TEACHING SLIDE RULE BY THE COUNTING METHOD

A Thesis

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Master of Arts

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by

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CHAPTER I

INTRODUCTION

The purpose of this work is to present a method of teaching slide rule operation which is superior to most methods now in use. Throughout this paper the method shall be referred to as the counting method. The most prominent features of the counting method are:

a. The problem of locating the decimal is completely integrated with the problem of determining the digits. It is usually treated as a completely separate operation.

b. Multiplication and division are treated as one step operations rather than three step procedures.

c. The notion of combined operations is treated first with binary multiplication and division as special cases. This more general approach is much more efficient and in the long run simpler than the conventional approach.

d. Decimal placement theory is simplified and greatly extended.

e. The method does not presuppose a knowledge of logarithms and has been taught successfully to students in high school, junior high and even in upper elementary school.

The section "Slide Rule in Texas" in Chapter I gives a brief history of the development of the counting method and describes slide rule competition in Texas. This competition served as the motivation for the development of the counting method and when its development was complete, became an ideal laboratory for the testing of its effectiveness. The cumulative results of these tests indicate that the counting method is superior to all methods known at this time. They also show that the counting method as well as many others are far superior to the estimation method which is the most commonly used method for teaching slide rule today.

Chapter IV presents the estimation method through a series of quotations from its advocates and the results of another experiment which again proves the inferiority of the estimation method.

Chapters II and III which present the counting method are written to the student. These instructions have been written in language simple enough to be understood by high school or junior high students, yet; they are extremely effective and completely general. The simplicity of presentation along with the large number of practice problems supplied greatly increase the student's chances to become a successful slide rule operator.

For the benefit of those who think the rules stated here should be proved to the student by logarithms before they are used, it should be emphasized that the rules stated are true statements which are the postulates for the counting method. By actual count, there are more assumptions which must be made to develop the theory of logarithms in a reasonable length of time than there are rules to accept when employing the counting method. The question then is not whether certain statements should be accepted without proof but which statements should be accepted without proof.

The next objection to the idea of teaching slide rule to students without a previous knowledge of logarithms is embodied in the statement, "They really do not understand what they are doing." This statement means that they place their trusts in a different set of postulates, and consequently, cannot give the objector an explanation which will satisfy him. Some of these objectors would have trouble explaining how junior high students who do not understand what they are doing can work problems twice as fast and more accurately than their critics; however, these students do understand what they are doing well enough to explain it to other students in such a way that these students can also work slide rule effectively.

Despite the differences of opinions concerning techniques, the long term goals for the student are the same.

Students who have learned slide rule by the counting method before being introduced to logarithms have in every case been exceptionally successful in the study of logarithms when they were presented in other mathematics courses. The fact that they could operate a slide rule efficiently gave them a greater interest and appreciation for logarithms than their classmates for they had applied logarithms hundreds of times and wondered what really made their slide rule work.

Another factor has proved to be surprisingly important. When a student becomes an efficient slide rule operator, because of this unusual skill, classmates and teachers begin to expect him to excel in other areas of mathematics as well. The student soon begins to believe this himself, and consequently, works extra so that he will not disappoint his teachers, his friends, or himself. The fruit of this extra work is additional knowledge which brings with it better grades. These better grades further convince him that he is an excellent mathematics student and by this time, he actually is. Thus, success breeds success, and in the long run, the student who learns slide rule by the counting method not only becomes competent in slide rule use and the understanding of logarithms, but in many other areas of mathematics as well.

The chief feature of this paper is an amplification of the theory by the presentation of an abundant number of carefully planned problems which will fix each idea in the student's mind to such an extent that he will be able to operate the slide rule with as much ease and confidence as he now does addition or multiplication. These problems have been designed to allow the student to progress slowly at first but to gradually grow into an extremely proficient slide rule operator. These problems will ground him in the fundamentals, then confront him with special cases. By the time he reaches the end of each section he will be prepared to easily and efficiently solve all problems of the type studied. Many new ideas are developed in the problems and the student who does not work the problems will find little meaning in what he has read, and consequently, will soon see no sense in the methods employed. The longer problem sets are divided into five parts, A, B, C, D, and E. The student with less time may work only the "A" problems.

SLIDE RULE IN TEXAS

In the fall of 1965, Jimmy Tang, a high school junior, completed a thirty minute college slide rule test with 100 per cent accuracy in two minutes and seventeen seconds. At the time, Jimmy was a student at The Kinkaid School, a private school in Houston, Texas. The test on which Jimmy demonstrated his amazing speed and accuracy had been constructed by Burt Frazer. Head of the Department of Engineering Graphics at the University of Houston. The test had been used for a number of years as a mid-semester examination in freshman Engineering Graphics taught by Frazer. It covered the operations that can be performed using the C, D, CF, DF, CIF, CI, DI, A, B, and K scales of a slide rule. In a discussion of Tang's feat, Frazer stated that his college students had always complained that the test was too long to be completed in the thirty minute time limit.

An article in The Houston Post quoted Tang as saying that he attributed his success to many hours of practice and the counting method of placing the decimal which was developed by his coach, Don Boles.

Exceptional skill with a slide rule is not at all unusual among high school students in Texas. The University Interscholastic League, referred to in Texas as the

UIL, has sponsored slide rule contests for the past twenty-three years. Programs designed to prepare students for competition in this contest have allowed hundreds of high school youngsters to develop skills in operating a slide rule which never cease to astound the college professors and professional engineers who are considered to be the slide rule experts. The following excerpts from the <u>Constitution and Contest Rules of the University Interscholastic League of 1969-1970</u> will familiarize one with the organization sponsoring this contest.

> What is now known as The University Interscholastic League was first organized in December, 1910, at the State Teachers' Meeting at Abilene. Each year since then it has been organized by a bureau of the Extension Division of The University of Texas at Austin. For the first year the League's activities were confined to debates among high schools affiliated with the University. The following year contests in declamation were added and membership in the League was thrown open to all the public schools of the state below college rank. Subsequently there were also added contests in various fields, suited to schools of different types, until the present schedule of contests was evolved.
>
> During the 1968-69 school year 2,954 schools registered for participation in League contests.
>
> The League covers a larger geographical area, serves more different types of public schools, schedules a greater variety of contests, holds larger meets and a larger number of meets and enjoys a greater school membership than any similar organization in the United States. Its purpose is to organize and direct, through the medium of properly supervised and controlled contests, desir-

able school activities, and thereby assist in preparing pupils for citizenship.[1]

About 1,000 of the UIL member schools participate in these slide rule contests each year. For the purpose of competition, these schools are divided into five classes or conferences according to school enrollment. Then, the approximately 200 schools in each class are divided geographically into four regions. Each of these regions will contain from thirty to eighty schools. The regions are then divided into districts so that each district will contain from six to ten schools. The ideal number of schools in a district is eight. The district is the basic unit for competition.

The district slide rule contest is held each April and is a part of the literary spring meet. At a district meet, the most academically talented students from each school compete in twelve literary events. These events include slide rule, number sense, science, typewriting, spelling, shorthand, ready writing, debate and four individual speech events. A trophy is awarded to the winning school, and medals are presented to the individual winners in each contest. These winners advance to the regional meet and the winners there qualify for the State contest.

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[1]Constitution and Contest Rules of the University Interscholastic League for 1969-1970 (Austin: The University of Texas, 1969), p. 6.

Any student who participates in the State meet becomes eligible for one of the scholarships offered by The League Foundation. This scholarship fund, established in 1962 in an effort to entice the academically talented students of Texas to attend college in their home state, now offers $200,000 in scholarships annually. The availability of these scholarships has greatly increased the interest in all UIL literary contests and is an important factor in the overall improvement of student performance on the slide rule test. To avoid any misunderstanding, it should be stated that the League Foundation is sponsored by private enterprise and is not a part of the UIL. The purpose of the league contests is not to screen scholarship winners.

Each UIL slide rule contest is conducted in accordance with the following rules.

> 1. Representation.--Each member high school in the University Interscholastic League is permitted three contestants in the district meet slide rule contest in the particular conference to which the school belongs.
>
> 2. Eligibility.--In addition to satisfying the eligibility requirements for literary contestants as set forth in Article VIII of the Constitution and Contest Rules, only pupils in the eighth, ninth, tenth, and eleventh grades in the eleven-grade school systems and only pupils in the ninth, tenth, eleventh, and twelfth grades in the twelve-grade school systems shall be permitted to enter this contest.

3. Contest Problems.--

a. All regular contest and tie-breaking contest problems and corresponding answering keys shall be provided by the State Office to the director general in a sealed envelope which shall not be opened until after the contestants are assembled and are ready to begin the contest.

b. The contest shall include all manner of problems involving only the following slide rule operations: multiplication, division, squares, cubes, square roots, cube roots, and placement of decimals.

c. The regular contest shall consist of 75 problems of such difficulty that only very seldom will a contestant complete them in the 30 minutes allowed. In each contest the problems will be simple at first and then become more complicated at a fairly uniform rate as one proceeds toward the end.

d. The tie-breaking contests shall consist of about 15 problems similar to those found in the regular contests. The time allotted for these tie-breaking contests is 10 minutes. These tie-breaking contest problems automatically accompany the regular problem envelopes, exception, see 5q.

4. Slide Rule Permitted.--Any type of standard slide rule without special accessories is permitted in the contest, whether it is straight or circular, wooden, plastic, or metallic. If the contestant desires, he may use more than one slide rule during the contest provided that each rule used is of standard make with no special accessories, such as additional indicators or special scales or markings. The use of any non-standard slide rule or non-standard equipment is prohibited.[2]

The rules for grading are somewhat complicated and are included as Appendix B.

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[2] Ibid., p. 93.

Each district contest serves as a field study which compares the teaching methods of the eight teachers assigned to prepare students for the district meet. Each teacher has a common goal. This goal is to teach his students to operate the slide rule as accurately and efficiently as possible. Once a teacher is given this assignment, he surveys the literature and chooses what he believes to be the best teaching method available. He is highly motivated to choose the most efficient method because he knows that his teaching will be evaluated at the end of the year by the performance of his students. This evaluation is unusual in that it is not made by the teacher himself but by an outside source. Furthermore, this is the only area of his teaching where his work can be compared to the work of his peers.

For twenty-three years 1000 coaches each year have worked on the problem of developing a better method for teaching slide rule. Many of those who have been more successful have shared with each other their successes and their failures. Each one tries to improve on the work that he has done in the past and at the same time to outdo all of the others. Each year a little more is learned and more progress is made toward finding the most efficient and effective way to teach slide rule use. Granted many of the

23,000 teacher-years of work have resulted in duplication of labor but many have not. This point is proved each year as the student performances improve, and the existing records of achievement are broken. Many excellent teaching methods have evolved from this contest. The performance of the students each year clearly proves that a tremendous advancement in slide rule teaching techniques has occurred as a result of the UIL Slide Rule Contest. Among the many slide rule experts who have developed through participation in UIL slide rule contests, those who have studied the counting method have been the most successful.

For the past ten years, these students have dominated slide rule competition in Texas. They have competed with and beaten a majority of times, each of the other better teams in the State.

Slide rule competition in Texas must be divided into two categories. Spring meet competition which includes the district, regional and State contests previously described and tournament competition.

In 1959, this investigator first coached students for a UIL district meet. These students were taught to place decimals by estimation as were each of the other students at this first district meet. They won first, second, and third places because they were able to use the

B and CI scales more efficiently than their opponents. The winning score was 83. However, at the regional meet these students were not even in the same class with the winners who scored about 200.

The next year, 1960, the students were again taught by the estimation method. Hundreds of hours were spent practicing and memorizing number facts such as squares, cubes, square roots, and cube roots of numbers. Many short cuts and tricks were learned both in manipulation of the rule and estimation of the decimal. The students worked very hard but seemed to reach a maximum of about 35 problems, and try as they would, they could not work any faster. They won first, second, and third in district and third, fourth and fifth at regional.

A discussion with Maurice Jones, coach of the second place winner in this contest, disclosed that his students used the characteristic or left extension method for locating the decimal. With this new idea in mind and a slide rule in hand, this investigator set out to discover for himself the rules of the characteristic method. This approach was necessary since none of the books available from the local library presented any method for placing the decimal in slide rule problems except the estimation method. The result, after many hours of effort, was the independent discovery of the right extension method.

Using this method in 1961 and 1962, students of this investigator won first, second, and third in district competition; first and second in regional; and, first and second at the State Meet each year. No student coached by this investigator was defeated at any level for this two year period, and scores reached the 270 level. During these two years much thought was given to the idea of simplifying this method; for even though the better students were very successful using it, other students found it difficult to master. The slide rule signaled a decimal change on about one-third of the operations which called for a lot of thinking. All these changes seemed unnecessary since the multiplication changes usually cancelled the division changes on the following step. A search for a signal to show the decimal changes which occurred less frequently than the slide extending to the right was made. With this goal in mind and after several intermediate steps, the recording one was discovered; that is, it was discovered that by following a particular set of rules all the decimal changes could be determined merely by noting each time the left index was used as a factor in an operation.

From 1963 through 1966, this investigator taught at The Kinkaid School, a private school in Houston. While there, the counting method was greatly improved. The most important improvement being that the decimal location was computed after each step rather than at the end of the problem. During this period, Jimmy Tang developed into the top slide rule operator in Texas. He set a State record by scoring 330 in a contest; raised this to 341; and then to 342. While at Kinkaid, the counting method was taught to the coach and students at Furr High School in Houston.

In 1966, this investigator moved to Andrews. During the past four years, the students there have won four district championships, four regional championships, and two State championships. The other two State championships were won by Furr High School whose students also used the counting method. Table I shows the UIL record of students taught by this investigator.

The other facet of slide rule competition in Texas is the invitational tournament. These invitational tournaments are team oriented, while the UIL meets recognize only individual achievement. Many of the better teams travel 400 to 600 miles to attend invitational tournaments. One of the big tournaments, the Pasadena tournament, last year drew seventy-eight schools and 1200 students. The Andrews tournament held in sparsely populated West Texas drew twenty-seven schools including six from San Antonio, 400 miles away. Because of the distance which schools travel,

TABLE I

RECORDS OF STUDENTS TAUGHT BY THIS INVESTIGATOR IN UIL COMPETITION

UIL DISTRICT CONTEST RECORDS

Year	1st	2nd	3rd	4th	5th	6th
1959	x	x	x			
1960	x	x	x			
1961	x	x	x			
1962	x	x	x			
1967	x	x	x			
1968	x	x	x			
1969	x	x	x			
1970	x	x	x			

UIL REGIONAL CONTEST RECORDS

Year	1st	2nd	3rd	4th	5th	6th
1960			x	x		
1961	x	x				
1962	x	x				
1967	x	x				
1968	x	x				
1969	x	x				
1970	x	x				

UIL STATE CONTEST RECORDS

Year	1st	2nd	3rd	4th	5th	6th
1961	x	x				
1962	x	x				
1967		x		x		
1968	x			x		
1969	x		x			
1970				x	x	

it is not uncommon for eight of the top ten teams in the State to attend the same tournament. There have been occasions when four State champions have competed in the same tournament. Thus, the invitational tournament truly tests the teaching program in each school.

In tournament competition over the past six years, Kinkaid and Andrews students have won fifty-six first places compared to ten for all the other schools in the State combined. In these tournaments, three first places are usually given. One is given to the student with the highest score. Another is given to the first year student with the highest score. The team score is the sum of four highest scores of individual contestants of which at least one must be a first year student. In team competition the Kinkaid and Andrews teams have won nineteen tournaments while the other teams have won three. First year students taught by the counting method have won first place in twenty tournaments compared to two for their opponents. In one of these two losses, a limit was placed on the number of problems which first year students could work. This nullified the advantage of speed which was the strong point of Andrews students applying the more efficient counting method. This loss is the only one Kinkaid-Andrews first year students have suffered in five years.

The preceding records show that the schools where students have been taught the counting method by this investigator have established the best competitive records among the 1,000 schools in Texas. More evidence will now be presented to demonstrate just how fast and accurate these students are.

In the summer of 1965, Martin Ambuhl, a junior from Kinkaid, attended a summer institute for gifted high school students. One of the seminars which he attended featured a lecture on slide rule operation by an expert representing the Pickett Slide Rule Company. Martin challenged him to demonstrate his techniques in a ten minute contest. The expert accepted the challenge. Not only was Martin more accurate than the expert, but he worked five times as many problems. In Texas, Martin had never won a slide rule contest.

This experiment, performed several times at Andrews High School, illustrates the speed which students can attain by the counting method. On each occasion, the experiment was conducted in the following manner.

The students in a class were told that they were to be a part of an experiment and were instructed to get out pencil and paper. When all the students were ready, each was issued a UIL slide rule test booklet. They were told that because of the cost of these booklets they were not to write in them but instead to copy on their own paper all the problems from the first page. Next, they were told to begin copying the problems. After thirty seconds, they were urged to hurry so that the experiment could be completed. They were again urged to hurry three or four more times during the next three minutes. Each student was told to raise his hand when he had finished copying the problems. After five minutes, all students were told to stop. At this point about one-fourth of them had finished copying the problems.

Three slide rule students were then brought into the room. They took the first page of this same test and finished in less than three minutes. This proved that problems like those on UIL slide rule contests could be worked with a slide rule quicker than they could be written. The students were then given a chance to write the problems at the same time that the slide rule students worked them. In every experiment the slide rule students won. Their answers were always checked and the average accuracy has been around 98 per cent. It has been 100 per cent in the majority of cases.

All of the preceding records and illustrations have been comparative. The following scores were made in competition by students who had been taught by the counting method. The scores are a matter of public record and will indicate exactly what the accomplishments of these students have been. Copies of these tests are available and may be obtained from the University Interscholastic League, Bureau of Public School Service, Box 8028, University Station, Austin, Texas 78712. The UIL tests are very similar to one another in difficulty. The sample test, Appendix A, and the instructions for graders, Appendix B, will allow any interested reader to determine the meaning of these scores. This method for evaluating the performances listed is suggested: First, work the problems on the test in the appendix for thirty minutes. This must be done in a quiet place without interruption and must be timed exactly. If when the first thirty minutes are spent, there are still problems unsolved, mark the last problem worked and continue working until all seventy-five are complete. Make a note of the total time elapsed. Next, study the instructions for graders in Appendix B and carefully score the paper by these instructions. Compare the speed and accuracy with these student performances, but when comparing your score to the scores in Table II, remember that these were high school students. These scores along with the tests upon which they were made furnish one with a precise measure of the effectiveness of the counting method.

TABLE II

SCORES MADE BY STUDENTS OF INVESTIGATOR IN COMPETITION

DISTRICT CHAMPIONS

Year	Student	School	Test	Score
1959	Newton Winski	Lancaster	109	83
1960	Weldon Mott	East Chambers	116	156
1961	Terry Canady	East Chambers	125	254
1962	Lois Broussard	East Chambers	132	295
1964	Jimmy Tang*	Kinkaid	147	244
1965	Jimmy Tang*	Kinkaid	S-1	330
1966	Jimmy Tang*	Kinkaid	S-4	337
1967	Ford Roberson	Andrews	168	350
1968	Ford Roberson	Andrews	175	354
1969	Patsy Shoffit	Andrews	183	349
1970	Rodney Cavett	Andrews	189	299

REGIONAL CHAMPIONS

Year	Student	School	Test	Score
1961	Terry Canady	East Chambers	127	244
1962	Lois Broussard	East Chambers	134	254
1964	Jimmy Tang*	Kinkaid	150	275
1965	Jimmy Tang*	Kinkaid	S-2	341
1966	Jimmy Tang*	Kinkaid	S-5	319
1967	Ford Roberson	Andrews	171	356
1968	Ford Roberson	Andrews	179	343
1968	Patsy Shoffit	Andrews	179	343
1969	Patsy Shoffit	Andrews	184	360
1970	Paula Manes	Andrews	190	318

*Jimmy Tang was the private school rather than the UIL champion.

TABLE II (continued)

STATE CHAMPIONS

Year	Student	School	Test	Score
1961	Terry Canady	East Chambers	128	261
1962	Linda Pickney	East Chambers	135	257
1964	Jimmy Tang*	Kinkaid	151	276
1965	Jimmy Tang*	Kinkaid	158	342
1966	Jimmy Tang*	Kinkaid	S-6	333
1968	Ford Roberson	Andrews	180	349
1969	Patsy Shoffit	Andrews	185	353

*Jimmy Tang was the private school rather than the UIL State champion.

A further breakdown of the times required to work parts of one of these tests may be helpful. The first page can be worked in two minutes and thirty seconds; the first two pages in six minutes; the first three in ten minutes; the first four in fourteen minutes and thirty seconds; the first five in nineteen minutes; the first six in twenty-three minutes; and, the entire test in twenty-eight minutes.

It has been shown that high school students using the counting method have defeated all opponents in competition; that they can work ten to fifteen times as fast as college engineering students; and that they can work difficult problems with a slide rule faster than these problems can be written. These facts along with the scores made by Jimmy Tang, Ford Roberson, Patsy Shoffit and many others prove that the counting method is a powerful and effective method for teaching slide rule use.

CHAPTER II

THE SYSTEM

In this chapter you shall learn to perform all manner of operations involving multiplication and division. When you have completed this chapter you should be able to work a problem like:

$$\frac{0.0615 \times 6.06 \times 52.9}{9.86 \times 7.98 \times 0.393 \times 0.155} \text{ .... } = \underline{\qquad\qquad}$$

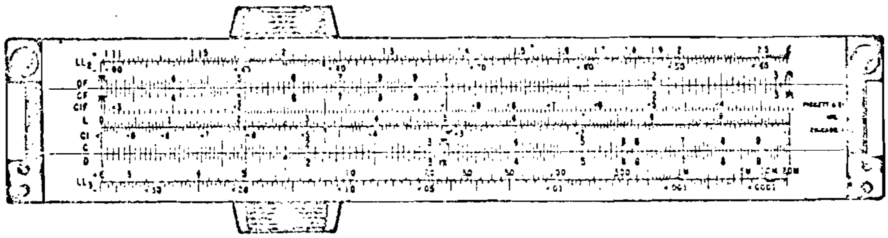
in a matter of seconds. A proficient slide rule operator can work this problem in less than 20 seconds.

The material which follows will be very easy but you must master each step or technique as soon as it is presented for each lesson presupposes a complete mastery of those which come before it. Study each lesson carefully and when you think you have mastered it, take the test which follows. If you do not have a slide rule handy get one now. It would be extremely foolish to attempt to read further without a slide rule in hand. Now we are ready for lesson one. Go slowly and carefully and refer to your own slide rule often.

LESSON I

THE PARTS OF A SLIDE RULE

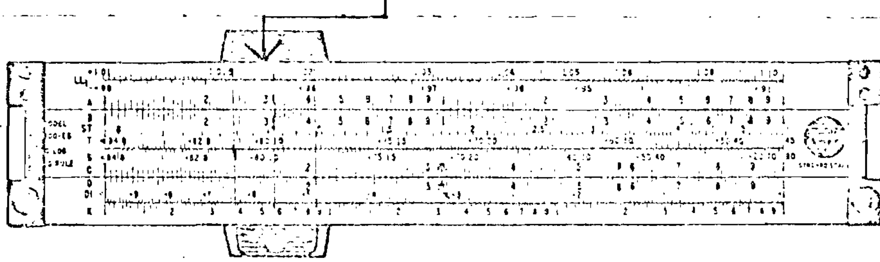
1. THIS IS A SLIDE RULE.



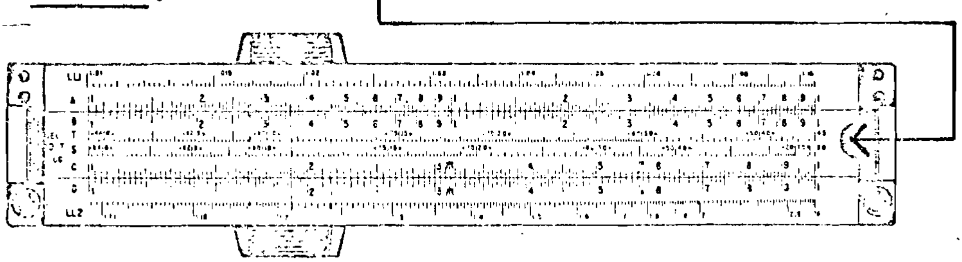
2. WE SHALL USE THE SLIDE RULE TO PERFORM TWO OPERATIONS - MULTIPLICATION AND DIVISION.

3. THE SLIDE RULE HAS TWO MOVABLE (OPERATING) PARTS. THEY ARE CALLED THE MULTIPLIER AND THE DIVIDER.

4. THIS IS THE MULTIPLIER. IT IS MOVED WHENEVER WE WANT TO MULTIPLY.



5. THIS IS THE DIVIDER. IT IS MOVED WHENEVER WE WANT TO DIVIDE.

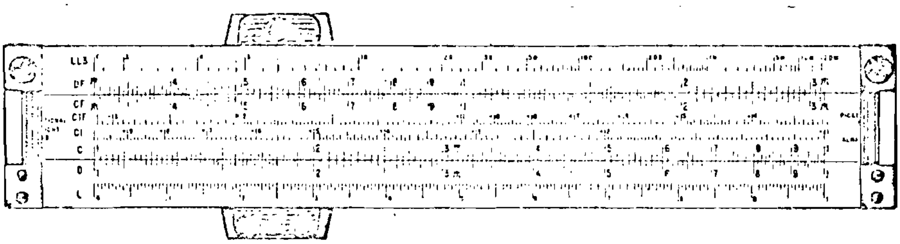


At this time you should pick up your slide rule. Move the multiplier and think multiply, then move the divider and think divide. The important point of this lesson is to train yourself to think multiply when the multiplier is moved and divide when the divider is moved. Read the numbered statements in Lesson I again very carefully. Now you are ready for Test I.

TEST I

<u>Directions</u>: Copy each sentence on your own paper and fill in the blanks.

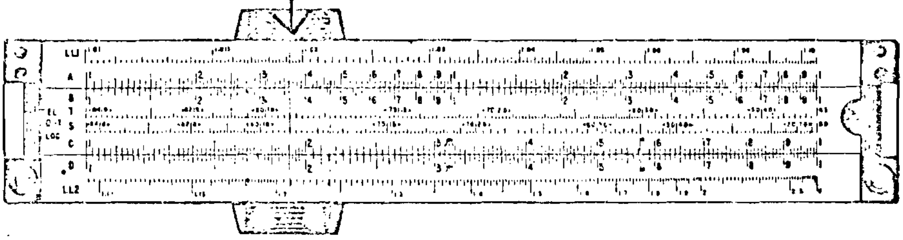
1. This is a __________ __________.



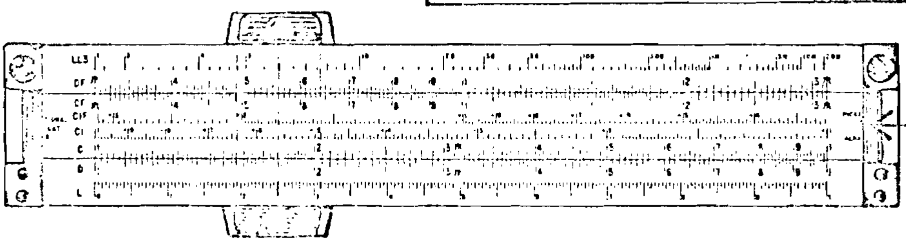
2. We shall use the slide rule to perform two operations - ________________ and ________________.

3. The slide rule has two movable (operating) parts. They are called the ________________ and the ________________.

4. This is the ______________. It is moved whenever we want to ___________.



5. This is the ______________. It is moved whenever we want to ___________.



You have finished Test I. That was easy wasn't it? If you missed any of the questions or had to look back it would be a good idea to read Lesson I again. If you got all the questions right on Test I, congratulations. Go to Lesson II.

LESSON II

MOTION RULES

In order to use the slide rule to multiply and divide, as one might expect, it is necessary to operate the movable parts of the rule, i.e. the multiplier and the divider. They must be manipulated in accordance with certain rules which we shall call "rules of motion." At this time you should learn these rules. They are:

M-1 MOVE THE MULTIPLIER FIRST.

M-2 MOVE THE MULTIPLIER AND DIVIDER ALTERNATELY.

M-3 MOVE THE MULTIPLIER LAST.

These rules are the essence of the counting method. In order to master this little set of rules the student needs to make a special effort at the beginning of each problem to remember rule M-1 (move the multiplier first) and at the end to remember rule M-3 (move the multiplier last). Experience has shown that rule M-3 is the rule which is most often overlooked or forgotten.

Rule M-2 (move the multiplier and divider alternately) is basic and is the one to concentrate on first. Pick up your slide rule so you can practice it.

Move the multiplier and pretend that you are multiplying by some number. Now immediately jerk your hand away from the multiplier and slap it onto the divider, move the divider as you think "divide." Very quickly move your hand back to the multiplier and multiply. Each time remember to think "multiply" as you move the multiplier and "divide" as you move the divider. Also, when you have completed a step, instantly move your hand to the other moving part. This will help you remember what to do next. If you have your hand on the multiplier, you should multiply next or if you are holding the divider, divide next. Practice this procedure several times.

In the practice you just completed you were multiplying and dividing with a slide rule. As you moved the parts of the slide rule you worked a problem like:

$$\frac{243 \times 618 \times 746}{394 \times 575} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = 494.5$$

but you probably did not know what problem you were working because you were not reading the scales as you were performing the operations. What you have learned to do so far is analogous to randomly punching the buttons on the keyboard of a calculating machine and correctly giving it instructions to multiply and divide. The machine would perform the operations and give the correct answer but you would not know what problem it had worked. Of course, this skill alone is useless, but in the next lesson you will learn to read the scales and soon be able to solve difficult multiplication and division problems at least twenty times as fast as someone who has only a pencil.

TEST II

Directions: Copy the following sentences on your paper and fill in the blanks with the correct words. Be sure to copy each sentence. These rules need to be memorized and copying the sentences will help you to learn them.

M-1 MOVE THE ______________ FIRST.

M-2 MOVE THE MULTIPLIER AND DIVIDER ____________.

M-3 MOVE THE ____________ LAST.

