A HANDBOOK OF NOMOGRAMS FOR ELEMENTARY PHYSICS FORMULAS

A THESIS

SUBMITTED TO THE DEPARTMENT OF

MATHEMATICS AND THE GRADUATE COUNCIL OF THE KANSAS STATE

TEACHERS COLLEGE OF EMPORIA IN PARTIAL FULFILLMENT OF

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are the one written by Brodetsky[2], professor of applied mathematics at Leeds University, and the one written by Mackey[3], Assistant Professor of Heat Power Engineering at Cornell University. Almack and Carr[4] have also written an interesting article on nomography, showing particularly how the nomogram may be applied to the field of education.

NOMOGRAPHY DEFINED

Nomography is the method of solving equations of a given type by means of a one graph diagram. Ever since Coordinate Geometry was invented by Descartes, the graphical method has been recognized as an important means of arriving at the solutions of various types of equations. However, when equations are solved by means of this graphical representation on a plane, the number of variables which can be used is limited to two. Also, such a method involves considerable inconvenience, for when several equations belong to a single general type it may be necessary to construct a separate graph for each equation. In nomography we have a means of getting around these limitations. By this device, not only is it possible to find the graphical solution of equations in which the number of variables exceeds two, but one graph may suffice for all equations of a given type, even though they be quite complicated. Take for example the formula for finding the number of calories of heat produced

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2 S. Brodetsky, <u>A First Course In Nomography</u>, London: G. Bell and Sons, LTD., 1925

3 Charles O. Mackey, <u>Graphical Solutions,</u> New York: John Wiley and Sons, Inc., 1936

4 John C. Almack and William G. Carr, " The Principle of the Nomograph in Education", in the <u>Journal of Educational Research</u>, Vol. XIV, December 1926, pp. 340 - 355

when an electric current is flowing, namely -

$$H = .24\ I^2\ R\ t$$

where H stands for the number of calories, I the current in amperes, R the resistance in ohms, and t the time in seconds. If it is desired to find the solution for a number of values for I, R, and t, it will be convenient to have one graph which can be used for any values for these variables which might occur.

Likewise, this same principle is applicable to the various algebraic equations and formulas used in the fields of science, education and industry. The nomogram is a certain, easy means of solving equations and proves to be helpful wherever the same operation has to be performed a great number of times. Calculations, which ordinarily would require a thorough knowledge of mathematical facts and principles, can be carried out by those whose working knowledge of mathematics is very limited.

NOMOGRAPHY EXPLAINED

In constructing a nomogram for an equation with two variables a' and b', two graduated straight lines (a) and (b) are drawn in such a way that, if a straight line is drawn joining the graduation a' on scale (a) to the graduation b' on scale (b), the line cuts a third graduated line (x) at the graduation x', where x' is the desired result. The lines (a), (b), and (x) may or may not be parallel, depending largely on the type of equation for which the nomogram is constructed. In this study all but one of the graphs are of the parallel line type.

Consider, for example, the nomogram for the equation $x = a + b$

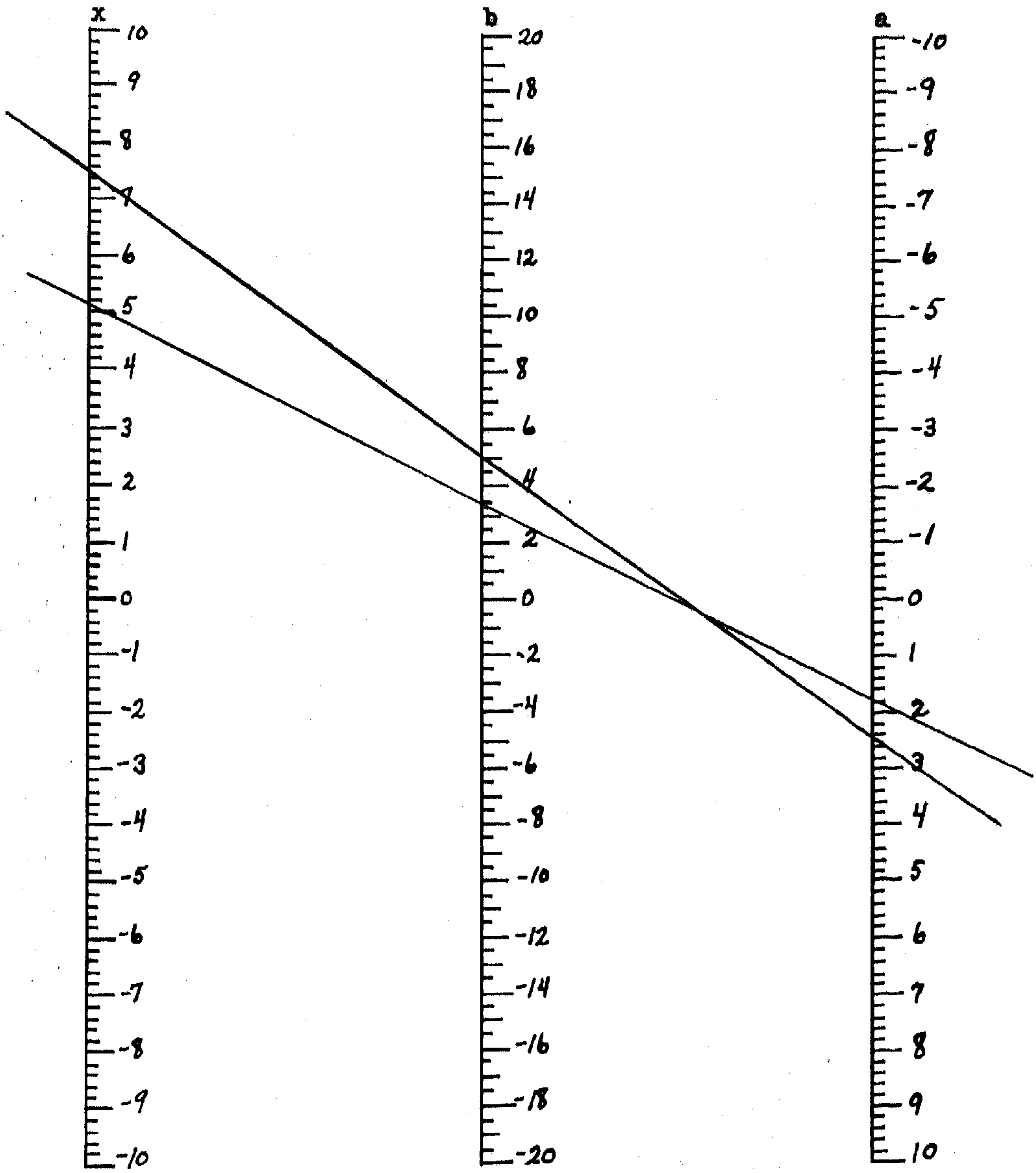

Fig. 1

$x = a + b$

in Fig. 1. It is recognized that a nomogram for such a simple operation as the addition of two quantities is perhaps of no practical value; however, it serves well to illustrate the fundamental ideas of nomography. Scales (a) and (x) are graduated from +10 to -10 while the (b) scale is graduated from +20 to -20. At first it may appear that, under these conditions, only small values for a' and b' might be used, and that if large values of these variables were involved, extremely long scales would be required. This is easily disposed of by taking out a common factor in some multiple of 10. Suppose that the two quantities, 255 and 492, are to be added. The point 2.55 is taken on the scale (a) and the point 4.92 on the scale (b), and the result, 7.47, is read on scale (x). Therefore the sum of 255 and 492 is 747. (See Fig. 1)

Very small quantities may be added by a similar method. If it is desired to find the sum of .0175 and .0337, the point 1.75 is taken on scale (a) and the point 3.37 on scale (b). The straight line which joins these two points cuts the (x) scale at the point 5.12. The required answer is therefore .0512.

If the quantities used are both negative or if one is negative and one positive, the method of procedure is the same as that just mentioned.

In the case of multiplication or division of two or more variables, the nomogram proves to be of considerably greater value. The general principle for the construction of the nomograms for these operations is the same as those for addition and subtraction, except that the scales are logarithmic scales. In order to find the product of two numbers it is necessary only to add the logarithms of the numbers to get the logarithm of the product; and to find the quotient of two numbers it is necessary only to find the difference of the logarithms. Accordingly, the nomograms for multiplication and division are really nomograms for addition and subtraction in type. This is illustrated in

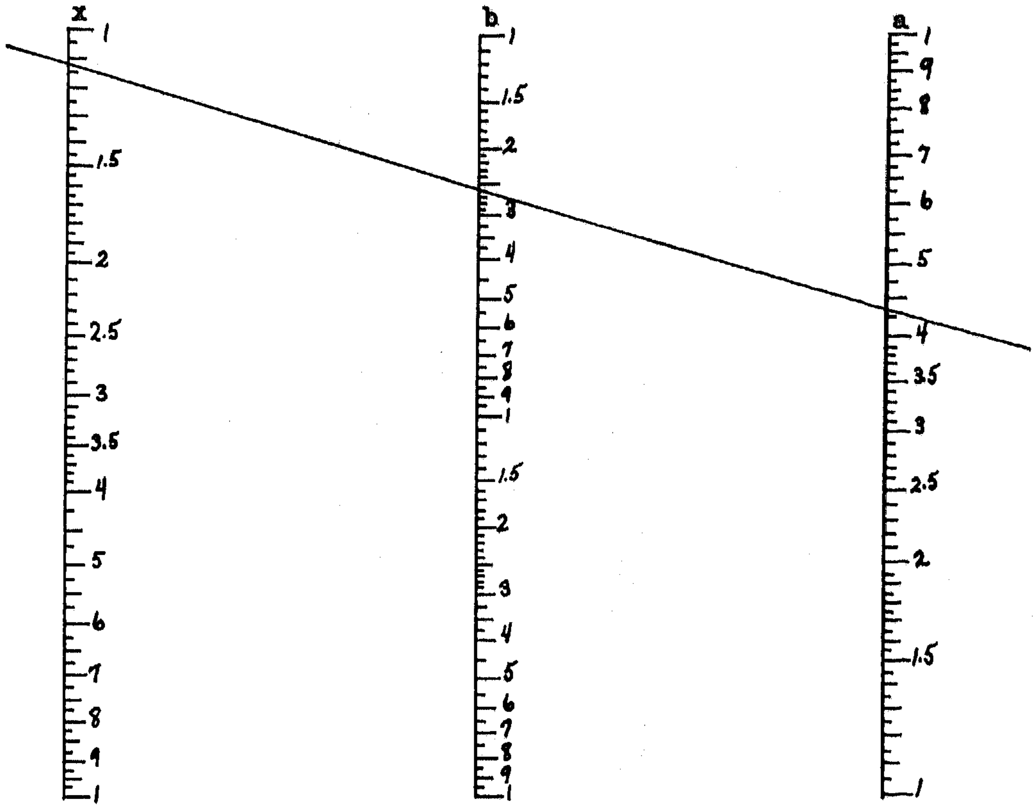

Fig. 2

x = a b

Fig. 2 which is a nomogram for the equation x = a b. Here scales (a) and (x) are graduated from 1 to 10 while scale (b) is graduated from 1 to 100. So, as in the case of addition, if it is desired to multiply 43 by 26, the point 4.3 is selected on scale (a), point 2.6 on scale (b), and the product of these two numbers is then given on the scale (x) by the point 11.18, and the answer is 1118. The same graduations would be used for any other product in which the factors are numbers having the same significant digits, 43 and 26. As in using the slide rule, the position of the decimal point is decided by a rough check.

