (polynomial, exponential, circular);
9. Recommendations of additional alternative units for Grade 12: either introductory probability with statistical applications, or an introduction to modern algebra.

School Mathematics Study Group. Another influential group, the School Mathematics Study Group (SMSG) was formed in 1958 under the leadership of Professor E. G. Begle of Yale University. Its fundamental aim was "to improve the teaching of mathematics in the secondary schools, to persuade more students to study more mathematics, and to ensure that the mathematics they study is appropriate to the world of

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[2]Commission on Mathematics, College Entrance Examination Board, Program for College Preparatory Mathematics and Appendices, (Princeton: Education Testing Service, 1959), p. xi.

today."[3] Through the activities of its summer writing groups, it has produced student textbooks and teacher commentaries for grades 7-12 and other books to aid the teacher in presenting the new material. Its work has served as a model for authors of commercial publishing houses to build and improve upon.

Cambridge Conference on School Mathematics. A third group, the Cambridge Conference on School Mathematics (CCSM), met in June, 1963, in Cambridge, Massachusetts. This group, composed of twenty-five professional mathematicians, met under the auspices of Educational Services, Incorporated, and The National Science Foundation. The purpose of the meeting was to consider problems of curricular reform for grades kindergarten through twelve. Their proposed thirteen-year program was the most ambitious of all. When leaving high school, the student would have a level of training comparable with a current three-year college program. This would include two years of calculus and one semester each of modern algebra and theory of probability. This would be made possible by the extension of many high school subjects down into the elementary grades. A spiral approach with the emphasis on discovery would be adapted and the traditional drill-for-drill's-sake would be abandoned.[4]

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[3]E.G. Begle, "The School Mathematics Study Group," The Bulletin of the National Association of Secondary-School Principals, XLIII, No. 247 (May, 1959), p. 27.

[4]Eugene D. Nickols, "The Many Forms of Revolution," The Continuing Revolution in Mathematics (Washington: The National Council of Teachers of Mathematics, 1968), p. vi.

II. PROGRAMS FOR GRADE TWELVE

There is no uniform secondary school mathematics program in the United States. There are variations among states, within states, and according to size of school. The grade placement and content of the course may vary. The name of the course may be the same but the actual content depends upon the teacher, the textbook used, and the course of study prescribed. It is not the name, but the treatment of the subject that distinguishes the "new mathematics" from the traditional.

There is more variation among the programs for the twelfth grade than at any other level. For college-bound seniors, Algebra I is usually taken in grade nine and Geometry in grade ten. In grade eleven the course will be Algebra II or an integrated course in algebra and trigonometry. But in the twelfth grade, many different courses are being taught and still others are in the planning stage. Much more content has been proposed than can be taught in any given program.

The programs proposed for grade twelve by the CEEB, the SMSG, and the CCSM are not in complete agreement. Both the CEEB and the SMSG would teach the trigonometry of triangles in grade eleven. For the first half of grade twelve, both groups would emphasize the treatment of functions. This would include circular, polynomial, exponential, and logarithmic functions. For the second half of grade twelve, the groups chose different courses. The SMSG proposed

teaching a course in matrix algebra while the CEEB recommended either probability and statistics or an introduction to modern algebra. The Cambridge Conference proposed analysis for grade twelve. Prior to his senior year, the student was to have completed algebra, geometry, probability, linear algebra, and one course in calculus. Calculus was to be started in the ninth grade.

Another proposal for grade twelve is the Advanced Placement Program. Four years of mathematics study is completed by the eleventh grade. In the twelfth grade, a college-level course in analytic geometry and calculus is presented to select students. These students then take nationally administered advanced placement examinations and either receive credit for a comparable college course or are excused from taking a required course.[5]

III. THE PROBLEM

Statement of the problem. It was the purpose of this study (1) to survey the twelfth grade mathematics programs in Kansas public high schools for the 1970-71 school year, (2) to examine the variations in regard to school size and preparation of teachers, (3) to compare what is being done in Kansas with the recommendations of national groups and, (4) to present opinions of teachers and leading mathematicians as to what course should be offered.

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[5]Kinsella, op. cit., pp. 23-25.

Source of materials for study. The material for this study was obtained through the use of questionnaires. Letters were sent to 325 teachers in 314 different high schools. These teachers had taught an advanced mathematics class in 1969-70. Their names were obtained from the 1969-70 Directory of Mathematics Teachers, compiled by Lucile Asher, Mathematics Consultant. Ten more letters were mailed to the mathematics departments of ten of the larger high schools. If a school did not offer advanced mathematics in 1969-70, a letter was not sent.

From the 335 letters that were sent out, 245 answers were received. This was 73 per cent. Information was received on 218 schools or 67 per cent of the schools contacted. There were 101 schools that did not respond.

The information on these 101 schools was obtained from the Principals Reports on file with the Kansas State Department of Education in Topeka. Ten of these schools did not have an advanced mathematics course in 1970-71. A check was also made on the schools who did not offer advanced mathematics in 1969-70. It was found that twelve of these schools were teaching a course in 1970-71. The information for these schools was also obtained from records in the Department of Education.

The information from the questionnaires and from the Education Department records has been combined to arrive at the total number of students in advanced mathematics. In all other cases the information was compiled separately and the

figures compared. It has been stated in the paper from which source the information was obtained.

CHAPTER II

STUDENTS IN ADVANCED MATHEMATICS

Part of the aim of the School Mathematics Study Group was "to persuade more students to study more mathematics." This is still a needed objective. Twelfth grade mathematics is not a popular course with students. In 1970-71, approximately 5,500 Kansas seniors were enrolled in an advanced mathematics course. This was only 16 per cent of the 34,534 seniors in Kansas public high schools.

I. RELATIONSHIP BETWEEN SCHOOL NUMBER AND STUDENT NUMBER

No advanced mathematics. In 65 schools, or 17 per cent of all Kansas public high schools, there were no advanced mathematics courses taught in 1970-71. This figure may seem high until it is compared to the percentage of students who had no opportunity to take an advanced course. The 65 schools had a total enrollment of 4,953 students in grades 10-12, which is only 4 per cent of all Kansas students. This difference can be accounted for by the fact that these 65 schools are among the smallest in the state.

Method of classification. In order to make comparisons by school size, some method of classifying schools was needed. The classification system of the Kansas State High School Activities Association has been used. This system is given in Table I.

TABLE I

CLASSIFICATION OF SCHOOLS BY NUMBER OF STUDENTS ENROLLED IN GRADES 10-12

CLASS	NUMBER OF STUDENTS
1-A	22 - 112
2-A	113 - 237
3-A	238 - 636
4-A	638 -1589
5-A	1642 -2561

Some idea of the relationship between number of schools and number of students is presented in Figures 1 and 2. The 5-A high schools have almost one-third (30 per cent) of the students but only represent one-twenty-fifth (4 per cent) of all schools. While on the other hand, the 1-A schools represent over two-fifths (42 per cent) of the schools while enrolling only 10 per cent of the students.