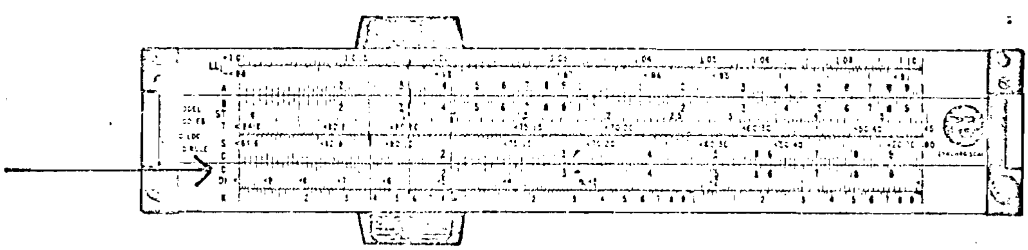
Look back and check your work. If you missed a question go back and reread the rules and try again. Most of the groundwork has now been laid. You will soon be ready to work problems.

LESSON III

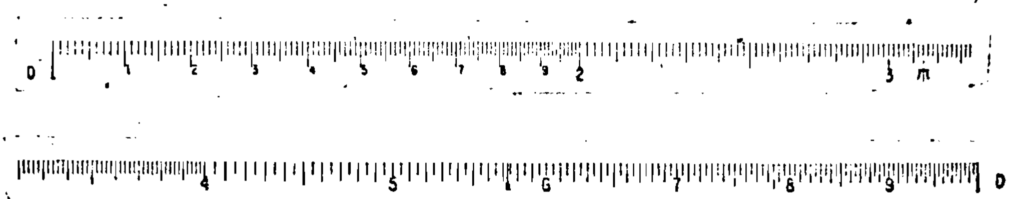
READING THE SCALES

In this lesson we hope to achieve two goals. You will learn to read certain numbers from the D scale and you will learn another little set of rules called the "scale rules." When these two tasks are accomplished you should be able by putting together the skills and techniques learned in Lessons I, II, and III to work many difficult problems involving multiplication and division with your slide rule.

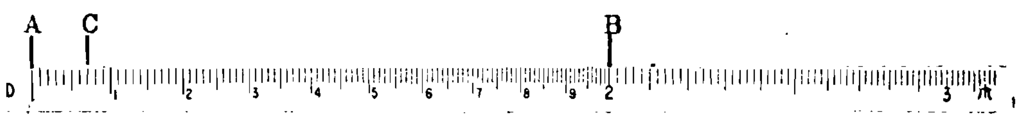
Locate the D scale on your slide rule. The figure below shows its usual location.



Now on your own slide rule focus your attention on the D scale. What you should see is shown below.

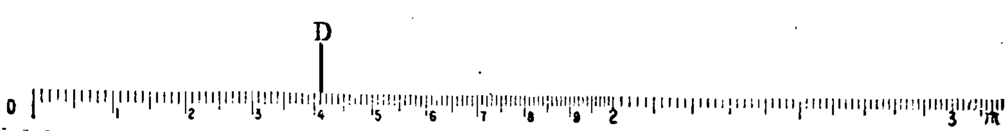


Remove the divider completely from the rule and place the hairline of the multiplier over the first mark located at the extreme left end of the D scale. It is identified by the large numeral 1 just below it. We shall call this position 100. (A in the figure below.) Now move the multiplier until the hairline is over the large numeral 2 which is about one-third of the way down the slide rule. We shall call this position 200. (B in the figure below.) Note that there are 100 spaces between the mark for 100 and the mark for 200. Thus, each mark on this section of the scale represents a number one more than the number that the mark before it represents. At this time familiarize yourself with this section of your rule. There are 101 different numbers which can be set on this interval of the D scale but the following three examples should be sufficient to point out the ease with which any one of them may be found.



Example one: Set 107 on your slide rule. (C in the figure above.) To find 107 you may start at 100 and count seven spaces to the right, however, you will find it quicker to go directly from 100 to 105 and then two more spaces to the right. This can be done easily since the mark for 105 is longer than any of the other marks between 100 and 110. Note the other marks of this length are graphs of the numbers 115, 125, 135,...195.

Example two: Set 141 on your slide rule. (D in the figure below.) To find 141, you could start at 100 and count 41 spaces to the right but this would be very tedious and subject to error. Note that between 100 and 200 in conjunction with every 10th mark there is a numeral smaller than the large 1 representing 100 or the large 2 representing 200. These numerals help us find the numbers 110, 120, 130,...190. The small numeral 1 is the graph of 110. The small 2 represents 120, etc. A great deal of care must be taken to avoid confusing these small numbers representing 110, 120, 130, etc. with the large numbers representing 100, 200, 300, etc. Remember, if a mark is between the graph of 100 and 200, its coordinate must be between 100 and 200. So, the easy way to find 141 is to go directly from 100 to 140 and then go one space to the right. (Remember that the mark for 140 is the long mark between 100 and 200 with the small numeral 4 next to it.)



Example three: Set 198 on your slide rule. (E in the figure above.) To set 198, find 200 and move two spaces to the left.

EXERCISE SET I

Directions: Copy the letters A through J on your paper. After each letter write the number associated with it. Refer to the diagram below.

ANSWERS EXERCISE SET I

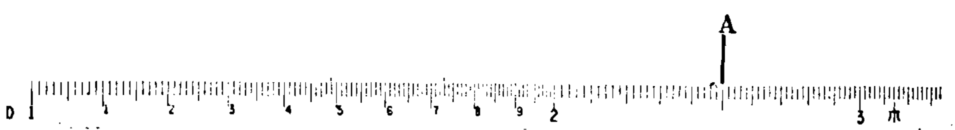
A. 117	F. 192
B. 138	G. 164
C. 176	H. 151
D. 103	I. 140
E. 185	J. 129

EXERCISE SET II

Directions: On your slide rule set each of the answers in Set I. Check by comparing your slide rule to the diagram.

Next we want to observe the section of the slide rule between 200 and 400. Move the multiplier of your slide rule until the hairline is over the large 3 which will be the first numeral to the right of 200. We will read this mark as 300. On down the scale you will see a large numeral 4. It denotes 400. There are 100 spaces between 200 and 400. Each of them has a value of two, thus the first mark to the right of 200 is 202. Notice that the marks for 210, 220,...380, 390 are longer than the marks around them. What number does the longest mark between 200 and 300 represent?

Example four: Set 250 on your slide rule. (See A in the figure below.) The coordinate of the longest mark between 200 and 300 is 250.

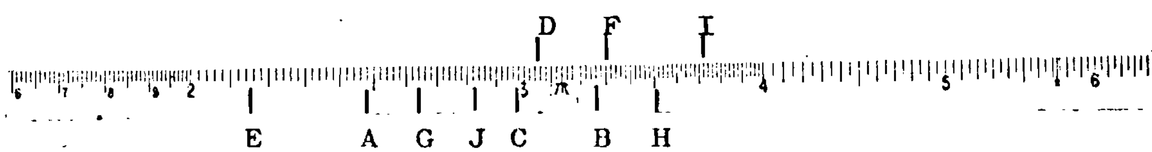


Example five: Set 304 on your slide rule. First find 300, then move two spaces to the right. As you do this think 300, 302, 304. One of the more common mistakes in reading or setting numbers is to forget that the graduations on this section of the rule represent two units rather than one.

Example six: Set 337 on your slide rule. Find 300, then 340 (the fourth ten mark), then 338 (one space to the left of 340), then 336 (one space to the left of 338). To set 337, position the hairline halfway between the 336 and the 338 marks.

EXERCISE SET III

Directions: Copy the letters A through J on your paper. After each letter write the number associated with it. Refer to the diagram below.



ANSWERS EXERCISE SET III

A. 248	F. 330
B. 326	G. 264
C. 298	H. 351
D. 305	I. 372
E. 215	J. 283

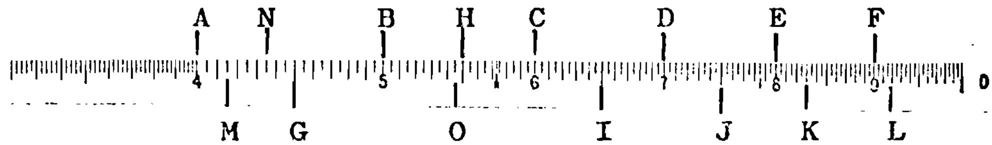
EXERCISE SET IV

Directions: On your slide rule set each of the answers in Set III. Check each setting by comparing your slide rule to the diagram.

On the remainder of the slide rule; that is, the part between 400 and 1000, each graduation represents a number which is five more than the one before it. Place the multiplier of your slide rule so that the hairline is over the mark for 400. Move it along the scale and read 405, 410, 415, 420, 425, 430, 435, 440, 445, 450, etc. Notice that the marks representing 410, 420,...980, 990 are longer than the marks representing 405, 415,...985, 995. Also, notice the marks for 450, 550, 650, 750, 850 and 950 are even longer than the 10 marks.

EXERCISE SET V

Directions: Copy the letters A through O on your paper. After each letter write the number associated with it. Refer to the diagram below.



ANSWERS EXERCISE SET V

A. 400	F. 900	K. 830
B. 500	G. 450	L. 920
C. 600	H. 550	M. 415
D. 700	I. 650	N. 435
E. 800	J. 750	O. 545

EXERCISE SET VI

Directions: On your slide rule set each of the answers in Exercise Set V. Check each setting by comparing your slide rule to the diagram.

The C scale of a slide rule is located on the divider. It is marked exactly like the D scale so you can now read both the C and D scales. All kinds of problems involving multiplication and division of numbers may be solved using only the C and D scales, but one must know when to set the number on the C scale or when to set it on the D scale. These questions are answered by the next set of rules. They will be referred to as the "scale rules."

S-1 SET THE FIRST FACTOR IN A PROBLEM ON THE D SCALE.

S-2 SET EACH FACTOR EXCEPT THE FIRST ON THE C SCALE.

S-3 READ THE ANSWER FROM THE D SCALE.

TEST III

Directions: Copy the following sentences on your paper and fill in the blanks with the correct words. Be sure to copy each sentence. These rules need to be memorized and copying the sentences will help you to learn them.

S-1 SET THE FIRST FACTOR IN A PROBLEM ON THE ___ SCALE.

S-2 SET EACH FACTOR EXCEPT THE FIRST ON THE ___ SCALE.

S-3 READ THE ANSWER FROM THE ___ SCALE.

At this time review each of the numbered rules. The problems in Lesson IV may be solved by applying these rules. In Exercise Set VII, there are 100 problems. You should work at least 20 of these before proceeding to the next lesson. Each practice problem that you work will make you a more efficient slide rule operator. For extra practice you may work some of the problems more than once. Always check your answer and rework each problem until you get exactly the same answer as the key or an answer which differs from it by no more than one in the last digit.

LESSON IV

MULTIPLICATION AND DIVISION

This lesson will require more time to complete than the first three combined. It consists of 100 problems to be solved. Completing this lesson forces a student to apply Rules M-1, M-3, S-1 and S-3 one hundred times each and Rules M-2 and S-2 two hundred times each. In addition, the C or D scale must be read four hundred times. Thus, to begin this assignment a student must have a good understanding of the first three lessons. When he correctly completes it he can be confident that he has completely mastered all the material up to this point. Since the motion rules and the scale rules must be used repeatedly in this exercise, they will be reviewed here.

M-1 MOVE THE MULTIPLIER FIRST.

M-2 MOVE THE MULTIPLIER AND DIVIDER ALTERNATELY.

M-3 MOVE THE MULTIPLIER LAST.

S-1 SET THE FIRST FACTOR IN A PROBLEM ON THE D SCALE.

S-2 SET EACH FACTOR EXCEPT THE FIRST ON THE C SCALE.

S-3 READ THE ANSWER FROM THE D SCALE.

EXERCISE SET VII

<u>Directions</u>: In the problems which follow, the C or D associated with each number indicates the scale upon

which it is to be set or read. The answers are given immediately after each of the first ten problems for the student's convenience. After getting the first ten problems right, the student should have enough confidence to work the next five or ten problems without checking. The answers to all the problems may be found at the end of this exercise set. First try these five problems.

1. $\dfrac{\overset{D}{300} \times \overset{C}{400}}{\underset{C}{200}}$ .... = $\overset{D}{600}$

2. $\dfrac{\overset{D}{300} \times \overset{C}{700}}{\underset{C}{500}}$ .... = $\overset{D}{420}$

3. $\dfrac{\overset{D}{600} \times \overset{C}{200}}{\underset{C}{400}}$ .... = $\overset{D}{300}$

4. $\dfrac{\overset{D}{650} \times \overset{C}{180}}{\underset{C}{540}}$ .... = $\overset{D}{217}$

5. $\dfrac{\overset{D}{210} \times \overset{C}{130}}{\underset{C}{175}}$ .... = $\overset{D}{156}$

If you were unable to work any one of the first five problems you should read the scale rules and motion rules again and carefully study these solutions making each setting on your own slide rule.

EXPLANATIONS FOR THE FIRST FIVE PROBLEMS

Problem 1: $\dfrac{\overset{D}{300} \times \overset{C}{400}}{\underset{C}{200}}$ .... = $\overset{D}{600}$

Explanation: First move the multiplier to the large numeral 3 on the D scale. You will find it about halfway down the scale. When you are satisfied that the hairline is directly over the mark for 300, move your hand quickly to the divider. At this point you look back at the problem to decide what to do next. You see that there are two operations left to perform: a multiplication by 400 and a division by 200. Since your hand is holding the divider you decide to divide by 200. You do this by moving the divider until 200 (the large 2) is directly under the hairline. When you are satisfied that the divider has been positioned correctly, move your hand to the multiplier. Look back at the problem and see that the only operation left is multiplication by 400. Your hand is on the multiplier so move it until the hairline is directly over 400 on the C scale. At this point the problem has been solved, if, you are able to answer these two questions yes:

1. Have you used every factor in the problem?

2. Did you multiply last?

Of course, in this case you can answer each of these questions "yes" so the solution is complete. Read your answer (600) from the D scale under the hairline of the multiplier.

Each of the problems in this practice set is solved just like problem 1. The above explanation can be made to fit any of the other problems in the set by replacing 300, 200 and 400 by the corresponding numbers from the problem you want to solve. A slightly shorter explanation of problem 2 follows.

Problem 2: $\dfrac{\overset{D}{300} \times \overset{C}{700}}{\underset{C}{500}}$ .... $= \overset{D}{420}$

Explanation: Take hold of the multiplier and move it to 300 on the D scale then place your hand on the divider. Divide by 500 by moving the divider until the 500 on the C scale is directly under the hairline. Place your hand on the multiplier. Move the multiplier until the hairline is directly above 700 on the C scale. Read the answer (420) from the D scale.

Problem 3: $\dfrac{\overset{D}{600} \times \overset{C}{200}}{\underset{C}{400}}$ .... $= \overset{D}{300}$

Explanation: Move the multiplier to 600 on the D scale. Move the divider to 400 on the C scale. Move the multiplier to 200 on the C scale. Read the answer (300) from the D scale.

$$\text{Problem 4:}\quad \frac{\overset{D}{650} \times \overset{C}{180}}{\underset{C}{540}} \;\ldots\ldots\ldots\ldots\ldots = \overset{D}{217}$$

Explanation:

1. Multiply by 650 D scale.

2. Divide by 540 C scale.

3. Multiply by 180 C scale.

4. Read Answer 217 D scale.

$$\text{Problem 5:}\quad \frac{\overset{D}{210} \times \overset{C}{130}}{\underset{C}{175}} \;\ldots\ldots\ldots\ldots\ldots = \overset{D}{156}$$

Explanation:

1. M (210) D

2. D (175) C

3. M (130) C

4. R.A. (156) D

Each of the previous explanations were designed to convey the same idea. Many of the later problems will be explained like problem 5. If later you cannot understand one of these explanations you may look back to the explanation of problem 4 to see what the symbols in 5 mean. If you still do not get it, look back to 3, then 2 and finally 1 for more explanation.

Example: M (278) C would mean move the Multiplier to the number 278 on the C scale.

In general the first letter will be used to represent the part of the slide rule to be moved. This, of course, will always be either an "M" if the multiplier is to be moved or a "D" if the divider is to be moved. Then, enclosed in parenthesis you will find the number to be set. Finally, the scale upon which the number is to be set will be designated by the letter following the parenthesis.

EXERCISE SET VII

6. $\dfrac{\overset{D}{214} \times \overset{C}{575}}{\underset{C}{398}}$ .... = $\overset{D}{309}$

7. $\dfrac{\overset{D}{625} \times \overset{C}{266}}{\underset{C}{725}}$ .... = $\overset{D}{229}$

8. $\dfrac{\overset{D}{260} \times \overset{C}{380}}{\underset{C}{440}}$ .... = $\overset{D}{225}$

9. $\dfrac{\overset{D}{350} \times \overset{C}{190}}{\underset{C}{270}}$ .... = $\overset{D}{246}$

10. $\dfrac{\overset{D}{188} \times \overset{C}{265}}{\underset{C}{342}}$ .... = $\overset{D}{146}$

11. $\dfrac{177 \times 815}{415}$ .... = __________

12. $\dfrac{231 \times 568}{354}$ .... = __________

13. $\frac{435 \times 923}{865}$ .... = ______

14. $\frac{753 \times 452}{820}$ .... = ______

15. $\frac{623 \times 470}{853}$ .... = ______

16. $\frac{457 \times 235}{681}$ .... = ______

17. $\frac{652 \times 308}{783}$ .... = ______

18. $\frac{450 \times 651}{320}$ .... = ______

19. $\frac{456 \times 425}{632}$ .... = ______

20. $\frac{235 \times 420}{682}$ .... = ______

21. $\frac{452 \times 785}{630}$ .... = ______

22. $\frac{102 \times 628}{534}$ .... = ______

23. $\frac{621 \times 812}{753}$ .... = ______

24. $\frac{647 \times 289}{453}$ .... = ______

25. $\frac{127 \times 862}{547}$ .... = ________

26. $\frac{358 \times 692}{764}$ .... = ________

27. $\frac{537 \times 684}{520}$ .... = ________

28. $\frac{321 \times 357}{452}$ .... = ________

29. $\frac{156 \times 943}{648}$ .... = ________

30. $\frac{608 \times 328}{456}$ .... = ________

31. $\frac{953 \times 732}{870}$ .... = ________

32. $\frac{721 \times 210}{286}$ .... = ________

33. $\frac{542 \times 351}{201}$ .... = ________

34. $\frac{452 \times 310}{248}$ .... = ________

35. $\frac{563 \times 305}{208}$ .... = ________

36. $\frac{235 \times 671}{345}$ .... = ________

37. $\frac{158 \times 403}{146}$ .... = ______

38. $\frac{758 \times 359}{641}$ .... = ______

39. $\frac{453 \times 453}{875}$ .... = ______

40. $\frac{854 \times 256}{453}$ .... = ______

41. $\frac{520 \times 863}{642}$ .... = ______

42. $\frac{302 \times 430}{156}$ .... = ______

43. $\frac{723 \times 420}{632}$ .... = ______

44. $\frac{231 \times 329}{251}$ .... = ______

45. $\frac{238 \times 238}{109}$ .... = ______

46. $\frac{357 \times 342}{256}$ .... = ______

47. $\frac{231 \times 405}{574}$ .... = ______

48. $\frac{628 \times 156}{203}$ .... = ______

49. $\frac{321 \times 246}{258}$ .... = ______

50. $\frac{230 \times 541}{458}$ .... = ______

51. $\frac{327 \times 857}{751}$ .... = ______

52. $\frac{235 \times 351}{420}$ .... = ______

53. $\frac{239 \times 402}{854}$ .... = ______

54. $\frac{301 \times 201}{560}$ .... = ______

55. $\frac{650 \times 328}{756}$ .... = ______

56. $\frac{253 \times 159}{230}$ .... = ______

57. $\frac{672 \times 158}{530}$ .... = ______

58. $\frac{651 \times 287}{530}$ .... = ______

59. $\frac{589 \times 457}{953}$ .... = ______

60. $\frac{157 \times 407}{256}$ .... = ______

61. $\frac{259 \times 157}{358}$ .... = ________

62. $\frac{246 \times 168}{374}$ .... = ________

63. $\frac{309 \times 463}{571}$ .... = ________

64. $\frac{736 \times 913}{846}$ .... = ________

65. $\frac{246 \times 208}{349}$ .... = ________

66. $\frac{510 \times 564}{289}$ .... = ________

67. $\frac{568 \times 283}{764}$ .... = ________

68. $\frac{432 \times 259}{563}$ .... = ________

69. $\frac{238 \times 632}{425}$ .... = ________

70. $\frac{178 \times 328}{259}$ .... = ________

71. $\frac{428 \times 357}{621}$ .... = ________

72. $\frac{436 \times 610}{529}$ .... = ________

73. $\frac{523 \times 128}{324}$ .... = ________

74. $\frac{287 \times 346}{952}$ .... = ________

75. $\frac{450 \times 435}{329}$ .... = ________

76. $\frac{756 \times 245}{548}$ .... = ________

77. $\frac{561 \times 653}{458}$ .... = ________

78. $\frac{456 \times 583}{532}$ .... = ________

79. $\frac{753 \times 158}{230}$ .... = ________

80. $\frac{423 \times 238}{654}$ .... = ________

81. $\frac{423 \times 536}{952}$ .... = ________

82. $\frac{843 \times 960}{940}$ .... = ________

83. $\frac{423 \times 706}{980}$ .... = ________

84. $\frac{453 \times 324}{258}$ .... = ________

85. $\frac{302 \times 752}{563}$ .... = __________

86. $\frac{320 \times 453}{568}$ .... = __________

87. $\frac{763 \times 159}{342}$ .... = __________

88. $\frac{308 \times 320}{915}$ .... = __________

89. $\frac{453 \times 435}{850}$ .... = __________

90. $\frac{756 \times 120}{235}$ .... = __________

91. $\frac{530 \times 248}{452}$ .... = __________

92. $\frac{532 \times 245}{456}$ .... = __________

93. $\frac{761 \times 530}{458}$ .... = __________

94. $\frac{723 \times 258}{563}$ .... = __________

95. $\frac{765 \times 161}{525}$ .... = __________

96. $\frac{118 \times 435}{378}$ .... = __________

97. $\frac{520 \times 216}{450}$ .... = ________

98. $\frac{198 \times 226}{113}$ .... = ________

99. $\frac{625 \times 725}{850}$ .... = ________

100. $\frac{366 \times 915}{845}$ .... = ________

ANSWERS EXERCISE SET VII

11. 348	27. 706	43. 480
12. 371	28. 254	44. 303
13. 464	29. 227	45. 520
14. 415	30. 437	46. 477
15. 343	31. 802	47. 163
16. 158	32. 529	48. 483
17. 256	33. 946	49. 306
18. 915	34. 565	50. 272
19. 307	35. 826	51. 373
20. 145	36. 457	52. 196
21. 563	37. 436	53. 113
22. 120	38. 425	54. 108
23. 670	39. 235	55. 282
24. 413	40. 483	56. 175
25. 200	41. 699	57. 200
26. 324	42. 832	58. 353

59. 282	73. 207	87. 355
60. 250	74. 104	88. 108
61. 114	75. 595	89. 232
62. 111	76. 338	90. 386
63. 251	77. 800	91. 291
64. 794	78. 500	92. 286
65. 147	79. 517	93. 881
66. 995	80. 154	94. 331
67. 210	81. 238	95. 235
68. 199	82. 861	96. 136
69. 354	83. 305	97. 250
70. 225	84. 361	98. 396
71. 246	85. 403	99. 533
72. 503	86. 255	100. 396

LESSON V

LOCATION OF THE DECIMAL

In the previous lessons you learned to set certain numbers on the slide rule. You set numbers such as 123, 456, 785, and 946; that is, you set three digit integers. But, where would you set 23.7, 12, 0.0145 or 1.97? In this lesson we shall learn to set these and all other positive numbers. We will also learn some of the theory for placing decimals in answers obtained by working problems with slide rule. Before discussing these ideas, let us consider how the problem of locating the decimal was approached in arithmetic.

Carefully study the solutions of these two arithmetic problems.

Problem 1	Problem 2
$\begin{array}{r} 32 \\ \underline{.21} \end{array}$	$\begin{array}{r} 32 \\ \underline{21} \end{array}$

Step 1

$\begin{array}{r} 32 \\ \underline{.21} \\ 2 \end{array}$	$\begin{array}{r} 32 \\ \underline{21} \\ 2 \end{array}$

Note that after Step 1, the partial solutions of these two problems are identical.

Step 2

Problem 1 | Problem 2

$$\begin{array}{ccc} & 3 & 2 \\ & .2 & 1 \\ \hline & 3 & 2 \end{array} \qquad\qquad \begin{array}{ccc} & 3 & 2 \\ & 2 & 1 \\ \hline & 3 & 2 \end{array}$$

Notice that after Step 2 is completed, the partial solutions are still identical.

Step 3

$$\begin{array}{ccc} & 3 & 2 \\ & .2 & 1 \\ \hline & 3 & 2 \\ & 4 & \end{array} \qquad\qquad \begin{array}{ccc} & 3 & 2 \\ & 2 & 1 \\ \hline & 3 & 2 \\ & 4 & \end{array}$$

Step 4

$$\begin{array}{ccc} & 3 & 2 \\ & .2 & 1 \\ \hline & 3 & 2 \\ 6 & 4 & \end{array} \qquad\qquad \begin{array}{ccc} & 3 & 2 \\ & 2 & 1 \\ \hline & 3 & 2 \\ 6 & 4 & \end{array}$$

Step 5

$$\begin{array}{ccc} & 3 & 2 \\ & .2 & 1 \\ \hline & 3 & 2 \\ 6 & 4 & \\ \hline \end{array} \qquad\qquad \begin{array}{ccc} & 3 & 2 \\ & 2 & 1 \\ \hline & 3 & 2 \\ 6 & 4 & \\ \hline \end{array}$$

Step 6

$$\begin{array}{ccc} & 3 & 2 \\ & .2 & 1 \\ \hline & 3 & 2 \\ 6 & 4 & \\ \hline & & 2 \end{array} \qquad\qquad \begin{array}{ccc} & 3 & 2 \\ & 2 & 1 \\ \hline & 3 & 2 \\ 6 & 4 & \\ \hline & & 2 \end{array}$$

Step 7

$$\begin{array}{ccc} & 3 & 2 \\ & .2 & 1 \\ \hline & 3 & 2 \\ 6 & 4 & \\ \hline & 7 & 2 \end{array} \qquad\qquad \begin{array}{ccc} & 3 & 2 \\ & 2 & 1 \\ \hline & 3 & 2 \\ 6 & 4 & \\ \hline & 7 & 2 \end{array}$$

After steps 3, 4, 5, 6, and 7, the partial solutions of the two problems are identical.

Step 8

Problem 1	Problem 2
$\begin{array}{r} 32 \\ \underline{.21} \\ 32 \\ \underline{64} \\ 672 \end{array}$	$\begin{array}{r} 32 \\ \underline{21} \\ 32 \\ \underline{64} \\ 672 \end{array}$

Even after Step 8, the partial solutions of the two problems are identical.

Step 9

$\begin{array}{r} 32 \\ \underline{.21} \\ 32 \\ \underline{64} \\ 6.72 \end{array}$	$\begin{array}{r} 32 \\ \underline{21} \\ 32 \\ \underline{64} \\ 672 \end{array}$

As you have just discovered, in arithmetic we multiply 32 by .21 in exactly the same way that we multiply it by 21, 2.1, or 210. The only difference occurs in the last step when the decimal location is determined by counting the places to the right of the decimal. The decimal is then placed by applying the following rule.

The number of digits to the right of the decimal in a product is equal to the sum of the number of digits to the right of the decimal in each of its factors.

Thus, we see that in arithmetic when numbers containing decimals are involved, we ignore the decimals and find the product of the resulting integers. This product correctly gives us the digits of the answer. We may then determine the decimal placement in the answer by counting the digits to the right of the decimal and pointing off that number of places in the product. This is almost exactly the way we handle decimal numbers when a problem is worked using a slide rule. Each number is read as a three digit integer regardless of where the decimal is placed. When the digits of the answer are found the placement of the decimal is determined by counting digits.

The principal difference between the slide rule method for finding the location of the decimal and the method used in arithmetic is this: The slide rule method requires that we count the digits to the left of the decimal while the arithmetic method requires that we count those to the right.

EXERCISE SET VIII

Directions: Take a sheet of paper or your slide rule and cover the numbers in the column on the right. As you read each decimal number in the column on the left, decide what three digit integer you would set on the slide rule to represent it. When you are certain of your answer, move the sheet down just one space and check.

Decimal Numbers	3 Digit Numbers
42.7	427
3.85	385
0.249	249
22	220
88	880
9	900
0.00245	245
0.0161	161
0.0004	400

In order to determine the decimal location in the more difficult slide rule problems it will be necessary to assign to each positive number an integer called the <u>count</u> of the number. We will first study a few examples and then state a precise definition for the count of a number.

Examples:

Number	Count
356,000	6
35,600	5
3,560	4
356	3
35	2
35.6	2
3.56	1
0.356	0

Number	Count
0.035,6	-1
0.003,56	-2
0.000,356	-3
0.000,035,6	-4

DEFINITION OF THE COUNT OF A NUMBER

THE COUNT OF A NUMBER GREATER THAN ONE IS THE NUMBER OF DIGITS TO THE LEFT OF THE DECIMAL.

THE COUNT OF A NUMBER LESS THAN ONE IS THE OPPOSITE (THE NEGATIVE) OF THE NUMBER OF ZEROES BETWEEN THE DECIMAL AND THE FIRST NON-ZERO DIGIT.

EXERCISE SET IX

Directions: Find the count of each of the following numbers.

1. 0.367
2. 2.15
3. 0.00724
4. 9.27
5. 62,300
6. 56.8
7. 0.0832
8. 3820
9. 218
10. 0.000425
11. 0.0000282
12. 0.00000772
13. 62.5
14. 5280
15. 3.79
16. 1.66
17. 39.2
18. 20.8
19. 6,830,000
20. 526

21. 2.38
22. 0.00114
23. 66.8
24. 1.75
25. 3.92
26. 0.416
27. 77.1
28. 0000446
29. 72
30. 55

ANSWERS EXERCISE SET IX

1. 0
2. 1
3. -2
4. 1
5. 5
6. 2
7. -1
8. 4
9. 3
10. -3
11. -4
12. -5
13. 2
14. 4
15. 1
16. 1
17. 2
18. 2
19. 7
20. 3
21. 1
22. -2
23. 2
24. 1
25. 1
26. 0
27. 2
28. -4
29. 2
30. 2

LESSON VI

THE COUNT OF A NUMBER

Many of the slide rule problems which you will encounter in practical applications are similar to this example.

$$\frac{52.8 \times 7.16 \times 4380}{.0416 \times 87.9} \ldots\ldots\ldots\ldots\ldots\ldots = 453{,}000$$

To solve this problem we carry out the multiplication and division with the slide rule as if the problem were:

$$\frac{528 \times 716 \times 438}{416 \times 879}$$

Then, we compute the decimal by applying the following rules. If the appropriate corrections are made so that decimal changes are included in these counts then:

D-1 THE COUNT OF A PRODUCT IS THE SUM OF THE COUNTS OF ITS FACTORS.