Consider now briefly, the case of successive multiplication or

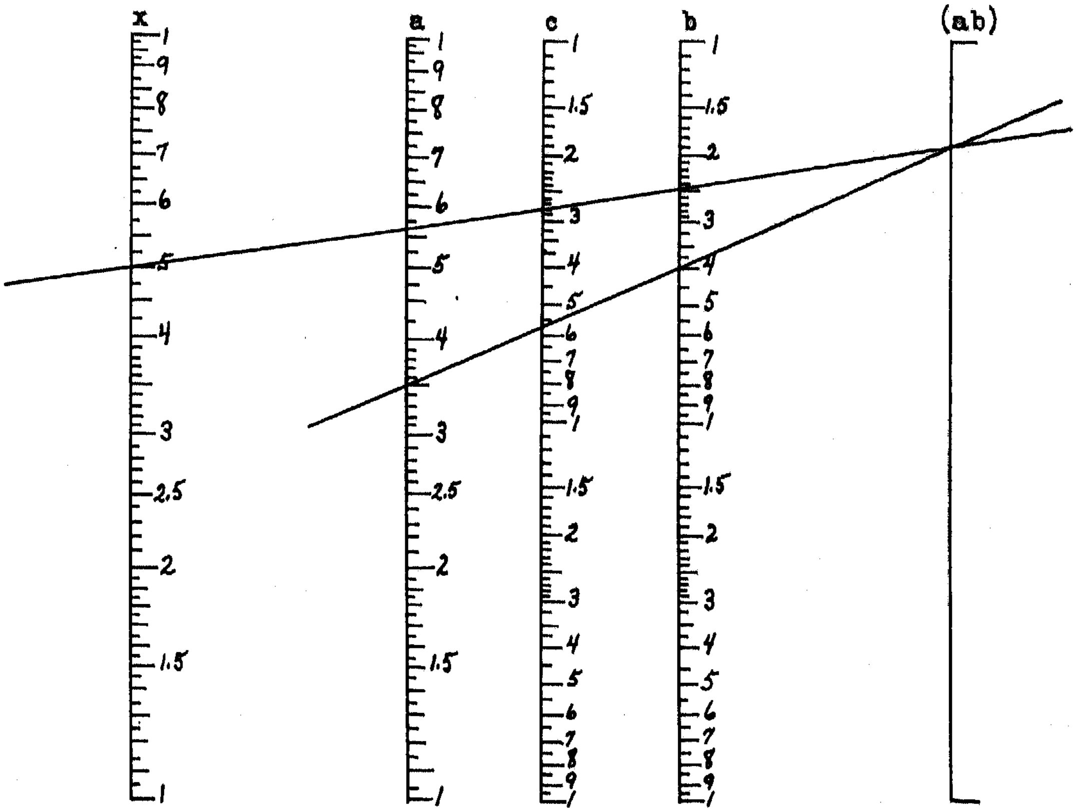

Fig. 3

$x = ab/c$

multiplication and division. Fig. 3 illustrates the nomogram for the equation $x = ab/c$. Here scales (a) and (x) are graduated, as before, from 1 to 10, scales (b) and (c) are graduated from 1 to 100, and scale (ab) is left ungraduated and is used only as a reference line. In a case such as is now being considered, we find first the point on the reference line (ab) which is the product of the value given on scales (a) and (b). This point on the reference line is then taken with the value on the scale (c) to give the result on scale (x). Consider for example the equation $x = \frac{35 \times 40}{28}$. The point 3.5 is located on scale (a), and a line is drawn through it and the point 4.0 on scale (b). This locates a point on the reference line (ab). The line through this point and through the point 2.8 on scale (c) determines the point 5.0 on scale (x). The solution of the equation is therefore x = 50.

Although the nomograms in this study are quite varied in type, the same general method is involved in their application.

ARRANGEMENT OF MATERIAL

The subject of Physics is divided into five general divisions and the material in this thesis has been arranged with these divisions in mind, giving one chapter for each division. The Physics texts, which were studied, varied somewhat in the order of the arrangement of material and the arrangement followed here is that used by Henderson[5]. The reason for selecting this particular text is that it is the present adopted for the stste of Kansas. The order is as follows:

1. Mechanics
   a. liquids
   b. gases
   c. solids
2. Heat
3. Electricity
4. Sound
5. Light

In Chapter VII are supplementary nomograms for a few of the more common equations. Although these are not specific formulas of Physics, the author feels they should be included in a study such as this.

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5 William D. Henderson, The New Physics In Everyday Life, Chicago: Lyons and Carnahan, 1935