II. NUMBER OF STUDENTS IN ADVANCED MATHEMATICS

Comparsion by school size. It was surprising to find that school size did not greatly effect the percentage of students enrolled in advanced mathematics courses. The number of students in mathematics courses was compared with the total number of students (grades 10-12) in the schools. The

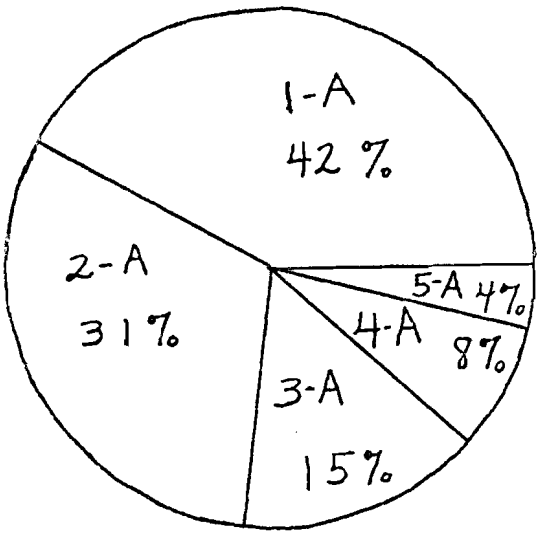

FIGURE 1

DISTRIBUTION OF KANSAS PUBLIC HIGH SCHOOLS AMONG THE FIVE CLASSIFICATIONS OF THE KANSAS HIGH SCHOOL ACTIVITIES ASSOCIATION

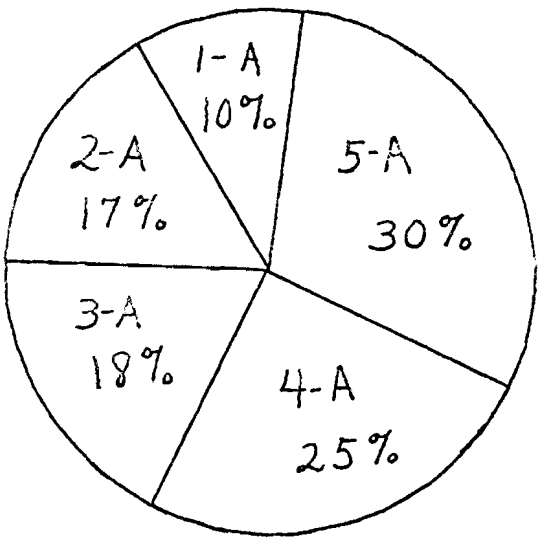

FIGURE 2

DISTRIBUTION OF HIGH SCHOOL STUDENTS IN KANSAS AMONG THE FIVE CLASSIFICATIONS OF THE KANSAS HIGH SCHOOL ACTIVITIES ASSOCIATION

data are presented in Table II. On the basis of this information, a prediction could be made that for any given high school in Kansas, between 4 and 5 per cent of all students will be enrolled in advanced mathematics classes.

Students in the various courses. Table III is presented to give an idea of the number of students in the various mathematics courses. This information was taken from computerized lists in the Curriculum Department of the Kansas Office of Education.

The total number of students in Table III is slightly higher than the total given in Table II. The number in Table II was compiled from questionnaire cards and included only senior students. If a school offered two one-semester courses, the number of students was counted only once. On the Curriculum Department's lists, there was no distinction made between courses of one or two semesters. Therefore if a student took two one-semester courses his senior year, he would have been counted twice. This would account for part of the difference.

Another possibility would be errors on the Principals Reports. There are only six listings for mathematics on the report--Algebra I, Algebra II, Basic Math, Plane Geometry, Solid Geometry, and Trigonometry. Other names may be typed in. In checking questionnaires against reports, it was

TABLE II

NUMBERS AND PERCENTAGES OF KANSAS HIGH SCHOOL STUDENTS ENROLLED IN ADVANCED MATHEMATICS CLASSES, 1970-71, BY SIZE OF SCHOOL

SIZE OF SCHOOL	TOTAL ENROLLMENT	STUDENTS IN ADV. MATH	PERCENTAGE ENROLLED
1-A	11,095	481	4.3
2-A	19,649	952	4.8
3-A	20,747	838	4.0
4-A	28,196	1,275	4.5
5-A	34,872	1,908	5.5
All schools	114,559	5,447	4.75

TABLE III

NUMBER OF STUDENTS IN ADVANCED MATHEMATICS COURSES IN KANSAS PUBLIC HIGH SCHOOLS, 1970-71 (FROM KANSAS STATE DEPARTMENT OF EDUCATION STATISTICS)

COURSE	NUMBER OF STUDENTS
Introduction to Analysis	1,279
Probability and Statistics	66
Solid Geometry	374
Computer Math	204
Introduction to Calculus	427
Calculus with Anal. Geometry	474
Analytic Geometry	95
Elementary Functions	151
Trigonometry	1,405
Algebra III	1,418
TOTAL	5,893

discovered that the course listed was not always the course taught. Also, in checking schools picked at random, it was found that some Algebra III courses were actually Algebra II. The number of students given in the table appears much too large to be the actual enrollment in Algebra III.

If the Algebra III enrollment is not considered, trigonometry is the most popular course with 1,405 students. Analysis is in second place with 1,279 students. The combined enrollments of Introduction to Calculus and Calculus with Analytic Geometry put the calculus course in third place with 901 students. The other courses have much smaller enrollments.

Since there is some doubt as to whether the course listed was the course taught, the conclusion reached from this information would concern only the number of students in advanced mathematics. Both the questionnaire results and the Education Department records reveal that around 16% of Kansas students are in advanced mathematics.

CHAPTER III

TYPES OF COURSES OFFERED AND TEXTBOOKS

There is a growing emphasis on the integration of secondary school mathematics courses. For example, twenty years ago solid geometry was a standard course for grade twelve. Today, plane and solid geometry are integrated in a tenth grade course. A new course for grade twelve, elementary functions, is an example of the integration of algebra and trigonometry. In this course, the trigonometric ratios are considered as functions.

Much of the integration and reorganization of course content is not apparent from looking at course titles. Also the frequent appearance of the term "mathematics" in twelfth year course titles evidences the trend to name courses without attempting to describe the content. These two facts make it difficult to classify courses according to content.