D-2 THE COUNT OF A FRACTION IS EQUAL TO THE COUNT OF THE NUMERATOR MINUS THE COUNT OF THE DENOMINATOR.

The above rules are generally true without the need for correction; however, there are special cases for which minor corrections must be made. These corrections are usually called decimal changes or simply changes. In the above problem, no changes occur, so the count of the answer is 6. The count of the numerator is 2 + 1 + 4 = 7. (D-1). The count of the denominator is -1 + 2 = 1. Thus, the count of the answer is 7 - 1 = 6. (D-2). However, in the problem 20 x 30 = 600 (done by arithmetic), the sum of the counts of the factors is 2 + 2 = 4, but the count of the product is 3. Hence, a correction of -1 is needed when rule D-1 is applied. This presents no problem. For in this lesson, none of the practice exercises involve problems where decimal changes occur, and in the next lesson, a very easy method is presented for spotting these changes. Because of its importance, we will preview the method of determining decimal changes at this time.

As we work certain problems in the next lesson, the motion rules will force us to multiply or divide by one. The one at the left end of the C scale is called the "recording one." That is, each time we multiply or divide by it, we record it and count it when placing the decimal. If these ones are recorded and counted then the decimal rules hold without exception.

Example one:

8.43 x 65.7 x 484 .... = 268
(without the decimal, of course)

The counts of the respective factors are 1, 2, and 3. Hence, the count of the product is 6 = 1 + 2 + 3.

Therefore: 8.43 x 65.7 x 484 .... = 268,000.

Example two:

0.0658 x 5.43 x 0.719 .... = 257
(without the decimal)

For the purpose of illustration, the count of each number will be written above it. A student should do this on the first five or ten problems so that if he makes a mistake in locating the decimal, he can easily see where the mistake was made. The work in finding the count of a product is so easy that this practice should be discontinued after a few problems have been worked correctly.

$$\overset{-1}{0.0658} \times \overset{+1}{5.43} \times \overset{+0}{0.719} \ldots\ldots\ldots\ldots\ldots = \overset{0}{0.257}$$

Example three:

$$\frac{\overset{1}{3.67} \times \overset{+0}{0.419} \times \overset{+2}{58.9}}{\underset{0}{0.198} \times \underset{+2}{78.5}} \ldots\ldots \frac{3}{2} \ldots\ldots\ldots = \overset{1}{5.83}$$

To locate the decimal in a problem like example three, notice that we add the counts of the factors of the numerator which is a product to get 3. Then, we add the counts of the factors of the denominator which is also a product and get 2. The count of the quotient or fraction is 3 - 2 = 1. Thus, we place the decimal between the 5 and 8 which is the only position in the number which will make the product (5.83) have a count of one.

Example four:

$$\frac{\overset{2}{38.7} \times \overset{+0}{0.489} \times \overset{+3}{395}}{\underset{2}{29.5} \times \underset{+3}{688}} \ldots\ldots \frac{5}{5} \ldots\ldots\ldots = \overset{0}{0.368}$$

It has been observed that when the count of a fraction is zero, the placement of a decimal is often forgotten. Of course, when the decimal is not placed, it is understood to be after the third digit which would make the count 3.

EXERCISE SET X

Directions: Copy these ten problems. Write the count of each factor above it. Write the count of the numerator above the division line and the count of the denominator below the line. Write the count of the answer above the three digit integer in the answer blank and correctly place the decimal.

1. $\frac{34.7 \times 285}{1.93}$ .... = 512

2. $\frac{49.4 \times 0.0486 \times 2.37}{27.7 \times 0.0749}$ .... = 274

3. $\frac{17.6 \times 0.231 \times 8.37}{2.51 \times 3.59}$ .... = 378

4. $\frac{0.0122 \times 46.7}{28.9}$ .... = 197

5. $\frac{47.6 \times 75.3 \times 19.8}{0.00399 \times 0.0418}$ .... = 426

6. $\frac{9.62 \times 7.13 \times 677}{6.95 \times 8.86}$ .... = 754

7. $\dfrac{0.0167 \times 72.3 \times 0.532}{8.19 \times 47.2}$ .... = .166

8. $\dfrac{0.00177 \times 14.3 \times 0.984}{0.0198 \times 0.00276}$ .... = 456

9. $\dfrac{4.33 \times 3.95 \times 329}{0.0521 \times 0.00637}$ .... = 170

10. $\dfrac{0.00214 \times 76.1 \times 0.00485}{3.22 \times 8.63}$ .... = 284

ANSWERS EXERCISE SET X

1. $\dfrac{\overset{2}{34.7} \times \overset{3}{285}}{\underset{1}{1.93}}$ .... $\genfrac{}{}{0pt}{}{5}{1}$... = $\overset{4}{5120}$

2. $\dfrac{\overset{2}{49.4} \times \overset{-1}{0.0486} \times \overset{1}{2.37}}{\underset{2}{27.7} \times \underset{-1}{0.0749}}$ .... $\genfrac{}{}{0pt}{}{2}{1}$... = $\overset{1}{2.74}$

3. $\dfrac{\overset{2}{17.6} \times \overset{0}{0.231} \times \overset{1}{8.37}}{\underset{1}{2.51} \times \underset{1}{3.59}}$ .... $\genfrac{}{}{0pt}{}{3}{2}$... = $\overset{1}{3.78}$

4. $\dfrac{\overset{-1}{0.0122} \times \overset{2}{46.7}}{\underset{2}{28.9}}$ .... $\genfrac{}{}{0pt}{}{1}{2}$... = $\overset{-1}{0.0197}$

5. $\dfrac{\overset{2}{47.6} \times \overset{2}{75.3} \times \overset{2}{19.8}}{\underset{-2}{0.00399} \times \underset{-1}{0.0418}}$ .... $\genfrac{}{}{0pt}{}{6}{-3}$... = $\overset{9}{426{,}000{,}000}$

6. $\dfrac{\overset{1}{9.62} \times \overset{1}{7.13} \times \overset{3}{677}}{\underset{1}{6.95} \times \underset{1}{8.86}}$ .... $\genfrac{}{}{0pt}{}{5}{2}$... = $\overset{3}{754}$

$$7.\quad \frac{\overset{-1}{0.0167} \times \overset{2}{72.3} \times \overset{0}{0.532}}{\underset{1}{8.19} \times \underset{2}{47.2}} \;\ldots\ldots\frac{1}{3}\ldots = \overset{-2}{0.00166}$$

$$8.\quad \frac{\overset{-2}{0.00177} \times \overset{2}{14.3} \times \overset{0}{0.984}}{\underset{-1}{0.0198} \times \underset{-2}{0.00276}} \;\ldots\ldots\frac{0}{-3}\ldots = \overset{3}{456}$$

$$9.\quad \frac{\overset{1}{4.33} \times \overset{1}{3.95} \times \overset{3}{329}}{\underset{-1}{0.0521} \times \underset{-2}{0.00637}} \;\ldots\ldots\ldots\ldots\frac{5}{-3}\ldots = \overset{8}{17{,}000{,}000}$$

$$10.\quad \frac{\overset{-2}{0.00214} \times \overset{2}{76.1} \times \overset{-2}{0.00485}}{\underset{1}{3.22} \times \underset{1}{8.63}} \;\ldots\frac{-2}{2}\ldots = \overset{-4}{0.0000284}$$

In problems like number 9, $\frac{4.33 \times 3.95 \times 329}{0.0521 \times 0.00637} =$, in which many of the factors have negative counts, the thought process is made much easier if the signs of the negative counts in the denominator are changed and these new counts are moved to the numerator. Instead of using the count of 0.0521 as -1 in the denominator, use it as +1 in the numerator. Since adding +1 to any number gives the same answer as subtracting -1 from that same number, this process is mathematically correct. Using this convention, the count in problem 9 would be computed in the following fashion.

$$\overset{+1}{\uparrow}\ \frac{\overset{+1}{4.33} \times \overset{+1}{3.95} \times \overset{+3}{329}}{0.0521 \times 0.00637}\ \overset{+2}{\uparrow} \;\ldots\ldots\ldots\frac{8}{0}\ldots = \overset{8}{17{,}000{,}000}$$

Essentially the same procedure can be followed when numbers in the numerator have negative counts. Reconsider problem 10.

$$\underset{+2}{\downarrow}\ \frac{\overset{2}{0.00214 \times 76.1 \times 0.00485}}{\underset{1 \qquad +1}{3.22 \times 8.63}}\ \underset{+2}{\downarrow} \quad \ldots\ldots\frac{2}{6}\ldots = \overset{-4}{0.0000284}$$

With this new procedure, only positive numbers need to be added and subtracted. After the count of the numerator and the count of the denominator have been determined and written above and below the division line, the decimal placement may be calculated by finding the difference of these numbers and counting this number of places to the right if the count of the numerator is greater or to the left if the count of the denominator is greater. An important point to remember is that to locate the decimal in an answer, start counting at the point just to the left of the first digit of the three digit slide rule answer. A summary of this procedure will read:

To avoid adding or subtracting negative numbers, replace the negative counts in the numerator with their absolute values and move them to the denominator; also, replace the negative counts in the denominator with their absolute values and move them to the numerator.

Try this new procedure on some of the practice problems in the next set. It will make the thinking much easier, and as a consequence, reduce the number of errors.

EXERCISE SET XI

Directions: Solve the following problems. After each set, check your answers. Any answer with the decimal correctly placed which is within two of the third significant figure is correct. Check the decimal location carefully and correct each error before going to the next set of problems.

A

1. $\frac{0.0400 \times 0.220}{2.02}$ .... = ________

2. $\frac{0.0173 \times 14.6}{2.15}$ .... = ________

3. $\frac{3.77 \times 0.416}{0.00293}$ .... = ________

4. $\frac{5.69 \times 14.2}{1.27}$ .... = ________

5. $\frac{0.00000377 \times 3.47}{0.026}$ .... = ________

6. $\frac{0.00192 \times 0.268}{4.92}$ .... = ________

7. $\frac{1.78 \times 0.0061}{2.37}$ .... = ________

8. $\frac{3.14 \times 3.20}{6.73}$ .... = ________

9. $\dfrac{0.279 \times 3.44}{62.4}$ .... = ________

10. $\dfrac{1.66 \times 3.14}{0.0297}$ .... = ________

11. $\dfrac{3.14 \times 364}{298}$ .... = ________

12. $\dfrac{42.6 \times 3.84}{373}$ .... = ________

13. $\dfrac{3.21 \times 0.00140}{32.0}$ .... = ________

14. $\dfrac{3.14 \times 1020}{0.00000600}$ .... = ________

15. $\dfrac{0.00410 \times 2.78}{61.0}$ .... = ________

16. $\dfrac{6.15 \times 0.124}{17.0}$ .... = ________

17. $\dfrac{5830 \times 72400 \times 0.0242}{811 \times 0.0859}$ .... = ________

18. $\dfrac{0.000743 \times 0.000452 \times 45500}{7350 \times 71700}$ ...... = ________

19. $\dfrac{64.1 \times 0.0000112 \times 632}{5850 \times 0.0147}$ .... = ________

20. $\dfrac{905 \times 116 \times 813}{523 \times 966{,}000}$ .... = ________

B

21. $\dfrac{425 \times 26.7}{58.0}$ .... = ____________

22. $\dfrac{3940 \times 284}{0.0000397}$ .... = ____________

23. $\dfrac{72.0 \times 0.289}{0.791}$ .... = ____________

24. $\dfrac{642 \times 14.8}{11.8}$ .... = ____________

25. $\dfrac{424 \times 0.600}{3.14}$ .... = ____________

26. $\dfrac{305 \times 112}{0.0300}$ .... = ____________

27. $\dfrac{0.0000267 \times 337}{826}$ .... = ____________

28. $\dfrac{83.2 \times 7.86}{876}$ .... = ____________

29. $\dfrac{6.17 \times 3.71}{258}$ .... = ____________

30. $\dfrac{13.2 \times 0.0461}{217}$ .... = ____________

31. $\dfrac{3.62 \times 5860}{0.222}$ .... = ____________

32. $\frac{3.14 \times 3.14}{65.0}$ .... = ________

33. $\frac{26.5 \times 734{,}000 \times 0.00215}{45.2 \times 0.217}$ .... = ________

34. $\frac{0.0173 \times 0.689 \times 86.4}{7920 \times 0.0146}$ .... = ________

35. $\frac{2.08 \times 3.55 \times 4.68}{0.227 \times 0.00352}$ .... = ________

36. $\frac{0.00617 \times 3.25 \times 0.0177}{480 \times 37.6}$ .... = ________

37. $\frac{0.364 \times 5.62 \times 4.88}{5.63 \times 2.68}$ .... = ________

38. $\frac{1.83 \times 2.48 \times 0.00173}{0.297 \times 48.7}$ .... = ________

39. $\frac{0.00382 \times 5.25 \times 7.63}{9.09 \times 2.11}$ .... = ________

40. $\frac{3.68 \times 0.00272 \times 81.6}{81.6 \times 9.93}$ .... = ________

C

41. $\frac{2.79 \times 0.00470}{2610}$ .... = ________

42. $\frac{13.7 \times 0.036}{473}$ .... = ________

43. $\frac{706 \times 866}{989}$ .... = ________

44. $\frac{27.0 \times 3.68}{0.000883}$ .... = ________

45. $\frac{413 \times 0.0297}{4.83}$ .... = ________

46. $\frac{872 \times 0.113}{0.260}$ .... = ________

47. $\frac{14.0 \times 79.0}{68}$ .... = ________

48. $\frac{28.0 \times 42.0}{83.0}$ .... = ________

49. $\frac{726 \times 38}{0.724}$ .... = ________

50. $\frac{41 \times 864}{0.0000496}$ .... = ________

51. $\frac{0.264 \times 0.00324 \times 12.4}{0.0162 \times 5060}$ .... = ________

52. $\frac{0.174 \times 3.66 \times 8.24}{3.86 \times 5.06}$ .... = ________

53. $\frac{0.0292 \times 2.11 \times 3.72}{1.48 \times 3.66}$ .... = ________

54. $\frac{0.00696 \times 0.392 \times 0.146}{2.66 \times 0.379}$ .... = ________

55. $\frac{0.0462 \times 0.137 \times 1.62}{1.83 \times 3060}$ .... = ________

56. $\frac{27 \times 491 \times 0.000660}{83 \times 2740}$ .... = ________

57. $\frac{4.68 \times 4.32 \times 5.05}{620{,}000 \times 3.70}$ .... = ________

58. $\frac{4.10 \times 2.68 \times 0.0410}{0.383 \times 0.0926}$ .... = ________

59. $\frac{3.33 \times 866 \times 414}{314 \times 723}$ .... = ________

60. $\frac{2.17 \times 3.60 \times 44.0}{0.0143 \times 3.14}$ .... = ________

D

61. $\frac{0.0240 \times 78.0}{32}$ .... = ________

62. $\frac{306 \times 4.92}{0.00929}$ .... = ________

63. $\frac{627 \times 562}{873}$ .... = ________

64. $\frac{414 \times 309}{211}$ .... = ________

65. $\frac{0.374 \times 496}{0.0223}$ .... = ________

66. $\dfrac{72.8 \times 0.0000200}{713}$ .... = ________

67. $\dfrac{42.6 \times 31.4 \times 3.03}{0.450 \times 8310}$ .... = ________

68. $\dfrac{2.67 \times 314 \times 21.0}{0.00370 \times 0.00274}$ .... = ________

69. $\dfrac{6.70 \times 0.523 \times 203}{97,400 \times 3,620,000}$ .... = ________

70. $\dfrac{0.00537 \times 4.93 \times 187}{0.0000502 \times 4.02}$ .... = ________

71. $\dfrac{3.90 \times 0.220 \times 3.14}{31.4 \times 13.6}$ .... = ________

72. $\dfrac{8.60 \times 14.2 \times 3760}{0.424 \times 7640}$ .... = ________

73. $\dfrac{39.0 \times 2.86 \times 3.42}{3.14 \times 31.4}$ .... = ________

74. $\dfrac{0.00620 \times 497 \times 13.8}{0.0800 \times 1.49}$ .... = ________

75. $\dfrac{926 \times 0.0000271 \times 14200}{7330 \times 0.0000271}$ .... = ________

76. $\dfrac{2170 \times 0.0927 \times 0.000610}{8340 \times 937}$ .... = ________

77. $\dfrac{291 \times 0.000610 \times 8340}{837 \times 291}$ .... = ________

78. $\dfrac{1700 \times 86.0 \times 0.000863}{926{,}000 \times 25.0}$ .... = __________

79. $\dfrac{7{,}320{,}000 \times 0.00436 \times 26.0}{0.661 \times 3.14}$ .... = __________

80. $\dfrac{0.927 \times 0.0647 \times 14300}{86200 \times 0.00723}$ .... = __________

E

81. $\dfrac{0.743 \times 21.6 \times 0.181}{3.64 \times 0.0397}$ .... = __________

82. $\dfrac{31.4 \times 0.00436 \times 8{,}320{,}000}{37.2 \times 4.68}$ .... = __________

83. $\dfrac{3.14 \times 0.000392 \times 864}{921 \times 336}$ .... = __________

84. $\dfrac{5.32 \times 4.83 \times 0.133}{3.14 \times 2.73}$ .... = __________

85. $\dfrac{2.85 \times 5430 \times 63700}{2.60 \times 7.70}$ .... = __________

86. $\dfrac{3.14 \times 0.0000864 \times 272}{399 \times 863}$ .... = __________

87. $\dfrac{28 \times 36 \times 44}{71 \times 12}$ .... = __________

88. $\dfrac{535 \times 170 \times 904}{799 \times 194}$ .... = __________

89. $\frac{971 \times 348 \times 258}{600 \times 893}$ .... = __________

90. $\frac{616 \times 719 \times 677}{611 \times 495}$ .... = __________

91 $\frac{705 \times 427 \times 605}{719 \times 304}$ .... = __________

92. $\frac{139 \times 640 \times 584}{339 \times 425}$ .... = __________

93. $\frac{945 \times 219 \times 211}{533 \times 676}$ .... = __________

94. $\frac{349 \times 730 \times 596}{295 \times 969}$ .... = __________

95. $\frac{809 \times 400 \times 547}{506 \times 502}$ .... = __________

96. $\frac{900 \times 852 \times 364}{782 \times 848}$ .... = __________

97. $\frac{141 \times 672 \times 749}{243 \times 598}$ .... = __________

98. $\frac{334 \times 428 \times 213}{150 \times 751}$ .... = __________

99. $\frac{774 \times 398 \times 328}{489 \times 770}$ .... = __________

100. $\frac{389 \times 862 \times 201}{363 \times 228}$ .... = __________

ANSWERS EXERCISE SET XI

A

1. 0.00436
2. 0.117
3. 535
4. 63.6
5. 0.000503
6. 0.000105
7. 0.00458
8. 1.49
9. 0.0154
10. 176
11. 3.84
12. 0.439
13. 0.000140
14. 534,000,000
15. 0.000187
16. 0.0449
17. 14,700
18. 2.90×10^{-11}
19. 0.00528
20. 0.169

B

21. 196
22. 2.82×10^{10}
23. 26.3
24. 805
25. 8.10
26. 1,140,000
27. 0.0000109
28. 0.747
29. 0.0887
30. 0.00280
31. 95,600
32. 0.152
33. 4260
34. 0.00891
35. 43,200
36. 1.97×10^{-8}
37. 0.619
38. 0.000543
39. 0.00798
40. 0.00101

C

41. 0.00000502
42. 0.00104
43. 618

44. 113,000

45. 2.54

46. 379

47. 16.3

48. 14.2

49. 38,100

50. 714,000,000

51. 0.000129

52. 0.275

53. 0.0423

54. 0.000395

55. 0.00000183

56. 0.0000385

57. 0.0000445

58. 12.7

59. 5.26

60. 7660

D

61. 0.0585

62. 162,000

63. 404

64. 606

65. 8320

66. 0.00000204

67. 1.08

68. 1.74×10^{9}

69. 2.02×10^{-9}

70. 24,500

71. 0.00631

72. 142

73. 3.87

74. 357

75. 1790

76. 1.57×10^{-8}

77. 0.0608

78. 0.00000545

79. 400,000

80. 1.38

E

81. 20.1

82. 6540

83. 0.00000344

84. 0.399

85. 49,200,000

86. 2.14×10^{-7}

87. 52.1

88. 530

89. 163

90. 991
91. 833
92. 361
93. 121
94. 531
95. 697
96. 421
97. 488
98. 270
99. 268
100. 814

LESSON VII

THE RECORDING ONE

The key to any system for placing the decimal in slide rule problems is the method used to determine the occurrence of decimal changes. And, in this area, the counting method is superior to other methods. The central theme of this lesson is the recording one. It is used to signal the slide rule operator when a decimal change has occurred. The previous lessons have all been written with this one in mind and practically all the work which follows will depend on these ideas about the recording one. Before this lesson, we had to be very careful in limiting the problems offered to a special type. After you have mastered this lesson, you will be able to efficiently solve all problems involving multiplication and division.

We shall begin this lesson with a definition.

DEFINITION: The 1 located at the left end of the C scale is called the recording one. Each time the recording one is used, we record it and count it in determining the location of the decimal.

Consider this example:

Example one: $\frac{34.3 \times 6.82}{1.18} =$

First, set 34.3 on the D scale and then divide by 1.18. Up to this point, the solution is just like those in Lessons IV and VI, but, when we try to multiply by 6.82, we realize that the multiplier cannot be moved to this position. Thus, we find ourselves confronted with a problem. The motion rules force us to multiply, yet, we are unable to reach the only unused factor of the numerator. The way out of this predicament is easy. We simply multiply by one. This leaves the digits of the answer unchanged. Since we are forced to multiply by the one at the left end of the C scale (the recording one), we must record a one and use it in counting the decimal. But, where do we record this one? Since we multiplied by it, we record it above the division line with the other factors of the numerator. The rules of motion now require that we divide. There are no unused factors in the denominator, so what do we divide by? Of course, the solution of this problem is easy because of the similarity to our last experience. We divide by one. This time we use the one on the right end of the C scale. This is <u>not</u> the recording one so we do not record it. Now the C scale is in a position which allows us to reach 6.82. We multiply by 6.82 and the problem is finished.

To summarize:

1. M (34.3) D
2. D (1.18) C
3. M (Recording one) C
4. D (one) C
5. M (6.82) C
6. R.A. (198) D

An asterisk has been placed in some of the problems to illustrate that a recording one has been recorded.

Example one:

$$\frac{34.3 \times {}^{*}6.82}{1.18} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{4}{1} \ldots = 198$$

From this example you can see how important this idea of multiplying by one really is. When used in conjunction with our motion rules, it forced us to change the position of the slide rule which would allow us to reach the next number and at the same time signal us that a decimal change was occurring.

You are now ready to work a problem of this type.

Example two:

$$\frac{28.9}{0.0425 \times 17.6} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = \_\_\_\_\_\_\_\_$$

As usual, begin by setting 28.9 on the D scale and dividing by 0.0425. The next step requires a little more thought. Clasp the multiplier tightly, and faithfully following the rules of motion, set out to multiply by the next number. You then see that there are no other numbers left in the numerator. So, what do you do? Multiply by 1.

Do you divide by one next as in example one? No. Notice the situation. At this time, you are required to divide and there is a number left in the denominator. That number is 17.6; consequently, you must divide by 17.6. Now every factor in the problem has been used but the problem is not finished. You must multiply one more time to satisfy rule M-3. (Move the multiplier last.) Multiply by 1. Since the recording one was used, you must record it and count it in placing the decimal.

$$\frac{28.9 \quad *}{0.0425 \times 17.6} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{3}{1} \ldots = 38.6$$

Work examples one and two several times with your slide rule. As each new situation arises, pause and think about it until you are sure that you know how to handle that situation. If necessary, refer to the summaries of the solutions of these two problems but work each one over until you need look only at the problem and your slide rule to solve it.

Summary for example two:

1. M (28.9) D	4. D (17.6) C
2. D (0.0425) C	5. M (Recording one) C
3. M (ONE) C	6. R.A. (38.6) D

Example three:

9.75 x 2.46 .... = __________

To work this problem, move the multiplier to 9.75 on the D scale. The motion rules then demand that you divide (M-2 Move the multiplier and divider alternately), but since there is no factor of the problem to divide by, you divide by one. In this situation you are able to divide either by the recording one or by the one at the right end of the C scale. Move the divider until the recording one approaches the hairline and at the same time keep an eye on the approximate position of 246. As the recording one is moved nearer the hairline the 246 goes beyond the end of the D scale. Thus, if you choose to divide by the recording one you will be unable to complete the next step. You are left with no alternative except to divide by the one at the right end of the C scale. This time you do not record it because it is not the recording one. Multiply by 2.46 and the problem is complete.

9.75 x 2.46 .... = 24.0

Summary:

1. M (9.75) C
2. D (one) C
3. M (2.46) C
4. R.A. (24.0) D

Each time it becomes necessary to divide by one, the next step will certainly be a multiplication. Look ahead and choose the one that will allow you to complete that step.

We have learned from these examples that whenever we are required to move the multiplier or divider and a factor cannot be reached (this occurs only when the multiplier is being moved) or when there are no unused factors, we multiply by one.

These ideas are of such importance that we will state them as rules at this time.

O-1 IF A FACTOR CANNOT BE REACHED WITH THE MULTIPLIER BECAUSE IT IS BEYOND THE END OF THE SCALE, MULTIPLY BY ONE.

O-2 IF THERE ARE NO UNUSED FACTORS IN THE DENOMINATOR AND THE RULES OF MOTION REQUIRE MOVEMENT OF THE DIVIDER, DIVIDE BY ONE. CHOOSE THE ONE WHICH WILL ALLOW THE NEXT STEP TO BE COMPLETED.

O-3 IF THERE ARE NO UNUSED FACTORS IN THE NUMERATOR AND THE RULES OF MOTION REQUIRE MOVEMENT OF THE MULTIPLIER, MULTIPLY BY ONE.

Study the ideas and examples in this section carefully. You should now be able to work any slide rule problem which involves multiplication and division. Most of the rest of this chapter has been designed to help you make these new ideas so familiar that you can use them automatically. This will require a lot of practice.

EXERCISE SET XII

Directions: Each of these problems includes an answer and a summary of its solution. Try the problem first without looking at the solution. If you work the problem and get the correct answer, glance quickly at the summary and proceed to the next problem. If you miss the answer, study the summary carefully and make each setting on your own slide rule. When you have finished the exercise, go back and rework each of the problems you missed on your first trial. In the problems below, an asterisk indicates that the recording one has been recorded.

1. $30.7 \underset{*}{x} 23.9$ ....$\frac{4}{1}$... = 734

   1. M (30.7) D
   2. D (Recording one) C
   3. M (23.9) C
   4. R.A. (734) D

2. $\frac{496}{7.15}$ ....$\frac{3}{1}$... = 69.4

   1. M (496) D
   2. D (7.15) C
   3. M (one) C
   4. R.A. (69.4) D

3. $\frac{7.88*}{153}$ ....$\frac{2}{3}$... = 0.0515

1. M (7.88) D

2. D (153) C

3. M (Recording one) C

4. R.A. (0.0515) D

4. $38.7 \times \underset{*}{2.06} \times 4.19$ ....$\underset{1}{4}$... = 334

1. M (38.7) D

2. D (Recording one) C

3. M (2.06) C

4. D (one) C

5. M (4.19) C

6. R.A. (334) D

5. $\dfrac{218 \overset{*}{\times} 795}{0.0143}$ ....$\underset{-1}{7}$... = 12,100,000

1. M (218) D

2. D (0.0143) C

3. M (Recording one) C

4. D (one) C

5. M (795) C

6. R.A. (12,100,000) D

6. $\dfrac{12.8 \times 11.3}{\underset{*}{495}}$ ....$\underset{4}{4}$... = 0.292

1. M (12.8) D

2. D (495) C

3. M (one) C

4. D (Recording one) C

5. M (11.3) C

6. R.A. (0.292) D

7. $\frac{47.6 \times 23.9^*}{1.98 \times 31.5}$ .... $\frac{5}{3}$... = 18.2

1. M (47.6) D

2. D (1.98) C

3. M (23.9) C

4. D (31.5) C

5. M (Recording one) C

6. R.A. (18.2) D

8. $\frac{0.0198 \times 3.75}{27.4 \times 5.32}$ .... $\frac{0}{3}$... = 0.000509

1. M (0.0198) D

2. D (27.4) C

3. M (3.75) C

4. D (5.32) C

5. M (one) C

6. R.A. (59.8) D

9. $\frac{38.6 \times 4.29 \times 1.85}{5.12 \quad *}$ .... $\frac{4}{2}$... = 59.8

1. M (38.6) D

2. D (5.12) C

3. M (4.29) C

4. D (Recording one) C

5. M (1.85) C

6. R.A. (59.8) D

10. $46.7 \times 2.84 \underset{*}{\times} 39.8 \times 3.15 \ldots\frac{6}{1}\ldots = 16600.$

1. M (46.7) D

2. D (one) C

3. M (2.84) C

4. D (Recording one) C

5. M (39.8) C

6. D (one) C

7. M (3.15) C

8. R.A. (16600) D

EXERCISE SET XIII

Directions: Most of the problems in this set will require multiplication or division by one. Be sure to record the recording one each time it is used. Be especially careful when you multiply last by the recording one. This is the situation in which students in the past have most often forgotten to record it. Do not multiply or divide by one unless you have no other alternative.