CHAPTER II

NOMOGRAMS
COVERING THE DIVISION OF
MECHANICS

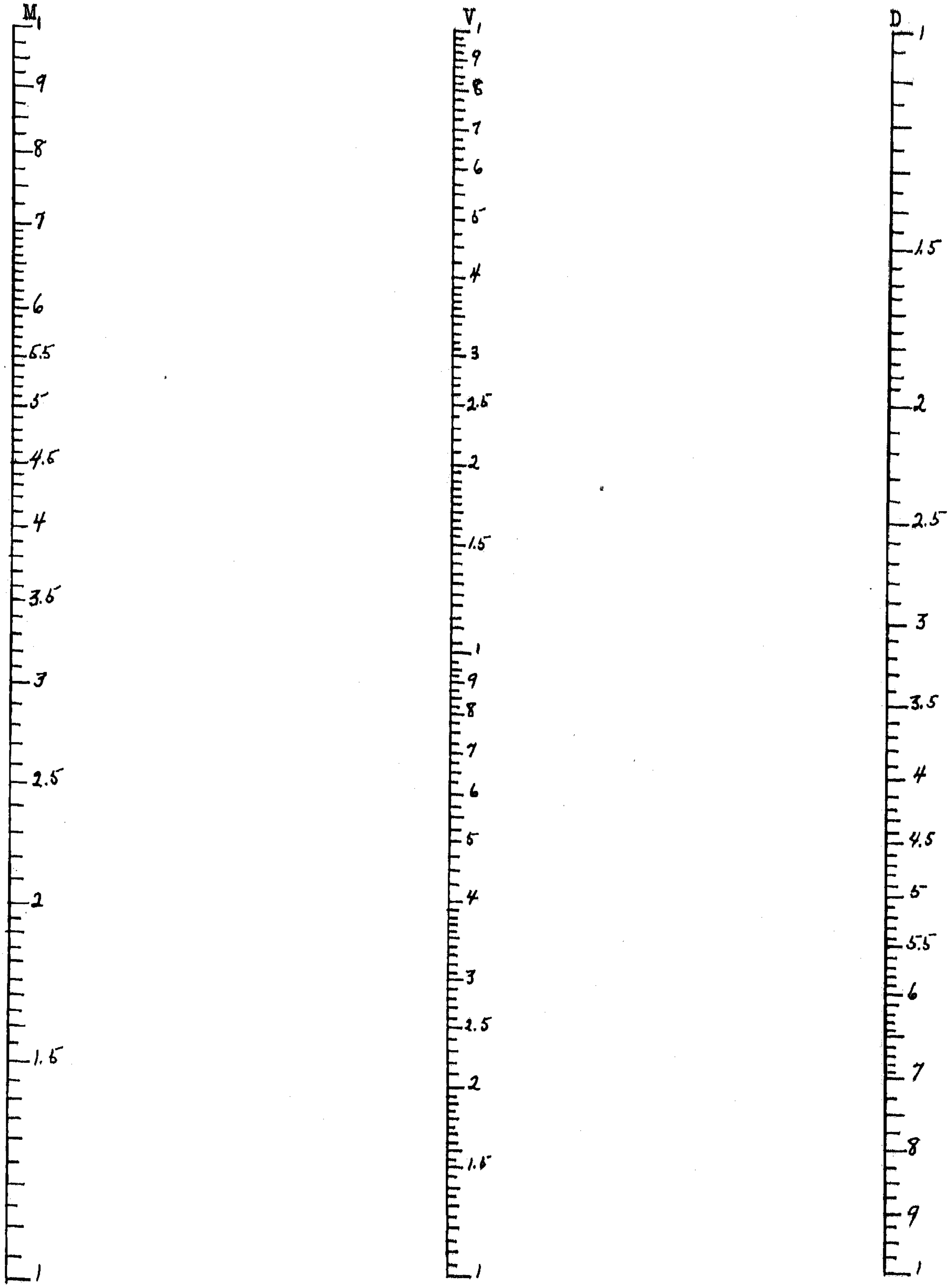


Fig. 4

M = D V

M = mass
D = density
V = volume

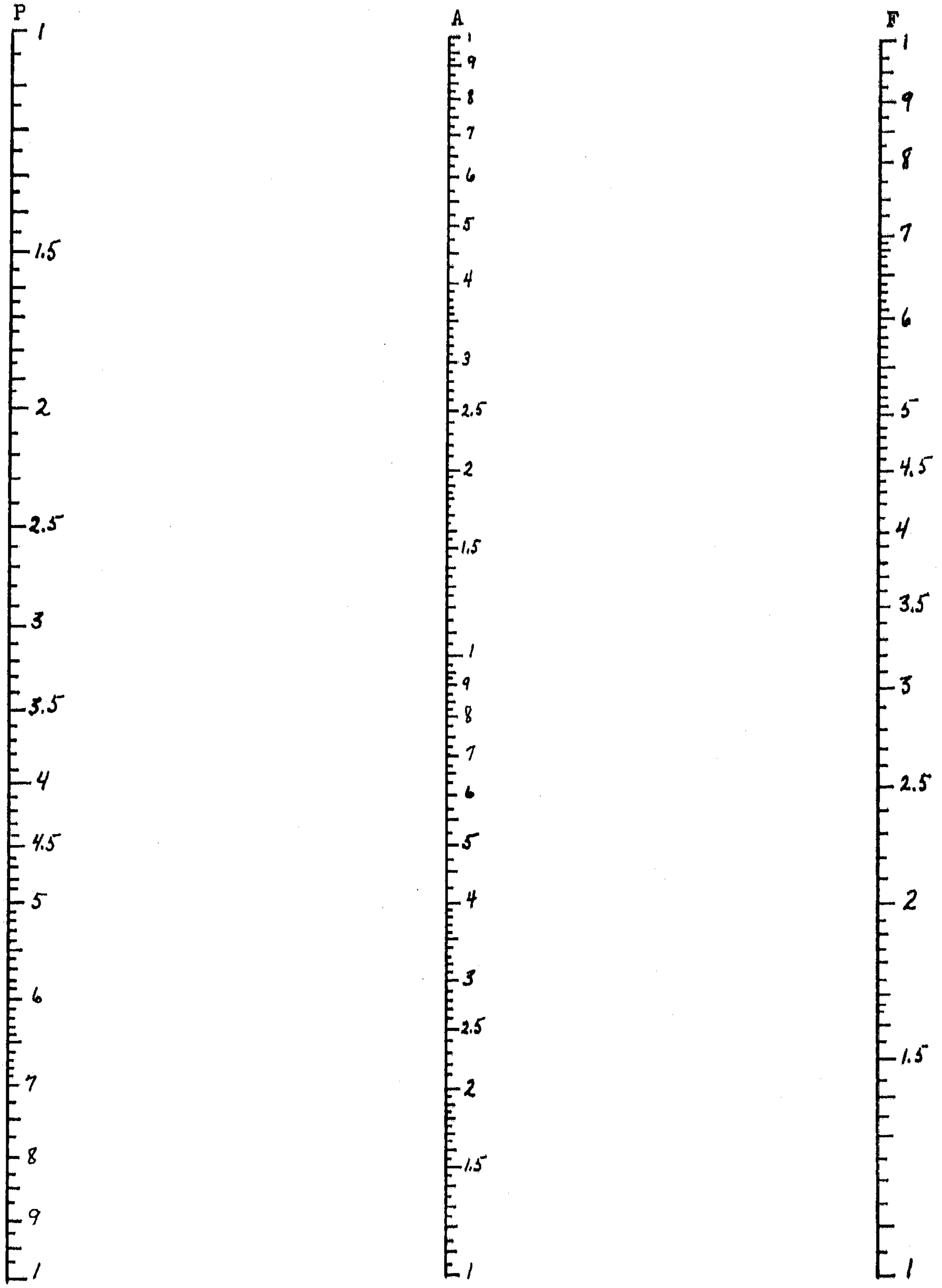


Fig. 5
P = F/A

P = pressure
F = force in pounds
A = area

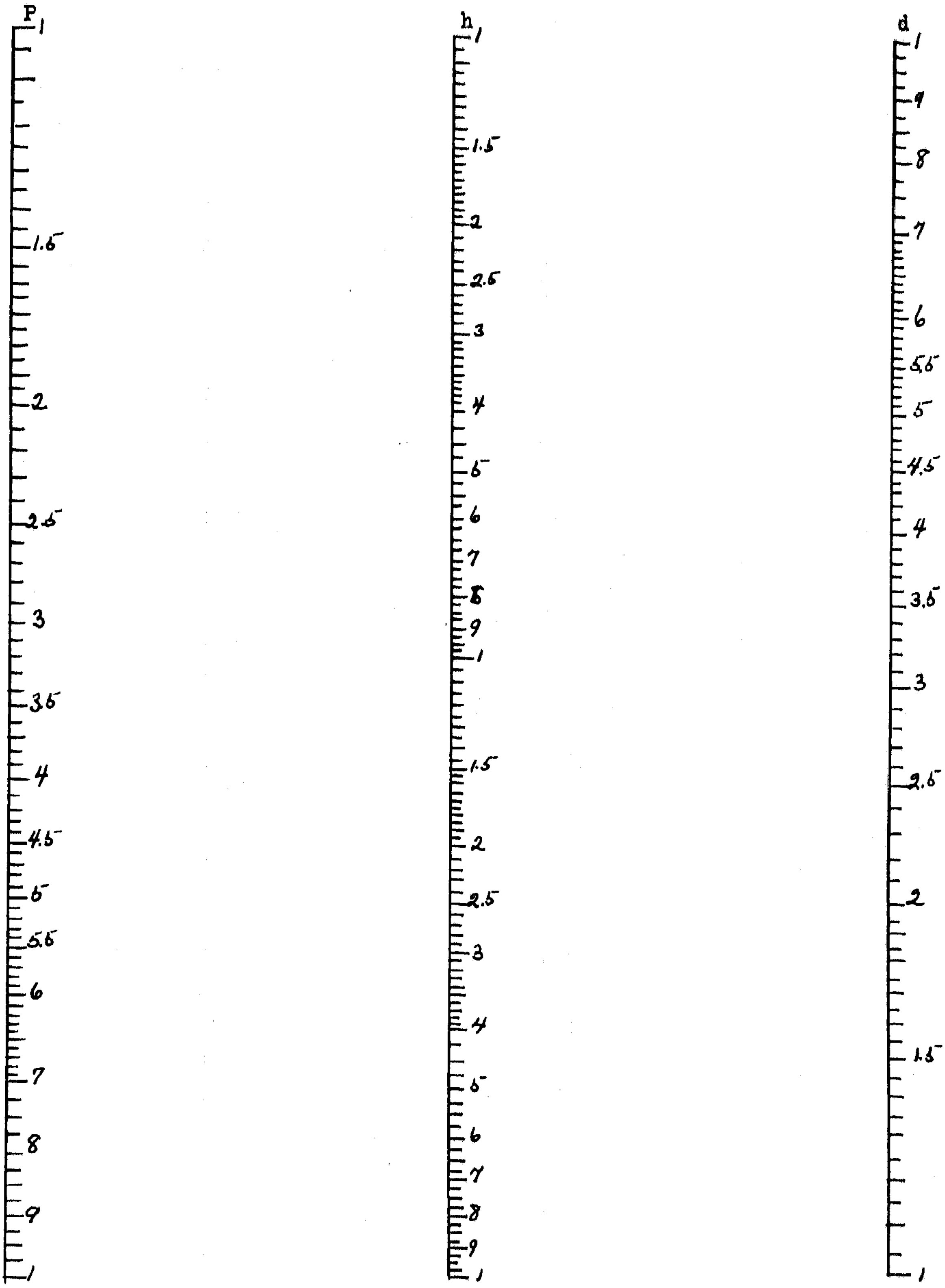


Fig. 6

P = h d

P = pressure
h = depth
d = density