I. COURSES TAUGHT IN KANSAS HIGH SCHOOLS

The questionnaire asked for the name of the advanced mathematics course taught. An accompanying letter explained that an advanced mathematics course was to be defined as a course presupposing three years of mathematics beginning with elementary algebra. Since many course titles are general terms, such as Senior Math or Advanced Math, the teacher

was also asked to give the name of the textbook used and to list briefly the material covered in the course.

The most popular name was Advanced Mathematics with 64 schools using this title. A close second was the term Senior Mathematics with 56 schools. Other schools also used general terms, such as Math IV, Modern Topics in Mathematics, or Principles of Mathematics. But more than one-half of the 218 schools surveyed did attempt to name the course according to content.

Results of the survey. Figure 3 gives the number of schools offering each course based on results of the survey. In some cases the textbook and list of material covered was used to classify the course. For example, to be classified as analytic geometry, the major part of a semester had to be spent studying analytic geometry. Several schools covered a variety of subjects, making it impossible to classify according to content. These courses are listed under combination.

Analysis was the most popular with 72 schools listing it as a full-year course while 23 taught it for one semester only. In most of these 23 schools, trigonometry was taught the first semester. This indicated that if trigonometry was integrated with algebra in the eleventh grade, then analysis became a full-year course.

Trigonometry was offered as a full-year course in 13 schools while 79 schools listed it as a one-semester course.

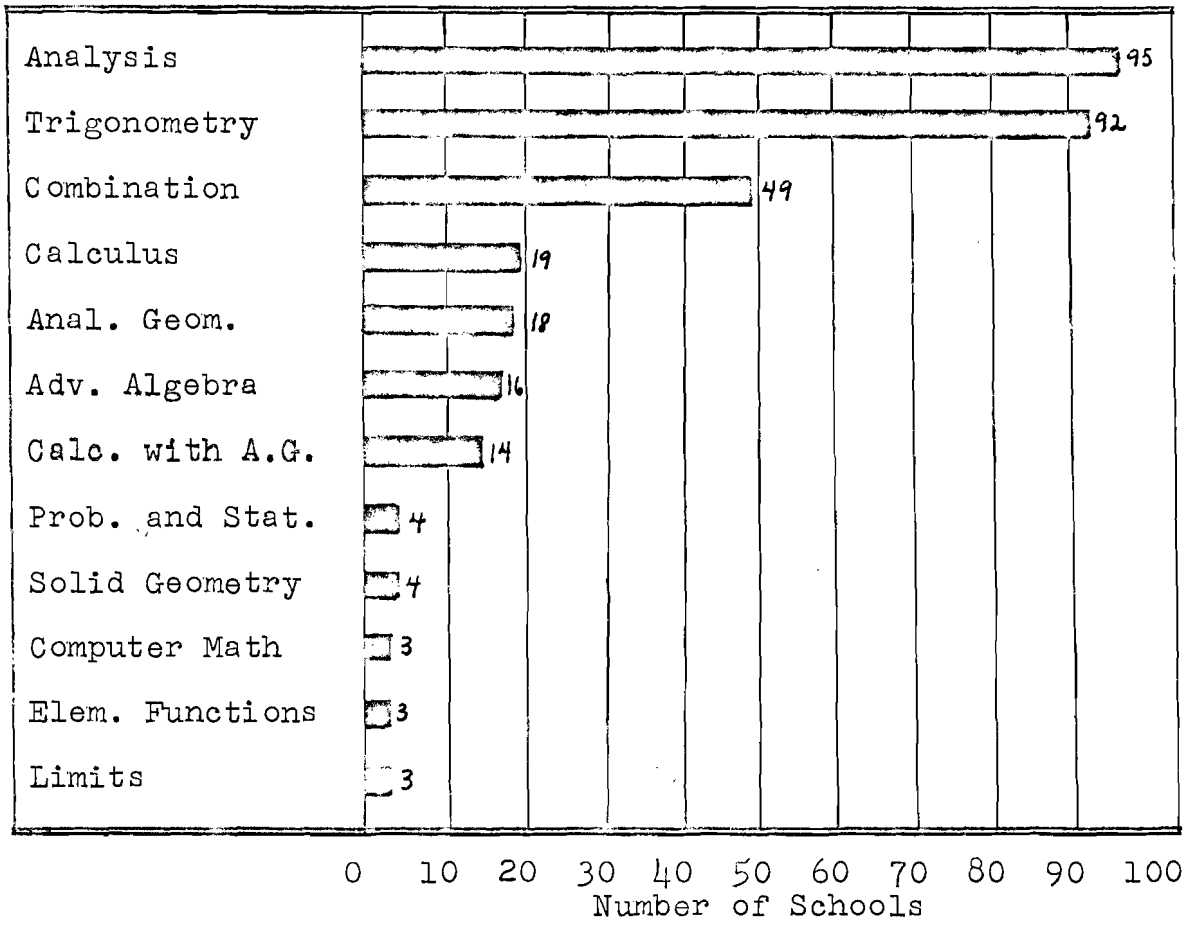

FIGURE 3

NUMBER OF KANSAS HIGH SCHOOLS OFFERING COURSES IN ADVANCED MATHEMATICS, 1970-71 (BASED ON A SURVEY OF 218 SCHOOLS)

It was noted that in some of the larger school systems in the state that trigonometry had disappeared as a separate course. It had been integrated with other courses.

Courses listed by Department of Education. Figure 4 gives the number of schools offering each course as listed with the Kansas State Department of Education. This includes all Kansas public high schools. In checking a few questionnaires picked at random, some discrepancies were found. For example, one school with the course title of Senior Math, using Modern Introductory Analysis as a textbook, was listed as calculus in the Education Department records. The main difference in the two tables is in the relationship of the analysis and trigonometry courses. This would suggest that some analysis courses are listed either as trigonometry or as third courses in algebra.