A

1. 792,000 x 0.326 .... = ____________
2. 68,200 x 0.0398 .... = ____________
3. 68.3 x 79.2 .... = ____________
4. 14.3 x 0.397 .... = ____________
5. 0.296 x 1.76 .... = ____________
6. 0.836 x 55.3 .... = ____________
7. 3.62 x 0.0314 .... = ____________
8. 73.6 x 4.06 .... = ____________
9. 0.663 x 2.91 .... = ____________
10. 8.32 x 14.2 .... = ____________

11. $\frac{1.60 \times 2.80}{89.9}$ .... = ________

12. $\frac{0.120 \times 0.190}{0.00382}$ .... = ________

13. $\frac{3.01 \times 1.62}{8.82}$ .... = ________

14. $\frac{0.0000278 \times 1.40}{9.90}$ .... = ________

15. $\frac{0.00861 \times 0.776}{3.20}$ .... = ________

16. $\frac{3.14 \times 0.170}{0.846}$ .... = ________

17. $\frac{0.875 \times 0.00260}{0.0137}$ .... = ________

18. $\frac{472 \times 861}{399}$ .... = ________

19. $\frac{803 \times 0.00420}{0.120}$ .... = ________

20. $\frac{1.04 \times 0.0000490}{762}$ .... = ________

B

21. 4.77 x 2.54 .... = ________

22. 2.38 x 1.68 .... = ________

23. 15.2 x 0.836 .... = ____________

24. 2.93 x 4.72 .... = ____________

25. 0.514 x 0.0267 .... = ____________

26. 82.6 x 2.37 .... = ____________

27. 3.28 x 56.7 .... = ____________

28. 0.00523 x 4.48 .... = ____________

29. $\frac{926 \times 8.77}{1.46}$ .... = ____________

30. $\frac{62.0 \times 84}{0.000186}$ .... = ____________

31. $\frac{0.486 \times 0.0329}{147}$ .... = ____________

32. $\frac{776 \times 913}{5670}$ .... = ____________

33. $\frac{3.79 \times 1.68}{7500}$ .... = ____________

34. $\frac{171 \times 9.88}{0.126}$ .... = ____________

35. $\frac{13.0 \times 142}{0.480}$ .... = ____________

36. $\frac{15.0 \times 1050}{8.72}$ .... = ____________

37. $\frac{1.92 \times 3.04}{846}$ .... = ___________

38. $\frac{723 \times 525 \times 446}{607 \times 248}$ .... = ___________

39. $\frac{953 \times 772 \times 903}{522 \times 506}$ .... = ___________

40. $\frac{918 \times 675 \times 694}{152 \times 707}$ .... = ___________

C

41. 994 x 0.0336 .... = ___________

42. 82.4 x 0.192 .... = ___________

43. 0.0925 x 73.6 .... = ___________

44. 3.95 x 0.00354 .... = ___________

45. 38200 x 0.448 .... = ___________

46. 3.91 x 52.5 .... = ___________

47. 0.0877 x 62.8 .... = ___________

48. 3.07 x 226 .... = ___________

49. 1.80 x 0.125 x 3.14 .... = ___________

50. 9.78 x 13.6 x 0.00162 .... = ___________

51. $\frac{574 \times 982 \times 327}{324 \times 916}$ .... = __________

52. $\frac{810 \times 626 \times 873}{246 \times 676}$ .... = __________

53. $\frac{783 \times 284 \times 641}{125 \times 398}$ .... = __________

54. $\frac{209 \times 417 \times 207}{875 \times 651}$ .... = __________

55. $\frac{478 \times 795 \times 447}{297 \times 270}$ .... = __________

56. $\frac{442 \times 707 \times 400}{200 \times 334}$ .... = __________

57. $\frac{884 \times 122 \times 441}{818 \times 979}$ .... = __________

58. $\frac{420 \times 201 \times 860}{518 \times 136}$ .... = __________

59. $\frac{170 \times 440 \times 202}{732 \times 994}$ .... = __________

60. $\frac{308 \times 744 \times 206}{809 \times 148}$ .... = __________

D

61. 27.6 x 0.154 .... = __________

62. 2.67 x 13.8 .... = __________

63. 0.114 x 44.7 .... = ____________

64. 0.0392 x 1.75 .... = ____________

65. 0.00503 x 6.31 .... = ____________

66. 0.0152 x 0.0407 .... = ____________

67. 2.64 x 2.06 .... = ____________

68. 1.88 x 0.363 .... = ____________

69. 0.824 x 3.33 .... = ____________

70. 2.84 x 0.535 .... = ____________

71. 0.0000448 x 3.79 x 4.86 .... = ____________

72. 0.0617 x 2.86 x 4.93 .... = ____________

73. 24.6 x 3.14 x 3.14 .... = ____________

74. $\dfrac{942 \times 165 \times 438}{730 \times 65.2}$ .... = ____________

75. $\dfrac{123 \times 519 \times 923}{878 \times 854}$ .... = ____________

76. $\dfrac{620 \times 669 \times 543}{942 \times 403}$ .... = ____________

77. $\dfrac{934 \times 595 \times 256}{243 \times 400}$ .... = ____________

78. $\frac{879 \times 624 \times 826}{453 \times 992}$ .... = ____________

79. $\frac{320 \times 445 \times 359}{971 \times 692}$ .... = ____________

80. $\frac{211 \times 821 \times 493}{123 \times 795}$ .... = ____________

E

81. 6.78 x 2.38 .... = ____________

82. 0.586 x 1.92 .... = ____________

83. 7.22 x 0.000324 .... = ____________

84. 78.6 x 2.59 .... = ____________

85. 0.0557 x 8.63 .... = ____________

86. 1.66 x 4.27 .... = ____________

87. 9.12 x 6.43 .... = ____________

88. $\frac{270 \times 975 \times 229}{697 \times 753}$ .... = ____________

89. $\frac{1950 \times 731 \times 350}{575 \times 687}$ .... = ____________

90. $\frac{173 \times 130 \times 505}{780 \times 571}$ .... = ____________

91. $\dfrac{909 \times 607 \times 663}{845 \times 886}$ .... = ____________

92. $\dfrac{164 \times 146 \times 541}{109 \times 702}$ .... = ____________

93. $\dfrac{995 \times 864 \times 493}{395 \times 949}$ .... = ____________

94. $\dfrac{517 \times 980 \times 967}{541 \times 322}$ .... = ____________

95. $\dfrac{53.2 \times 0.143 \times 8620}{924 \times 75.2}$ .... = ____________

96. $\dfrac{0.0000196 \times 893 \times 0.0470}{0.00163 \times 360}$ .... = ____________

97. $\dfrac{0.302 \times 5180 \times 0.000611}{15100 \times 20100}$ .... = ____________

98. $\dfrac{93600 \times 0.0338 \times 0.000874}{0.0000124 \times 0.00474}$ .... = ____________

99. $\dfrac{25.4 \times 0.000497 \times 0.222}{12.1 \times 0.00171}$ .... = ____________

100. $\dfrac{505 \times 81.9 \times 212}{31.6 \times 493}$ .... = ____________

ANSWERS EXERCISE SET XIII

A

1. 258,000
2. 2170
3. 5410
4. 5.68

5. 0.521
6. 46.2
7. 0.114
8. 299
9. 1.93
10. 118
11. 0.0498
12. 5.97
13. 0.553
14. 0.00000393
15. 0.00209
16. 0.631
17. 0.166
18. 1020
19. 28.1
20. 6.69×10^{-8}

B

21. 12.1
22. 4.00
23. 12.7
24. 13.8
25. 0.0137
26. 196
27. 186
28. 0.0234
29. 5560
30. 28,000,000
31. 0.000109
32. 125
33. 0.000849
34. 13,400
35. 3850
36. 1810
37. 0.00689
38. 1120
39. 2520
40. 4000

C

41. 33.4
42. 15.8
43. 6.81
44. 0.0140
45. 17,100
46. 205
47. 5.51
48. 694
49. 0.707
50. 0.215

51. 621
52. 2660
53. 2870
54. 31.7
55. 2120
56. 1870
57. 59.4
58. 1030
59. 20.8
60. 394

D

61. 4.25
62. 36.8
63. 5.10
64. 0.0686
65. 0.0317
66. 0.000619
67. 5.44
68. 0.682
69. 2.74
70. 1.52
71. 0.000825
72. 0.870
73. 243
74. 1430
75. 78.6
76. 593
77. 1460
78. 1010
79. 76.1
80. 873

E

81. 16.1
82. 1.13
83. 0.00234
84. 204
85. 0.481
86. 7.09
87. 58.6
88. 115
89. 1260
90. 25.5
91. 489
92. 169
93. 1130
94. 2810
95. 0.944
96. 0.000140

97. 3.15×10^{-9}

98. 40,700,000

99. 0.135

100. 563

LESSON VIII

KEEPING THE DECIMAL

Few people realize just how much practice is necessary to master the basic slide rule skills, consequently, one of the most difficult tasks that a student is asked to perform is the job of patiently practicing each skill until it is completely mastered before attempting another.

The first seven sections of this chapter contain all the information and techniques necessary to enable one to solve multiplication and division problems with a slide rule easily and efficiently. These techniques can be mastered only by practicing each step until it becomes automatic. If you have solved correctly every problem in sections one through seven, you have mastered these techniques and are an efficient slide rule operator. You may become more efficient by studying the next two sections. In this section and the next, you will learn to keep the decimal and to use the CI scale. These techniques will not allow you to work any problems you cannot already work but they will enable you to work problems at a rate 20 to 25 per cent faster than your current speed.

If you have not worked all the problems in the first seven sections, more practice will improve your speed much more than employing these new ideas. Furthermore, these methods are effective only if their user has completely mastered all the preceding work. If you have not practiced until each step is automatic, these new procedures will prove to be confusing.

The short cut of this section is called keeping the decimal. It will allow you to work about 20 per cent faster after you have mastered it. It reduces the time spent in computing the decimal to zero. The idea for keeping the decimal is very simple. Each time you set a number, think its count and add it algebraically to the count you already have in mind. Then, remember the subtotal only. These additions and subtractions are performed while you are moving the multiplier or divider to the proximity of the number. Thus, you are using time which was previously wasted to do the very important job of placing the decimal.

You must not forget the accumulated count. Think this count audibly so you can remember it while you are reading and preparing to set the next number. Your thinking during the process of solving a problem should go something like this: Look at each factor just once, remember its digits as you add its count to the one already in mind. If you must look at the number a second time to determine its count after you have set it on your slide rule, you have defeated the purpose of carrying the decimal.

At all times keep the count in your mind and the digits on your slide rule. Once you start using this procedure, never work a problem without keeping the count. This procedure will seem hard at first because of the extra thinking it requires, but remember, you can master it.

Example one:

$$\frac{5.72 \times 81.4 \times 0.0339}{0.00821 \times 1.43} \text{ .... } = \_\_\_\_\_\_\_\_$$

Look at 5.72 and think "one" as you move the multiplier to 5.72 on the D scale. Divide by 0.00821, think "3" (1+2). Multiply by 81.4 and think "5" (3+2). Next divide by 1.43 and think "4" (5-1). Since you cannot reach 0.0339, multiply by the recording one and think "5" (4+1). Then, divide by one and think "5" again. Finally, multiply by 0.0339 and think "4" (5-1). Read the answer 1340.

Summary:

Motion	Decimal Location
1. M (5.72) D	1
2. D (0.00821) C	3 = 1+2
3. M (81.4) C	5 = 3+2
4. D (1.43) C	4 = 5-1
5. M (Recording one) C	5 = 4+1
6. D (one) C	5
7. M (0.0339) C	4 = 5-1
8. R.A. (1340) D	4

Study the summaries of the next four examples carefully. Then work out each problem keeping the decimal as you go.

Example two:

$$\frac{14.7 \times 5.09 \times 10.4}{9.42 \times 0.0472} \text{ .... } = \underline{\hspace{3cm}}$$

Summary:

Motion	Decimal Location
1. M (14.7) D	2
2. D (9.42) C	1 = 2-1
3. M (one) C	1
4. D (0.0472) C	2 = 1+1
5. M (5.09) C	3 = 2+1
6. D (Recording one) C	2 = 3-1
7. M (10.4) C	4 = 2+2
8. R.A. (1750) D	4

Example three:

$$\frac{1.63 \times 4.17 \times 6,370,000}{7.31 \times 4.65} \text{ .... } = \underline{\hspace{3cm}}$$

Summary:

Motion	Decimal Location
1. M (1.63) D	1
2. D (7.31) C	0 = 1-1
3. M (one) C	0
4. D (4.65) C	-1 = 0-1
5. M (4.17) C	0 = -1+1

Motion	Decimal Location
6. D (one) C	0
7. M (6,370,000) C	7 = 0+7
8. R.A. (1,270,000) D	7

Example four:

$\frac{81.6 \times 3.75 \times 441}{16.5 \times 2.33}$ .... = ________

Summary:

Motion	Decimal Location
1. M (81.6) D	2
2. D (16.5) C	0 = 2-2
3. M (Recording one) C	1 = 0+1
4. D (2.33) C	0 = 1-1
5. M (3.75) C	1 = 0+1
6. D (one) C	1
7. M (441) C	4 = 1+3
8. R. A. (3510) D	4

Example five:

$\frac{0.0837 \times 3.82 \times 5.12}{1.67 \times 3.54}$ .... = ________

Summary:

Motion	Decimal Location
1. M (0.0837) D	-1
2. D (1.67) C	-2 = -1-1
3. M (Recording one) C	-1 = -2+1
4. D (3.54) C	-2 = -1-1

Motion	Decimal Location
5. M (3.82) C	-1 = -2+1
6. D (one) C	-1
7. M (5.12) C	0 = -1+1
8. R.A. (0.277) D	0

EXERCISE SET XIV

Directions: Keep the decimal count as you work the following problems. Look at each factor just once and add its count to the accumulated count as you move the multiplier or divider toward the number.

A

1. $\dfrac{25.6 \times 812}{752 \times 328}$ .... = ________

2. $\dfrac{86.4 \times 23.4}{50.5 \times 26.2}$ .... = ________

3. $\dfrac{7.30 \times 1.45}{86.4 \times 140}$ .... = ________

4. $\dfrac{86.2 \times 143}{0.383 \times 245}$ .... = ________

5. $\dfrac{5.13 \times 42.8}{130 \times 784}$ .... = ________

6. $\dfrac{5.94 \times 10.6}{742 \times 521}$ .... = ________

7. $\frac{42.3 \times 0.752}{0.105 \times 12.3}$ .... = ________

8. $\frac{7.15 \times 51.2}{74.0 \times 52.3}$ .... = ________

9. $\frac{45.2 \times 23.0}{56.1 \times 752}$ .... = ________

10. $\frac{42,000 \times 561}{1.56 \times 75.2}$ .... = ________

11. $\frac{432 \times 4200}{854 \times 5.12}$ .... = ________

12. $\frac{456 \times 4.23}{4.03 \times 45.2}$ .... = ________

13. $\frac{85.4 \times 62.3}{75.4 \times 5.89}$ .... = ________

14. $\frac{45.2 \times 52.0}{35.7 \times 861}$ .... = ________

15. $\frac{12.5 \times 1.23}{45.0 \times 45.8}$ .... = ________

16. $\frac{68.4 \times 78.2}{45.8 \times 53.1}$ .... = ________

17. $\frac{94.0 \times 21.8}{41.2 \times 56.4}$ .... = ________

18. $\frac{38.2 \times 53.5}{46.7 \times 32.9}$ .... = ________

19. $\frac{34.0 \times 52.8}{42.1 \times 54.7}$ .... = __________

20. $\frac{563 \times 0.130}{532 \times 0.165}$ .... = __________

B

21. $\frac{0.550 \times 12.5}{68.7 \times 568}$ .... = __________

22. $\frac{9,520,000 \times 852}{7520 \times 86,200}$ .... = __________

23. $\frac{0.426 \times 0.404}{3.82 \times 0.260}$ .... = __________

24. $\frac{101 \times 0.0426}{303 \times 2.17}$ .... = __________

25. $\frac{1.81 \times 1.51}{3.14 \times 5.27}$ .... = __________

26. $\frac{1.44 \times 0.0617}{3900 \times 0.0372}$ .... = __________

27. $\frac{14.8 \times 3.62}{0.121 \times 39.2}$ .... = __________

28. $\frac{2.78 \times 0.0496}{8.27 \times 1.49}$ .... = __________

29. $\frac{16.2 \times 0.226}{4.37 \times 9.28}$ .... = __________

30. $\dfrac{32.6 \times 0.774}{5.23 \times 8.28}$ .... = ________

31. $\dfrac{5.43 \times 1.63}{9.28 \times 0.0296}$ .... = ________

32. $\dfrac{378 \times 442}{5.26 \times 77.9}$ .... = ________

33. $\dfrac{48.2 \times 369}{27.9 \times 4.82}$ .... = ________

34. $\dfrac{1.13 \times 2.06}{50.5 \times 0.372}$ .... = ________

35. $\dfrac{6.79 \times 16.2}{0.0000296 \times 83700}$ .... = ________

36. $\dfrac{1420 \times 30.7}{92.6 \times 0.0287}$ .... = ________

37. $\dfrac{1.39 \times 4.18}{12.5 \times 2760}$ .... = ________

38. $\dfrac{82.1 \times 0.496}{29.6 \times 3.79}$ .... = ________

39. $\dfrac{0.0488 \times 0.617}{42.8 \times 3.16}$ .... = ________

40. $\dfrac{9.82 \times 4.06}{0.362 \times 1.72}$ .... = ________

C

41. $\frac{14.8 \times 1.92}{3.09 \times 20.4}$ .... = __________

42. $\frac{6.37 \times 2.48}{4.73 \times 16.2}$ .... = __________

43. $\frac{0.00372 \times 1.29}{2.78 \times 205}$ .... = __________

44. $\frac{327{,}000 \times 1.68}{42.8 \times 3.66}$ .... = __________

45. $\frac{483 \times 792}{0.284 \times 67.2}$ .... = __________

46. $\frac{4.12 \times 2.98}{1.45 \times 3.78}$ .... = __________

47. $\frac{5.04 \times 0.000882}{92.7 \times 156}$ .... = __________

48. $\frac{181{,}000 \times 368{,}000}{2.81 \times 0.144}$ .... = __________

49. $\frac{217 \times 6.42}{1410 \times 0.728}$ .... = __________

50. $\frac{0.00526 \times 149}{117 \times 8.36}$ .... = __________

51. $\frac{2.15 \times 2.62}{7.49 \times 8.25}$ .... = __________

52. $\dfrac{0.00368 \times 9.22}{426 \times 714}$ .... = __________

53. $\dfrac{13.4 \times 0.161}{796 \times 21.6}$ .... = __________

54. $\dfrac{1.75 \times 2.69}{1.28 \times 12.7}$ .... = __________

55. $\dfrac{0.662 \times 1.49}{10.4 \times 0.0128}$ .... = __________

56. $\dfrac{13.9 \times 0.462}{12.8 \times 0.669}$ .... = __________

57. $\dfrac{0.809 \times 0.716}{4.82 \times 0.337}$ .... = __________

58. $\dfrac{0.0428 \times 0.276}{1.82 \times 6.87}$ .... = __________

59. $\dfrac{3.86 \times 0.119}{5.05 \times 0.374}$ .... = __________

60. $\dfrac{2.87 \times 0.0864}{0.862 \times 1.43}$ .... = __________

D

61. $\dfrac{2.77 \times 11.3}{1.78 \times 129}$ .... = __________

62. $\dfrac{2.63 \times 2.81}{41.8 \times 3.82}$ .... = __________

63. $\frac{7.11 \times 7.06}{2.86 \times 1.45}$ .... = ________

64. $\frac{0.0396 \times 2.82}{4.76 \times 0.0229}$ .... = ________

65. $\frac{3.64 \times 278}{62.8 \times 2.08}$ .... = ________

66. $\frac{3.64 \times 4.28}{8.62 \times 3.61}$ .... = ________

67. $\frac{5.27 \times 6.08}{1.77 \times 372}$ .... = ________

68. $\frac{865 \times 372}{2.68 \times 26.0}$ .... = ________

69. $\frac{1.49 \times 3.22}{0.343 \times 0.0179}$ .... = ________

70. $\frac{306 \times 0.00892}{3.14 \times 1.27}$ .... = ________

71. $\frac{0.000426 \times 0.432}{6.30 \times 0.0000723}$ .... = ________

72. $\frac{2.70 \times 0.274}{362{,}000 \times 46.8}$ .... = ________

73. $\frac{12.0 \times 5280}{0.00716 \times 0.300}$ .... = ________

74. $\frac{82.0 \times 2.54}{0.875 \times 9.86}$ .... = ________

75. $\dfrac{256 \times 64.0}{220 \times 0.125}$ .... = ________

76. $\dfrac{273 \times 1760}{0.866 \times 0.414}$ .... = ________

77. $\dfrac{1.73 \times 365}{0.00390 \times 360}$ .... = ________

78. $\dfrac{1.66 \times 1.24 \times 1.39}{0.00297 \times 0.0461 \times 6230}$ .... = ________

79. $\dfrac{7610 \times 3.14 \times 392}{445 \times 0.0392 \times 537}$ .... = ________

80. $\dfrac{6.71 \times 3.49 \times 1.74}{8.52 \times 8.77 \times 5.43}$ .... = ________

E

81. $\dfrac{6.21 \times 9.06 \times 3.14}{392 \times 846 \times 2{,}700{,}000}$ .... = ________

82. $\dfrac{84.0 \times 0.283 \times 0.128}{0.376 \times 492 \times 863}$ .... = ________

83. $\dfrac{3.62 \times 0.417 \times 0.0628}{0.00634 \times 2.84 \times 0.961}$ .... = ________

84. $\dfrac{0.0272 \times 0.00552 \times 0.000432}{3.74 \times 5.05 \times 8320}$ ...... = ________

85. $\dfrac{0.493 \times 862{,}000 \times 0.0000551}{2.17 \times 1.44 \times 2.79}$ ...... = ________

86. $\dfrac{0.0163 \times 0.148 \times 3.61}{4.86 \times 0.0828 \times 0.0428}$ .... = ________

87. $\dfrac{1.17 \times 24.6 \times 0.927}{3.14 \times 37{,}200 \times 0.0000296}$ .... = ________

88. $\dfrac{7.26 \times 0.0437 \times 0.876}{24.0 \times 792 \times 3680}$ .... = ________

89. $\dfrac{2130 \times 0.392 \times 83.7}{0.0290 \times 0.290 \times 1.76}$ .... = ________

90. $\dfrac{14.8 \times 0.276 \times 0.423}{3.14 \times 4.27 \times 0.0190}$ .... = ________

91. $\dfrac{0.876 \times 14.8 \times 79{,}200}{0.0000370 \times 29.0 \times 0.479}$ .... = ________

92. $\dfrac{0.0362 \times 27.0 \times 86.0}{34.0 \times 51.0 \times 0.926}$ .... = ________

93. $\dfrac{1.74 \times 0.866 \times 144}{1.73 \times 0.00362 \times 0.00497}$ .... = ________

94. $\dfrac{0.0136 \times 0.0874 \times 0.00000100}{3.14 \times 0.0726 \times 0.942}$ ..... = ________

95. $\dfrac{39.2 \times 76.4 \times 0.0726}{0.942 \times 8.28 \times 7.07}$ .... = ________

96. $\dfrac{64{,}200 \times 327 \times 816}{0.00286 \times 28.3 \times 16.0}$ .... = ________

97. $\dfrac{31.4 \times 1.74 \times 4.29}{2.76 \times 4.83 \times 8.67}$ .... = ________

98. $\dfrac{0.00323 \times 3.45 \times 0.720}{0.460 \times 0.390 \times 0.392}$ .... = ____________

99. $\dfrac{6.44 \times 7.45 \times 1.06}{15.2 \times 0.00412 \times 19.2}$ .... = ____________

100. $\dfrac{0.440 \times 3.14 \times 14.6}{0.0000226 \times 0.927 \times 9.46}$ .... = ____________

ANSWERS EXERCISE SET XIV

A

1. 0.0843
2. 1.53
3. 0.000875
4. 131
5. 0.00215
6. 0.000163
7. 24.6
8. 0.0946
9. 0.0246
10. 201,000
11. 415
12. 10.6
13. 12.0
14. 0.0765
15. 0.00746
16. 2.20
17. 0.882
18. 1.33
19. 0.780
20. 0.834

B

21. 0.000176
22. 12.5
23. 0.173
24. 0.00654
25. 0.165
26. 0.000612
27. 11.3
28. 0.0112
29. 0.0903
30. 0.583
31. 32.2

32. 408

33. 132

34. 0.124

35. 44.4

36. 16,400

37. 0.000168

38. 0.363

39. 0.000223

40. 64.0

C

41. 0.451

42. 0.206

43. 8.42×10^{-6}

44. 3510

45. 20,000

46. 2.24

47. 3.07×10^{-7}

48. 1.65×10^{11}

49. 1.36

50. 0.000801

51. 0.0912

52. 1.12×10^{-7}

53. 0.000125

54. 0.290

55. 7.41

56. 0.750

57. 0.357

58. 0.000945

59. 0.243

60. 0.201

D

61. 0.136

62. 0.0463

63. 12.1

64. 1.02

65. 7.75

66. 0.501

67. 0.0487

68. 4620

69. 781

70. 0.684

71. 0.404

72. 4.73×10^{-8}

73. 29,500,000

74. 24.1

75. 596

76. 1,340,000

77. 450

78. 3.35

79. 1000

80. 0.100

E

81. 1.97×10^{-10}

82. 0.0000191

83. 5.48

84. 4.13×10^{-13}

85. 2.69

86. 0.506

87. 7.72

88. 3.97×10^{-9}

89. 4,720,000

90. 6.78

91. 2.00×10^{9}

92. 0.0523

93. 6,970,000

94. 5.54×10^{-9}

95. 3.94

96. 1.32×10^{10}

97. 2.03

98. 0.114

99. 42.3

100. 102,000

LESSON IX

THE CI SCALE

Many of the problems which you have recently worked required frequent multiplications and divisions by one. Some of these movements are unavoidable, but efficiency demands that, whenever possible, a factor be used each time a motion is made. Use of the CI scale eliminates many of these unnecessary movements. The CI or C inverted scale is located on the divider just above the C scale. On many slide rules it is printed in red so that it will not be confused with the C scale. The graduations on the CI scale are similar to those on the C and D scales. The important difference is that the CI is read from right to left rather than from left to right like the C and D scales. The CI scale along with the C scale may be used in finding reciprocals of numbers.

Set each of the following numbers on the CI scale and read its reciprocal from the C scale. Be sure to remember that the CI scale is read from right to left.

Number (CI)	Reciprocal (C)
1. 23.2	$\frac{1}{23.2}$ ....$\frac{1}{2}$.. = 0.0431
2. 0.0035	$\frac{1}{0.0035}$ ....$\frac{3}{0}$.. = 286

Number (CI)	Reciprocal (C)

Number (CI)	Reciprocal (C)
3. 5	$\frac{1}{5}$ ....$\frac{1}{1}$.. = 0.200
4. 3.19	$\frac{1}{3.19}$ ....$\frac{1}{1}$.. = 0.313
5. 2	$\frac{1}{2}$ ....$\frac{1}{1}$.. = 0.500
6. 6780	$\frac{1}{6780}$ ....$\frac{1}{4}$.. = 0.000147
7. 0.000375	$\frac{1}{0.000375}$ ......$\frac{4}{0}$.. = 2670
8. 26100	$\frac{1}{26100}$ ....$\frac{1}{5}$.. = 0.0000383

To correctly place the decimals, look at the problems as they are written in column 2. In each case, subtract the count of the denominator which varies from the count of the numerator whose count is always one.

EXERCISE SET XV

Directions: Set each number on the CI scale and read its reciprocal on the C scale.

Number	Reciprocal
1. 487	$\frac{1}{487}$ .... = ________
2. 2.96	$\frac{1}{2.96}$ .... = ________
3. 0.185	$\frac{1}{0.185}$ .... = ________

Number	Reciprocal	
4. 0.0258	$\frac{1}{0.0258}$ .... =	________
5. 17.6	$\frac{1}{17.6}$ .... =	________
6. 95.3	$\frac{1}{95.3}$ .... =	________
7. 0.000,359	$\frac{1}{0.000359}$ .... =	________
8. 26,800	$\frac{1}{26,800}$ .... =	________
9. 146	$\frac{1}{146}$ .... =	________
10. 2980	$\frac{1}{2980}$ .... =	________

ANSWERS EXERCISE SET XV

1. 0.00205
2. 0.338
3. 5.41
4. 38.8
5. 0.568
6. 0.0105
7. 2790
8. 0.0000373
9. 0.00685
10. 0.000336

Some slide rules have a DI scale on the body which corresponds to the D scale as the CI does the C. This DI scale is simply a D scale inverted. Reciprocals may be found by setting numbers on the DI scale and reading their reciprocals from the D. If your slide rule has a DI scale work the last 18 problems over by setting the numbers on the DI and reading the reciprocals from the D. It should be pointed out at this time that since reciprocals are two numbers whose product is one, if a number is set on the C scale, its reciprocal will appear on the CI; and, if a number is set on the D scale, its reciprocal will appear on the DI.

In mathematics another name for reciprocal is multiplicative inverse and often division is defined as the inverse of multiplication. Thus, in arithmetic one learns that dividing by a number is equivalent to multiplying by its multiplicative inverse. The example $3 \div 1/2$ becomes much easier if we change it to 3×2. Analogously, $15 \times 1/3$ becomes $15 \div 3$. We thus see that there are times when it is easier to divide by the reciprocal of a number than to multiply by the number. There are other times when it is easier to multiply by the reciprocal of a number than to divide by that number. These ideas allow us to use the CI scale as a short cut in many slide rule problems.

First, let us see how the CI is used to make multiplication easier.

Example one:

$$2 \times 3 \times 4 = 2 \div 1/3 \times 4 = \frac{2 \times 4}{1/3}$$

Solution:

1. M (2) D
2. D (3) CI — This automatically places 1/3 or .333
3. M (4) C — on the C scale.
4. R.A. (24) D

Work this problem without using the CI scale; then work it using the CI scale. Do this several times and notice the difference in the amount of slide rule movement required by each method.

In practice, we do not rewrite the problem or think about the reciprocal being on the C scale. We merely apply the following rule.

CI-1 TO MULTIPLY BY A NUMBER WHEN THE DIVIDER IS BEING MOVED, SET THE NUMBER ON THE CI SCALE.

You might remember this rule by thinking "the nverted scale is used to perform the inverse operation."

<u>Example two</u>:

3.18 x 0.467 x 18.5 .... = ___________

Solution:

1. M (3.18) D
2. D (0.467) CI
3. M (18.5) C
4. R.A. (27.5) D

Note when comparing the solution of this problem ısing the CI scale to the solution without the CI scale, we see that when the CI scale is not used a recording one is required.