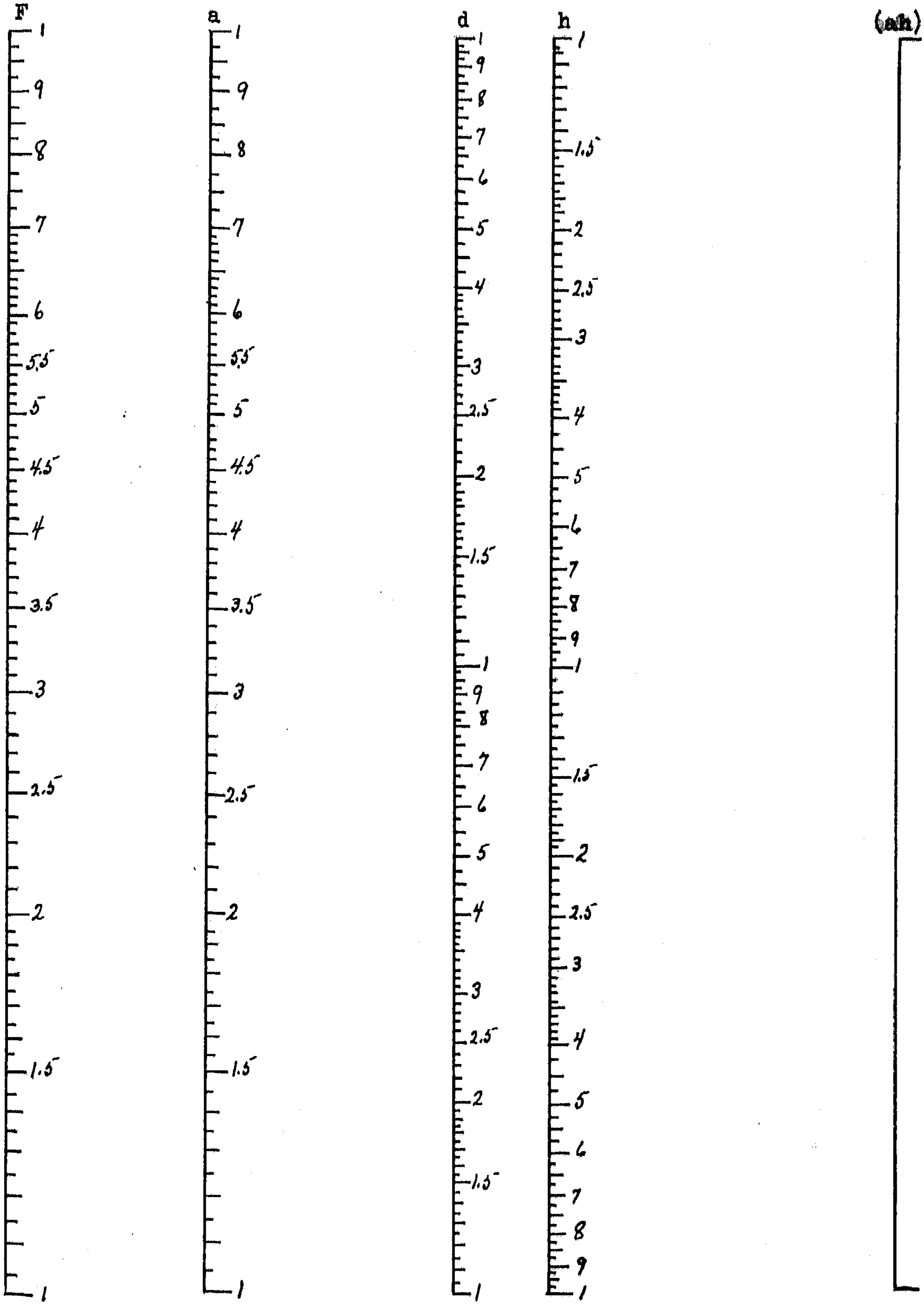


Fig. 7

F = a h d

F = force
a = area
d = density
h = depth

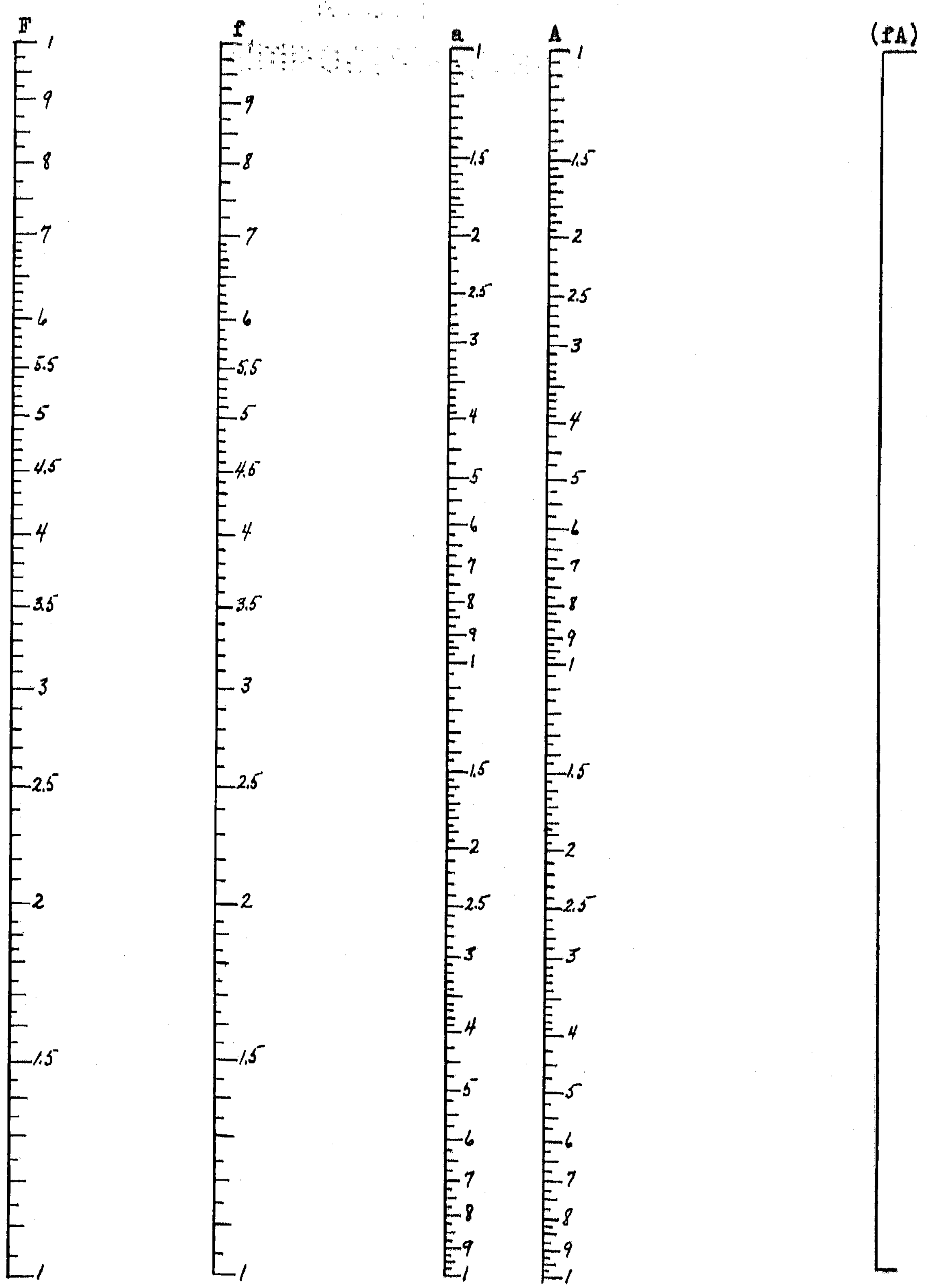


Fig. 8

$$F = f\ A/a$$

F = force on large piston of hydraulic press
f = force on small piston of hydraulic press
A = area of large piston
a = area of small piston

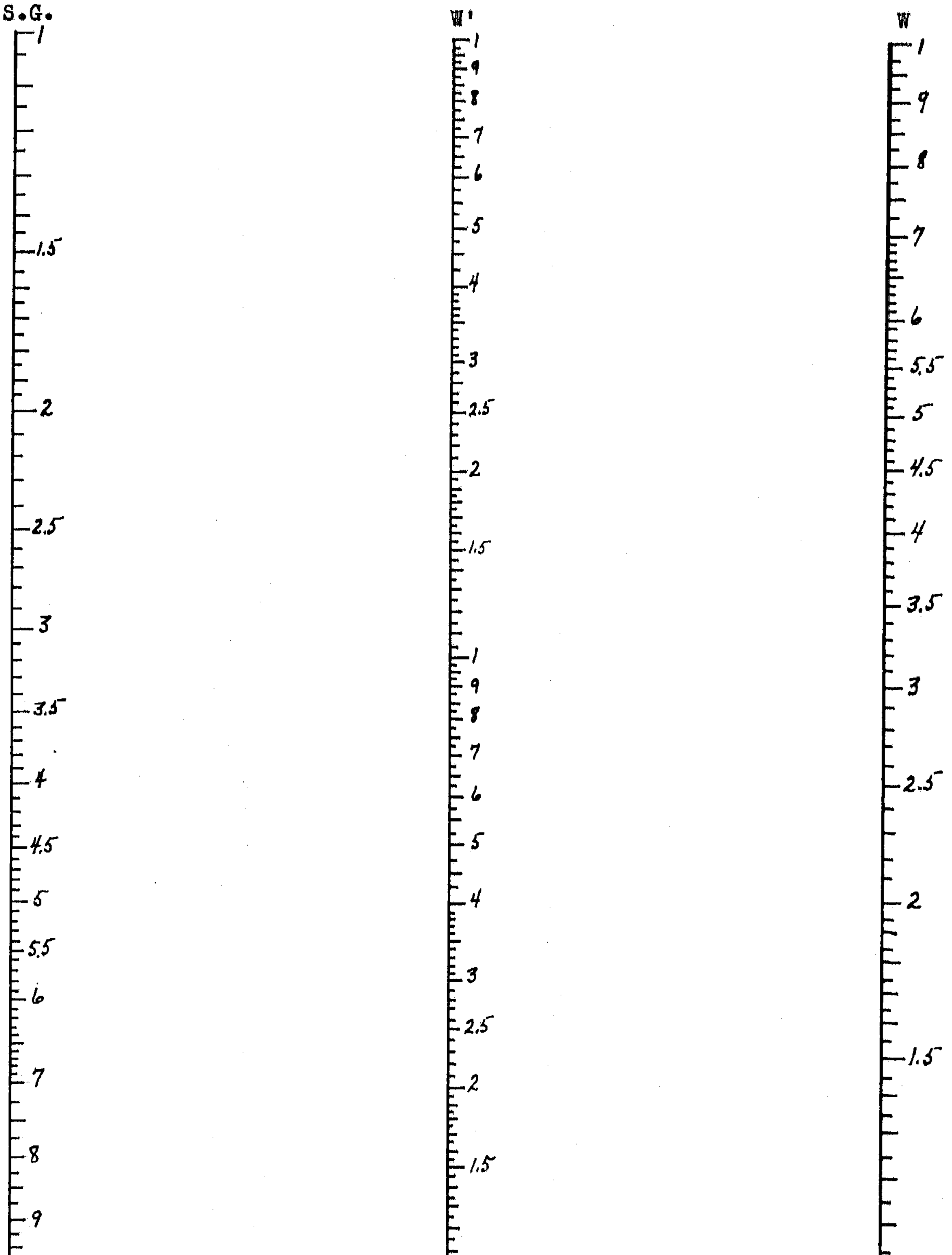


Fig. 9

S.G.= W/W'