II. TEXTBOOKS

The choice of a textbook greatly influences the approach and presentation of course content. A list of the most frequently used textbooks is presented in Table IV. By far the most popular textbook was Modern Introductory Analysis by Dolciani, Beckenbach, Donnelly, Jorgensen, and Wooten and published by Houghton-Mifflin. This book, used by 77 schools, emphasizes the treatment of functions. Polynomial,

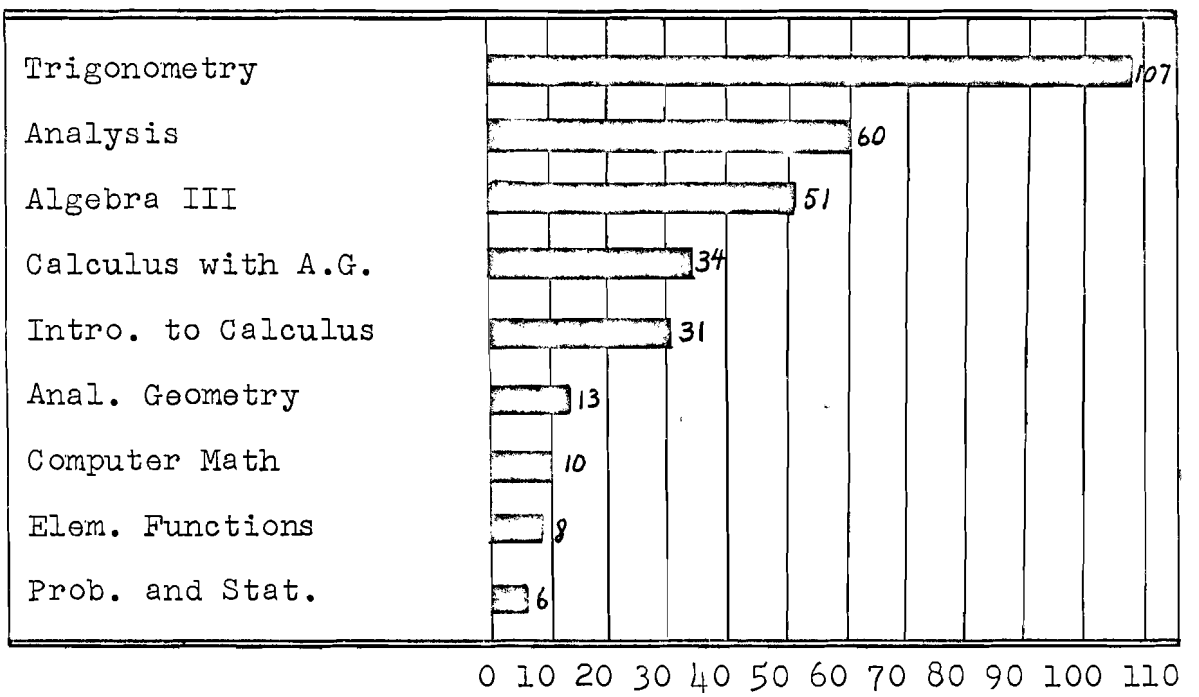

FIGURE 4

NUMBER OF KANSAS HIGH SCHOOLS OFFERING COURSES IN ADVANCED MATHEMATICS, 1970-71, AS LISTED WITH THE STATE DEPARTMENT OF EDUCATION

TABLE IV

TEXTBOOKS FOR ADVANCED MATHEMATICS AND THE NUMBER OF KANSAS HIGH SCHOOLS IN WHICH THEY ARE USED (BASED ON SURVEY OF 218 SCHOOLS, 1970-71)

TEXTBOOK	PUBLISHER	NO. OF SCHOOLS
Modern Introductory Analysis	Houghton-Mifflin	77
Advanced High School Mathematics	Merrill	26
Foundations of Advanced Math.	American	17
Modern Trigonometry	Ginn	21
Modern Trigonometry	Houghton-Mifflin	20
Modern Alg. and Trig., Book 2	Houghton-Mifflin	9
Anal. Geom. and Calculus	Prentice-Hall	8
Elementary Functions	Addison-Wesley	4
College Trigonometry	MacMillan	5
Analytic Geometry	MacMillan	4
Principles of Mathematics	McGraw & Hill	5
Pre-Calculus	Addison-Wesley	4

exponential, logarithmic, circular, and trigonometric functions are covered. There is also a chapter for each of the following topics--vectors, complex numbers, analytic geometry, solid geometry, induction, and probability.

For trigonometry, two texts were equally popular. Twenty-one schools used Ginn Publishing Company's Modern Trigonometry by Welchons, Pearson, and Krickenberger, while twenty schools listed Houghton-Mifflin's Modern Trigonometry by Wooten, Beckenbach, and Dolciani. Nine other schools said they were using the unfinished sections on trigonometry in Houghton-Mifflin's textbook for Algebra II.

Twenty-six schools indicated they were using Merrill's Advanced High School Mathematics by Vannatta, Carraham, and Fawcett. American Book Company's Foundations of Advanced Mathematics by Kline, Oesterle, and Wilson was listed by seventeen schools. All other textbooks listed were used by fewer than ten schools.

There was very little agreement on a calculus text. For the 33 courses of calculus and calculus with analytic geometry, ten different textbooks were used. The most popular was Analytic Geometry and Calculus by Schock and Warshaw. It was listed by eight schools.

III. COMPARISON OF RESULTS WITH RECOMMENDED PROGRAMS

On the questionnaires, the teachers were asked to

list the material covered in their courses. This information is given in Table V. Any topic that was listed 9 or more times has been included. Some of these topics are general such as trigonometry, calculus, or analytic geometry. Others are more specific and would be included in the broader terms. These have been tabulated exactly as listed on the questionnaire cards.

It is evident that the recommendations of the CEEB and the SMSG are reflected in Kansas mathematical programs. Recalling the CEEB's nine-point program, point eight calls for emphasis on elementary functions in grade 12. Nearly every teacher listed functions, either the broad term or specific types. Other points in the recommended program called for the treatment of inequalities, use of unifying ideas such as sets, variables, functions, and relations, and appreciation of mathematical structure illustrated by properties of sets of numbers. A check of the list of topics indicates that these ideas have begun to permeate the Kansas programs.

But on the other hand, some of the recommendations have met with only limited success. Point seven called for trigonometry in grade 11. In the survey, almost every teacher listed trigonometry as a topic covered in the twelfth grade and 92 separate courses were offered. The recommended probability and statistics course has been used

TABLE V

TOPICS LISTED BY TEACHERS AS BEING INCLUDED IN THEIR ADVANCED MATHEMATICS COURSES

TOPIC	NUMBER OF TIMES LISTED
Trigonometry	156
Analytic Geometry	106
Functions	96
Vectors	90
Calculus	75
Limits	70
Probability and Stat.	67
Sequences and series	53
Complex numbers	50
Matrices	41
Induction	37
Circular functions	36
Fields	34
Derivatives	31
Sets and set theory	31
Theory of equations	29
Advanced algebra	26
Log and exponential functions	23
Logic	23
Graphing	22
Conic sections	19
Logarithms	16
Trigonometric functions	13
Slide rule	12
Inequalities	11
Number theory	11
Polar coordinates	11
Determinants	9
Analysis	9
Permutations and combinations	9
Solid Geometry	9

only sparingly. According to Department of Education records, there were six courses offered with an enrollment of 66 students. But 67 teachers indicated they were teaching some probability and statistics. Some teachers noted that two or three weeks were devoted to probability.