Solution of example two without the CI scale:

1. M (3.18) D
2. D (one) C
3. M (0.467) C
4. D (Recording one) C
5. M (18.5) C
6. R.A. (27.4) D

Not only in this case but in almost every case, use of the CI scale replaces a recording one. This knowledge along with a consideration of the method for finding the count of a reciprocal should help the student understand the necessity for the next rule.

CI-2 MOVE THE DECIMAL ONE PLACE TO THE LEFT IN EACH NUMBER SET ON THE CI SCALE AND PROCEED AS USUAL.

<u>Example two:</u>

$$\overset{1}{3.18} \times \overset{-1}{0.467} \times \overset{2}{18.5} \ \ldots\ldots\ldots\ldots\ldots \underset{1}{3} \ldots = \overset{2}{27.4}$$

EXERCISE SET XVI

<u>Directions:</u> Solve each of the following problems using the CI scale.

A

1. 0.0447 x 0.0160 x 0.293 .... = ____________

2. 0.0157 x 3.14 x 0.00762 .... = ____________

3. 0.130 x 6.44 x 0.282 .... = ____________

4. 0.184 x 3.14 x 2.86 .... = ____________

5. 0.0379 x 1.45 x 5.27 .... = ____________

6. 0.000337 x 13.6 x 77.2 .... = ____________

7. 4.82 x 3.66 x 0.0145 .... = ____________

8. 0.392 x 1.74 x 0.00200 .... = ____________

9. 7,210,000 x 0.144 x 0.000000770 .. = ____________

10. 48.2 x 0.337 x 6.25 .... = ____________

11. 0.223 x 12.9 x 0.990 .... = ____________

12. 0.714 x 2.61 x 6.78 .... = ____________

13. 14.6 x 15.2 x 1.33 .... = ____________

14. 4.32 x 7.08 x 3.88 .... = ____________

15. 2.77 x 1.05 x 3.98 .... = ____________

B

16. 0.0000737 x 1.68 x 3.41 .... = ____________

17. 0.277 x 3.88 x 1.45 .... = ____________

18. 1.77 x 0.0362 x 0.00524 .... = ____________

19. 0.736 x 2.55 x 3.92 .... = ____________

20. 3.68 x 1.42 x 0.250 .... = ____________

21. 0.136 x 9.27 x 6.17 .... = ____________

22. 0.146 x 0.837 x 0.526 .... = ____________

23. 48.2 x 36.6 x 0.145 .... = ____________

24. 4.66 x 3.25 x 14.8 .... = ____________

25. 67.8 x 26.1 x 0.0714 .... = ____________

26. 0.112 x 1.40 x 7.60 .... = ____________

27. 3.14 x 3.827 x 0.227 .... = ____________

28. 0.000217 x 0.0000278 x 2.50 ...... = ____________

29. 19.6 x 0.120 x 0.0278 .... = ____________

30. 0.684 x 0.770 x 4.62 .... = ____________

C

31. 1.62 x 3.77 x 3.79 .... = __________

32. 3.96 x 4.25 x 0.00392 .... = __________

33. 0.0149 x 4.72 x 3.14 .... = __________

34. 0.0630 x 7.24 x 14.7 .... = __________

35. 3.14 x 0.0362 x 4.97 .... = __________

36. 0.114 x 3.14 x 0.442 .... = __________

37. 3.14 x 1.69 x 1.62 .... = __________

38. 18.1 x 0.492 x 0.000339 .... = __________

39. 0.228 x 4.62 x 2.94 .... = __________

40. 0.0617 x 3.14 x 0.000449 .... = __________

41. 9.72 x 3.14 x 0.227 .... = __________

42. 0.0000666 x 1.47 x 0.229 .... = __________

43. 8.21 x 0.027 x 0.149 .... = __________

44. 0.445 x 22.7 x 0.000446 .... = __________

45. 0.360 x 87.4 x 1.98 .... = __________

ANSWERS EXERCISE SET XVI

A

1. 0.000210
2. 0.000376
3. 0.236
4. 1.65
5. 0.290
6. 0.354
7. 0.256
8. 0.00136
9. 0.799
10. 102
11. 2.84
12. 12.6
13. 295
14. 119
15. 11.6

B

16. 0.000422
17. 1.56
18. 0.000336
19. 7.36
20. 1.31
21. 7.78
22. 0.0643
23. 256
24. 224
25. 126
26. 1.19
27. 2.73
28. 1.51×10^{-8}
29. 0.0654
30. 2.43

C

31. 23.1
32. 0.0660
33. 0.221
34. 6.71
35. 0.565
36. 0.158
37. 8.60
38. 0.00302
39. 3.10
40. 0.0000870
41. 6.93

42. 0.0000224

43. 0.0330

44. 0.00451

45. 62.3

The CI scale is just as useful in division.

<u>Example</u> <u>three</u>:

$\frac{3}{4 \times 2}$ .... $= \frac{3 \times 1/2}{4}$

$\frac{3 \times 1/2}{4}$ .... $= .375$

Solution:

1. M (3) D
2. D (4) C
3. M (2) CI Which automatically places 1/2 or
4. R.A. (.375) D .5 on the C scale

CI-3 TO DIVIDE BY A NUMBER WHEN THE MULTIPLIER IS BEING MOVED, SET THE NUMBER ON THE CI SCALE.

You may remember this rule by thinking "the inverted scale is used to perform the inverse operation."

<u>Example</u> <u>four</u>:

$\frac{28.7}{0.0375 \times 2.96}$ .... $= 259$

Solution:

1. M (28.7) D
2. D (0.0375) C
3. M (2.96) CI
4. R.A. (259) D

The decimal is handled as before. It is moved one place to the left in 2.96, the number set on the CI scale.

After learning to use the CI scale you should be careful not to use it unnecessarily. Each time you move the multiplier or divider, try to use a factor from the problem. If possible, use the C scale, otherwise, use the CI. In some cases, use of the CI scale makes the problem easier, and in others, it makes it harder. You should use the CI scale only to avoid multiplying or dividing by one. This makes the problem easier.

EXERCISE SET XVII

Directions: Use the CI scale whenever it makes a problem easier.

A

1. $\dfrac{7.45}{0.332 \times 5.080}$ . . . . = ____________

2. $\dfrac{0.0296}{37.4 \times 6.52}$ . . . . = ____________

3. $\dfrac{0.399}{4.06 \times 0.0193}$ . . . . = ____________

4. $\dfrac{779}{3.04 \times 0.0178}$ . . . . = ____________

5. $\dfrac{0.276}{0.0295 \times 3.44}$ . . . . = ____________

6. $\dfrac{0.234}{3.86 \times 0.0512}$ .... = ____________

7. $\dfrac{3.07}{8.65 \times 0.931}$ .... = ____________

8. $\dfrac{44.2}{3.98 \times 6.14}$ .... = ____________

9. $\dfrac{33.2}{7920 \times 0.000526}$ .... = ____________

10. $\dfrac{1.73}{0.0293 \times 3.53}$ .... = ____________

11. $\dfrac{4.61}{77.8 \times 79.3}$ .... = ____________

12. $\dfrac{5.93}{6.42 \times 0.0372}$ .... = ____________

13. $\dfrac{8.41}{1.83 \times 2.04}$ .... = ____________

14. $\dfrac{5.14}{0.0000392 \times 6{,}720{,}000}$ .... = ____________

15. $\dfrac{0.112}{3.67 \times 453}$ .... = ____________

16. $\dfrac{2.09}{2.76 \times 52.8}$ .... = ____________

17. $\dfrac{3.62}{55.4 \times 0.637}$ .... = ____________

18. $\frac{926}{78.3 \times 83.2}$ .... = ________

19. $\frac{9.61}{5.42 \times 33.8}$ .... = ________

20. $\frac{0.292}{523 \times 2.68}$ .... = ________

B

21. $\frac{2.73}{3.99 \times 4.28}$ .... = ________

22. $\frac{0.178}{0.00496 \times 5060}$ .... = ________

23. $\frac{3.27}{8.42 \times 0.283}$ .... = ________

24. $\frac{7.42}{8.03 \times 0.614}$ .... = ________

25. $\frac{3.75}{0.0482 \times 55.3}$ .... = ________

26. $\frac{8.61}{47.2 \times 1.83}$ .... = ________

27. $\frac{0.687}{3.38 \times 6.78}$ .... = ________

28. $\frac{2.24}{6.37 \times 4.07}$ .... = ________

29. $\dfrac{0.0526}{0.728 \times 0.632}$ .... = ________

30. $\dfrac{43.6}{0.00486 \times 0.0671}$ .... = ________

31. $\dfrac{4.44}{0.0000228 \times 36{,}400}$ .... = ________

32. $\dfrac{0.296}{3.67 \times 4.88}$ .... = ________

33. $\dfrac{1.27}{5.08 \times 9.21}$ .... = ________

34. $\dfrac{0.000714}{963{,}000 \times 10.1}$ .... = ________

35. $\dfrac{86.4}{0.0833 \times 546}$ .... = ________

36. $\dfrac{2.38}{4.23 \times 0.182}$ .... = ________

37. $\dfrac{0.0432}{0.0000783 \times 1040}$ .... = ________

38. $\dfrac{64.2}{3.38 \times 58.6}$ .... = ________

39. $\dfrac{0.662}{0.322 \times 0.873}$ .... = ________

40. $\dfrac{784}{0.0634 \times 2.81}$ .... = ________

C

41. $\dfrac{6.27}{4.88 \times 2.16}$ .... = ________

42. $\dfrac{0.338}{1.64 \times 0.293}$ .... = ________

43. $\dfrac{5.07}{3.90 \times 4.68}$ .... = ________

44. $\dfrac{5.09}{3.72 \times 0.618}$ .... = ________

45. $\dfrac{0.524}{0.00864 \times 4.68}$ .... = ________

46. $\dfrac{2.71}{0.142 \times 376}$ .... = ________

47. $\dfrac{28{,}400}{1.66 \times 0.576}$ .... = ________

48. $\dfrac{0.0000006}{0.0274 \times 0.154}$ .... = ________

49. $\dfrac{2.51}{3.68 \times 1.28}$ .... = ________

50. $\dfrac{317}{0.119 \times 0.0427}$ .... = ________

51. $\dfrac{0.82}{28.0 \times 3.14}$ .... = ________

52. $\frac{0.000228}{2.70 \times 0.360}$ .... = ________

53. $\frac{0.00465}{0.993 \times 0.278}$ .... = ________

54. $\frac{0.160}{3.14 \times 32.9}$ .... = ________

55. $\frac{3.72}{0.285 \times 0.00196}$ .... = ________

56. $\frac{3.14}{0.161 \times 0.640}$ .... = ________

57. $\frac{15.0}{1050 \times 8.72}$ .... = ________

58. $\frac{4920}{32.0 \times 51.0}$ .... = ________

59. $\frac{6.10}{0.288 \times 876}$ .... = ________

60. $\frac{0.0200}{876 \times 4920}$ .... = ________

ANSWERS EXERCISE SET XVII

A

1. 4.42
2. 0.000121
3. 5.09
4. 14,400
5. 2.72
6. 1.18
7. 0.381
8. 1.81
9. 7.97

10. 16.7
11. 0.000747
12. 24.8
13. 2.25
14. 0.0195
15. 0.0000674
16. 0.0143
17. 0.103
18. 0.142
19. 0.0525
20. 0.000208

B

21. 0.160
22. 0.00709
23. 1.37
24. 1.50
25. 1.41
26. 0.0997
27. 0.0300
28. 0.0864
29. 0.114
30. 134,000
31. 5.35
32. 0.0165
33. 0.0271
34. 7.34×10^{-11}
35. 1.90
36. 3.09
37. 0.531
38. 0.324
39. 2.35
40. 4400

C

41. 0.595
42. 0.703
43. 0.278
44. 2.21
45. 13.0
46. 0.0508
47. 29,700
48. 0.000142
49. 0.533
50. 62,400
51. 0.00933
52. 0.000235
53. 0.0168
54. 0.00155
55. 6660
56. 30.5
57. 0.00164

58. 3.01

59. 0.0242

60. 4.64×10^{-9}

LESSON X

CHAPTER SUMMARY

The preceding nine sections present the counting method for teaching slide rule operation. They are written to the student who has no previous knowledge of slide rule. In this section, all the rules and definitions of this chapter are listed. They may be used as a reference or review for the student who has completed the first nine sections but has not used the slide rule enough to maintain his acquired skill. Also, they may be used by someone already familiar with the slide rule to compare the counting method to the one he is accustomed to using.

DEFINITIONS

Multiplier. The multiplier is one of the two moving parts of the slide rule. It is called the multiplier rather than the indicator or cursor because it is the part which is moved whenever a multiplication is performed.

Divider. The divider is the moving part of a slide rule which is moved whenever a division is performed. It is commonly called the slide.

Recording one. The recording one is commonly called the left index. It is located at the left end of the C scale. Each time the recording one is used, it is recorded and the count is used in determining the location of the decimal.

Count. The count of a positive number is an integer which describes the decimal location in that number. The count of a number greater than one is the number of digits to the left of the decimal. The count of a number less than one is the opposite (the negative) of the number of zeroes between the decimal and the first non-zero digit.

Keeping the Decimal. The technique of computing the decimal location while each step is being performed so that the decimal location as well as the significant digits of an answer may be written immediately after the last factor has been set is called keeping the decimal.

RULES

M-1 MOVE THE MULTIPLIER FIRST.

M-2 MOVE THE MULTIPLIER AND DIVIDER ALTERNATELY.

M-3 MOVE THE MULTIPLIER LAST.

S-1 SET THE FIRST FACTOR IN A PROBLEM ON THE D SCALE.

S-2 SET EACH FACTOR EXCEPT THE FIRST ON THE C SCALE.

S-3 READ THE ANSWER FROM THE D SCALE.

O-1 IF A FACTOR CANNOT BE REACHED WITH THE MULTIPLIER BECAUSE IT IS BEYOND THE END OF THE SCALE, MULTIPLY BY ONE.

O-2 IF THERE ARE NO UNUSED FACTORS IN THE DENOMINATOR AND THE RULES OF MOTION REQUIRE MOVEMENT OF THE DIVIDER, DIVIDE BY ONE. CHOOSE THE ONE WHICH WILL ALLOW THE NEXT STEP TO BE COMPLETED.

O-3 IF THERE ARE NO UNUSED FACTORS IN THE NUMERATOR AND THE RULES OF MOTION REQUIRE MOVEMENT OF THE MULTIPLIER, MULTIPLY BY ONE.

D-1 THE COUNT OF A PRODUCT IS THE SUM OF THE COUNTS OF ITS FACTORS.

D-2 THE COUNT OF A FRACTION IS EQUAL TO THE COUNT OF THE NUMERATOR MINUS THE COUNT OF THE DENOMINATOR.

CI-1 TO MULTIPLY BY A NUMBER WHEN THE DIVIDER IS BEING MOVED, SET THE NUMBER ON THE CI SCALE.

CI-2 MOVE THE DECIMAL ONE PLACE TO THE LEFT IN EACH NUMBER SET ON THE CI SCALE AND PROCEED AS USUAL.

CI-3 TO DIVIDE BY A NUMBER WHEN THE MULTIPLIER IS BEING MOVED, SET THE NUMBER ON THE CI SCALE.

EXERCISE SET XVIII

A

1. $\dfrac{6.76 \times 3.22 \times 0.000173}{0.0497 \times 0.772 \times 0.00321}$ .... = ________

2. $\dfrac{2.60 \times 8.00 \times 0.619}{0.0515 \times 2.75 \times 3.14}$ .... = ________

3. $\dfrac{0.292 \times 3{,}740{,}000 \times 872}{5{,}160{,}000 \times 0.209 \times 0.0000527}$ .... = ________

4. $\dfrac{0.0140 \times 0.235 \times 0.000232}{1.64 \times 0.00496 \times 2.54}$ .... = ________

5. $\dfrac{0.390 \times 0.227 \times 0.946}{0.922 \times 2.78 \times 9.45}$ .... = ________

6. $\dfrac{3.79 \times 0.00464 \times 2.22}{3.14 \times 0.0375 \times 0.194}$ .... = ________

7. $\dfrac{12.0 \times 19.6 \times 472}{321 \times 2.60 \times 3.70}$ .... = ________

8. $\dfrac{4.92 \times 0.872 \times 0.0503}{3.75 \times 38.2 \times 31{,}300}$ .... = ________

9. $\dfrac{36.7 \times 2.78 \times 0.643}{15.2 \times 9460 \times 3.14}$ .... = ________

10. $\dfrac{0.729 \times 0.463 \times 0.276}{14.2 \times 62.3 \times 0.921}$ .... = ________

11. $\frac{0.518 \times 2.93 \times 6.48}{1.27 \times 5.39 \times 0.624}$ .... = ________

12. $\frac{0.875 \times 0.375 \times 3.14}{9.62 \times 0.866 \times 1.73}$ .... = ________

13. $\frac{0.0100 \times 3.14 \times 3.75}{2.06 \times 4.12 \times 3.98}$ .... = ________

14. $\frac{0.105 \times 0.736 \times 2.45}{0.250 \times 1.46 \times 256}$ .... = ________

15. $\frac{0.320 \times 0.569 \times 0.0000827}{0.115 \times 1.80 \times 0.352}$ .... = ________

16. $\frac{0.924 \times 0.637 \times 0.00626}{3.14 \times 3.14 \times 0.394}$ .... = ________

17. $\frac{1.70 \times 2.43 \times 0.357}{0.220 \times 1.66 \times 3.59}$ .... = ________

18. $\frac{7280 \times 3.21 \times 3.14}{4280 \times 0.776 \times 3.24}$ .... = ________

19. $\frac{22.3 \times 41.7 \times 8.25}{0.828 \times 0.135 \times 2.48}$ .... = ________

20. $\frac{0.0000663 \times 484 \times 12.0}{0.0234 \times 479 \times 326}$ .... = ________

B

21. $\frac{0.0498 \times 0.236 \times 2.69}{0.776 \times 0.0812 \times 0.00704}$ .... = ________

22. $\dfrac{3.14 \times 100 \times 3.68}{4.92 \times 3.78 \times 4.69}$ .... = ____________

23. $\dfrac{76.0 \times 0.462 \times 0.0397}{8,340,000 \times 1.82 \times 2.37}$ .... = ____________

24. $\dfrac{0.427 \times 68.0 \times 0.00497}{14.2 \times 0.0815 \times 38.0}$ .... = ____________

25. $\dfrac{467 \times 0.290 \times 0.625}{146 \times 12.0 \times 0.0000392}$ .... = ____________

26. $\dfrac{0.886 \times 36.4 \times 3.14}{4.28 \times 0.00193 \times 12.0}$ .... = ____________

27. $\dfrac{2.49 \times 0.367 \times 1.48}{0.922 \times 3.26 \times 0.0492}$ .... = ____________

28. $\dfrac{0.283 \times 4.66 \times 3.14}{1.14 \times 7.64 \times 6.25}$ .... = ____________

29. $\dfrac{3.76 \times 4.92 \times 28.0}{471 \times 78.0 \times 42.0}$ .... = ____________

30. $\dfrac{86.0 \times 98.0 \times 2.66}{1.45 \times 3.08 \times 5.17}$ .... = ____________

31. $\dfrac{422 \times 863 \times 725}{326 \times 3.14 \times 3.00}$ .... = ____________

32. $\dfrac{1.49 \times 2.60 \times 3.50}{0.0000226 \times 39.2 \times 0.00239}$ .... = ____________

33. $\dfrac{0.143 \times 0.000292 \times 0.464}{0.829 \times 0.172 \times 0.545}$ .... = ____________

34. $\dfrac{474 \times 0.0100 \times 0.828}{0.909 \times 736{,}000 \times 52.7}$ .... = ________

35. $\dfrac{723 \times 525}{446 \times 607 \times 248}$ .... = ________

36. $\dfrac{535 \times 170}{904 \times 799 \times 194}$ .... = ________

37. $\dfrac{971 \times 348}{258 \times 600 \times 893}$ .... = ________

38. $\dfrac{953 \times 772}{903 \times 522 \times 506}$ .... = ________

39. $\dfrac{918 \times 675}{694 \times 152 \times 707}$ .... = ________

40. $\dfrac{574 \times 982}{327 \times 324 \times 916}$ .... = ________

C

41. $\dfrac{616 \times 719}{677 \times 611 \times 495}$ .... = ________

42. $\dfrac{810 \times 526}{873 \times 246 \times 676}$ .... = ________

43. $\dfrac{783 \times 284}{641 \times 125 \times 398}$ .... = ________

44. $\dfrac{209 \times 417}{207 \times 875 \times 651}$ .... = ________

45. $\frac{478 \times 795}{447 \times 297 \times 270}$ .... = ________

46. $\frac{705 \times 427}{605 \times 719 \times 304}$ .... = ________

47. $\frac{139 \times 640}{584 \times 339 \times 425}$ .... = ________

48. $\frac{442 \times 707}{400 \times 200 \times 334}$ .... = ________

49. $\frac{945 \times 219}{211 \times 533 \times 676}$ .... = ________

50. $\frac{349 \times 730}{596 \times 295 \times 969}$ .... = ________

51. $\frac{809 \times 400}{547 \times 506 \times 502}$ .... = ________

52. $\frac{900 \times 852}{364 \times 782 \times 848}$ .... = ________

53. $\frac{884 \times 122}{441 \times 818 \times 979}$ .... = ________

54. $\frac{141 \times 672}{749 \times 243 \times 598}$ .... = ________

55. $\frac{420 \times 201}{860 \times 518 \times 136}$ .... = ________

56. $\frac{170 \times 440}{202 \times 732 \times 994}$ .... = ________

68. $\dfrac{270 \times 975}{229 \times 697 \times 753}$ .... = ________

69. $\dfrac{1950 \times 731}{350 \times 575 \times 687}$ .... = ________

70. $\dfrac{173 \times 130}{505 \times 780 \times 571}$ .... = ________

71. $\dfrac{909 \times 607}{663 \times 845 \times 886}$ .... = ________

72. $\dfrac{164 \times 146}{541 \times 109 \times 702}$ .... = ________

73. $\dfrac{995 \times 864}{493 \times 395 \times 949}$ .... = ________

74. $\dfrac{517 \times 980}{967 \times 541 \times 322}$ .... = ________

75. $\dfrac{53.2 \times 0.143}{8620 \times 924 \times 75.2}$ .... = ________

76. $\dfrac{5830 \times 72{,}400}{0.00242 \times 811 \times 0.0859}$ .... = ________

77. $\dfrac{0.0000196 \times 893}{0.00470 \times 0.0163 \times 360}$ .... = ________

78. $\dfrac{0.000743 \times 0.000452}{45{,}500 \times 7350 \times 71{,}700}$ .... = ________

79. $\dfrac{0.302 \times 4180}{0.000611 \times 15{,}100 \times 20{,}100}$ .... = ________

57. $\dfrac{334 \times 428}{213 \times 150 \times 751}$ .... = ________

58. $\dfrac{774 \times 398}{328 \times 489 \times 770}$ .... = ________

59. $\dfrac{389 \times 862}{201 \times 363 \times 228}$ .... = ________

60. $\dfrac{308 \times 744}{206 \times 809 \times 148}$ .... = ________

D

61. $\dfrac{942 \times 165}{438 \times 730 \times 65.2}$ .... = ________

62. $\dfrac{123 \times 519}{923 \times 878 \times 854}$ .... = ________

63. $\dfrac{620 \times 669}{543 \times 942 \times 403}$ .... = ________

64. $\dfrac{934 \times 595}{256 \times 243 \times 400}$ .... = ________

65. $\dfrac{879 \times 624}{826 \times 453 \times 992}$ .... = ________

66. $\dfrac{320 \times 445}{359 \times 971 \times 692}$ .... = ________

67. $\dfrac{211 \times 821}{493 \times 123 \times 795}$ .... = ________

80. $\dfrac{93{,}600 \times 0.0338}{0.000874 \times 0.0000124 \times 0.00474}$ ... = ____________

ANSWERS EXERCISE SET XVIII

A		B	
1.	30.6	21.	71.3
2.	29.0	22.	13.2
3.	16,800,000	23.	3.87×10^{-8}
4.	0.0000369	24.	0.00328
5.	0.00346	25.	1230
6.	1.71	26.	1020
7.	36.0	27.	9.15
8.	4.81×10^{-8}	28.	0.0761
9.	0.000145	29.	0.000336
10.	0.000114	30.	971
11.	2.30	31.	86,000
12.	0.0715	32.	6,400,000
13.	0.00349	33.	0.000249
14.	0.00203	34.	1.11×10^{-7}
15.	0.000207	35.	0.00565
16.	0.000948	36.	0.000649
17.	1.12	37.	0.00244
18.	6.82	38.	0.00308
19.	27,700	39.	0.00831
20.	0.000105	40.	0.00581

C

41. 0.00216
42. 0.00293
43. 0.00697
44. 0.000739
45. 0.0106
46. 0.00228
47. 0.00106
48. 0.0117
49. 0.00272
50. 0.00150
51. 0.00233
52. 0.00318
53. 0.000305
54. 0.000871
55. 0.00139
56. 0.000509
57. 0.00596
58. 0.00249
59. 0.0202
60. 0.00929

D

61. 0.00746
62. 0.0000922
63. 0.00201
64. 0.0223
65. 0.00148
66. 0.000590
67. 0.00359
68. 0.00219
69. 0.0103
70. 0.000100
71. 0.00111
72. 0.000578
73. 0.00465
74. 0.00301
75. 1.27×10^{-8}
76. 2.50×10^{9}
77. .635
78. 1.40×10^{-20}
79. 0.00681
80. 6.16×10^{13}

CHAPTER III

POWERS AND ROOTS

In Chapter III, the A, B, and K scales are introduced. With these scales, problems involving squares, cubes, square roots, and cube roots can be solved. When these scales are combined with the C, D, and CI scales, many combinations of operations can be performed. All the problems in Appendix B can be solved using only these scales.

Some slide rules have multiple section extended scales for powers and roots. These scales are also discussed briefly. On slide rules such as the Pickett Model N-3 which has the A, B, and K power scales as well as 20 inch square root and 30 inch cube root scales, fourth, sixth, and ninth powers and roots may be read directly with one setting.

The problems of this chapter involve only one step operations. These should help to bridge the gap between problems such as those on page one of a UIL test and those on pages two through seven. Authorities have estimated that about 80 per cent of all practical problems can be solved using only the scales discussed in Chapter II. When scales discussed in this chapter are included, the figure may be raised to 95 per cent.

LESSON I

SQUARES

In many practical problems the same factor is used repeatedly. One example of this is the problem of finding the area of a square given its side. The problem of using a factor twice has been so often illustrated by this geometrical application that mathematicians have come to refer to the product obtained when a number is used as a factor twice as the square of the number. An analogous statement may be made about cubes.

Of course, we can find $(62.5)^2$ on a slide rule by simply multiplying 62.5 by 62.5, but since this type problem is encountered repeatedly, slide rule designers have built scales which allow us to perform the squaring operation in one step. To see how this can be done, take your slide rule and set 2 on the D scale by moving the multiplier until the hairline is directly over 2. Look at the A scale and see that 4 is directly under the hairline. Move to 3 on the D and read 9 from the A. Next, set 4 on the D scale and read 16 from the A. Continue this procedure setting 5, 6, 7, 8, and 9 on the D scale and reading 25, 36, 49, 64, and 81 from the A scale. As you examine the A scale, notice that it is a double scale. (A property of logarithms guarantees us that if one logarithmetic scale is a double scale, the numbers read from it will be the squares of the numbers read from the regular logarithmic scale.)

The section of the scale upon which a number is set or read will be important later, therefore, we must name the two sections of the A scale. Starting from the left and going to the right, just as we read, we will name the left most section A1 and the other A2. Thus, in the previous exercise we read 4 and 9 from A1, and 16, 25, 36, 49, 64, and 81 from A2.

Since each section of the A scale is only half as long as the D scale, readings from the A scale are not as accurate as those from the D. They are, however, accurate enough for most practical problems. By carefully inspecting your slide rule you will discover that the smallest graduation on the A scale counts two between 100 and 200; five between 200 and 500; and ten between 500 and 1000.

A more precise statement of the squaring process will now be given. To find the square of a number, set the number on the D scale and read its square from the A scale. The count of the square of a number is twice the count of the number if the square is read from scale A2. If the square is read from scale A1, the count of the square is twice the count of the number less one.

As you practice squaring numbers using the A and D scales, you should check the first ten problems by multiplication using each number as a factor twice. As you check these problems by multiplication, notice that each time a recording one is used to complete the multiplication, the square will be read from the left end of the A scale (A1), and each time the recording one is not used, the square will fall on the right end of the A scale (A2).

EXERCISE SET XIX

Directions: Square each of the following numbers using the D and A scales. Notice which section of the A scale the square is read from so the decimal may be easily placed. Check the first ten problems by multiplication using the C and D scales.

1. $(812)^2$ ..... = __________.
2. $(23.4)^2$ .... = __________.
3. $(1.45)^2$ .... = __________.
4. $(143)^2$ ..... = __________.
5. $(42.8)^2$ .... = __________.
6. $(10.6)^2$ .... = __________.
7. $(0.752)^2$ ... = __________.
8. $(51.2)^2$ .... = __________.
9. $(23.0)^2$ ... = __________.
10. $(561)^2$ .... = __________.
11. $(4200)^2$ ... = __________.
12. $(62.3)^2$ ... = __________.
13. $(52.0)^2$ ... = __________.
14. $(1.23)^2$ ... = __________.
15. $(78.2)^2$ ... = __________.
16. $(21.8)^2$ ... = __________.

17. $(53.5)^2$ ... = ____________.

18. $(52.8)^2$ ... = ____________.

19. $(0.130)^2$ .. = ____________.

20. $(12.5)^2$ ... = ____________.

21. $(852)^2$ .... = ____________.

22. $(0.404)^2$ .. = ____________.

23. $(0.0426)^2$. = ____________.

24. $(1.51)^2$ ... = ____________.

25. $(0.0617)^2$. = ____________.

26. $(3.62)^2$ ... = ____________.

27. $(0.0496)^2$. = ____________.

28. $(0.226)^2$ .. = ____________.

29. $(0.774)^2$ .. = ____________.

30. $(1.63)^2$ ... = ____________.

31. $(442)^2$ .... = ____________.

32. $(369)^2$ .... = ____________.

33. $(2.06)^2$ ... = ____________.

34. $(162)^2$ .... = ____________.

35. $(307)^2$ .... = ____________.

36. $(4.18)^2$ ... = ____________.

37. $(0.496)^2$ .. = ____________.