S.G.= specific gravity of object
W = weight of object
W'= weight of an equal volume of water

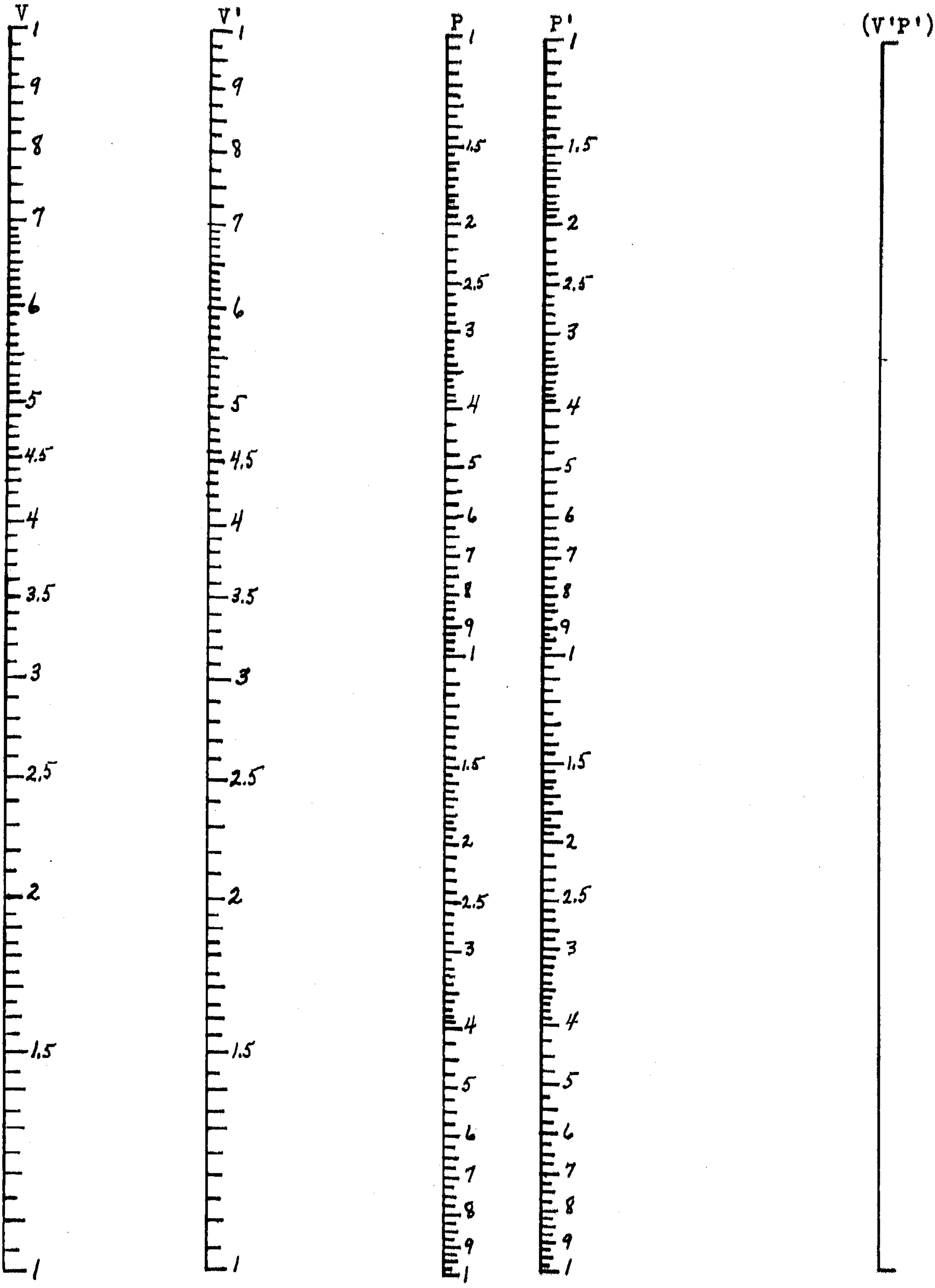


Fig. 10
$V = V'P'/P$

V = first volume
V'= second volume
P = first pressure
P'= second pressure

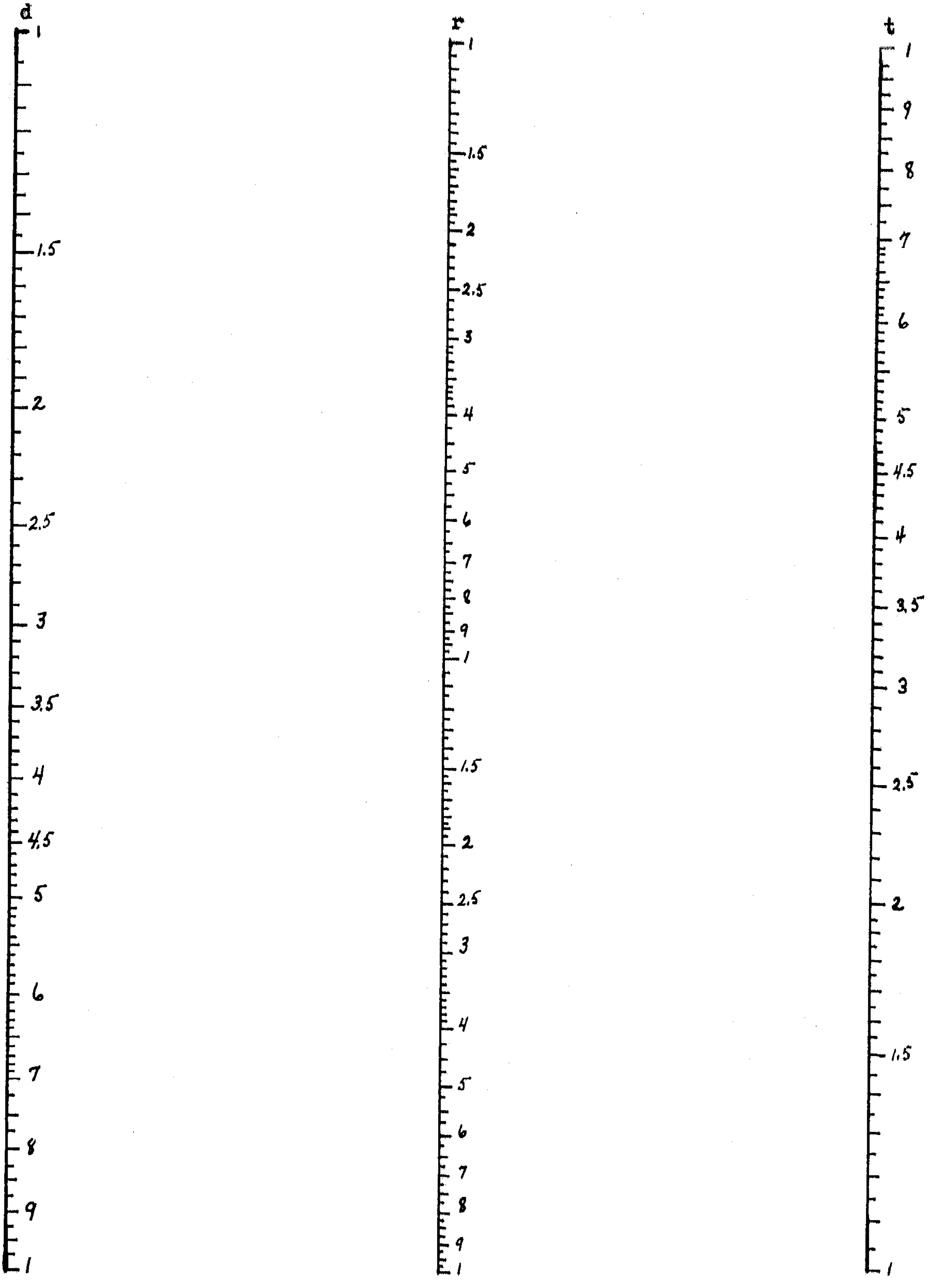


Fig. 11

$d = r\,t$

d = distance
r = rate
t = time

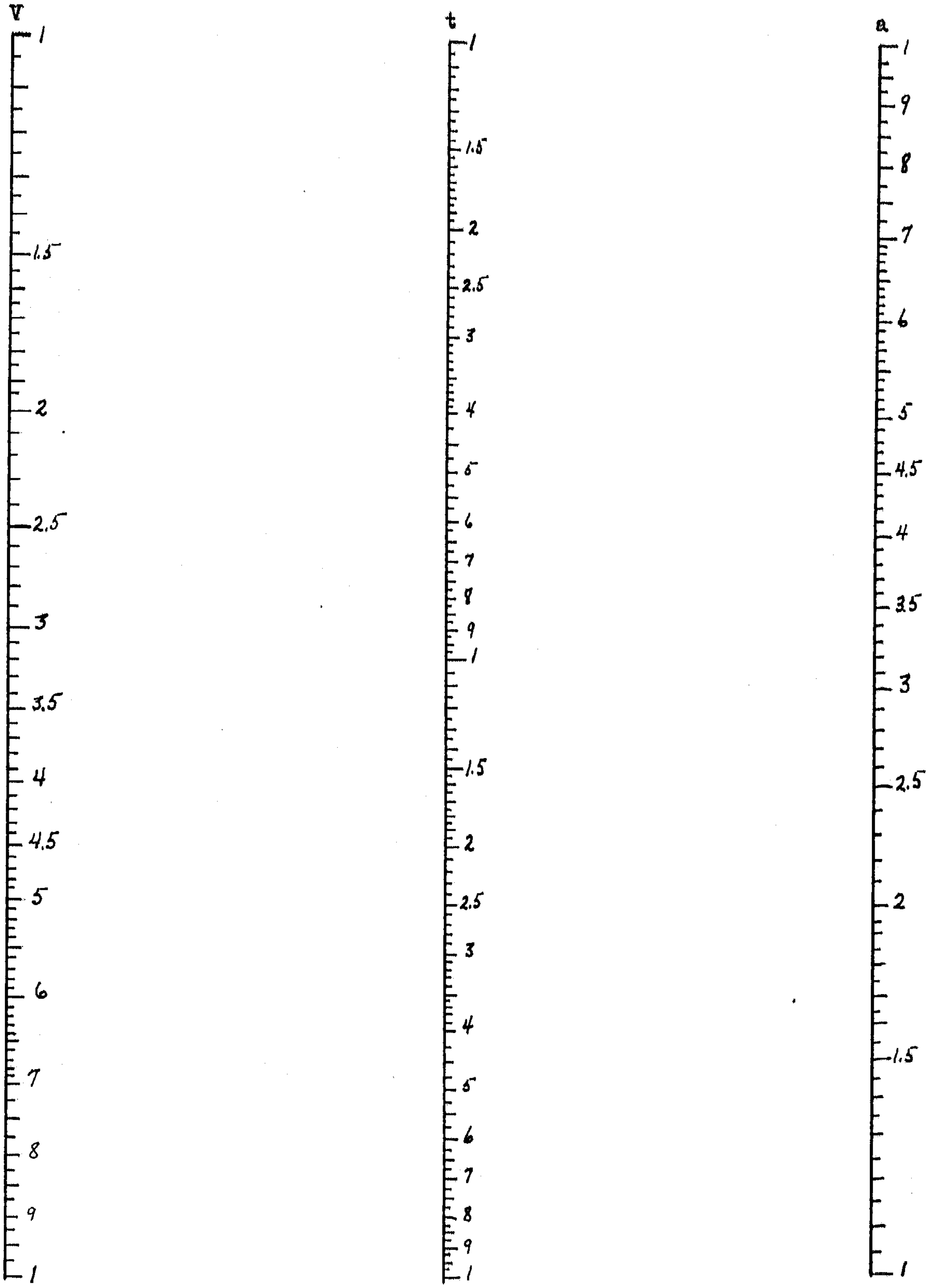


Fig. 12

V = a t

V = velocity
a = acceleration
t = time in seconds

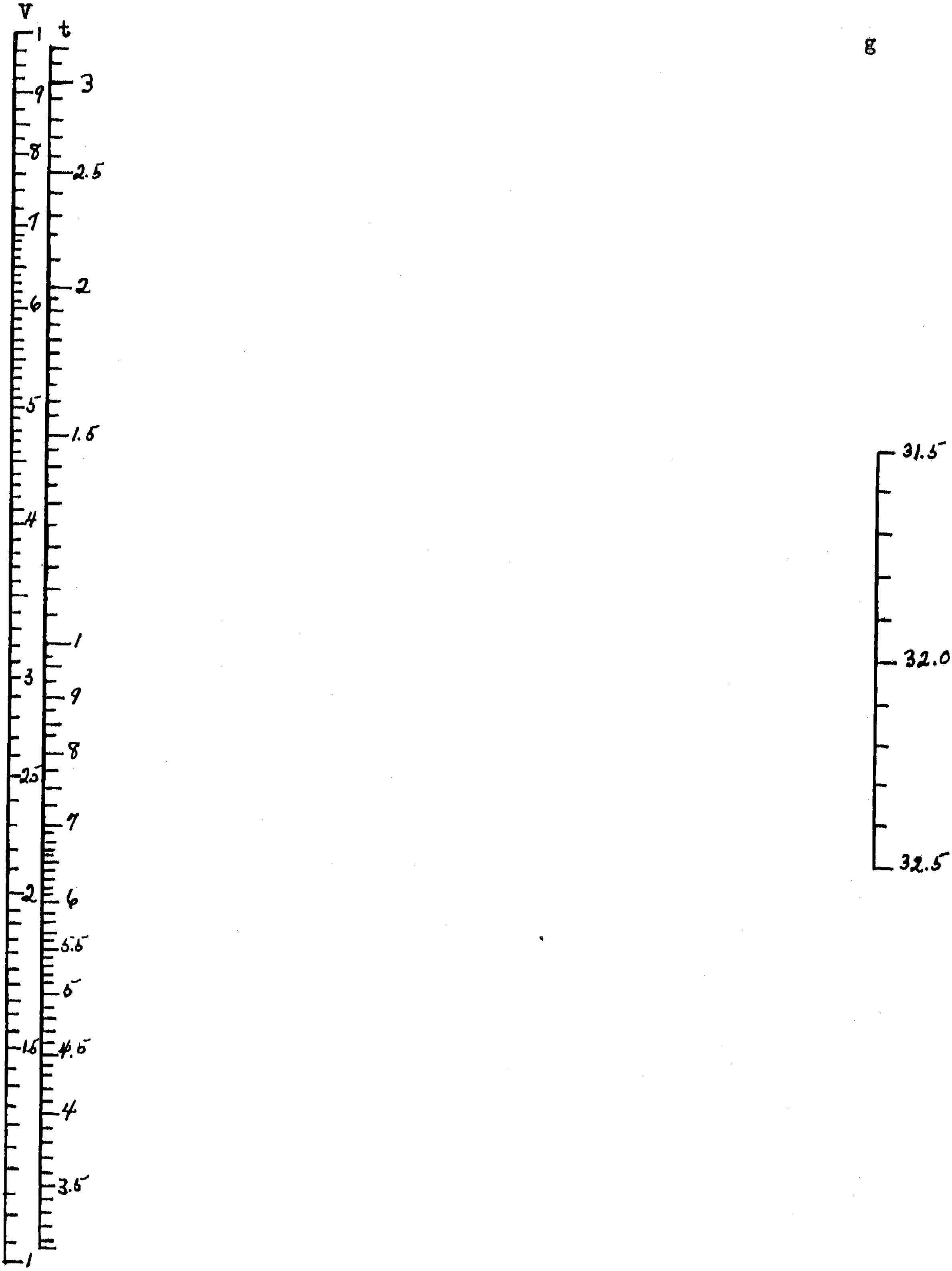


Fig. 13

V = g t

V = velocity
g = acceleration of gravity
t = time in seconds

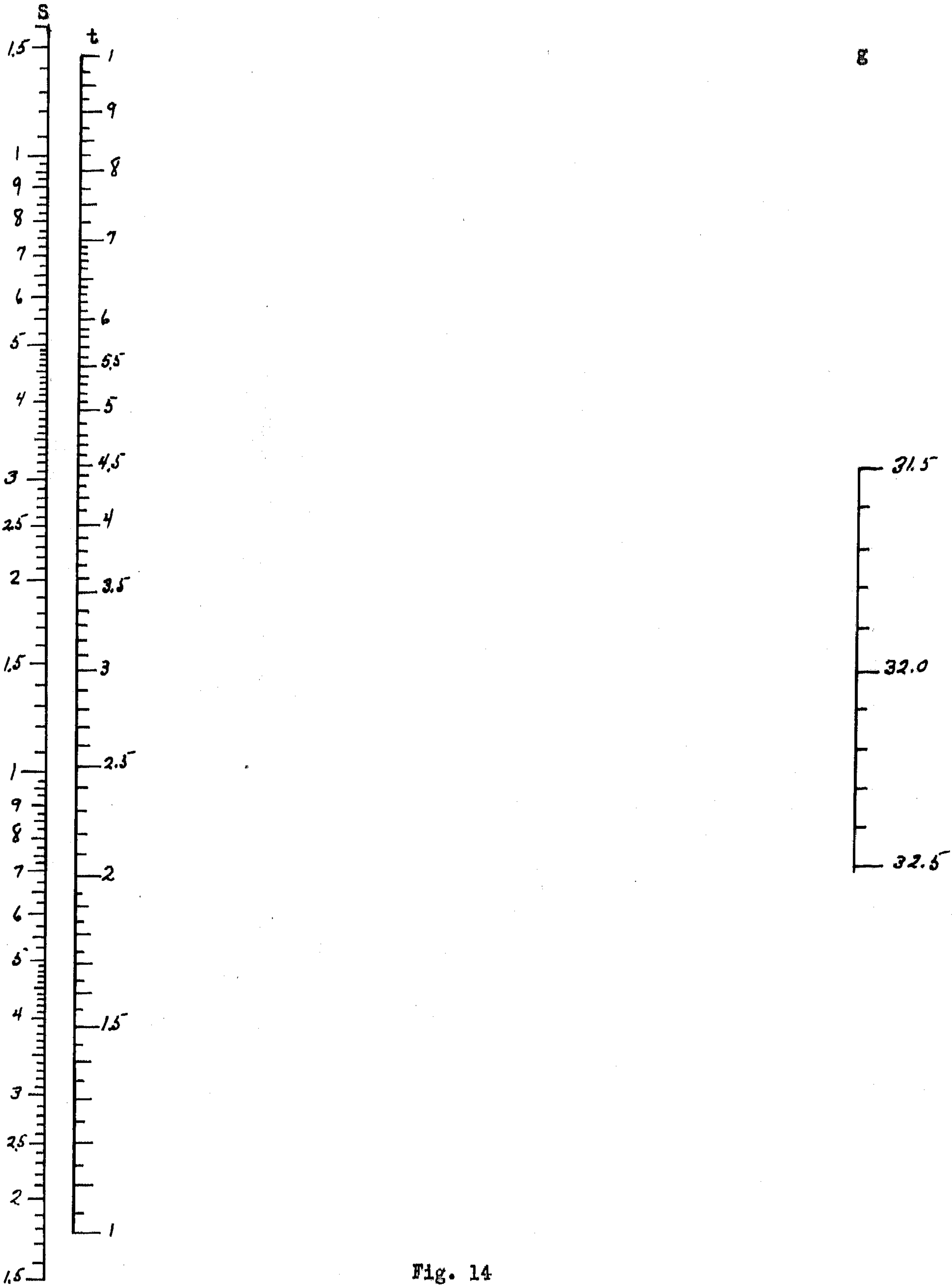


Fig. 14

$$S = \frac{1}{2} g t^2$$

S = distance
t = time in seconds