The alternate recommendation, introduction to modern algebra, has met with some success. Although there were only 16 separate courses in advanced algebra, many algebraic topics are included in the list. This indicated that this subject also has been integrated into the twelfth grade course.

Calculus has become one of the more popular twelfth grade subjects. Almost every teacher stated that the subject was at least introduced. The Commission of Mathematics of the CEEB recommended that calculus in the high school be restricted to the advanced placement program.

The influence of the School Mathematics Study Group is most evident in choice of textbooks. Although only two schools indicated they were using SMSG texts, the most popular textbooks were Houghton-Mifflin's. Nearly all of Houghton-Mifflin's writers have been members of the SMSG, have participated in SMSG writing projects, or have served as consultants for teachers of SMSG materials. This line of textbooks reflects the SMSG viewpoint.

CHAPTER IV

TEACHER PREPARATION

The effectiveness of a high school mathematics program depends to a large extent upon the teacher of mathematics. The teacher must have an adequate background in abstract algebra, modern geometry, logic, number theory, linear algebra, principles of mathematics, probability, and modern analysis. He must have a broad and deep understanding of mathematics and must realize that the ability to discover and to prove are just as essential as competence in techniques.

The leaders of the movement to change high school mathematics curriculum realized that the change would be slow unless the preparation of the teachers was improved. A serious weakness lay in the teachers' mathematical backgrounds. A U. S. Office of Education study based on a sample of 799 teachers who taught one or more courses in mathematics in 1957-58 revealed that the average number of semester hours was 23. Only 61 per cent of the teachers had studied the calculus or a more advanced course.[6]

I. RECOMMENDATIONS OF M.A.A.

In 1960 the Mathematical Association of America (M.A.A.)

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[6]Kinsella, op. cit., p. 97.

presented its recommendations for the training of mathematics teachers at every level, from the primary grades through the college sophomore year.[7] The key idea was that the amount of mathematics preparation should increase with the level of mathematics to be taught. To teach at the third level, grades 9-12, the teacher should have 33 semester hours of mathematics. This should include 9 in analysis, 6 in geometry, 6 in abstract algebra, 6 in probability and statistics, and 6 electives. To teach at the fourth level, grades 12-14, or teaching such courses as elements of calculus, linear algebra, and probability, the teacher should have 54 semester hours. The additional hours should include 3 more hours of both analysis and geometry and 15 more hours of electives. These recommendations were approved by the National Association of State Directors of Teacher Education and Certification and have helped upgrade the preparation of high school mathematics teachers.[8]

II. PREPARATION OF KANSAS TEACHERS

The preparation of Kansas teachers compares favorably with the recommendations of the M.A.A. This

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[7]Mathematical Association of America, "Recommendations of the Mathematical Association of America for the Training of Teachers of Mathematics," The American Mathematical Monthly, LXVII, No. 10 (December, 1960), pp. 982-91.

[8]Kinsella, op. cit., p. 95.

conclusion was based on returned questionnaires received from 239 teachers of advanced mathematics in Kansas high schools. These teachers were teaching subjects on the third and fourth levels of the M.A.A. report.

Number of semester hours. The average number of hours of the Kansas teachers was 52. In grouping the teachers by school size, the results were as expected. The teachers in 1-A schools averaged almost 44 hours while those in 5-A schools had 80 hours. The teachers in 2-A and 3-A schools had about the same amount of preparation. The largest jump was from 4-A to 5-A schools, going from 57 to 80 hours.

The minimum mandatory requirement for teacher certification in mathematics in Kansas is 18 hours. Only 2 of the 239 teachers had the minimum requirement. Forty of the teachers, or about 17 per cent, were under the 33 hours recommended by the M.A.A. for grades 9-12. Of these 40 teachers, 32 were teaching in 1-A or 2-A high schools.

When questionnaires are sent out, there are always some that are never returned. The reasons for not completing and returning are never known. A study was made to see if there would be any difference in preparation between the group of teachers who returned the cards and those who did not.

There were 108 teachers of advanced mathematics in the schools which did not respond. Their names were taken from

the Principals Reports on file at the Kansas State Department of Education. The number of hours in mathematics comes from records in the Teacher Certification section. This will be the number of hours the teacher had when the certificate was granted. Therefore, it is possible that this number is not current. These teachers averaged 43.5 hours which is 8.5 hours less than the teachers in the other group. The largest differences were between the teachers in the 3-A group and the 5-A group. In the 3-A schools, the drop was nearly 23 hours while the 5-A schools dropped 25 hours. Table VI compares the hours of the two groups.

TABLE VI

AVERAGE NUMBER OF SEMESTER HOURS IN MATHEMATICS OF KANSAS TEACHERS WHO WERE TEACHING ADVANCED MATHEMATICS COURSES IN 1970-71

GROUP	1-A	2-A	3-A	4-A	5-A	ALL SCHOOLS
Teachers who returned questionnaires	43.7	51.8	51.9	56.8	80.3	52.0
Teachers who did not return questionnaires	41.0	36.6	29.0	55.0	55.4	43.5

Year of last mathematics course. The rapid change in mathematics has made it essential that teachers continually update their education. The teachers questioned were asked to give the year they had taken their last mathematics course. It was found that 12 teachers were currently en-

rolled in a course and 55 others had completed courses in 1970. Only 3 teachers, or slightly more than one per cent, had taken their last course prior to 1960.

The data in regards to year of last mathematics course are presented in Figures 5 and 6. Figure 5 shows the number of teachers who had their last course in the given year. In Figure 6, the average number of years since the last course is compared according to school size. The slight difference among schools indicates that school size had no bearing on teachers returning to the college classroom.

III. SUMMARY

The results from the survey indicate that Kansas teachers are very well-prepared. The average for the teachers questioned was 52 hours. This is only 2 hours under the 54 recommended by the M.A.A. for teachers of advanced mathematics. The average was lowered by teachers in the 1-A schools. In the smaller schools, the teacher may not have majored or minored in mathematics, but is teaching the course because no better qualified teacher is available. Omitting the 1-A teachers, the average would be 56 hours.