38. $(0.617)^2$ .. = ____________.

39. $(4.06)^2$ ... = ____________.

40. $(1.92)^2$ ... = ____________.

41. $(2.48)^2$ ... = ____________.

42. $(1.29)^2$ ... = ____________.

43. $(1.68)^2$ ... = ____________.

44. $(792)^2$ .... = ____________.

45. $(2.98)^2$ ... = ____________.

46. $(0.000882)^2$ = ____________.

47. $(36{,}800)^2$. = ____________.

48. $(6.42)^2$ ... = ____________.

49. $(149)^2$ .... = ____________.

50. $(2.62)^2$ ... = ____________.

ANSWERS EXERCISE SET XIX

1. 659,000
2. 548
3. 2.10
4. 20,400
5. 1830
6. 112
7. 0.566
8. 2620
9. 529
10. 315,000
11. 17,600,000
12. 3880
13. 2700
14. 1.51
15. 6120
16. 475
17. 2860
18. 2790
19. 0.0169
20. 156
21. 726,000
22. 0.163
23. 0.00181
24. 2.28
25. 0.00381
26. 13.1
27. 0.00246
28. 0.0511
29. 0.599
30. 2.66
31. 195,000
32. 136,000
33. 4.24
34. 26,200
35. 94,200
36. 17.5
37. 0.246
38. 0.381
39. 16.5
40. 3.69
41. 6.15
42. 1.66
43. 2.82
44. 627,000
45. 8.88
46. 7.78×10^{-7}
47. 1.35×10^{9}
48. 41.2

49. 22,200 50. 6.86

In the discussions which follow, it will be necessary to refer to "the digits of a number." This expression will mean the three digit integer between 100 and 1000 which is set on the slide rule to represent each decimal number. You may have noticed as you worked the previous problem set that the squares of numbers whose digits were in the interval from 101 to 316 inclusive were read from A1, while the squares of numbers whose digits were in the 317 to 999 interval were read from A2. Thus, the decimal could have been placed in each of the previous fifty squares by doubling the count of the number if the digits were more than 316, and by doubling the count and subtracting one if the digits were 316 or less. By remembering the digits of the last number whose square will fall on A1 (316), we can always compute the count of a square without referring to the slide rule. In practice, many problems involve multiplications and divisions first and then the squaring of the answer. When this case arises, we will save time by obtaining the decimal change from the slide rule and reading the A scale only rather than reading the D first to determine if the digits are more or less than 316 and then reading the answer from the A scale.

We may simplify the rule for the count of a square by restating it as follows:

D-3 THE COUNT OF THE SQUARE OF A NUMBER IS TWICE THE COUNT OF THE NUMBER LESS THE NUMBER OF (COMPLETE) SCALES TO THE RIGHT.

Thus, if a square falls on A1, there is one (complete) scale to the right, and if it falls on A2, there are no scales to the right.

A number may also be squared by setting it on the C scale and reading its square from the B. This is a very useful procedure in a problem such as:

$$\frac{54.2 \times (1.23)^2 \times 13.2}{0.0291 \times 9.93} \ldots\ldots\ldots\ldots\ldots\ldots = \underline{\qquad\qquad}.$$

After dividing by 0.0291, move the multiplier to 1.23 on the C and read $(1.23)^2$ or 1.51 on the B. Immediately move the multiplier to 1.51 on the C. Thus, we multiplied by 1.51, the square of 1.23, almost as easily as we handled the regular numbers. Only a slight pause at 1.23 on the C scale was required to read 1.51 on the B as we moved the multiplier to 1.51.

Some slide rules have an extended scale twice as long as the D. An extended scale must be put on the slide rule in sections. The first or upper section contains the numbers whose digits range from 100 to 316; the lower section contains the numbers whose digits range from 317 to 999. If the hairline is set on a number on the extended scale its square may be read under the hairline from the D.

When the extended scale and the D scale are used to square numbers, the count of the square is twice the count of the number less the scales below. The scales below, of course, refer to the sections of extended scale which are physically below the one on which the number is set.

EXERCISE SET XX

<u>Directions</u>: Square each of the following numbers using the B and C scales. If your slide rule has an extended scale repeat the exercise using the extended scale and the D scale.

1. $(4.23)^2$ .... = ________.	10. $(0.0864)^2$. = ________.
2. $(9.22)^2$ .... = ________.	11. $(11.3)^2$ ... = ________.
3. $(0.161)^2$ ... = ________.	12. $(2.81)^2$ ... = ________.
4. $(2.69)^2$ .... = ________.	13. $(7.06)^2$ ... = ________.
5. $(1.49)^2$ .... = ________.	14. $(2.82)^2$ ... = ________.
6. $(0.462)^2$ ... = ________.	15. $(278)^2$ .... = ________.
7. $(0.716)^2$ ... = ________.	16. $(4.28)^2$ ... = ________.
8. $(0.276)^2$ ... = ________.	17. $(60.8)^2$ ... = ________.
9. $(0.119)^2$ ... = ________.	18. $(372)^2$ .... = ________.

19. $(32.2)^2$ ... = ____________.

20. $(0.00892)^2$ = ____________.

21. $(0.432)^2$ .. = ____________.

22. $(0.274)^2$ .. = ____________.

23. $(5280)^2$ ... = ____________.

24. $(2.54)^2$ ... = ____________.

25. $(64.0)^2$ ... = ____________.

26. $(1760)^2$ ... = ____________.

27. $(365)^2$ .... = ____________.

28. $(81.0)^2$ ... = ____________.

29. $(1.24)^2$ ... = ____________.

30. $(3.14)^2$ ... = ____________.

31. $(3.49)^2$ ... = ____________.

32. $(9.06)^2$ ... = ____________.

33. $(0.283)^2$ .. = ____________.

34. $(0.417)^2$ .. = ____________.

35. $(0.00552)^2$ = ____________.

36. $(862{,}000)^2$ = ____________.

37. $(0.148)^2$ .. = ____________.

38. $(24.6)^2$ ... = ____________.

39. $(0.0437)^2$. = ____________.

40. $(0.392)^2$ .. = ____________.

41. $(0.0276)^2$. = ____________.

42. $(14.8)^2$ ... = ____________.

43. $(27.0)^2$ ... = ____________.

44. $(0.866)^2$ .. = ____________.

45. $(0.0874)^2$. = ____________.

46. $(76.4)^2$ ... = ____________.

47. $(327)^2$ .... = ____________.

48. $(1.74)^2$ ... = ____________.

49. $(3.45)^2$ ... = ____________.

50. $(7.45)^2$ ... = ____________.

ANSWERS EXERCISE SET XX

1. 17.9
2. 85.0
3. 0.0259
4. 7.24
5. 2.22
6. 0.213
7. 0.513
8. 0.0762
9. 0.0142
10. 0.00746
11. 128
12. 7.90
13. 49.8
14. 7.95
15. 77,300
16. 18.3
17. 3700
18. 138,000
19. 1040
20. 0.0000796
21. 0.187
22. 0.0751
23. 27,900,000
24. 6.45
25. 4100
26. 3,100,000
27. 133,000
28. 6560
29. 1.54
30. 9.87
31. 12.2
32. 82.1
33. 0.0801
34. 0.174
35. 0.0000305
36. 7.43×10^{11}
37. 0.0219
38. 605
39. 0.00191
40. 0.154
41. 0.000762
42. 219
43. 729
44. 0.750
45. 0.00764
46. 5840
47. 107,000
48. 3.03
49. 11.9
50. 55.5

LESSON II

CUBES

Often in mathematics, especially in problems involving volume, a factor must be repeated three times. Just as with the square, the process of using a number as a factor three times has been so often illustrated by the formula for the volume of a cube given its edge that the product obtained by using a number as a factor three times is referred to as the cube of that number.

A number may be cubed by using it as a factor three times. This is particularly easy when the digits of the number belong to the 216-464 interval and the CI scale is used.

<u>Example</u> <u>one</u>:

$(33.5)^3$ .... = 33.5 x 33.5 x 33.5

Summary:

Motion	Count
1. M (33.5) D	2
2. D (33.5) CI	3
3. M (33.5) C	5
4. R.A. (37,600) D	5

A very useful fact to remember is that when the digits of a number are less than 216 the multiplication is made easier by using the recording one twice rather than the CI scale.

<u>Example two</u>:

$(149)^3$ .... = 149 x 149 x 149

Summary:

Motion	Count
1. M (149) D	3
2. D (Recording one) C	2
3. M (149) C	5
4. D (Recording one) C	4
5. M (149) C	7
6. R.A. (3,310,000) D	7

When the digits of a number exceed 464 the multiplication is easier if the right index is used twice.

<u>Example three</u>:

$(6.89)^3$ .... = 6.89 x 6.89 x 6.89

Summary:

Motion	Count
1. M (6.89) D	1
2. D (one) C	1
3. M (6.89) C	2
4. D (one) C	2
5. M (6.89) C	3
6. R.A. (327) D	3

This three to five step process is not necessary to find the cube of a number. Almost all slide rules have a triple scale called the K scale designed in such a way that when a number is set on the D scale its cube will be on the K. This is accomplished by placing each number y on the K scale at a distance proportioned to 1/3 the log y while the numbers on the D scale are located at distances proportional to their logarithms. Thus, when the hairline falls across a number on the D scale and a number on the K scale the following equations hold.

$$\frac{1}{3} \log y = \log x$$

$$\log y = 3 \log x$$

$$\log y = \log x^3$$

$$y = x^3$$

Thus, if the multiplier is moved to 2 on the D scale, 8, the cube of 2, will be directly under the hairline on the K scale. In fact, the letter "K" used to designate the K scale is the first letter of Kubus, the German word for cube.[1] Take your slide rule and verify that $2^3 = 8$ by setting 2 on the D scale and reading 8 from K1. Set 3 on the D and read 27 from K2. Next, set 4 on the D scale and read 64 from K2. Set 5, 6, 7, 8, and 9 on the D scale and read 125, 216, 343, 512, and 729 from K3.

If later you are not sure how to cube a number repeat this routine. Observing that settings of 2, 3, and 4 on the D scale yield readings of 8, 27, and 64 on the K often gives

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[1]Isaac Asimov, An Easy Introduction to the Slide Rule (Greenwich, Connecticut: Fawcett Publications, Inc., 1965), p. 150.

a student more confidence in the procedure than proving the logarithmic equation on the preceding page. A more precise statement of the procedure for cubing a number follows:

To cube a number, set the number on the D scale and read its cube on the K.

The count of the cube of a number is three times the count of the number if the cube is read from K3; it is three times the count of the number less one if the cube is read from K2; and, it is three times the count of the number less two if the cube is read from K1. This rule may be simplified to read:

D-4 THE COUNT OF THE CUBE OF A NUMBER IS THREE TIMES THE COUNT OF THE NUMBER MINUS THE NUMBER OF (COMPLETE) SCALES TO THE RIGHT.

Notice the similarity between this rule and the one for squares. The count of the square of a number is two times the count of the number minus the number of scales to the right. If your slide rule has an extended scale three times as long as the D scale, a number may be cubed by setting it on the extended scale and reading the cube from the D scale. If the extended scale and D scale are used, the count of the cube of a number is three times the count of the number less the sections of the extended scale below the one on which the number is set.

EXERCISE SET XXI

<u>Directions</u>: Cube each of the following numbers using the D and K scales. Check your answers by multiplication using the D, C, and CI scales.

1. $(752)^3$ ... = __________.
2. $(50.5)^3$ .. = __________.
3. $(86.4)^3$ .. = __________.
4. $(.383)^3$ .. = __________.
5. $(130)^3$ ... = __________.
6. $(742)^3$ ... = __________.
7. $(0.105)^3$. = __________.
8. $(74.0)^3$ .. = __________.
9. $(56.1)^3$ .. = __________.
10. $(1.56)^3$ .. = __________.
11. $(854)^3$ ... = __________.
12. $(4.03)^3$ .. = __________.
13. $(75.4)^3$ .. = __________.
14. $(35.7)^3$ .. = __________.
15. $(45.0)^3$ ... = __________.
16. $(45.8)^3$ ... = __________.
17. $(41.2)^3$ ... = __________.
18. $(46.7)^3$ ... = __________.
19. $(42.1)^3$ ... = __________.
20. $(532)^3$ .... = __________.
21. $(68.7)^3$ ... = __________.
22. $(7520)^3$ ... = __________.
23. $(3.82)^3$ ... = __________.
24. $(303)^3$ .... = __________.
25. $(3.14)^3$ ... = __________.
26. $(3900)^3$ ... = __________.
27. $(0.121)^3$ .. = __________.
28. $(8.27)^3$ ... = __________.

29. $(4.37)^3$ .. = __________.

30. $(5.23)^3$ .. = __________.

31. $(9.28)^3$ .. = __________.

32. $(5.26)^3$ .. = __________.

33. $(27.9)^3$ .. = __________.

34. $(50.5)^3$ .. = __________.

35. $(0.000296)^3$ = __________.

36. $(92.6)^3$ .. = __________.

37. $(12.5)^3$ .. = __________.

38. $(29.6)^3$ .. = __________.

39. $(42.8)^3$ .. = __________.

40. $(0.326)^3$ .. = __________.

41. $(3.09)^3$ ... = __________.

42. $(4.73)^3$ ... = __________.

43. $(2.78)^3$ ... = __________.

44. $(42.8)^3$ ... = __________.

45. $(0.284)^3$ .. = __________.

46. $(1.45)^3$ ... = __________.

47. $(92.7)^3$ ... = __________.

48. $(2.81)^3$ ... = __________.

49. $(1410)^3$ ... = __________.

50. $(117)^3$ .... = __________.

ANSWERS EXERCISE SET XXI

1. 425,000,000
2. 129,000
3. 645,000
4. 0.0562
5. 2,200,000
6. 409,000,000
7. 0.00116
8. 405,000
9. 177,000
10. 3.80
11. 623,000,000
12. 65.5
13. 429,000
14. 45,500

15. 91,100

16. 96,100

17. 69,900

18. 102,000

19. 74,600

20. 151,000,000

21. 324,000

22. 4.25×10^{11}

23. 55.7

24. 27,800,000

25. 31.0

26. 5.93×10^{10}

27. 0.00177

28. 566

29. 83.5

30. 143

31. 799

32. 146

33. 21,700

34. 129,000

35. 2.59×10^{-9}

36. 794,000

37. 1950

38. 25,900

39. 78,400

40. 0.0474

41. 29.5

42. 106

43. 21.5

44. 78,400

45. 0.0229

46. 3.05

47. 797,000

48. 22.2

49. 2.80×10^{9}

50. 1,600,000

LESSON III

SQUARE ROOTS

For every problem whose solution involves finding the square of a number there is an inverse problem which requires finding the square root. In the section on squares, we began by setting 2 on the D scale and reading its square, 4, from A1. Now set 4 on the A1 and read 2 from the D scale. We already know that the root 2 appears on the D scale while the root's square falls on the A scale. Our problem now is to find the root given the square. That is, we are asked to find the square's root. Over the years, mathematicians have shortened this to square root. To solve the problem of finding the square root of a number, we think of the number as a square and set it on the scale from which the squares thus far have been read, the A scale, and find the root on the D scale.

This presents a slight problem. Since the A scale has two sections we must decide whether to set the given number on A1 or A2. From the lesson on squaring, we remember that squares with even counts were read from A2 while squares with odd counts were read from A1. Thus, if the count of a number is even, set it on A2 and read its square root from the D scale. Earlier we doubled the count of the root to get the count of the square.

Obviously, we may now divide the count of the number (square) by two to obtain the count of the square root (root). If the count of a number (square) is odd, we set it on A1 and read the square root (root) from the D scale. Remember that squares obtained odd counts by having the counts of their roots doubled and diminished by one because the square was read from A1. To reverse this procedure and find the count of the square root of a number, add one to the count of the square and divide by two.

The following statements which are equivalent to the above procedure are easier to use and remember. Thus, we will make them our rules. Before we state these rules we must present a few definitions. The first significant digit of a number is the first (reading from left to right) non-zero digit. The second significant digit is the digit immediately following the first. Assume in this discussion that every number contains at least two significant digits. We begin <u>at the decimal</u> and separate the number into groups of two until the first significant digit is included in a group. The group containing the first significant digit is called the key group.

R-1 IF THE KEY GROUP CONTAINS ONE SIGNIFICANT DIGIT, SET THE NUMBER ON A1. IF THE KEY GROUP CONTAINS TWO SIGNIFICANT DIGITS, SET THE NUMBER ON A2. READ THE SQUARE ROOT FROM THE D SCALE.

D-5 THE COUNT OF THE SQUARE ROOT OF A NUMBER GREATER THAN ONE IS THE NUMBER OF GROUPS TO THE LEFT OF THE DECIMAL.

D-6 THE COUNT OF THE SQUARE ROOT OF A NUMBER LESS THAN ONE IS THE OPPOSITE OF THE NUMBER OF GROUPS OF ZEROES BETWEEN THE DECIMAL AND THE FIRST NON-ZERO GROUP.

In the problems which follow the notation $(2.86)^{\frac{1}{2}}$ will indicate the principal square root of 2.86.

Example one:

$(28700)^{\frac{1}{2}}$. = __________.

Start at the decimal which is understood to be after the second zero and separate the number into groups of two. Be certain to leave two digits in the group next to the decimal.

Thus, we have $(\underline{2}'87'00)^{\frac{1}{2}}$. = __________.
The group which has been underlined is the key group. Since it contains one significant digit we set the number on A1 and read its square root from the D scale. The count of the square root is three because there are three groups to the left of the decimal. The square root thus contains one digit for each group.

$(2'87'000)^{\frac{1}{2}}$ ..... = 169.

<u>Example</u> <u>two</u>:

$(286,000)^{\frac{1}{2}}$ .. = __________.

$(\underline{28}'60'00)^{\frac{1}{2}}$. = 535.

The key group in example two contains two significant digits so we set 286 on A2. The answer 535 is read from the D scale. The count is again 3. A summary of this problem would read:

Motion	Count
1. M $(\underline{28}'60'00)$ A2	6
2. R.A. (535) D	3

<u>Example</u> <u>three</u>:

$(38.3)^{\frac{1}{2}}$ ..... = __________.

$(\underline{38}.3)^{\frac{1}{2}}$ ..... = 6.19.

Summary:

Motion	Count
1. M (38.3) A2	2
2. R.A. (6.19) D	1

<u>Example</u> <u>four</u>:

$(4060)^{\frac{1}{2}}$ ..... = __________.

$(\underline{40}'60)^{\frac{1}{2}}$ .... = 63.8.

Summary:

Motion	Count
1. M (4060) A2	4
2. R.A. (63.8) D	2

Notice that zero in the key group is a significant digit if it follows the first significant digit.

<u>Example</u> <u>five</u>:

$(0.283)^{\frac{1}{2}}$ .... = __________.

$(0.\underline{28}'3)^{\frac{1}{2}}$ ... = 0.532.

Summary:

Motion	Count
1. M (0.283) A2	0
2. R.A. (0.532) D	0

Remember the grouping begins at the decimal and may be discontinued when the group with the first significant digit is reached. In this example only one group, the group with the digits 28, needs to be marked. Because there are two significant figures in this group, 283 must be set on A2. Since the number of groups of zeroes between the decimal and the first non-zero group is zero, the count of the square root is zero.

Example six:

$(0.0462)^{\frac{1}{2}}$ ... = __________.

$(0.\underline{04}'62)^{\frac{1}{2}}$ .. = 0.215.

Summary:

Motion	Count
1. M (0.0462) A1	-1
2. R.A. (0.215) D	0

The key group in this example contains the digits 04; since the zero comes before the first significant figure it is not counted as a significant figure. Hence, the key group has one significant digit and 462 must be set on A1. The count is zero since we do not have a group of zeroes between the decimal and the first non-zero group.

Example seven:

$(0.000527)^{\frac{1}{2}}$. = ________.

$(0.00'\underline{05}'27)^{\frac{1}{2}} = 0.0230.$

Summary:

Motion	Count
1. M (0.000527) A1	-3
2. R.A. (0.0230) D	-1

EXERCISE SET XXII

Directions: Find the square roots of each of the following numbers by setting the number on the A and reading its square root from the D. Repeat each problem by setting the number on the B and reading the square root from the C. If your slide rule has an extended scale, set each number on the D and read its square root from the extended scale. When the root is to be read from the extended scale, read it from the upper scale if there is one significant digit in the key group and from the lower scale if there are two.

1. $(0.00256)^{\frac{1}{2}}$ = __________.

2. $(86.4)^{\frac{1}{2}}$ ... = __________.

3. $(7.30)^{\frac{1}{2}}$ ... = __________.

4. $(86.2)^{\frac{1}{2}}$ ... = __________.

5. $(5.13)^{\frac{1}{2}}$ ... = __________.

6. $(0.0594)^{\frac{1}{2}}$. = __________.

7. $(0.00423)^{\frac{1}{2}}$ = __________.

8. $(7.15)^{\frac{1}{2}}$ ... = __________.

9. $(45.2)^{\frac{1}{2}}$ ... = __________.

10. $(42,000)^{\frac{1}{2}}$. = __________.

11. $(432)^{\frac{1}{2}}$ .... = __________.

12. $(456)^{\frac{1}{2}}$ .... = __________.

13. $(85.4)^{\frac{1}{2}}$ ... = __________.

14. $(45.2)^{\frac{1}{2}}$ ... = __________.

15. $(12.5)^{\frac{1}{2}}$ ... = __________.

16. $(68.4)^{\frac{1}{2}}$ ... = __________.

17. $(94.0)^{\frac{1}{2}}$ ... = __________.

18. $(38.2)^{\frac{1}{2}}$ ... = __________.

19. $(34.0)^{\frac{1}{2}}$ ... = __________.

20. $(563)^{\frac{1}{2}}$..... = __________.

21. $(0.550)^{\frac{1}{2}}$ .. = __________.

22. $(9,520,000)^{\frac{1}{2}}$= __________.

23. $(0.426)^{\frac{1}{2}}$ .. = __________.

24. $(101)^{\frac{1}{2}}$ .... = __________.

25. $(1.81)^{\frac{1}{2}}$ ... = __________.

26. $(1.44)^{\frac{1}{2}}$ ... = __________.

27. $(14.8)^{\frac{1}{2}}$ ... = __________.

28. $(2.78)^{\frac{1}{2}}$ ... = __________.

29. $(16.2)^{\frac{1}{2}}$ ... = __________.

30. $(32.6)^{\frac{1}{2}}$ ... = __________.

31. $(5.43)^{\frac{1}{2}}$ ... = __________.

32. $(378)^{\frac{1}{2}}$ .... = __________.

33. $(48.2)^{\frac{1}{2}}$ ... = __________.

34. $(1.13)^{\frac{1}{2}}$ ... = __________.

35. $(6.79)^{\frac{1}{2}}$ ... = ________.

36. $(1420)^{\frac{1}{2}}$ ... = ________.

37. $(1.39)^{\frac{1}{2}}$ ... = ________.

38. $(82.1)^{\frac{1}{2}}$ ... = ________.

39. $(0.0488)^{\frac{1}{2}}$. = ________.

40. $(9.82)^{\frac{1}{2}}$ ... = ________.

41. $(14.8)^{\frac{1}{2}}$ ... = ________.

42. $(6.73)^{\frac{1}{2}}$ ... = ________.

43. $(0.00372)^{\frac{1}{2}}$ = ________.

44. $(327,000)^{\frac{1}{2}}$ = ________.

45. $(483)^{\frac{1}{2}}$ .... = ________.

46. $(4.12)^{\frac{1}{2}}$ ... = ________.

47. $(5.04)^{\frac{1}{2}}$ ... = ________.

48. $(181,000)^{\frac{1}{2}}$ = ________.

49. $(217)^{\frac{1}{2}}$ .... = ________.

50. $(0.00526)^{\frac{1}{2}}$ = ________.

ANSWERS EXERCISE SET XXII

1. 0.0506
2. 9.30
3. 2.70
4. 9.28
5. 2.26
6. 0.244
7. 0.0650
8. 2.67
9. 6.72
10. 205
11. 20.8
12. 21.4
13. 9.24
14. 6.72
15. 3.54
16. 8.27
17. 9.70
18. 6.18
19. 5.83
20. 23.7
21. 0.742
22. 3090
23. 0.653
24. 10.0

25. 1.35
26. 1.20
27. 3.85
28. 1.67
29. 4.02
30. 5.71
31. 2.33
32. 19.4
33. 6.94
34. 1.06
35. 2.61
36. 37.7
37. 1.18
38. 9.06
39. 0.221
40. 3.13
41. 3.85
42. 2.59
43. 0.0610
44. 572
45. 22.0
46. 2.03
47. 2.24
48. 425
49. 14.7
50. 0.0725

LESSON IV

CUBE ROOTS

The methods for finding cube roots are analogous to the methods for finding square roots. Each rule or statement in the lesson on square roots will apply to cube roots if the word two is replaced by three and the word square is replaced by cube each time they appear. Thus, in this lesson it will be sufficient to state the rules and give the summaries for a few example problems.

The student who seeks an explanation of these rules may refer to the last lesson and make the suggested substitutions. These definitions are given to clarify the following rules. The first significant digit of a number is its first non-zero digit. The second and third significant digits are the digits immediately to the right of the first significant digit. To find the cube root of a number, begin at the decimal and separate the digits of the number into groups of three until the first significant digit is included in a group. For this discussion we assume that each number has at least three significant digits. The group containing the first significant digit is called the key group.

R-2 IF THE KEY GROUP CONTAINS ONE SIGNIFICANT DIGIT, SET THE NUMBER ON K1. IF THE KEY

GROUP CONTAINS TWO, SET THE NUMBER ON K2. IF IT CONTAINS THREE, SET THE NUMBER ON K3. READ THE CUBE ROOT FROM THE D SCALE.

D-7 THE COUNT OF THE CUBE ROOT OF A NUMBER GREATER THAN ONE IS THE NUMBER OF GROUPS TO THE LEFT OF THE DECIMAL.

D-8 THE COUNT OF THE CUBE ROOT OF A NUMBER LESS THAN ONE IS THE NUMBER OF GROUPS OF ZEROES BETWEEN THE DECIMAL AND THE FIRST NON-ZERO GROUP.

Example one:

$(97,400)^{1/3}$ .... = __________.

$(\underline{97}'400)^{1/3}$ .... = 46.0

Summary:

Motion	Count
1. M (97,400) K2	5
2. R.A. (46.0) D	2

Example two:

$(0.0000502)^{1/3}$ ...... = __________.

$(0.000'\underline{050}'2)^{1/3}$ .... = 0.0369

Summary:

Motion	Count
1. M (0.0000502) K2	-4
2. R.A. (0.0369) D	-1

<u>Example</u> <u>three</u>:

$(7330)^{1/3}$ ...... = __________.

$(\underline{7}'330)^{1/3}$ ..... = 19.4

Summary:

Motion	Count
1. M (7330) K1	4
2. R.A. (19.4) D	2

<u>Example</u> <u>four</u>:

$(0.324)^{1/3}$ ..... = __________.

$(0.\underline{324}')^{1/3}$ .... = 0.687

Summary:

Motion	Count
1. M (0.324) K3	0
2. R.A. (0.687) D	0

<u>Example</u> <u>five</u>:

$(3.14)^{1/3}$ ...... = __________.

$('\underline{3}.14)^{1/3}$ ..... = 1.46

Summary:

Motion	Count
1. M (3.14) K1	1
2. R.A. (1.46) D	1

<u>Example</u> <u>six</u>:

$(0.000929)^{1/3}$ .. = __________.

$(0.000'\underline{929})^{1/3}$. = 0.0976

Summary:

Motion	Count
1. M (0.000929) K3	-3
2. R.A. (0.0976) D	-1

<u>Example seven</u>:

$(0.0170)^{1/3}$ .... = _________.

$(0.\underline{017}'0)^{1/3}$ ... = 0.257

Summary:

Motion	Count
1. M (0.0170) K2	-1
2. R.A. (0.257) D	0

If your slide rule has a 30 inch extended scale, the cube root may be found by setting the number on the D scale and reading the cube root from the extended scale. The root is read from the upper section of the extended scale if there is one significant digit in the key group; from the middle section if there are two; and, from the lower section if there are three.

EXERCISE SET XXIII

<u>Directions</u>: Find the cube roots of the following numbers by setting the number on the proper section of the K scale and reading the root from the D scale. If your rule has a 30 inch extended scale, repeat each problem

setting the number on the D scale and reading the root from the proper section of the extended scale.

1. $(328)^{1/3}$ ... = ________.	16. $(56.4)^{1/3}$ .. = ________.
2. $(26.2)^{1/3}$ .. = ________.	17. $(45.2)^{1/3}$ .. = ________.
3. $(140)^{1/3}$ ... = ________.	18. $(32.9)^{1/3}$ .. = ________.
4. $(245)^{1/3}$ .. = ________.	19. $(54.7)^{1/3}$ .. = ________.
5. $(784)^{1/3}$ ... = ________.	20. $(0.165)^{1/3}$. = ________.
6. $(521)^{1/3}$ ... = ________.	21. $(568)^{1/3}$ ... = ________.
7. $(12.3)^{1/3}$... = ________.	22. $(86,200)^{1/3}$ = ________.
8. $(52.3)^{1/3}$ .. = ________.	23. $(0.206)^{1/3}$. = ________.
9. $(752)^{1/3}$ ... = ________.	24. $(2.17)^{1/3}$ .. = ________.
10. $(75.2)^{1/3}$ .. = ________.	25. $(5.27)^{1/3}$ .. = ________.
11. $(5.12)^{1/3}$ .. = ________.	26. $(0.0372)^{1/3}$ = ________.
12. $(5.89)^{1/3}$ .. = ________.	27. $(39.2)^{1/3}$ .. = ________.
13. $(861)^{1/3}$ ... = ________.	28. $(1.49)^{1/3}$ .. = ________.
14. $(45.8)^{1/3}$ .. = ________.	29. $(9.28)^{1/3}$ .. = ________.
15. $(53.1)^{1/3}$ .. = ________.	30. $(8.28)^{1/3}$ .. = ________.

31. $(0.0296)^{1/3}$ = __________.

32. $(77.9)^{1/3}$ .. = __________.

33. $(4.82)^{1/3}$ .. = __________.

34. $(0.372)^{1/3}$. = __________.

35. $(83,700)^{1/3}$ = __________.

36. $(0.0287)^{1/3}$ = __________.

37. $(2760)^{1/3}$ .. = __________.

38. $(3.79)^{1/3}$ .. = __________.