g = acceleration of gravity

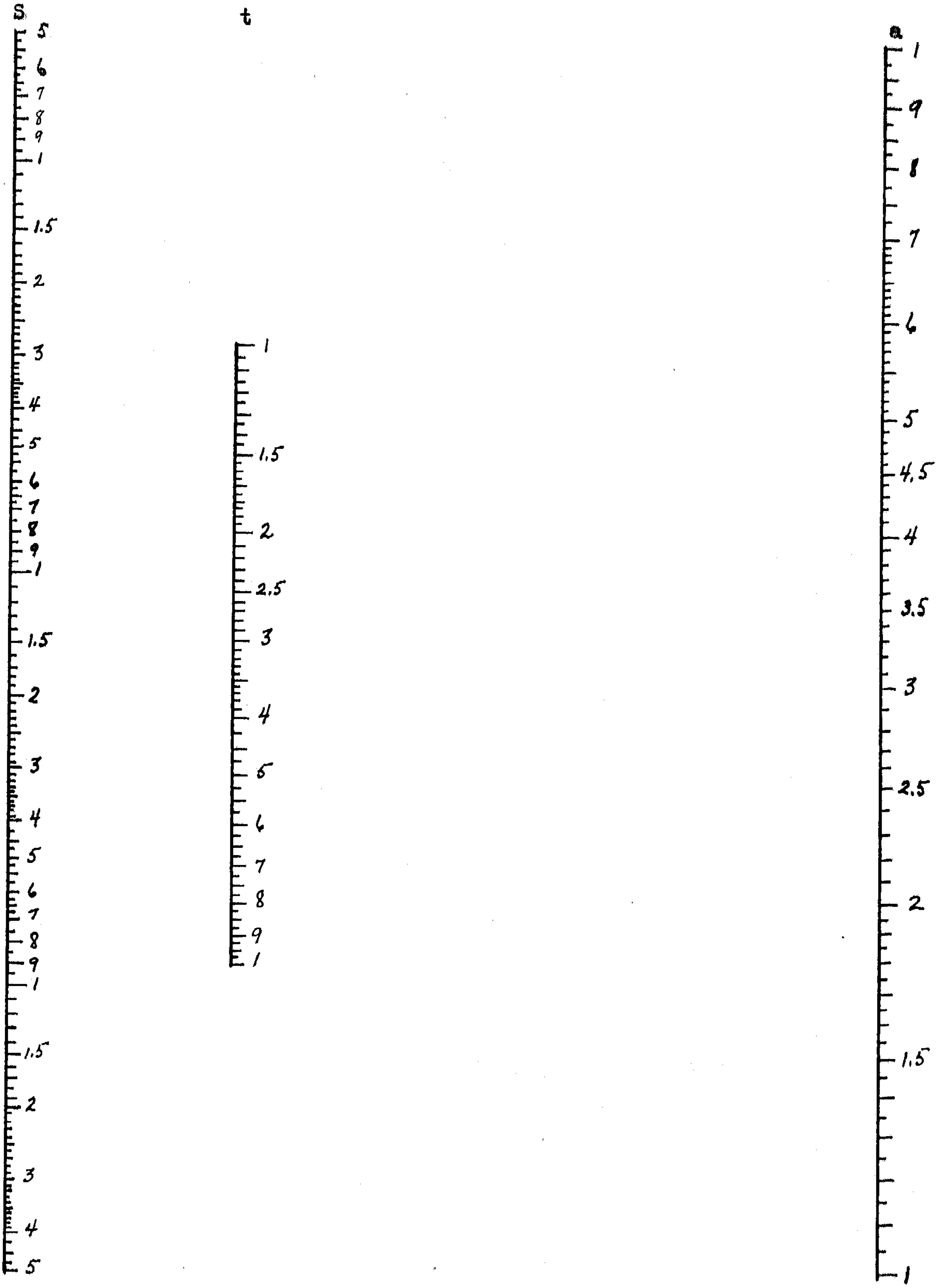


Fig. 15

$$S = \frac{1}{2} a t^2$$

S = distance
a = acceleration
t = time in seconds

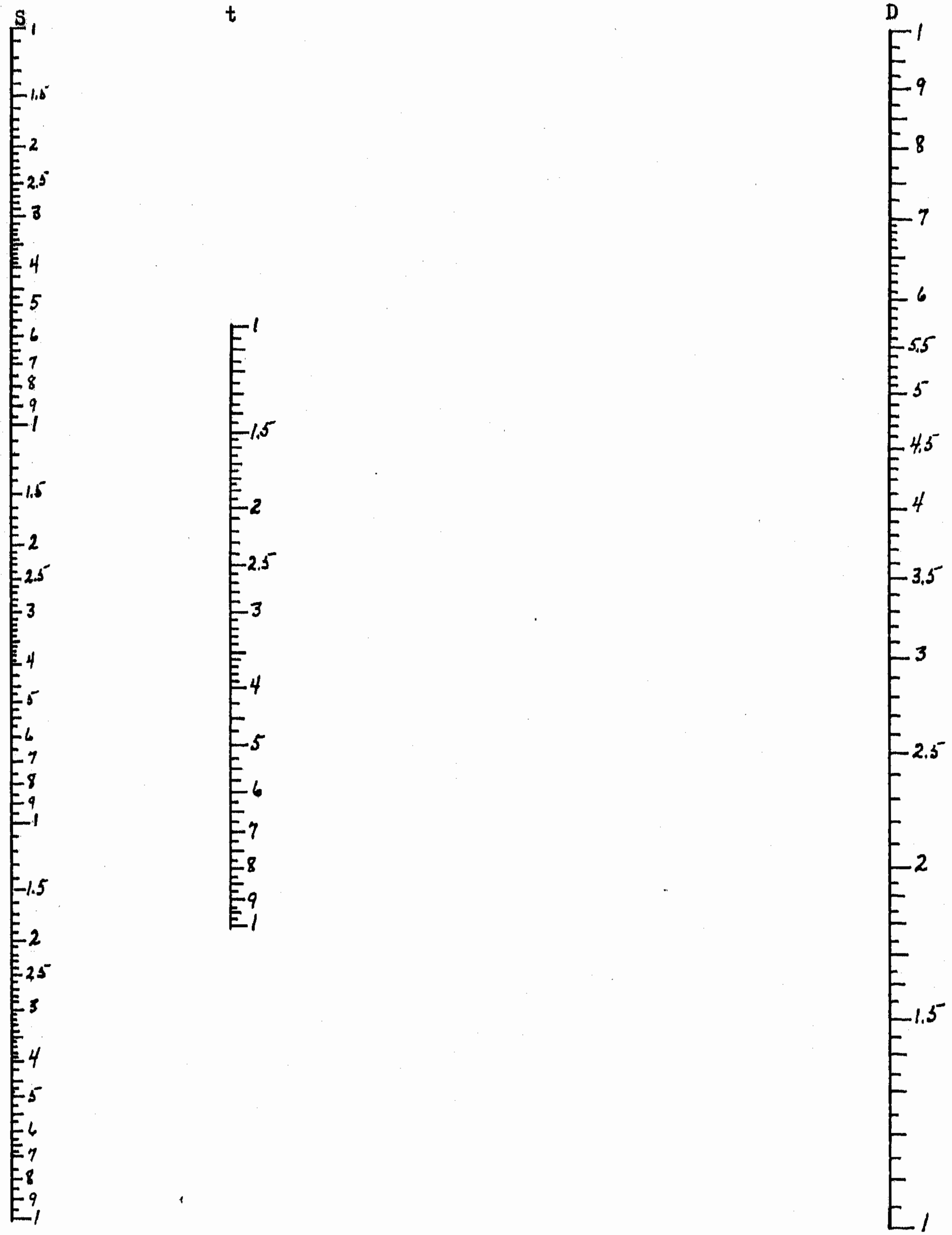


Fig. 16

$$S = D t^2$$

S = total distance
D = distance an object falls during the first second
t = time in seconds

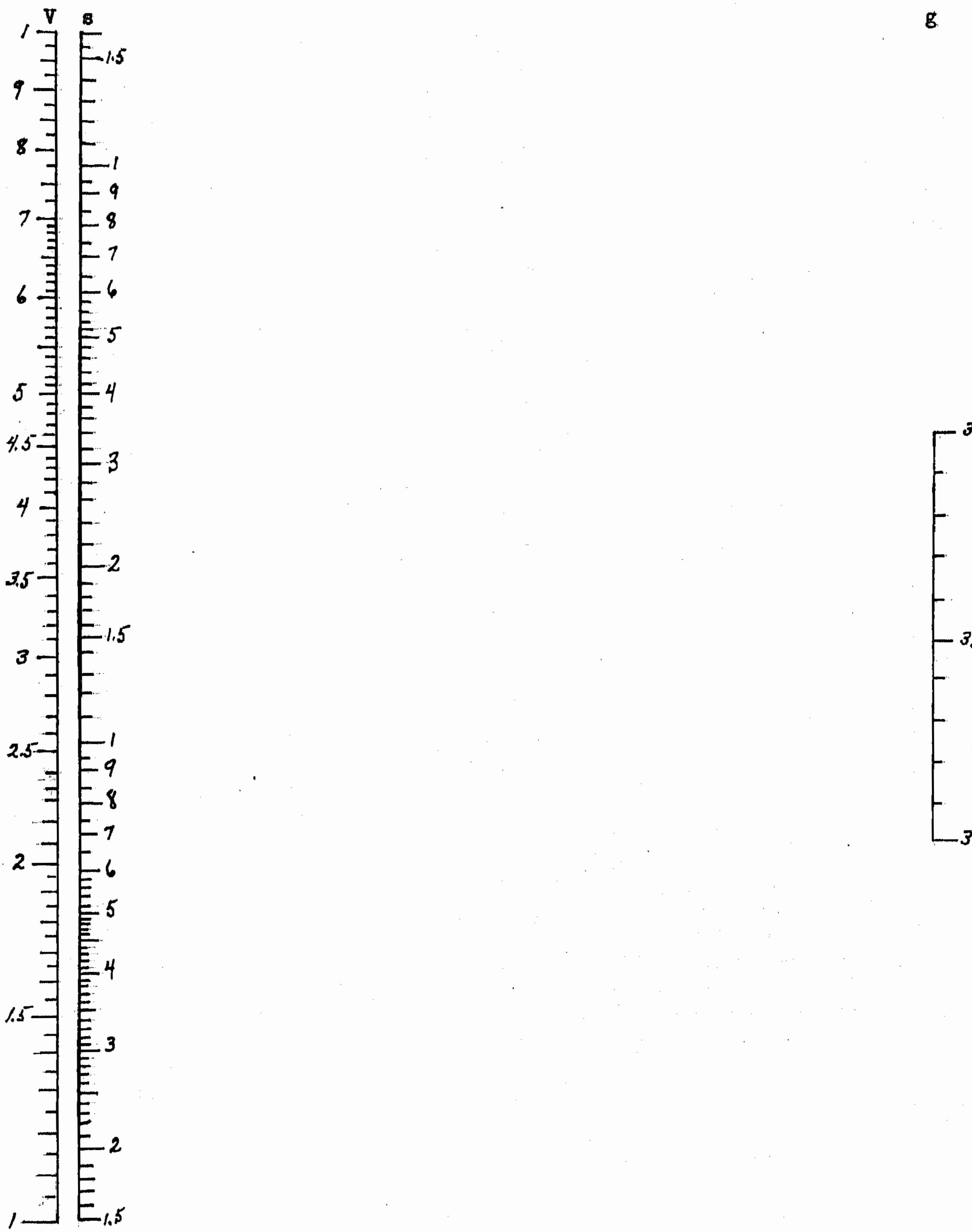
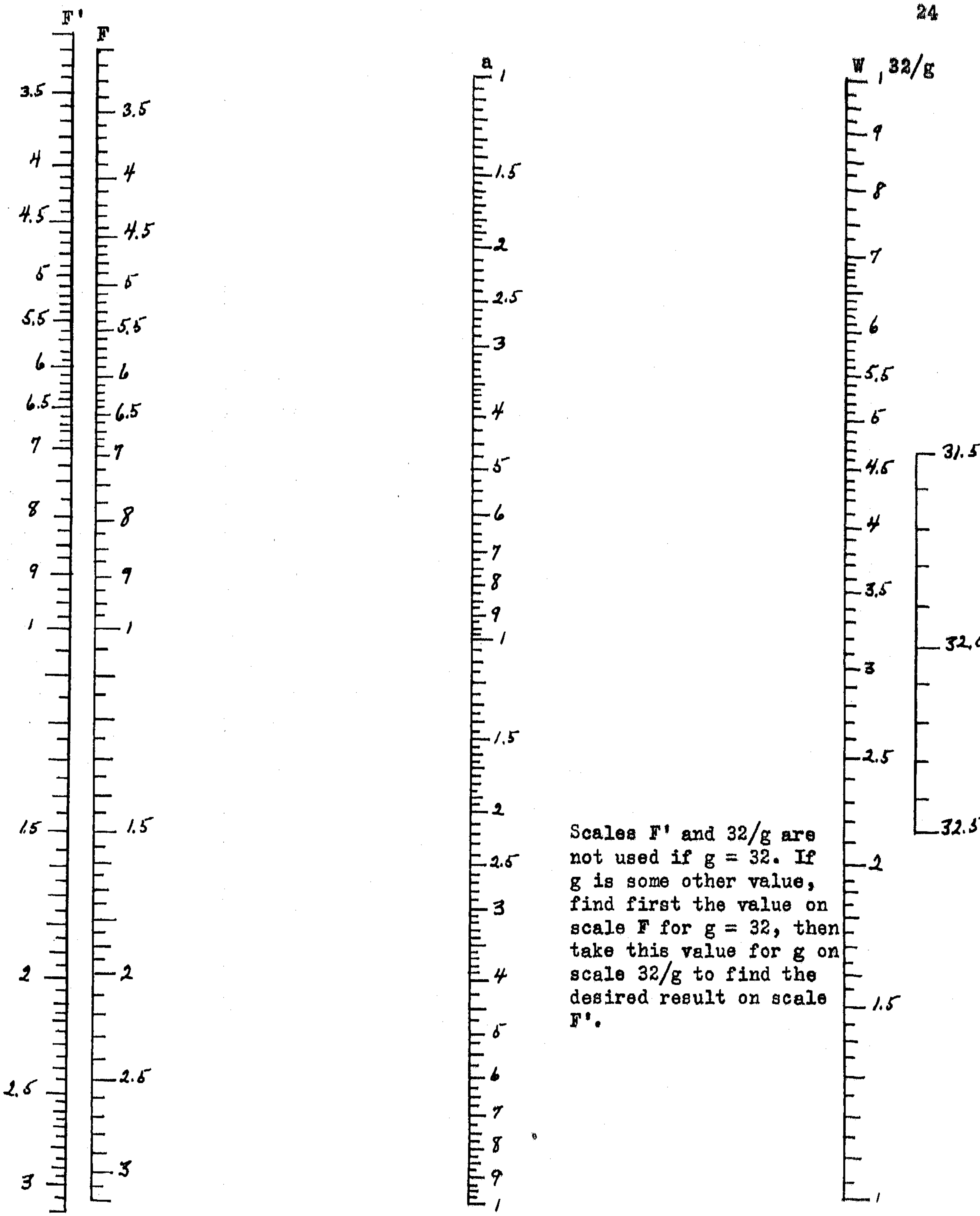


Fig. 17

$$V^2 = 2\ g\ s$$

V = velocity
s = distance
g = acceleration of gravity



Scales F' and 32/g are not used if g = 32. If g is some other value, find first the value on scale F for g = 32, then take this value for g on scale 32/g to find the desired result on scale F'.