The survey also indicated that Kansas teachers are concerned with keeping their training updated. More than 87 per cent have taken courses in the five-year period

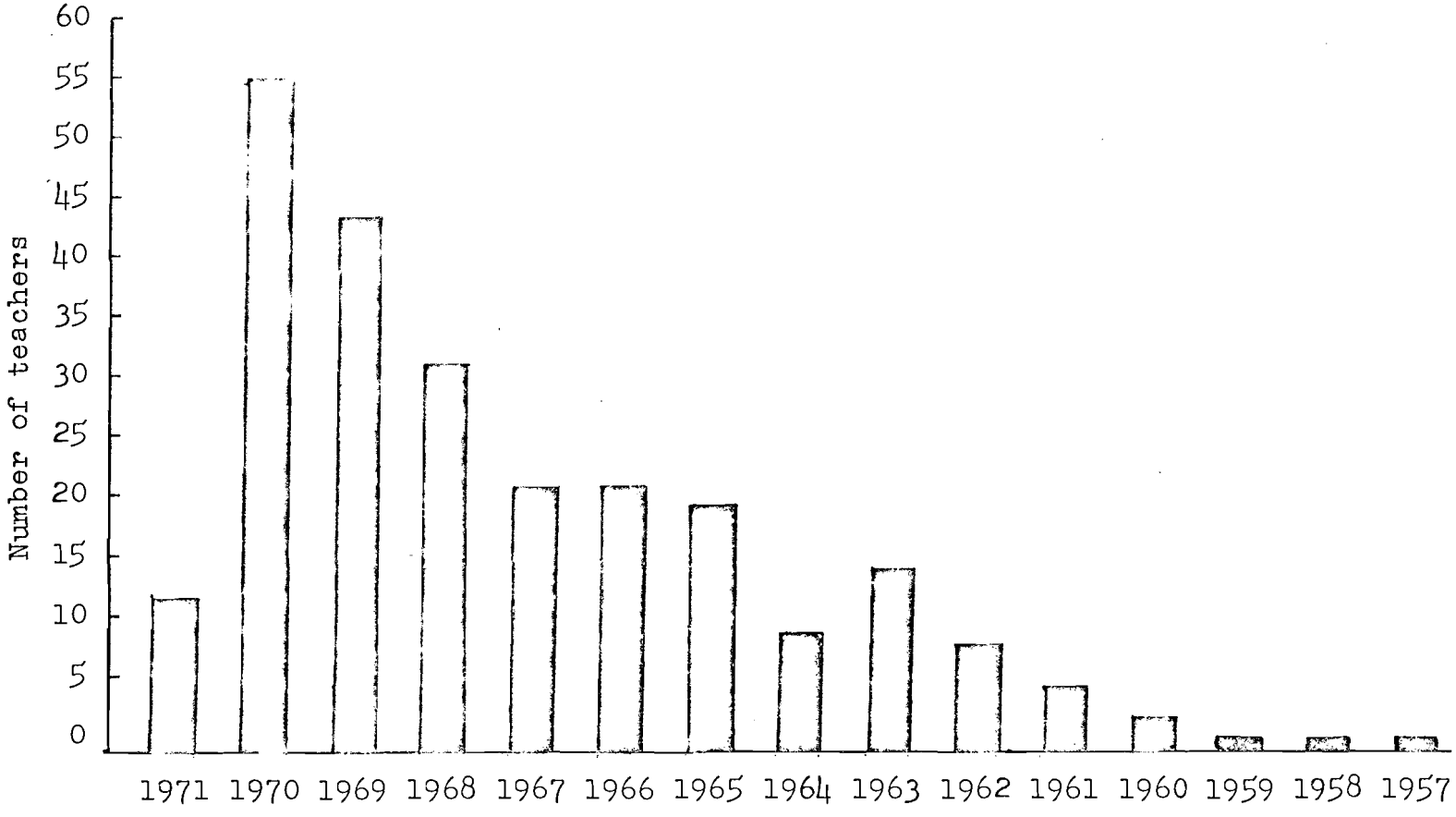

FIGURE 5

NUMBER OF KANSAS TEACHERS OF ADVANCED MATHEMATICS WHO TOOK THEIR LAST MATHEMATICS COURSE IN THE GIVEN YEAR, 1957-1971*

*Based on a survey of 239 teachers

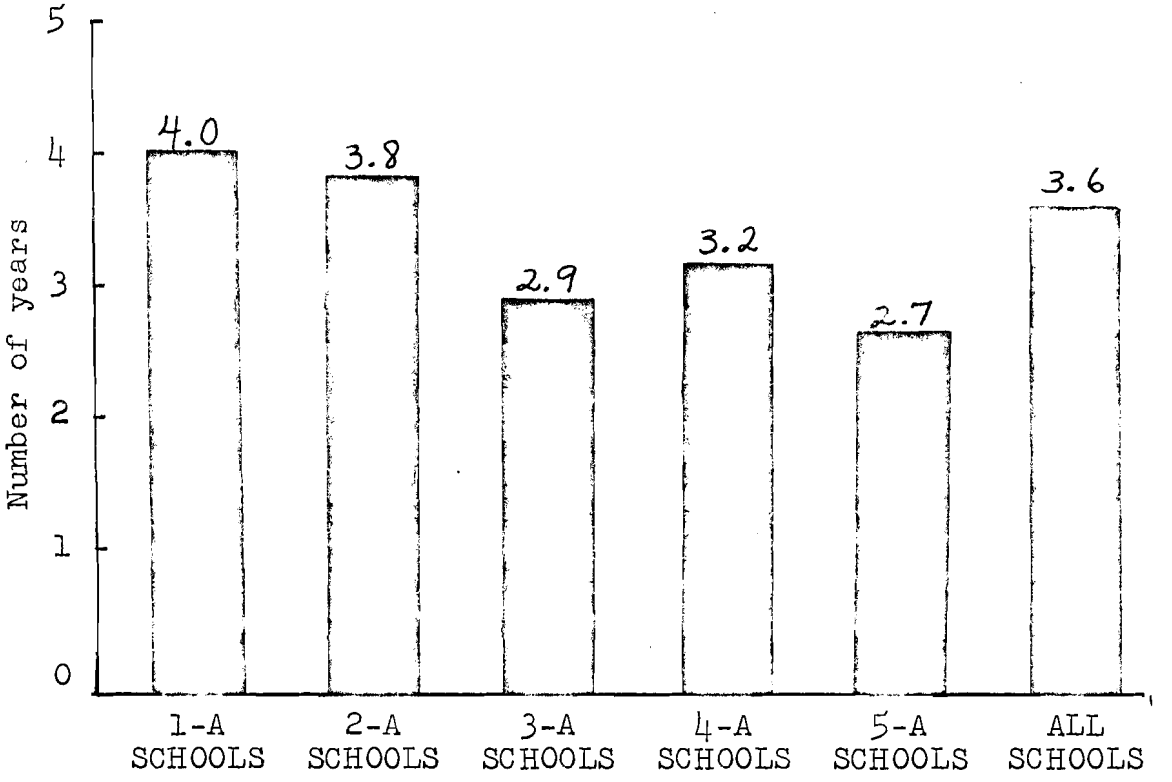

FIGURE 6

COMPARISON BY SCHOOL SIZE OF THE AVERAGE NUMBER OF YEARS SINCE LAST MATHEMATICS COURSE WAS TAKEN BY TEACHERS OF ADVANCED MATHEMATICS IN KANSAS HIGH SCHOOLS, 1970-71

beginning with 1965. The teachers are also interested in course content. The next chapter will deal with the opinions of teachers concerning the twelfth grade program.