39. $(3.16)^{1/3}$ .. = __________.

40. $(1.72)^{1/3}$ .. = __________.

41. $(20.4)^{1/3}$ .. = __________.

42. $(16.2)^{1/3}$ .. = __________.

43. $(205)^{1/3}$ ... = __________.

44. $(3.66)^{1/3}$ .. = __________.

45. $(67.2)^{1/3}$ .. = __________.

46. $(3.78)^{1/3}$ .. = __________.

47. $(156)^{1/3}$ ... = __________.

48. $(0.144)^{1/3}$. = __________.

49. $(0.728)^{1/3}$. = __________.

50. $(8.36)^{1/3}$ .. = __________.

ANSWERS EXERCISE SET XXIII

1. 6.90
2. 2.97
3. 5.19
4. 6.26
5. 9.22
6. 8.05
7. 2.31
8. 3.74
9. 9.09
10. 4.22
11. 1.72
12. 1.81
13. 9.51
14. 3.58
15. 3.76
16. 3.83

17. 3.56
18. 3.20
19. 3.80
20. 0.548
21. 8.28
22. 44.2
23. 0.591
24. 1.29
25. 1.74
26. 0.334
27. 3.40
28. 1.14
29. 2.10
30. 2.02
31. 0.309
32. 4.27
33. 1.69
34. 0.719
35. 43.7
36. 0.306
37. 14.0
38. 1.56
39. 1.47
40. 1.20
41. 2.73
42. 2.53
43. 5.90
44. 1.54
45. 4.07
46. 1.56
47. 5.38
48. 0.524
49. 0.900
50. 2.03

CHAPTER IV

THE ESTIMATION METHOD

The chief advantage of the counting method over other methods for slide rule operation is its more efficient system for placing decimals. The purpose of this chapter is to present the most popular of these methods, the estimation method. In this presentation, the estimation method will be presented through a series of quotations. The comments which follow these quotations are generally hostile since this investigator rates the estimation method as the weakest of all methods now in use. Every effort has been made to present the entire discussion of decimal placement from each source quoted. In most cases, this is easy since the advocates of the estimation method generally attempt to solve the problem of placing the decimal by ignoring it. A careful study of these quotations should give the reader a complete picture of the argument favoring the estimation method.

The first quotation in this series is a rather long one from John P. Ellis' book, The Theory and Operation of the Slide Rule. Section eight of his book entitled, "Locating the Decimal Point" gives an excellent presentation of the rough estimate method and the standard number method.

As has been stated before, the slide rule adds and subtracts only the mantissas of logarithms of numbers. The answer from a slide rule solution thus contains the proper significant figures, but the decimal point must be located by another means.

Probably the two most useful procedures are:

1. Rough Estimate Method. Each factor in the problem is rounded off to one or two significant figures and an approximate answer is obtained (usually by mental calculation). The decimal point is located in the significant-figure answer obtained from the slide rule so that the true and approximate answers are nearly equal.

Example:

$$X = 1.1 \times 87 \times 470$$
$$= 45$$

(significant figures only from the slide rule).

Approximately,

$$X \doteq 1 \times 80 \times 500$$
$$\doteq 40{,}000$$

Since the answer must be approximately 40,000, and the significant figures in the answer are 45, then the answer must be 45,000.

The rough estimate method is very useful and rapid for simple problems, such as have been encountered up to this time. It is recommended for such situations. It is subject, however, to severe limitations if the factors of a problem are very large (say, 17,500,000) or very small (say, 0.000049).

2. Standard Number Method. This method is applicable to any problem and is most suitable for problems containing very large or very small numbers. Each factor in a problem is changed to standard number form and an answer obtained (often by rough estimate).

Example:

$$X = 47{,}000 \times 0.0013 \times 72$$
$$= 44$$

(significant figures only from the slide rule).

Express the factors in standard number form:

$$\begin{aligned} X &= 4.7 \times 10^4 \times 1.3 \times 10^{-3} \times 7.2 \times 10 \\ &= (4.7 \times 1.3 \times 7.2) \times 10^2 \\ &\doteq 5 \times 1 \times 7 \times 10^2 \\ &\doteq 35 \times 100 \doteq 3500 \end{aligned}$$

Since the answer is approximately 3500 and the significant figures are 44, the answer must be 4400.

There are other methods in use for locating the decimal point, probably the most important of which is the so-called characteristic method. It is a theoretically correct method (just as is the standard number method), but it does not give the beginner the training in approximate calculation that is acquired in using the methods described above. For those who are interested in learning the characteristic method of decimal point location, see Appendix VII.[1]

Ellis also indicates a knowledge of the characteristic method of decimal location and gives its rules in the appendix. The fact that he does not understand that it is superior to the methods he chooses shows that he has not taken the trouble to master it or has never had occasion to work problems of such difficulty to make its advantage evident. The characteristic method is dismissed on the grounds that it does not give the beginner the training in approximate calculation that is acquired in using the other methods. This same logic would dismiss the use of logarithms as a theoretically correct method of multiplication

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[1]John P. Ellis, The Theory and Operation of the Slide Rule (New York: Dover Publications, Inc., 1961), pp. 12-13.

which does not give the beginner training in arithmetic. A slide rule student using an advanced system for placing the decimal does not need training in approximation by arithmetic.

It is interesting to carefully study Ellis' two example problems. He gives just two significant figure accuracy in his answers. This is certainly correct if one considers his data approximate. It is impossible to avoid noticing that on the slide rule before rounding the answers are 4.50×10^4 and 4.40×10^3. The chances of both the third significant figures being zero are about one in one hundred. Thus, the digits of the examples appear to be "rigged" to come out nicely. This makes one suspect that maybe the problems are also "rigged" so that the decimal will work out easily when the approximation method is used.

The following quotations are from Ira Ritow's book, The Complete Book of Slide Rule Use. Since this paper attacks rather than supports each of these works, fairness dictates that each author's entire statement concerning decimal placement be quoted.

> As a result, the slide rule does not give decimal places. Methods have been devised to keep track of the decimal place during slide rule manipulation, but although ingenious, they are easily remembered wrong and few people use them.[2]

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[2]Ira Ritow, The Complete Book of Slide Rule Use (Garden City, New York: Doubleday and Co., Inc., 1963), p. 7.

The slide rule does give decimal places just as it gives significant figures. However, in each case it is the responsibility of the operator to recognize them.

LOCATING THE DECIMAL POINT. The easiest and most foolproof method to locate the decimal point in the answer to a slide rule problem is to solve the problem approximately with a pencil and paper. For example, if the slide rule is used to solve

$$(20.7 \div 825) \times 14.03 = ?$$

the slide rule answer is 3502, but where does the decimal point go? If the problem is approximately solved as

$$(20 \div 800) \times 15 = 20 \times 15 \div 800 = 300 \div 800 = \text{about } 0.4$$

then it is obvious that the slide rule answer must be 0.3502.

A trick to make the approximate solving of a problem easier is to write the problem in fraction form and then cancel like factors in the top and bottom of the fraction. In the case of the above example the fraction form is:

$$(20.7 \div 825) \times 14.03 = \frac{20.7 \times 14.03}{825}$$

or approximately:

$$\frac{20 \times 15}{800} = \frac{2\cancel{0} \times 15}{80\cancel{0}} = \frac{\cancel{2} \times 15}{\cancel{80}\ 40} = \frac{15}{40} = \text{about } \frac{16}{40} = \frac{4}{40} = 0.4$$

A variation of the approximate method of locating the decimal point is to rewrite all the numbers in approximate form as simple numbers multiplied by powers of ten. In the above example this technique leads to:

$$(20.7 \div 825) \times 14.03 = ((2 \times 10) \div (8 \times 10^2)) \times (1.5 \times 10) = (2 \times 10 \div 8) \times 10^{-2} \times 1.5 \times 10 = (2 \div 8) \times 1.5 \times 10^{1-2+1} = (3/8) \times 10^0 = 3/8 = 0.375$$

= approximately 0.4. [3]

---

[3] Ibid. p. 23.

Locating the decimal point by solving the problem approximately with a pencil and paper is neither easy nor foolproof. The results of the experiment conducted in the Math 525 class at K.S.T.C. proved that it is not easy when the students tested spent 10 minutes on 15 simple problems.[4] Andrews students can both work these problems and determine the decimals in less than three minutes. Of this three minutes the part of the time used in determining the decimal location is around fifteen seconds. The results of the K.S.T.C. experiment also proved that this method was not foolproof when the participating students placed more decimals incorrectly than correctly. Note that the answer to Ritow's example problem should be 0.3520 or 0.352 rather than 0.3502.

An important consideration is brought out in the Dietzgen slide rule instructions manual when the statement is made that in practical problems the location of the decimal is obvious. However, an investigation of engineering problems shows that most of these problems are of about the same order of difficulty as the initial UIL contest problems. A list of practical problems may be found in <u>The Slide Rule</u> by Joseph Arnold, pages 143-178.[5] It may be true that over

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[4] A more detailed discussion of this experiment is given on page 198.

[5] Joseph Norman Arnold, <u>The Slide Rule</u> (New York: Prentice-Hall, 1954), p. 17.

half of all practical problems are so simple that the decimal location is obvious, but a student's preparation should allow him to work all slide rule problems that he might encounter. If he knows too much, then nothing is lost, however, if he knows too little, all is lost.

> In slide rule working significant figures only are considered, and the position of the decimal point is found from a rough estimate of the size of the answer. In practical problems the number of figures is obvious.[6]

Another slide rule manual has this to say about placing decimals:

> At first practice a few very simple multiplications until you gain skill. Learn to estimate the products and to place the decimal points correctly by inspection.[7]

The authors of the *K & E Slide Rule Manual* offer these rules for locating decimals in multiplication, division, and combination of operations problems.

> Rule. To find the product of two numbers, disregard the decimal points, opposite either of the numbers on the D scale set the index of the C scale, push the hairline of the indicator to the second number on the C scale, and read the answer under the hairline of the D scale. The decimal point is placed in accordance with the result of a rough calculation.

> Rule. To find the quotient of two numbers, disregard the decimal points, opposite the numerator

---

[6] *Dietzgen Slide Rule Instructions No. 1778 Redi-Log* (New York) Eugene Dietzgen, Inc.), p. 4.

[7] *Introducing the Slide Rule* (Rev. Ed.; Wabash, Indiana: Wabash Instruments and Specialities Co., Inc., 1943), p. 10.

on the D scale set the denominator on the C scale, opposite the index of the C scale read the quotient on the D scale. The position of the decimal point is determined from information gained by making a rough calculation.

Rule. To compute a number defined by a series of multiplications and divisions:

(a) arrange the expression in fractional form with one more factor in the numerator than in the denominator, (1 may be used if necessary.)

(b) push the hairline to the first number in the numerator on the DI scale.

(c) using the C or CI scale take the other numbers alternately, drawing each number of the denominator under the hairline, and pushing the hairline to each number of the numerator,

(d) read the answer on the D scale,

(e) to get a rough approximation, compute the value of the expression obtained by replacing each number of the given expression by a convenient approximate number involving one, or at most two, significant figures.[8]

Isaac Asimov, one of today's most popular writers, recommends the estimation method for locating the decimal. His book is delightfully easy to read, but a student certainly would not be able to operate a slide rule after reading it. On examining his example problem, one wonders if he is always able to read four significant figures from his slide rule or if he obtained the digit 5 in his answer

[8] Lyman M. Kells, Ph.D., Willis F. Kern, James D. Bland, K & E Slide Rule Manual (New York: Kueffel & Esser Co., 1955), pp. 8, 10, 27.

484.5 by some other method. The following excerpts identify Asimov's position on decimal location.

> If we go by the slide rule we deal only with digit-combinations and all the examples above boil down to 15 x 323 = 4845. The decimal point--that side issue--is for us to handle.
>
> There is, however, no need to feel aggrieved, for, though placing the decimal point may be a tedious necessity, it is not difficult. Occasionally, a bit of carelessness will result in a misplaced decimal point, but this can happen even in pencil-and-paper calculations. The correct response to carelessness is a sober determination to be careful, that's all.
>
> The best way to place the decimal point is to consider the problem and substitute, for the numbers involved, similar numbers that are particularly easy to handle. Such similar numbers will give you an answer that is wrong, of course, but one that is close enough to the right answer to have the decimal point in the same place. You will have an answer that is of the same order of magnitude.
>
> Now let's look at one of the problems in the list given on page 88, say, 15 x 32.3. We can convert 15 to 20 and 32.3 to 30. The numbers are changed but not the order of magnitude. It is easy to multiply 20 by 30 in our head. The answer is 600. That is not the answer we are looking for, but it is the same order of magnitude as the answer. If the slide rule tells us that the digit-combination of the product is 4845, then to make that the same order of magnitude as 600 we must write it 484.5. We know then that 15 x 32.3 = 484.5.[9]

The remainder of the references given here repeat these same ideas. They are given as further proof that the

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[9]Isaac Asimov, <u>An Easy Introduction to the Slide Rule</u> (Greenwich, Connecticut: Fawcett Publications, Inc., 1965), pp. 88-90.

estimation method is the most popular one now being taught. The experiments to which Arnold refers were carried out by C. N. Schuster.[10] Here are Arnold's methods for decimal location.

> If the decimal point is determined independently of the slide rule, a single logarithmic cycle (1 to 10) is sufficient for all problems in division and multiplication. And although decimal point determination initially appears to be quite a chore, a systematic method and practice make it very little trouble.
>
> Several methods for placing the decimal point have been devised. It is not intended to include all of these methods because to do so might be more confusing than helpful. Experiments in teaching slide rule operation by different methods have shown that about one third of the errors of beginners occur in misplacing the decimal point. Also, a considerable reduction in the number of errors results if instruction is given in the "standard number" method. This method and one which may be called the "similar simple number scheme" are presented here.[11]

Several programmed texts on slide rule have recently appeared. The Tutor Text Series offers as its contribution a book entitled, The Slide Rule by Robert Saffold and Ann Smalley. They do not explicitly state that they use the standard number method but all of the example problems employ this or the simple number method in their solutions. The

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[10] Carl N. Schuster, A Study of the Problems in Teaching the Slide Rule (New York: Bureau of Publications, Columbia University, 1940), p. 43.

[11] Joseph Norman Arnold, The Slide Rule (New York: Prentice-Hall, 1954), p. 17.

following excerpt shows how the example problem is solved.

> Factoring a number so that it can be written as the product of some number between 1 and 10 multiplied by a power of 10 is called writing the number in scientific notation. With this notation it is possible to do many very involved calculations quickly and easily.
>
> For example, consider the problem
>
> $$\frac{285 \times 3{,}569 \times 92}{9{,}472} = ?$$
>
> We begin by writing in scientific notation each of the numbers involved:
>
> $$\frac{2.85 \times 10^2 \times 3.57 \times 10^3 \times 9.2 \times 10^1}{9.47 \times 10^3}$$
>
> (3.569×10^3 has been rounded off to 3.57×10^3, and 9.472 has been rounded off to 9.47×10^3, because ordinarily the slide rule can't be read accurately to more than three significant figures.)
>
> Next, we make a rough estimate of the final answer, by combining the powers of 10, rounding off each of the other factors to the nearest integer, and performing the indicated multiplication and division. Since 2.85 is almost 3, 3.57 is closer to 4 than to 3, 9.2 is just over 9, and 9.47 is closer to 9 than to 10, we get
>
> $$\frac{3 \times 10^2 \times 4 \times 10^3 \times 9 \times 10^1}{9 \times 10^3} = \frac{3 \times 4 \times \cancel{9} \times 10^6}{\cancel{9} \times 10^3} =$$
>
> 12×10^3, or 1.2×10^4. So the complete answer should be somewhere near 1.2×10^4. Complete the problem on your slide rule, and select the correct answer below.[12]

---

[12] Robert Saffold, Ann Smalley, The Slide Rule (Garden City, New York: Doubleday and Co., Inc., 1962), p. 82.

Fredrick Post Company came out with a programmed text entitled Learn Basic Slide Rule on Your Own. They apply essentially the same techniques as Saffold and Smalley. An example problem from this book follows.

Study the following procedures, then use them to solve the practice problem.

Example: $\dfrac{26 \times 79{,}800 \times .00633}{.0081 \times 7{,}800{,}000} =$

1. Express each factor in scientific notation ..

.... $\dfrac{2.6 \cdot 10^{1} \times 7.98 \cdot 10^{4} \times 6.33 \cdot 10^{-3}}{8.1 \cdot 10^{-3} \times 7.8 \cdot 10^{6}} =$

2. Write the numerator and denominator in their simplest forms .... $\dfrac{2.6 \times 7.98 \times 6.33 \cdot 10^{2}}{8.1 \times 7.8 \cdot 10^{-3}} =$

3. Subtract the denominator's exponent from the numerator's exponent ..... $\dfrac{2.6 \times 7.98 \times 6.33 \cdot 10^{-1}}{8.1 \times 7.8} =$

4. Round the factors and multiply ....

.... $\dfrac{3 \times \cancel{8} \times 6}{8 \times \cancel{8}} \cdot 10^{-1} =$

5. Determine the estimated answer ....

.... $\dfrac{18}{8} \cdot 10^{-1} = .2$

6. Calculate .... Slide rule answer: .208.

...When multiplying or dividing a combination of four or more numbers, it is impractical to attempt to formulate a set of procedures to prevent running "off scale." Therefore, the remaining four pages of this lesson are designed to allow you to solve typical problems following accepted procedures. You

> should, however, use your knowledge of scientific notation to determine the decimal point location before solving the problem with the slide rule.[13]

Mittlestadt's book, _Basic Slide Rule Operation_, gives a simple two step procedure for locating the decimal point. Unfortunately, the rough calculation proves to be rougher for students than most authors are willing to admit.

Mittlestadt's solution to the decimal location problem follows.

> Now what do we do if decimal points are involved? These can be easily placed in the answer if you follow two simple steps:
>
> 1. Make the calculation on your slide rule without regard to the decimal points.
>
> 2. Then, by rough calculation, determine the location of the decimal point in the answer.[14]

Graesser follows the crowd when he makes these three statements concerning decimal location.

> Rough estimates are also the best means of fixing the decimal point in many results obtained with the slide rule.
>
> ...This sequence of figures is all that we need since we shall fix the decimal point independently of the slide rule by means of a rough calculation.
>
> ...We find the figures of the dividend and divisor on D and C, respectively, no matter where their

---

[13] _Learn Basic Slide Rule on Your Own_ (Chicago: The Frederick Post Company, 1967), pp. 88,82.

[14] W. S. Mittelstadt, _Basic Slide Rule Operation_ (New York: McGraw Hill Book Company, 1964), p. 37.

> decimal points may be, and then fix the decimal point in the quotient by a rough calculation using 3.6.[15]

Finally the Pickett slide rule manual states this rule for locating the decimal in combination of operation problems.

> Rule. To determine the decimal point, first express the numbers in standard form. Carry out the indicated operations of multiplication or division, using the laws of exponents to combine the exponents until a single power of 10 is indicated. If desired, write out the resulting number, using the final exponent of 10 to determine how far, and in what direction, the decimal point in the coefficient should be moved.[16]

Decimal location plays such a vital role in any method of teaching slide rule operation that teaching methods may be classified in accordance with the decimal placing techniques they recommend. The next experiment was designed to test claims made by proponents of the estimation method. The estimation method could well be called the mantissa method or the halfway method for it completely ignores those properties of logarithms and exponents which allow the decimal to be calculated by addition and subtraction of integers.

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[15] R. F. Graesser, Understanding The Slide Rule (Patterson, New Jersey: Littlefield, Adams and Company, 1963), pp. 22, 37-38.

[16] Maurice L. Hartung, How To Use Log Log Slide Rules (Chicago: Pickett, Inc., 1953), p. 27.

The following experiment proves the inefficiency of the estimation method of placing decimals. Thirty-three students in Math 525 at Kansas State Teachers College in Emporia were given a test in approximating the decimal location in simple slide rule problems. The three significant digits of each answer were given and their instructions were to place the decimal in the answers by estimation. This, according to the slide rule authorities, would be an easy job. But, the experience of this investigator had made him doubt that to an average student this job would seem easy. The students in this test group were better prepared than the average for this job. All thirty-three were college graduates; eight had master's degrees; thirty taught mathematics; and twenty taught their students estimation.

These were the results. The average teacher attempted only fifteen problems in ten minutes and placed fewer than seven decimals correctly. The accuracy of the entire group was 44 per cent. In comparison, the average slide rule student in Andrews using the counting method will develop to the point where he can work thirty-five problems in ten minutes with above 90 per cent accuracy. The results of this experiment prove that a student taught to operate a slide rule by the counting method can both work the problem and place the decimal quicker than he could place the decimal by estimation if he were given the digits. On problems which are as difficult as those on UIL slide rule tests, the counting method is as much of an advancement over the estimation method as the slide rule is over the pencil.

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BIBLIOGRAPHY

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APPENDIX A

Do NOT unfold this paper until

the Contest Director

gives you permission to do so!

THE UNIVERSITY INTERSCHOLASTIC LEAGUE

Slide Rule Contest

Number 185

Conference: ....

Contestant's Serial Number: ....

Date of Contest: ....

Location of Contest: ....

Contestant's Net Score: ....

$22.8 \times 3.41 \times 0.667$ ------------------------- equals ____________

$0.0414 \times 13.2 \times 15.4$ ------------------------ equals ____________

$7070 \times 1.83 \times 0.715$ ------------------------- equals ____________

$6.47 \times 0.00228 \times 0.136$ ---------------------- equals ____________

$\frac{715 \times 2.86}{31.5}$ ---------------------------------- equals ____________

$\frac{4.26}{3.11 \times 17.2}$ --------------------------------- equals ____________

$\frac{0.516 \times 0.337}{2.81 \times 6.04}$ ------------------------------- equals ____________

$\frac{7.52 \times 3.01 \times 0.444}{0.838}$ ------------------------- equals ____________

$\frac{0.0692}{1.43 \times 0.227 \times 1.45}$ ------------------------- equals ____________

$\frac{9.08 \times 3.76 \times 0.647}{0.373}$ ------------------------- equals ____________

$\frac{8.11 \times 0.374 \times 0.637}{0.191 \times 5260 \times 38.2}$ ------------------------ equals ____________

$\frac{92.8 \times 3.67 \times 8.08 \times 3.21}{0.00337 \times 608{,}000}$ ------------------- equals ____________

$\frac{0.0309 \times 12.1 \times 62.4 \times 828}{12{,}300 \times 0.000232 \times 0.0728}$ ------------------ equals ____________

$\frac{52.5 \times 3.82 \times 0.219 \times 4.04}{13.6 \times 8.01 \times 92.6 \times 0.981}$ ------------------ equals ____________

$$\frac{\sqrt{25.6} \times 7.22 \times 0.0704}{0.00884 \times 267 \times 5280}$$ ---------------------- equals ________

$$\frac{1.7 \times \sqrt{0.171} \times 0.0335}{.866 \times 0.505 \times \sqrt{2.91}}$$ ---------------------- equals ________

$$\frac{(2.97)^2 \times 5.06 \times 0.000392}{16.1 \times 0.00922 \times \sqrt{476}}$$ -------------------- equals ________

$$1.6 \times \sqrt{0.0837} \times 2.66 \times 3.82$$ -------------- equals ________

$$\frac{\sqrt{71.5} \times 428 \times 0.0619}{8.2 \times (52.2)^2 \times 0.307}$$ ---------------------- equals ________

$$\frac{.86 \times (0.0225)^2 \times 1.79}{.0361 \times \sqrt{5.01} \times 28.6}$$ ---------------------- equals ________

$$\sqrt{29{,}700} \times 0.0468 \times (0.00913)^2$$ -------------- equals ________

$$\frac{9.6 \times (14{,}300)^2 \times 0.0000191}{30.4 \times 14.6 \times 2.77 \times 1.99}$$ ---------------- equals ________

$$\sqrt{0.000721} \times 0.0833 \times (2610)^2$$ -------------- equals ________

$$\frac{1.98 \times (0.0361 \times 47{,}200)^2}{.14 \times \sqrt{7.28} \times 13.8 \times 72.6}$$ ----------------- equals ________

$$\frac{(3.84)^3 \times 0.00273 \times 4.28}{6.02 \times \sqrt{1.45} \times 2.37}$$ -------------------- equals ________

$$\frac{14.7 \times 62.4 \times 5280}{(4.07 \times 3.25)^3 \times (2.16)^2}$$ -------------------- equals ________

$$\frac{\sqrt{7.14 \times 0.226} \times 0.00771}{32.6 \times 0.0402 \times 3940}$$ -------------------- equals ________

$$\frac{\sqrt[3]{2.86} \times 0.117 \times (0.0183)^3}{27.6 \times 13.4 \times 0.0000282}$$ ---------------- equals ______________

$$\frac{86.3 \times 1.44}{(12.6 \times 0.523)^2} \times \sqrt{\frac{0.391}{0.0146}}$$ ---------------- equals ______________

$$\frac{72.4 \times 0.276}{0.357 \times 64.4} \times \sqrt{\frac{21,300}{428 \times 397}}$$ ---------------- equals ______________

$$\sqrt[3]{0.615 \times 4.83} \times (1.82 \times 3.96)^3$$ ------------- equals ______________

$$\frac{(62,500,000 \times 0.0224)^2 \times 0.0343}{7.86 \times (88.4)^3 \times 1.92 \times 4.63}$$ ------------- equals ______________

$$(3.85 \times 0.661)^3 \times \left[\frac{864 \times 281}{352 \times 673}\right]^2$$ -------------- equals ______________

$$\sqrt[3]{\frac{6.42}{38,100}} \times \frac{\sqrt{0.000782 \times 4.69}}{2.88 \times 1.54 \times 3.65}$$ -------------- equals ______________

$$\sqrt{\frac{0.000627}{312 \times 456}} \times \frac{(2.65 \times 7.88)^2}{0.00000297}$$ --------------- equals ______________

$$\frac{(\pi \times 3.81 \times 5280 \times 1760)^2}{(325 \times 617 \times 0.515 \times 0.927)^3}$$ ---------------- equals ______________

$$\left[\frac{3.79 \times 4200}{61.7 \times 582}\right]^2 \times \sqrt{\sqrt{1.79 \times 8.24}}$$ ----------- equals ______________

$$\frac{\sqrt{0.396 \times 0.473} \times \pi^2 \times 2.86}{4.82 \times 0.00815 \times (7.74 \times 3.68)^3}$$ ------------- equals ______________

$$\sqrt{\frac{8.19 \times 46.3 \times 0.275}{14,600 \times 204 \times \sqrt{\pi}}} \times \left[\frac{0.827}{0.0431}\right]^2$$ ---------- equals ______________

$$\frac{25{,}000 \times (0.119 \times 0.037)^3 \times 1.88}{\pi \times 6.15 \times 105 \times 1.23 \times 2.68}$$ ------------- equals ______________

$$\sqrt[3]{3.82 \times (0.0269 \times \pi)^2} \times \left[(0.0827)^2\right]^2$$ ------- equals ______________

$$\left[\frac{0.888 \times 3.75}{6.25 \times 4.08}\right]^3 \times \frac{39{,}200{,}000}{(864 \times 392)^2}$$ --------------- equals ______________

$$\frac{\sqrt{1.77 \times 0.862 \times 15.7 \times 3.66}}{4.92 \times 5.08 \times 1.79 \times 25.4}$$ ---------------- equals ______________

$$\sqrt{\frac{19.7 \times 0.118}{3.26 \times 0.0175}} \times \sqrt[3]{\frac{\pi^2 \times \sqrt{0.227}}{0.00463 \times 1.97}}$$ -------- equals ______________

$$\frac{(24 \times 365 \times 1.02)^2 \times 0.00000637}{5280 \times 12 \times 2.54 \times (0.0429)^3}$$ ------------- equals ______________

$$\left[\frac{0.392}{10.4}\right]^2 \times \sqrt[3]{\frac{27.6}{38{,}400}} \times \sqrt{\frac{38.6}{75 \times \pi}}$$ ------------ equals ______________

$$\left[\frac{14.8 \times 1.71 \times 62.4}{93{,}200{,}000 \times 0.866}\right]^2 \times \sqrt{\frac{42.5}{0.00373}}$$ ---------- equals ______________

$$\frac{\sqrt{\sqrt{\pi} \times 14.7} \times 0.221 \times 0.00393}{(100 \times \pi)^2 \times (0.0042 \times 0.0396)^3}$$ ------------ equals ______________

$$\sqrt{\sqrt{(16.1 \times 0.456)^3} \times 3.82 \times 0.132}$$ --------- equals ______________

$$\frac{0.356 \times \pi^3 \times (9.21 \times 0.0066)^2}{(2.46)^2 \times 19.4 \times 0.11 \times 0.396}$$ -------------- equals ______________

$$\sqrt{\frac{0.269}{24 \times 5.2}} \times \frac{(4.8 \times 6.25 \times 0.0428)^2}{0.327 \times 0.464 \times 0.928}$$ --------- equals ______________

$$\left[\frac{0.0883 \times 5.26}{92.7 \times 0.00348}\right]^2 \times \frac{1.73 \times 0.0225}{(1.04 \times 0.271)^3}$$ ---------- equals ______________

$$\frac{8.29 \times \sqrt{0.446 \times \sqrt{0.409 \times 13.6}}}{\left[(6.23 \times 0.0921)^2 \times 4.26 \times 1.75\right]^2}$$ ----------- equals ______________

$$\sqrt{\sqrt{\frac{17 \times \pi}{0.000828}}} \times \left[\frac{785}{362}\right]^3 \times \left[\frac{\pi \times 5.06}{19 \times 3.21}\right]^2$$ ------ equals ______________

$$\sqrt{\frac{0.025 \times 236{,}000}{13.2 \times 282 \times \sqrt{\pi}}} \times \left[\frac{729 \times 462}{(1.68 \times 324)^2}\right]^2$$ ---- equals ______________

$$\frac{(2.78 \times 62.5)^3}{39{,}200{,}000} \times \left[\frac{7.12 \times \pi^2}{4.66 \times 327}\right]^2$$ -------------- equals ______________.