Fig. 18

$F = W a/g$ for $g = 32$

F = force
W = weight
a = acceleration
g = acceleration of gravity

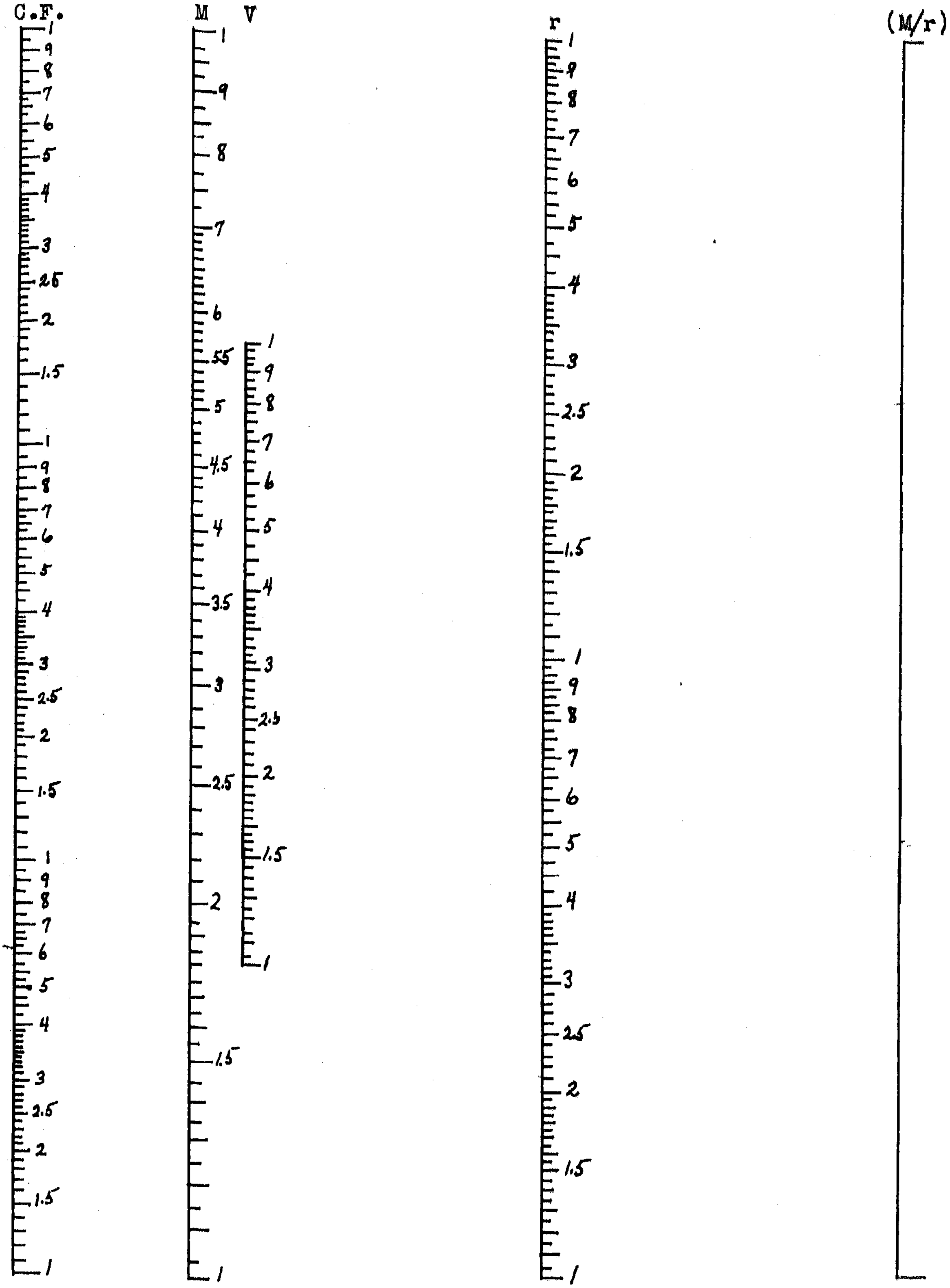


Fig. 19

C.F. = Mv^2/r

C.F. = centrifugal force
M = mass
v = velocity
r = radius

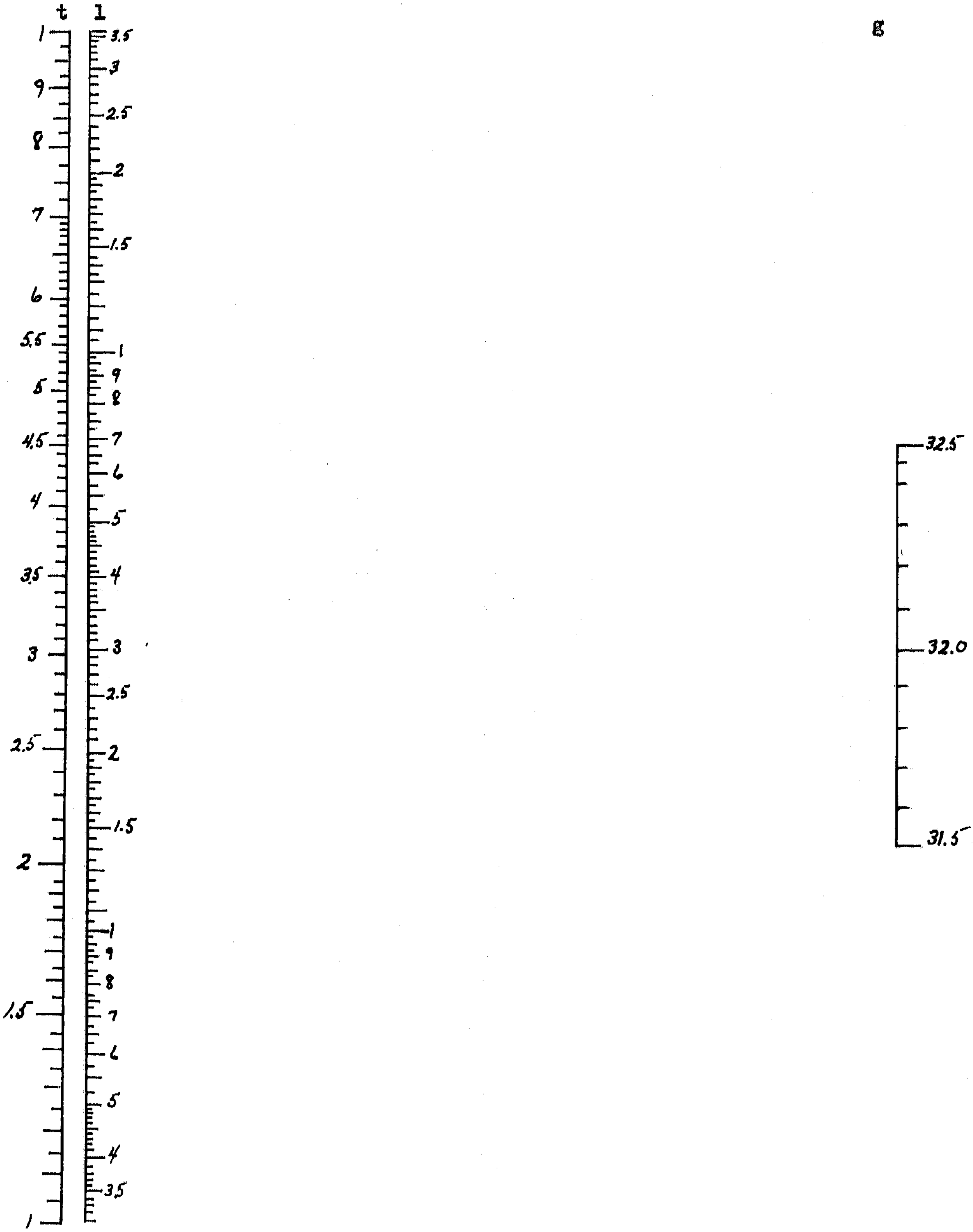


Fig. 20

$$t = \pi\sqrt{l/g}$$

t = time of one vibration of a pendulum
l = length of pendulum
g = acceleration of gravity