CHAPTER V

WHICH TWELFTH GRADE PROGRAM IS BEST?

In no other secondary mathematics course does the opinion of the teacher influence the content as much as in a twelfth grade course. This was evident from answers given on the questionnaires. Such expressions as "sections from other texts which I feel are needed" or "selected topics from my college texts" appeared on the cards. This indicated that most teachers were free to pick and choose content and textbooks. The content, length of time for each topic, and textbooks were selected upon the basis of what each teacher felt was most important.

I. OPINIONS GIVEN ON SURVEY

The teachers were asked this question, "In your opinion, which course best prepares students for college mathematics?" Several choices were listed and a space was provided for any additional choice to be written in. More than one-half of the teachers did not limit themselves to one choice. They believed that a combination of subjects should be taught. In compiling the data, the opinions of the 113 teachers who did list only one choice were given separate treatment. This information is presented in Table VII. The chosen subject, the number of teachers selecting

it, and the average number of semester hours in mathematics are all given. It was noted that the ten teachers selecting calculus had from 9-17 fewer hours than the teachers in the other groups.

TABLE VII

OPINIONS OF 113 KANSAS TEACHERS REGARDING WHICH COURSE BEST PREPARES STUDENTS FOR COLLEGE MATHEMATICS AND AVERAGE NUMBER OF SEMESTER HOURS IN MATHEMATICS FOR EACH GROUP

SUBJECT CHOSEN	NO. OF TEACHERS	AVE. NO. OF SEM. HOURS
Analysis	46	52 hours
Trigonometry	32	48 hours
Functions	19	53 hours
Calculus	10	39 hours
Analytic Geometry	6	56 hours

A compilation was also made for all subjects based on all 239 questionnaires. No distinction was made between one choice or several choices. Each choice was counted once. Table VIII gives the total number of times each subject was checked. With the exception of algebra, the two subjects with the fewest votes were calculus and probability and statistics. The algebra votes do not present a true picture since algebra had not been included on the card, and these 22 votes were write-ins.

TABLE VIII

SUBJECTS CONSIDERED ESSENTIAL IN PREPARATION FOR COLLEGE MATHEMATICS (BASED ON A SURVEY OF 239 KANSAS TEACHERS)

SUBJECT	NUMBER OF TIMES CHECKED	SUBJECT	NUMBER OF TIMES CHECKED
Trigonometry	131	Limits	54
Analysis	99	Calculus	49
Functions	88	Prob. & Stat.	26
Analytic Geometry	75	Algebra	22

The consensus of the teachers was that no one course was best, but that a twelfth grade course should include introductions to several topics. Trigonometry, analysis, and functions were considered essential while a course in probability and statistics could be omitted as far as college preparation was concerned. Only about one-fifth of the teachers believed that some calculus was necessary. This means that the majority of the teachers hold the viewpoint that pre-college mathematics is pre-calculus mathematics.

II. CURRENT LITERATURE

Calculus in the high school. The question that often arises in discussing the twelfth grade program is "Should calculus be taught in the high school?" This question must be answered by the individual school on the basis of whether

or not the school has an adequate number of students who can profitably study a minimum of one year of calculus, and whether there is a qualified teacher for the course. Jerry McIntosh and Philip Peak of Indiana University give some guidelines for the school considering teaching calculus. If the school has suitable students and teacher for the course, if the school's enrollment is about 1,000, and if it offers a comprehensive instructional program, then it can provide a quality course in calculus. From 4 to 10 per cent of the seniors in an academic high school should probably be enrolled in calculus.[9]

Albert A. Blank of the Courant Institute of Mathematical Sciences, New York, claims that we no longer need to argue the case for calculus. The case has been made and people are convinced. He then gives the following reasons for calculus to be in the high school:[10]

> Calculus is a natural cap to the high school curriculum. It reinforces by utilization all the concepts and techniques learned earlier. In its manipulative and problem-solving aspects the calculus is entirely in the spirit of secondary mathematics. No alternative to the calculus is superior for opening so many avenues to higher mathematics, to physical sciences and technology, and even to the biological, management, and social sciences.

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[9]Jerry McIntosh and Philip Peak, "Material of Instruction," The Continuing Revolution in Mathematics (Washington: The National Council of Teachers of Mathematics, 1968), p. 157.

[10]Albert A. Blank, "The Case for Calculus," The Twelfth-Grade Pre-College Mathematics Program (Washington: The National Council of Teachers of Mathematics, 1965), p. 13.

Dr. Blank, who helped write the SMSG calculus text, believes that the high school textbook should be rigorous, yet not ignore applications. He claims that the SMSG text could serve as a honors course in many colleges.

Arthur P. Mattuck of the Massachusetts Institute of Technology disagrees with this viewpoint. He believes that a first course in high school calculus is no place for mathematical rigor. He states that a course does not have to be rigorous to be good mathematics. He further states that a good pre-calculus course should come first. He suggests a course in analytic geometry and elementary functions. This course would give the student a broad experience with functions and variables to enrich and make the calculus course more meaningful. Calculus should be taught only if there is still time after the student has an adequate background in pre-calculus mathematics.[11]

Probability and statistics. A high school course in probability has many supporters. Even severe critics of changes in school mathematics will concede the importance of this subject for our time. The Commission on Mathematics of the CEEB believed that for college-capable students with interests in the biological, behavioral, and social sciences,

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[11]Arthur P. Mattuck, "Some Remarks About the Calculus Course for Grade Twelve," The Twelfth Grade Pre-College Mathematics Program (Washington: The National Council of Teachers of Mathematics, 1965), pp. 16-17.

probability and statistics had more interest value and applications than courses slanted in the direction of calculus and analysis.

Franklin A. Graybill of Colorado State University discussed probability and statistics for grade twelve at the joint meeting of the Mathematical Association of America and the National Council of Teachers of Mathematics in 1965. He recommended that if there were a choice between teaching probability and calculus, probability should be taught. He also recommended that probability, based on set theoretical ideas only, not be taught as a separate course, but included in an enriched course in mathematics. Descriptive statistics could also be included in some other course. He did recommend that in some of the larger schools, probability and statistics be made available to twelfth grade students as an elective.[12]

A course covering several topics. Several schools have experimented with a course which consists of a series of units having little if any logical or organic interrelation. One model for such a course is the Mathematics 12X Program, developed by the State Department of Education

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[12]Franklin A. Graybill, "Probability and Statistics for Grade 12," The Twelfth Grade Pre-College Mathematics Program (Washington: The National Council of Teachers of Mathematics, 1965), pp. 28-29.

of the State of New York. The intent of such a course is to open doors and to indicate vistas in higher mathematics.[13]

McIntosh and Peak would disagree with the teaching of many topics. They claim that the most important consideration in selecting mathematics content for the last year of school mathematics is that it provides for and encourages study of a topic in depth. The student should become increasingly concerned with axioms, undefined terms, definitions, and theorems and with the interdependence of structural elements as a man-made check and balance system.[14] They recommended the following as topics that lend themselves to study in depth; probability and statistics, algebra structures, linear algebra, and geometry. The geometry course should be an axiomatic appraisal of several kinds of geometry.