$$\frac{\sqrt{\frac{1.45}{0.282}} \times \sqrt[3]{\frac{9.37 \times 0.00462}{0.538 \times 63{,}700}}}{\pi^3 \times 2.36 \times 14 \times 2.81 \times 127}$$ ---------------- equals ______________

$$\frac{1.09 \times 0.0962}{\left[\frac{1.44 \times 2.68}{3.79 \times 8.21}\right]^2} \times \left[\left[\frac{0.337 \times 65}{3.29 \times 2.77}\right]^3\right]^2$$ ----------- equals ______________

$$\frac{\sqrt{(0.00792 \times 32.6)^3 \times 0.68 \times 3.22}}{\sqrt[3]{\pi} \times (14.2 \times 0.179 \times 0.236)^2}$$ ----------- equals ______________

$$\sqrt{\frac{\sqrt{\frac{2.7 \times 6.4}{0.0000045}}}{0.0000000396}} \times \frac{\left[\frac{\pi^2}{72}\right]^2 \times \frac{17}{64}}{782{,}000{,}000}$$ ---------------- equals ________________

$$\left[\frac{3.82}{\left[\frac{6.08}{0.99}\right]^2}\right]^3 \times \left[\frac{14.7 \times 62.4 \times 144}{5280 \times 12 \times \pi}\right]^2$$ ------------ equals ________________

$$\frac{\sqrt{\sqrt{\sqrt{8.75 \times 0.39} \times 1.36} \times 0.0843}}{\left[\left[(0.308 \times 2.8)^2 \times 7.45\right]^2 \times 0.143\right]^2}$$ ---------- equals ________________

$$\frac{\left[\frac{3.16}{0.45}\right]^3 \times \left[\frac{37 \times 16}{43 \times 86}\right]^2 \times \frac{\sqrt[3]{\pi}}{\pi^2}}{14.7 \times 1760 \times \sqrt{0.316 \times 0.055}}$$ ---------------- equals ________________

$$\left[\frac{876 \times 0.0412}{\sqrt{\frac{19.8 \times 4.66}{6.04 \times 7.21}}}\right]^3 \times \left[\frac{3.09 \times 6.41}{\sqrt[3]{\frac{25 \times 0.673}{4.21 \times 7.7}}}\right]^2$$ --------- equals ________________

$$\left[\frac{\frac{92.4 \times 1.37}{2.6 \times 0.278}}{\left[\frac{14.9 \times 0.303}{5.26 \times \pi}\right]^2}\right]^2 \times \sqrt{\sqrt{\frac{84 \times 0.61}{0.225 \times 763}}}$$ ------ equals ________________

$$\frac{\sqrt{\sqrt{\frac{3.6 \times 7.7}{0.0000275}}} \times \left[\frac{13}{16} \times 2.79\right]^3}{(4.88 \times 5280 \times 12)^2 \times 3.84}$$ -------------- equals ________________

$$\frac{\sqrt[3]{\frac{6.28 \times \pi}{0.42 \times 3.16}}}{\frac{69{,}200{,}000}{278 \times 846}} \times \frac{\pi^3}{12} \times \frac{\left[\frac{2.16}{3.77}\right]^2}{0.00818}$$ ---------------- equals ________________

$$\frac{\sqrt{\sqrt{2.17} \times 0.424}}{\left[\frac{2.84 \times 1.66}{3.71 \times 4.08}\right]^2} \times \left[\frac{(3.16)^2}{\sqrt[3]{4.89}}\right]^2$$ ------------- equals ____________

$$\frac{\left[\frac{1.73 \times 0.414}{13.6 \times 0.866}\right]^3 \times \sqrt[3]{39.4 \times 2.54}}{(2.06 \times 0.0115)^2 \times 19.7 \times 3680}$$ ------------- equals ____________

$$\sqrt{\frac{\sqrt{\frac{64}{2.16}}}{\sqrt[3]{\frac{9.6}{0.414}}}} \times \frac{\left[\frac{364}{722}\right]^2}{\frac{1.63}{0.808}} \times \frac{7}{16}$$ ------------------- equals ____________

$$\frac{\sqrt[3]{\sqrt[3]{\pi} \times 5.27} \times 0.0936 \times 12}{\sqrt{\frac{2.83 \times 6.45}{\sqrt{19.6 \times 0.335}}} \times \left[\frac{8.23 \times 1.55}{\pi \times 7.06}\right]^2}$$ ---------- equals ____________

$$\left[\frac{\frac{4.28}{3.69}}{\left[\frac{5.42}{8.37}\right]^3}\right]^2 \times \frac{\sqrt{\sqrt{\pi}}}{\left[\frac{1.72}{4.68}\right]^3} \times \frac{\sqrt[3]{\frac{17}{3.28}}}{0.774}$$ ------------- equals ____________

$$\frac{\frac{826 \times 3970}{0.000000327}}{\left[\frac{10.7 \times 3.79}{0.00000882}\right]^2} \times \sqrt{\frac{\sqrt[3]{\frac{2.66}{0.927}}}{0.0000262}}$$ --------------- equals ____________

$$\frac{\sqrt[3]{(37{,}800{,}000 \times 676)^2 \times 0.00042}}{\left[\frac{2.7 \times 3.68}{0.001 \times 1.92}\right]^2 \times \pi^3 \times 92 \times 0.04}$$ ----------- equals ____________

$$\frac{\sqrt{\sqrt{\sqrt{19.3}}}}{\left[\frac{0.00427}{1.3 \times 1.8}\right]^3} \times \left[\frac{\frac{2.77}{3.64}}{\frac{19.9}{32.6}}\right]^2 \times \frac{\sqrt[3]{\pi}}{\frac{6.11}{4.82}}$$ ------------- equals ____________

SLIDE RULE ANSWER KEY #185

1.	5.19×10^{1}	5.17×10^{1}	to	5.21×10^{1}
2.	8.42	8.40		8.44
3.	9.25×10^{3}	9.23×10^{3}	to	9.27×10^{3}
4.	2.01×10^{-3}	1.99×10^{-3}	to	2.03×10^{-3}
5.	6.49×10^{1}	6.47×10^{1}	to	6.51×10^{1}
6.	7.96×10^{-2}	7.94×10^{-2}	to	7.98×10^{-2}
7.	1.02×10^{-2}	1.00×10^{-2}	to	1.04×10^{-2}
8.	1.20×10^{1}	1.18×10^{1}	to	1.22×10^{1}
9.	1.47×10^{-1}	1.45×10^{-1}	to	1.49×10^{-1}
10.	5.92×10^{1}	5.90×10^{1}	to	5.94×10^{1}
11.	5.03×10^{-5}	5.01×10^{-5}	to	5.05×10^{-5}
12.	4.31	4.29	to	4.33
13.	9.30×10^{4}	9.28×10^{4}	tp	9.32×10^{4}
14.	1.79×10^{-2}	1.77×10^{-2}	to	1.81×10^{-2}
15.	2.06×10^{-4}	2.04×10^{-4}	to	2.08×10^{-4}
16.	4.03×10^{-1}	4.01×10^{-1}	to	4.05×10^{-1}
17.	5.40×10^{-3}	5.38×10^{-3}	to	5.42×10^{-3}
18.	1.22×10^{2}	1.20×10^{2}	to	1.24×10^{2}
19.	7.01×10^{-3}	6.99×10^{-3}	to	7.03×10^{-3}
20.	1.91×10^{-3}	1.89×10^{-3}	to	1.93×10^{-3}
21.	6.72×10^{-4}	6.70×10^{-4}	to	6.74×10^{-4}
22.	1.59×10^{2}	1.57×10^{2}	to	1.61×10^{2}
23.	1.52×10^{4}	1.50×10^{4}	to	1.54×10^{4}
24.	3.46×10^{2}	3.44×10^{2}	to	3.48×10^{2}
25.	3.85×10^{-2}	3.83×10^{-2}	to	3.87×10^{-2}
26.	4.49×10^{2}	4.47×10^{2}	to	4.51×10^{2}
27.	1.90×10^{-6}	1.88×10^{-6}	to	1.92×10^{-6}
28.	9.76×10^{-5}	9.74×10^{-5}	to	9.78×10^{-5}
29.	1.48×10^{1}	1.46×10^{1}	to	1.50×10^{1}
30.	3.08×10^{-1}	3.06×10^{-1}	to	3.10×10^{-1}
31.	5.38×10^{2}	5.36×10^{2}	to	5.40×10^{2}
32.	1.39×10^{3}	1.37×10^{3}	to	1.41×10^{3}
33.	1.73×10^{1}	1.71×10^{1}	to	1.75×10^{1}
34.	2.07×10^{-4}	2.05×10^{-4}	to	2.09×10^{-4}
35.	9.75×10^{3}	9.73×10^{3}	to	9.77×10^{3}
36.	1.41×10^{1}	1.39×10^{1}	to	1.43×10^{1}
37.	3.85×10^{-1}	3.83×10^{-1}	to	3.87×10^{-1}
38.	1.35×10^{-2}	1.33×10^{-2}	to	1.37×10^{-2}
39.	1.64	1.62	to	1.66
40.	6.00×10^{-7}	5.98×10^{-7}	to	6.02×10^{-7}
41.	1.41×10^{-5}	1.39×10^{-5}	to	1.43×10^{-5}
42.	7.61×10^{-7}	7.59×10^{-7}	to	7.63×10^{-7}
43.	8.24×10^{-3}	8.22×10^{-3}	to	8.26×10^{-3}
44.	5.12×10^{1}	5.10×10^{1}	to	5.14×10^{1}
45.	4.00×10^{1}	3.98×10^{1}	to	4.02×10^{1}
46.	5.15×10^{-5}	5.13×10^{-5}	to	5.17×10^{-5}
47.	4.09×10^{-8}	4.07×10^{-8}	to	4.11×10^{-8}
48.	9.76×10^{3}	9.74×10^{3}	to	9.78×10^{3}
49.	2.27	2.25	to	2.29
50.	7.98×10^{-3}	7.96×10^{-3}	to	8.00×10^{-3}
51.	5.44×10^{-1}	5.42×10^{-1}	to	5.46×10^{-1}
52.	3.60	3.58	to	3.62

53.	1.41			1.39			to	1.43		
54.	1.10	x	10^{1}	1.08	x	10^{1}	to	1.12	x	10^{1}
55.	1.22			1.20			to	1.24		
56.	2.85	x	10^{-4}	2.83	x	10^{-4}	to	2.87	x	10^{-4}
57.	6.70	x	10^{-8}	6.68	x	10^{-8}	to	6.72	x	10^{-8}
58.	1.31	x	10^{3}	1.29	x	10^{3}	to	1.33	x	10^{3}
59.	3.68	x	10^{-1}	3.66	x	10^{-1}	to	3.70	x	10^{-1}
60.	1.42	x	10^{-6}	1.40	x	10^{-6}	to	1.44	x	10^{-6}
61.	4.57	x	10^{-4}	4.55	x	10^{-4}	to	4.59	x	10^{-4}
62.	1.90	x	10^{-2}	1.88	x	10^{-2}	to	1.92	x	10^{-2}
63.	3.86	x	10^{-4}	3.84	x	10^{-4}	to	3.88	x	10^{-4}
64.	9.26	x	10^{6}	9.24	x	10^{6}	to	9.28	x	10^{6}
65.	4.07	x	10^{6}	4.05	x	10^{6}	to	4.09	x	10^{6}
66.	1.01	x	10^{-9}	9.90	x	10^{-10}	to	1.03	x	10^{-9}
67.	8.67	x	10^{-1}	8.65	x	10^{-1}	to	8.69	x	10^{-1}
68.	2.82	x	10^{2}	2.80	x	10^{2}	to	2.84	x	10^{2}
69.	2.57	x	10^{-5}	2.55	x	10^{-5}	to	2.59	x	10^{-5}
70.	7.62	x	10^{-2}	7.60	x	10^{-2}	to	7.64	x	10^{-2}
71.	2.51			2.49			to	2.53		
72.	1.44	x	10^{3}	1.42	x	10^{3}	to	1.46	x	10^{3}
73.	1.10	x	10^{2}	1.08	x	10^{2}	to	1.12	x	10^{2}
74.	2.13	x	10^{-4}	2.11	x	10^{-4}	to	2.15	x	10^{-4}
75.	4.28	x	10^{8}	4.26	x	10^{8}	to	4.30	x	10^{8}

APPENDIX B

APPENDIX B

RULES FOR THE UIL SLIDE RULE CONTEST

5. Conducting the Contest and Determining the Winners.--All slide rule contests shall be conducted under the following regulations:

a. Contest graders in addition to the contest directors, shall be secured from competent and unbiased citizens. These should be chosen in advance of the meet to enable the graders to familiarize themselves with the contest rules and methods of grading. In advance of the contest, "Instructions for Graders of the Slide Rule Contest" should be obtained from the University Interscholastic League, Bureau of Public School Service, The University of Texas, Austin, Texas 78712. The sample test and its sample grading are included in these instructions which all graders should study and understand before the contest, in addition to reading the slide rule contest rules appearing in this Constitution and Contest Rules. A copy of these instructions will accompany each set of slide rule contest papers to enable the graders to reread the rules during the contest and have for ready reference during the grading of the contest papers.

b. Since the performance of a slide rule contestant depends largely upon his comfort during the contest, care should be taken in the selection of a contest room and its equipment. The contest room shall be adequate in size for the comfort of all contestants and shall be selected with quietness of location and excellence of lighting as prime factors. During the actual conduct of the slide rule contest, no other contest or other activity shall be permitted to take place in the contest room. Tables or desks with accompanying armless chairs (not stools) are to be used, if at all available, for the contestants; these should not be of grade school size but should comfortably accommodate high school contestants. Since a very small minority of the contestants prefer classroom armchairs to desks or tables, a few classroom armchairs should be provided in addition to the desks or tables.

c. Subject to its availability, at a position easily seen by all contestants, a large electric clock shall be provided to indicate silently the remaining time in the contest. No oral time warnings, blackboard indications, or any other type of remaining time notations shall be employed. If all the contestants agree to its absence, this clock may be omitted.

d. At least 30 minutes before the actual beginning of the contest, the contestants, coaches, graders, and other interested individuals shall be gathered together in the contest room, and the rules shall be reviewed. Free asking of questions shall be permitted to see that all concerned are agreed as to the manner of conducting the contest, the point system of grading, the method of breaking ties, and all other items concerning the contest. If a conference precedes a contest within 24 hours, and if the rules are reviewed at this conference, a pre-contest review may be eliminated. However, a question period immediately preceding the contest shall be permitted.

e. When the contest is about to begin, all individuals with the exception of the contestants, the slide rule contest director, and one grader shall be excluded from the contest room; throughout the actual contest, only these individuals last mentioned may remain in the contest room. The other graders (if there be any) or other individuals (if there be no available graders for this duty) shall be stationed outside the contest room to act as sergeants-at-arms to effect quietness throughout the actual conduct of the contest.

f. The envelope containing the slide rule contest literature shall be opened and the sheet or sheets for tabulation of results shall be removed. These sheets provide a method of preserving the identity of the contestants. The contest director shall number the folded contest papers on the outside and keep memoranda on above mentioned result sheets of the name, address, and school of each contestant to correspond to the numbers respectively assigned, so that at the close of the contest the papers may be identified readily.

g. The contestants shall be given orally the following last-minute instructions:

(1) Write your answers above the line following the word equals.
(2) No oral time warnings or blackboard time tabulations shall be given; if you desire to see the amount of remaining time in the contest, you may refer to the large electric clock or to your own watch.
(3) If you finish the contest before the end of the allotted time, remain at your seat and retain your paper until told to do otherwise. You may use this time to check your answers if you desire.
(4) Keep your papers folded at all times except when told to do otherwise; this is particularly important while the contest papers are being distributed and before the signal to begin the contest has been given.
(5) If you are in the process of actually writing down an answer when the quitting signal is given, you may complete writing down the digits of your answer, however, you will not be permitted to determine the decimal placement unless you already know its location before the quitting signal is given.
(6) In solving combination problems involving successive steps, it is permissible for you to write down any successive results. In fact, you may place as many notations as you desire anywhere on the contest paper with the exception of the answer spaces which are reserved for answer only. You may not use any additional scratch paper.
(7) Use either the actual decimal point to indicate decimal placement or you may use powers of ten. Both methods may be used on the same paper. If the answer is a whole number it may be written

without a decimal point indicated but with its location understood to be at the right side.

(8) Remember that if you skip a problem, you will be penalized one point.

(9) During the contest proper, no questions may be asked or answered.

h. Hand out the contest papers by orally calling out the serial numbers and having each contestant identify his. Warn the contestants that the contest is about to begin.

i. Give the signal starting the contest in a manner that is well understood by all contestants. In a clear voice announce, "The contest is about to begin. Get ready. Unfold your paper now and begin."

j. Give the "Instructions for Graders of the Slide Rule Contests," enclosed in the contest package, to the grader in order that he may refresh himself concerning the grading of the contest.

k. Exactly 30 minutes after the beginning signal has been given, give the signal ending the contest. Announce clearly, "Stop. Fold your papers immediately and turn them in to me."

l. Answer no questions concerning the contest at this time.

m. Exclude all individuals from the room with the exception of the slide rule contest director and the graders. This applies to contestants, coaches, parents, friends, and all other individuals.

n. Remove the answer key from the contest envelope and proceed to grade the contest papers. Allow adequate time for careful, accurate grading of the papers; do not sacrifice accuracy for speed. Double check the grading to be sure that no errors have been made.

o. Record the net scores on the outside of each contest paper.

p. First place goes to the contestant making the highest net grade; second place goes to the contestant making the next highest net grade; third place to the next highest and so on.

q. In the event two or more contestants are tied for first, second, and/or third place in the regional or State Meet, call into the room those contestants involved in the ties and give them the 10 minute tie-breaking contest provided in the contest envelope. Follow items 5a through 5p listed above insofar as they apply to the tie-breaking contest, permitting each contestant to be identified by the same number as that which he used for the regular contest. In the event a tie still remains after the first tie-breaking contest has been given and graded, an additional tie-breaking contest shall be given to only those concerned with the remaining tie; the slide rule contest director shall determine the 15 additional problems to be given on this additional 10 minute tie-breaking contest, if he wishes, he may take 15 problems from the regular 30 minute contest for the additional tie-breaker. This process shall continue until no ties remain in the first three places of the contest. As a matter of interpretation, if two individuals are tied for first place on the regular 30 minute contest, the one receiving the higher net grade on the tie-breaking contest receives first place and the other receives second place in the general contest. If two individuals are tied for third place on the general 30 minute contest, after distribution of first and second place, either by no ties existing or by the results of the tie-breaking contest, the individual making the higher grade on the tie-breaking contest shall receive third place, and the other individual shall not place at all. No ties in first, second or third place shall be resolved in district competition.

r. After all papers have been graded and no ties remain in the first three places, completely fill out the remaining blanks on the tabulation of results sheet. Prepare a list of winners and their schools. Exception, see Rule 5p.

s. Gather all contestants, graders, coaches, and other interested parties in the contest room and announce the winners of the contest, that is, the names of the contestants making the first three places and their net scores. Contestants should be permitted to examine their papers and check the answers if they desire, but they are not permitted to retain them. Permit discussion concerning the contest. If it is evident that any errors have been made, correct them and be sure that all contestants are informed of their correction.

t. Gather all used contest papers so that none may be retained by the contestants, coaches, or other interested parties; these shall be destroyed.

u. The instructions for graders shall be retained for future use of the slide rule coaches.

v. The tabulation of results sheet and the list of winners and their schools shall be given to the director general.

6. Grading the Contests.--Adequate time for careful, accurate grading shall be taken. Accuracy shall not be sacrificed for speed. The State Office of the University Interscholastic League will provide the graders with a list of the correct answers; this list will be included with each regular contest or tie-breaking contest envelope. The grading of all papers and the determination of the net grades shall be double-checked to reduce the possibility of errors.This checking is best done by having the graders exchange papers and grade them a second time in such a way as to avoid being influenced by the first grading. If the two differ the graders should confer and agree on a final grade for the paper.

For the sake of uniformity and freedom from argument, all grading shall be done according to answers written on the answer key, unless the contest director desires to contact the state slide rule director by telephone for a corrected answer. If the latter option is selected, the corrected answer must be received from the state slide rule director as soon after the completion of the slide rule contest as practical but in no case later than four hours after the completion of the contest. In the absence of a corrected answer from the state slide rule director, all grading must be done according to the answer printed on the answer key, even in the case of an obviously incorrect answer if such ever is the case. Any telephone calls to the state slide rule director will be made at neither his expense nor that of the University Interscholastic League.

The papers of all contestants in the district, regional, and state slide rule contest and in all tie-breaking contests shall be graded uniformly on the following basis:

a. The first significant digit is defined as that digit other than zero which first occurs in the number. The first significant digit of 83.4 is 8. The first significant digit of 0.00428 is 4.

b. Three significant digit accuracy shall be required on each problem. If the answer of a problem has just one or two significant digits, the addition of two or one zeros should be used to indicate accuracy to three significant figures. If the answer is 25, to indicate three significant figures it should be written as 25.0. If the answer is 0.04, to indicate three significant figures it should be written as 0.0400.

c. If any digits on the right of the first three significant digits in the answer are not zeros, 1 point shall be subtracted from the score for that problem if the score otherwise is a positive value.

d. The gross grade is the addition of positive points. The negative or subtractive grading system shall not be used except in the two cases mentioned in Paragraphs 6e and 6m below.

e. The maximum point value for any answer is 5 points; the maximum amount that can be subtracted for any one answer is 1 point as indicated in Paragraphs 6e and 6m below.

f. If the first significant digit in the contestant's answer is incorrect according to the range of acceptable answers given in the answer key, no positive credit shall be given for this problem; in this case 1 point shall be deducted from the contestant's gross grade. See Paragraph 6n below.

g. If only the first significant digit in the answer is correct according to the range of acceptable answers given in the answer key and if the decimal point is placed correctly, a value of 3 points shall be given for the problem.

h. If only the first two significant digits in the answer are correct according to the range of acceptable answers given in the answer key and if the decimal point is placed correctly, a value of 4 points shall be given for the problem.

i. If each of the first three significant digits in the answer is correct according to the range of acceptable answers given in the answer key and if the decimal point is placed correctly, a value of 5 points shall be given for the problem.

j. If only the first significant digit in the answer is correct according to the range of acceptable answers given in the answer key and if the decimal point is placed incorrectly or omitted when necessary, a value of 1 point shall be given for the problem.

k. If the first two significant digits in the answer are correct according to the range of acceptable answers given in the answer key and if the decimal point is placed incorrectly or omitted when necessary, a value of 2 points shall be given for the problem.

l. If each of the first three significant digits in the answer is correct according to the range of acceptable answers given in the answer key and if the decimal point is placed incorrectly or omitted when necessary, a value of 3 points shall be given for the problem.

m. An answer must be written in the space provided to the right of the problem before any credit shall be given for the problem; in this case where the answer has been written but in the incorrect place, the problem shall be counted as having been skipped; see Paragraph 6n below.

n. The sum of the points awarded for each problem shall constitute the gross score of the contestant. From the gross score, 1 point shall be deducted for each problem skipped and 1 point shall be deducted for each answer in which the first significant digit is incorrect according to the range of answers given in the answer key; the latter previously had been mentioned in Paragraph 6e. Those problems occurring after the last problem solved or attempted are not considered skipped; hence no deduction shall be made for them. In the case of Paragraph 6e where an attempt at a solution has been made or where the solution has been determined but neither the attempt nor the solution are written in the proper place for answer, the problem shall be considered skipped and one point shall be deducted.

o. An illegible figure shall be counted as an incorrect digit. To determine whether or not a figure is illegible, place a blank piece of white paper on either side of it, thus separating it from its context, and then if the grader cannot identify the figure, it shall be counted as an incorrect digit.

7. Qualification.--District winners in each conference qualify for regional meets and regional winners for the State Meet in accordance with the schedules provided in Rules 18 and 22 of the Spring Meet Plan.

8. Graders.--A committee of competent and unbiased graders shall be appointed by the director general of the meet to grade the papers produced in the contest and to report grades to the contest director. This committee generally should have three members, but in cases of very small number of slide rule contestants in any one contest, one or two graders may be all that are necessary. In cases where there are large numbers of slide rule contestants, more than

three graders may be used. The contest director should choose the graders in advance of the meet and should use every effort to induce the graders to familiarize themselves with the rules. See Paragraph 5a.

<u>Constitution and Contest Rules of the University Interscholastic League for 1969-1970</u> (Austin: The University of Texas, 1969), pp. 94-100.

APPENDIX C

QUESTIONNAIRE

NAME ____

SCHOOL ____

NUMBER OF YEARS TEACHING EXPERIENCE ________________

GRADE LEVEL TAUGHT _______________________________

HIGHEST DEGREE EARNED ____________________________

TOTAL NUMBER OF HOURS CREDIT IN COLLEGE MATH _________

TOTAL NUMBER OF GRADUATE HOURS IN MATH ______________

DO YOU TEACH YOUR STUDENTS ESTIMATION? ______________

ARE YOU FAMILIAR WITH LOGARITHMS? __________________

DO YOU KNOW HOW TO OPERATE A SLIDE RULE? ____________

1. $2.81 \times 0.494 \times 3.21$ ------------------------- equals 446

2. $16.7 \times 4.32 \times 0.0261$ ------------------------- equals 188

3. $3740 \times 8.21 \times 0.0115$ ------------------------- equals 353

4. $6.71 \times 0.00815 \times 3.27$ ------------------------- equals 179

5. $\frac{15.5 \times 0.637}{0.418}$ ------------------------- equals 236

6. $\frac{37.8}{2.61 \times 0.837}$ ------------------------- equals 173

7. $\frac{9.05 \times 0.0581}{3.65 \times 0.727}$ ------------------------- equals 198

8. $\frac{2.11 \times 7.68 \times 3.42}{5280}$ ------------------------- equals 105

9. $\frac{0.00816}{0.0416 \times 3.72 \times 1.09}$ ------------------------- equals 484

10. $\frac{2.77 \times 3640}{1.66 \times 39.2 \times 40.1}$ ------------------------- equals 386

11. $\frac{1.76 \times 0.415 \times 3.29}{6.17 \times 9.54 \times 13.2}$ ------------------------- equals 309

12. $\frac{2.77 \times 8.16 \times 0.525 \times 19.6}{32{,}500 \times 0.00868}$ ------------------------- equals 824

13. $\frac{5.75 \times 0.396 \times 4260}{727 \times 0.165 \times 0.00329}$ ------------------------- equals 246

14. $\frac{2.99 \times 0.00365 \times 4.82 \times 717}{0.0525 \times 6.19 \times 0.000615}$ ------------------------- equals 189

$$\frac{0.216 \times \sqrt{13.7} \times 4.66}{0.0804 \times 2.75 \times 1.82}$$ ------------------------ equals 926

$$\frac{15.2 \times 0.00516 \times \sqrt{3.04}}{\sqrt{3.25} \times 14.9 \times 0.0373}$$ ---------------------- equals 136

$$\frac{(11.2)^2 \times 0.365 \times 19{,}200}{21.5 \times \sqrt{274} \times 3620}$$ --------------------- equals 682

$$\sqrt{4.81} \times 0.293 \times (704)^2 \times 0.115$$ -------------- equals 366

$$\frac{\sqrt{17.6} \times 1.82 \times 0.0515 \times 1.66}{0.00392 \times (20.4)^2 \times 0.0815}$$ --------------- equals 491

$$\frac{(0.00372)^2 \times \sqrt{46{,}300} \times 1.29}{1.55 \times 60.3 \times 1.27 \times 0.112}$$ ----------------- equals 289

$$\sqrt{0.000335} \times 2.83 \times (0.00169)^2$$ --------------- equals 148

$$\frac{22.4 \times 30.5 \times 2{,}730{,}000}{\sqrt{15.8} \times (7270)^2 \times 0.0873}$$ ------------------ equals 102

$$2.99 \times \sqrt{1.43 \times 7.86} \times 0.00815$$ ---------------- equals 817

$$\frac{(32.6 \times 0.413)^2 \times 0.000719}{4.82 \times 3.66 \times 2.05 \times 3.11}$$ ------------------ equals 116

$$\frac{(2.79)^3 \times 4.68 \times 0.337}{\sqrt{0.0816 \times 42.5} \times 6.05}$$ --------------------- equals 304

$$\frac{(0.0279)^3 \times 3.62 \times 5040}{(286 \times 19.8)^2 \times 0.0352}$$ ---------------------- equals 351

$$\frac{72.8 \times \sqrt{3.61 \times 0.0495}}{2.16 \times (0.374)^2 \times 2640}$$ ------------------------ equals 386

8. $$\frac{\sqrt[3]{1.29} \times 32.2 \times 52.8}{(0.000726 \times 529)^2 \times 0.392}$$ ------------------ equals 320

9. $$\frac{(2.96 \times 0.412)^2}{1.86 \times 0.0297} \times \sqrt{\frac{4.27}{0.883}}$$ ------------- equals 592

0. $$\frac{\sqrt[3]{1.79 \times 4.28} \times 0.0000427}{(0.0167 \times 0.225)^2 \times 0.528}$$ ------------- equals 113

1. $$(0.00398 \times 218)^3 \times \sqrt{8640 \times 0.0326}$$ ---------- equals 110

2. $$\frac{28.6 \times 0.0274}{\sqrt{0.112 \times 3.68}} \times \frac{(2.67 \times 0.529)^2}{3.27 \times 16.8}$$ ------------ equals 443

3. $$\frac{\sqrt[3]{0.00237 \times 0.0194} \times 627{,}000}{(1.45 \times 3.29 \times 16.7)^3 \times 2.14}$$ --------------- equals 208

4. $$\left[\frac{0.297 \times 3420}{61.5 \times 3.92}\right]^2 \times \left[\frac{1.72 \times 496}{378 \times 2.55}\right]^3$$ ------------- equals 123

5. $$\left[\frac{4.77 \times 0.256}{3.57 \times 4.92}\right]^2 \times \frac{\sqrt{4.22 \times 8.61}}{(2.04 \times 6.15)^3}$$ ------------ equals 148

6. $$\frac{\pi \times 0.866 \times 0.707 \times 5280}{\sqrt[3]{1.77 \times 2.09} \times (86.4 \times 17.7)^2}$$ ------------- equals 281

7. $$\sqrt{\frac{0.000428}{0.721 \times 4.16}} \times \frac{(3.75 \times 0.446)^3}{16.2 \times 0.0917}$$ ---------- equals 376

8. $$\frac{\sqrt{2.79 \times 0.00747 \times \pi} \times 43{,}400}{3.92 \times 55 \times (24 \times 0.00272)^2}$$ -------------- equals 121

9. $$\frac{\sqrt{\sqrt{82{,}600{,}000 \times 0.114} \times 0.000404}}{5.66 \times 19.2 \times 75 \times \pi \times 9.2}$$ -------- equals 473