The teacher of advanced mathematics must face and answer the question of what to teach in the senior year. Much more content has been proposed than can possibly be taught. The answer can be found by carefully considering all factors involved. What is the ability of the students? What are their plans for the future? What type of mathe-

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[13]Julius H. Hlavaty and Harry D. Ruderman, "How Provide for the Mathematically Talented?", The Continuing Revolution in Mathematics (Washington: The National Council of Teachers of Mathematics, 1968), p. 108.

[14]McIntosh and Peak, op. cit., p. 158.

matical background do they have? How much time is provided? In what areas is the teacher best qualified? What is the philosophy of the mathematics department? The answers to these questions and others will aid the teacher in planning and teaching a twelfth grade course.

CHAPTER VI

SUMMARY AND CONCLUSIONS

It has always been assumed in American education that public schools exist to serve the needs of all the children of all the people. It is doubtful if the new high school mathematics programs are in agreement with this aim. These programs are definitely concerned with the college-capable student. It has been estimated that over the nation as a whole, the college-capable constitute about one-fourth of the students.[15] And even among this group, there are different levels of ability. They will not all learn mathematics at the same rate.

Schools have partially solved this problem by offering two or more "tracks." The needs of the most talented segment of the twelfth grade are met by the Advanced Placement Program. These students are recognized early and are placed in an accelerated program. When they enter college, they will have completed a year or more of calculus. Another program is provided for the college-bound student who plans to begin his college work with calculus. And still another "track" is followed by the under-achiever in mathematics.

But this solution does not help the smaller high schools. It is impossible to offer two "tracks" when the

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[15]Kinsella, op. cit., p. 101.

average enrollment of the twelfth grade course is four students. One writer claimed that a high school needed 1,000 students to offer a quality course in calculus.[16] In Kansas, there are 359 public high schools with enrollments under 1,000. While 1,000 students may be too high, there must be enough students to provide two programs so the talented student can complete his pre-calculus work before the twelfth grade.

Using this reasoning, it would seem best for small high schools to offer a pre-calculus course. In Kansas 1-A schools, the average enrollment in advanced mathematics was 4.5 students. In 2-A schools, it was 8.7 students. And even in a small class, there will be different levels of ability. The teacher can best decide what the course content should be. He knows the ability and background of the students.

The twelfth grade course must include adequate work with functions and variables to give the student the needed experience and store of examples for his calculus course. A common pitfall is including too many topics in the twelfth grade course. It is possible that in trying to cover so much material in a year's time the student does not have a thorough understanding or "good feeling" for any of it.

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[16] McIntosh and Peak, loc. cit.

I. SUMMARY OF SURVEY RESULTS

The data from the survey indicated that some of the recommendations of the national groups in mathematics are reflected in the Kansas program. Some of the recommended topics, such as functions, inequalities, vectors, and complex numbers have been integrated into the courses. And as recommended by the Commission on Mathematics of the CEEB, the objective of most Kansas programs is to prepare the student for college mathematics at the level of calculus.

The most popular courses for the twelfth grade in Kansas were analysis and trigonometry. Trigonometry still remains as a separate course. This is true, not only in Kansas but in other states.[17] The major problem seems to be lack of time in the junior year to cover extensively both algebra and trigonometry. Perhaps in the future, trigonometry will be presented in the elementary grades and spiraled teaching will eliminate the necessity of a separate course.

Kansas has around 5,500 students enrolled in advanced mathematics or 16 per cent of all seniors. This number compares favorably with the estimate given by James Bryant Conant. He wrote in a national article in 1959:

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[17] John W. Alspaugh, R.D. Kerr and Robert E. Reys, "Curriculum Change in Secondary School Mathematics," School Science and Mathematics, (February, 1970), pp. 171-173.

> I am convinced that on a national basis something like 15 per cent of the youth of high school age have the ability to study effectively and rewardingly advanced mathematics, science, and a foreign language.[18]

Kansas teachers of advanced mathematics are well-prepared. They have an average of 52 semester hours in mathematics and more than 96 per cent have taken mathematics courses within the past ten years. The size of school does affect the number of hours, with teachers in the 5-A schools averaging 36 hours more than the teachers in the 1-A schools. However the size of school did not seem to have any bearing on taking additional work in mathematics.

The majority of the teachers favored a twelfth grade course that covers a variety of topics. The emphasis should be on functions, but a solid background in trigonometry was deemed essential.

II. SUGGESTIONS FOR FURTHER STUDY

This study of twelfth grade mathematics was considered from the teacher's viewpoint. It was concerned not only with kinds of courses and number of students, but with the preparation and opinions of the teachers. An effective course is judged by the knowledge and skill of the students who complete it. If the question, "Which course most adequately prepares students for college mathematics?" is to be answered, then it should be considered from the student's viewpoint.

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[18] James Bryant Conant, "A Hard Look at Our High Schools," Look, 23 (February 3, 1959), pp. 31-32.

A suggested study would be a survey of college freshmen in mathematics courses. This would be made during the second semester and would include only those freshmen who had graduated from Kansas high schools. They would be asked to list the courses taken in high school and to give the name of the high school so comparison could be made by school size. From a list of topics common to twelfth grade mathematics, the student would check the ones covered and indicate to what extent. He would also be asked the grade received in his high school course.

In relation to his college work, the student would be asked the grade received and to list any problems he had encountered. In regard to his high school courses, the student would be asked to evaluate (1) the amount of applications of mathematical ideas, (2) the amount of time spent in acquiring skills in computation, and (3) the rigor of the course. He also would be asked for any suggestions he might have for changes in the high school program.

A survey of this nature would give some indication as to whether the present program is fulfilling its purpose.

The twelfth grade mathematics program will continue to change. There will be more experimental texts and teaching. The advanced thinking of the Cambridge Conference Report may well be the guideline for the future. At any rate, downward movement of course content can be expected. And if

the present trend continues, the mathematics courses will be less compartmentalized, more integrated and spiraled. The college freshman of the future may begin his college work with courses his father took at the graduate level.

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