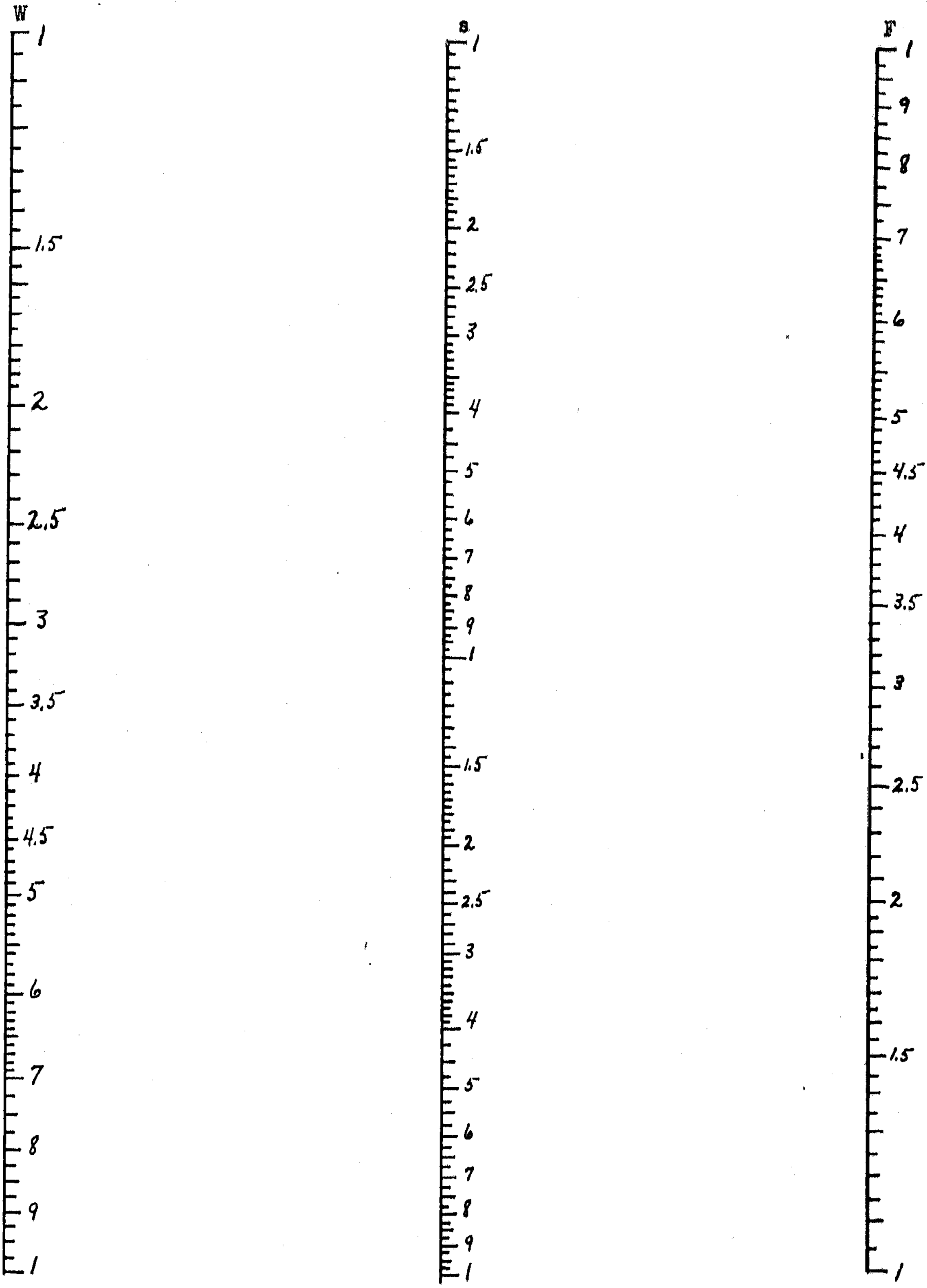


Fig. 21

W = F s

W = work in foot pounds
F = force in pounds
s = distance

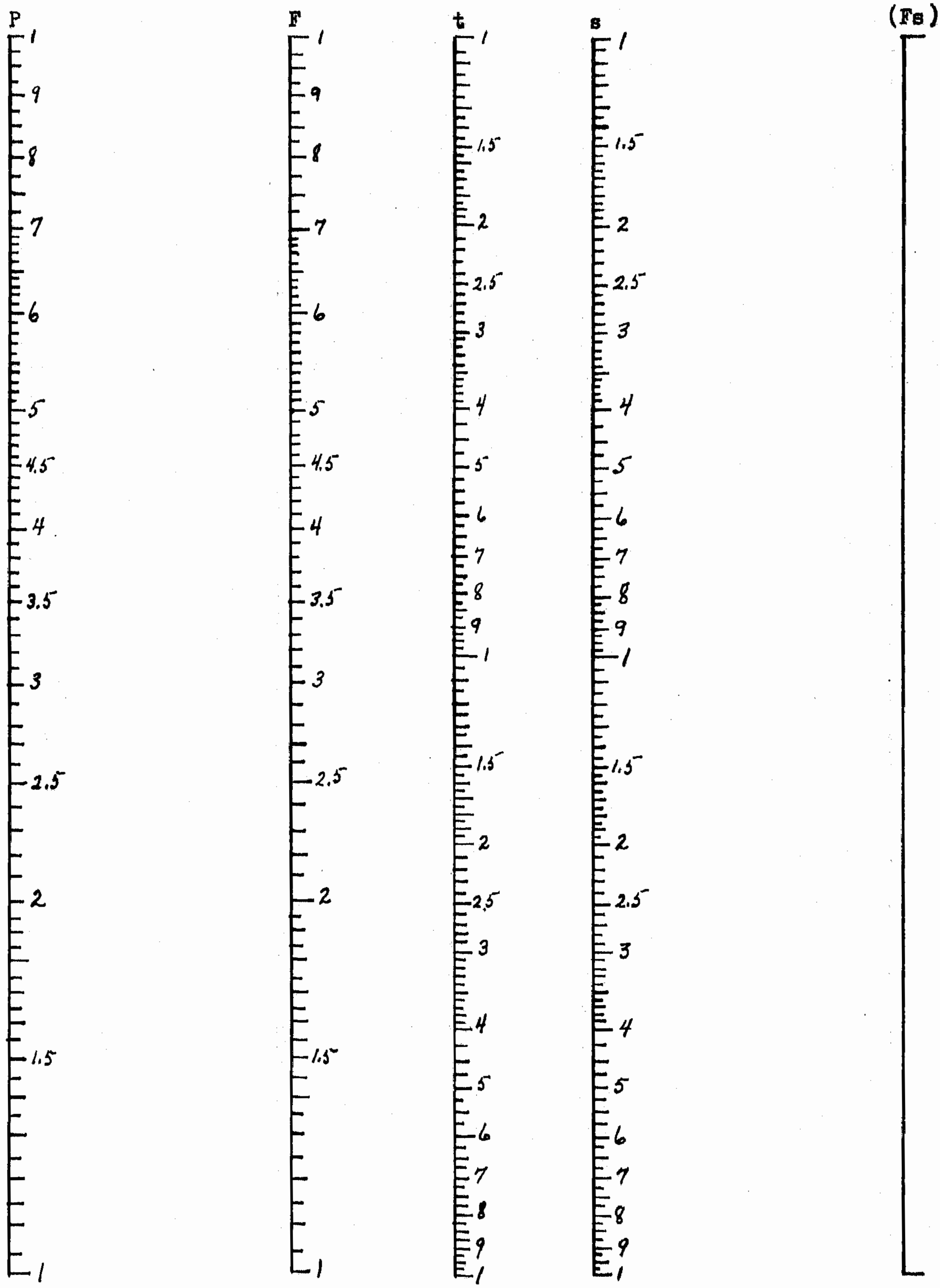


Fig. 22

$P = Fs/t$

P = power
F = force
s = distance
t = time

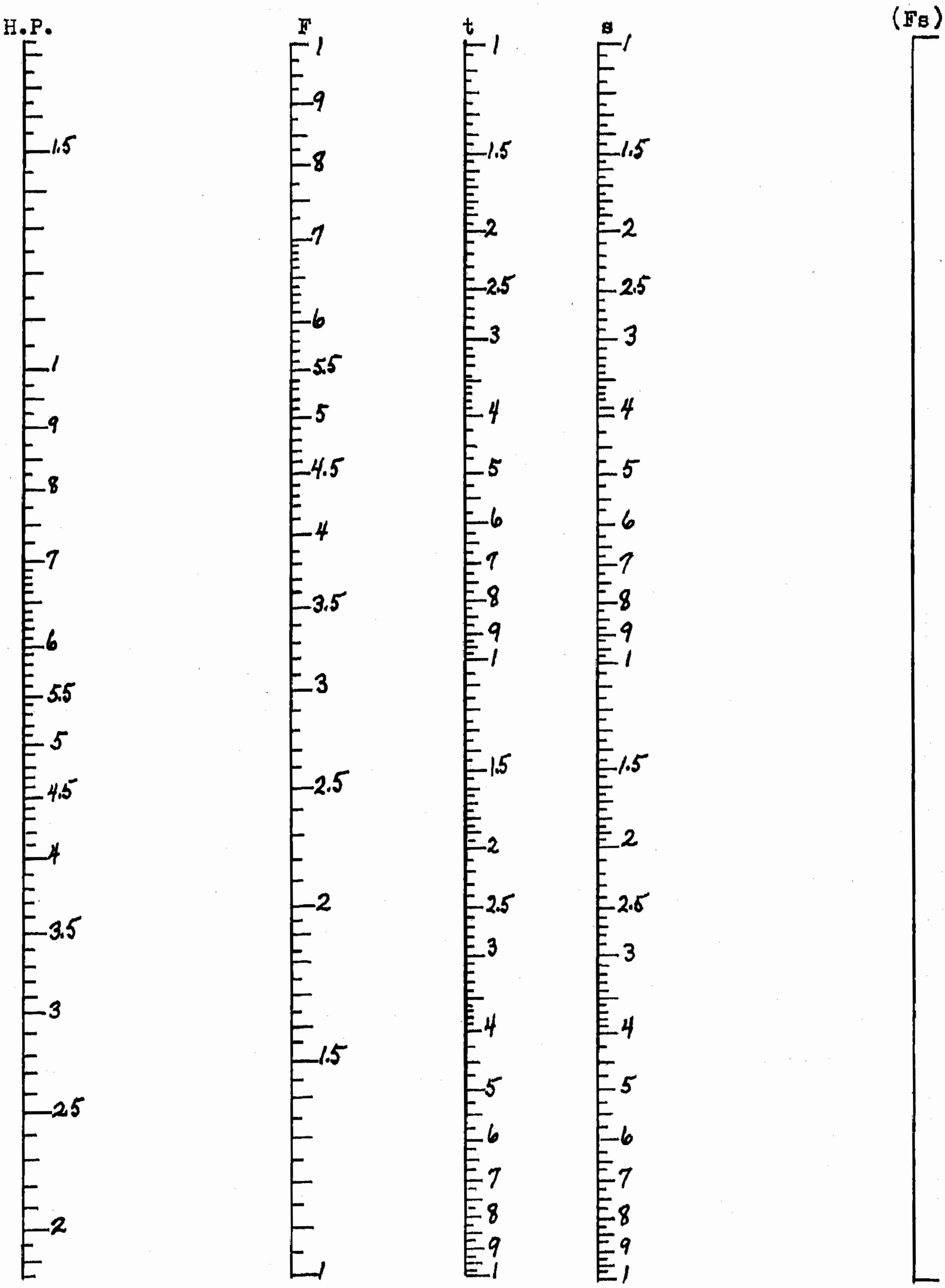


Fig. 23

H.P.= Fs/550 t

H.P.= horse power
F = force in pounds
s = distance in feet
t = time in seconds

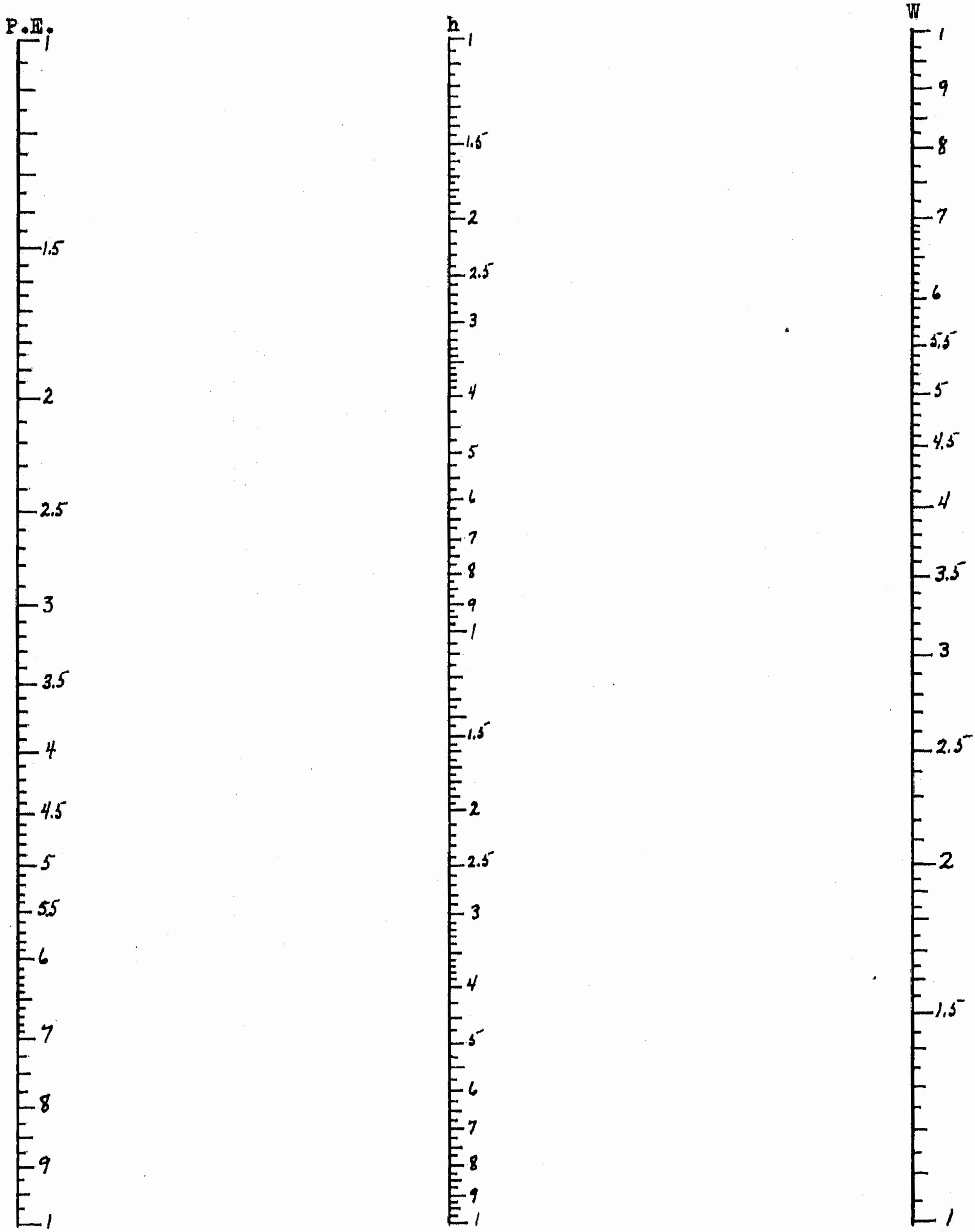


Fig. 24

$$P.E. = W\ h$$

P.E.= potential energy
W = weight
h = heigth

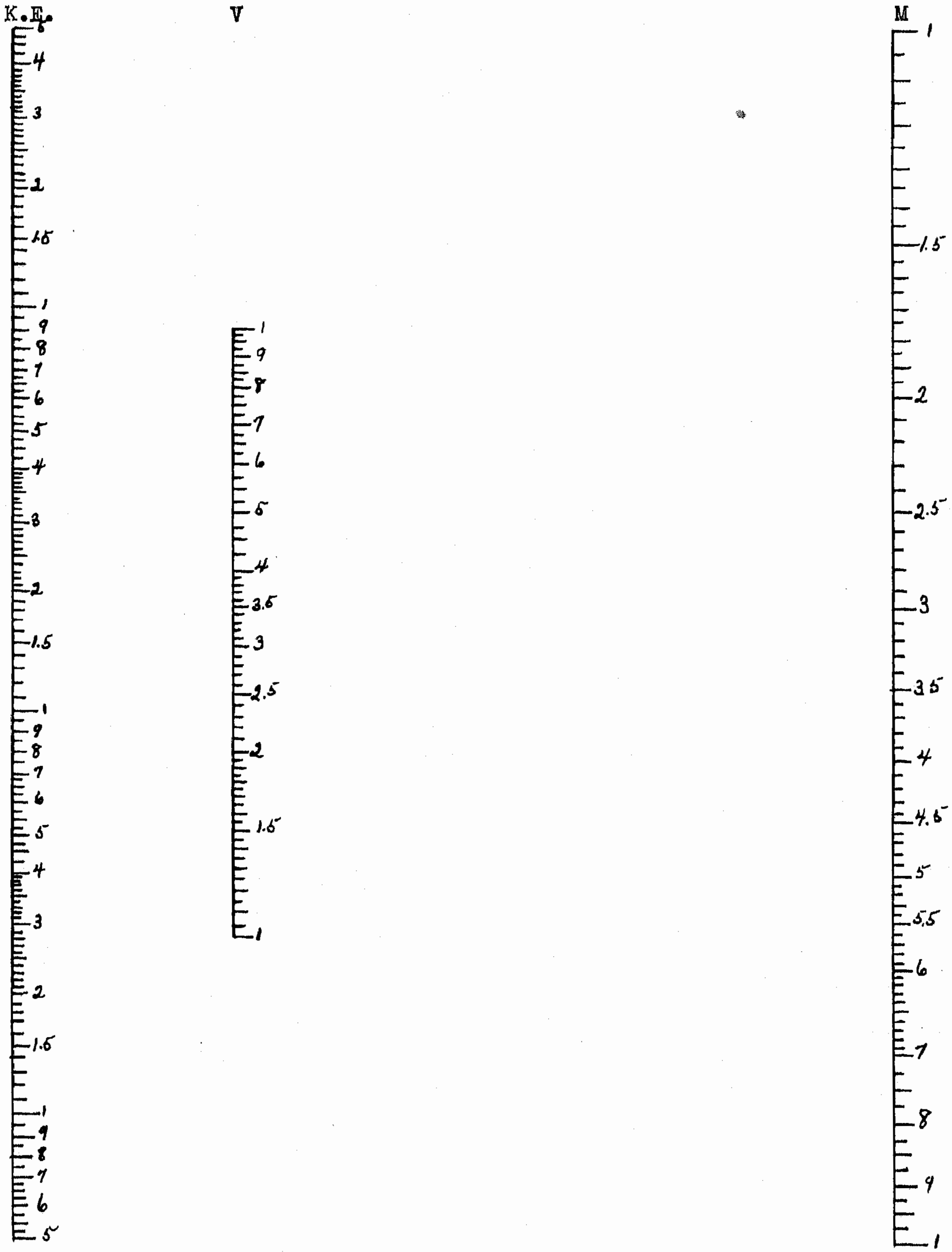


Fig. 25

$$K.E. = \frac{1}{2} M V^2$$

K.E.= kinetic energy
M = mass
V = velocity

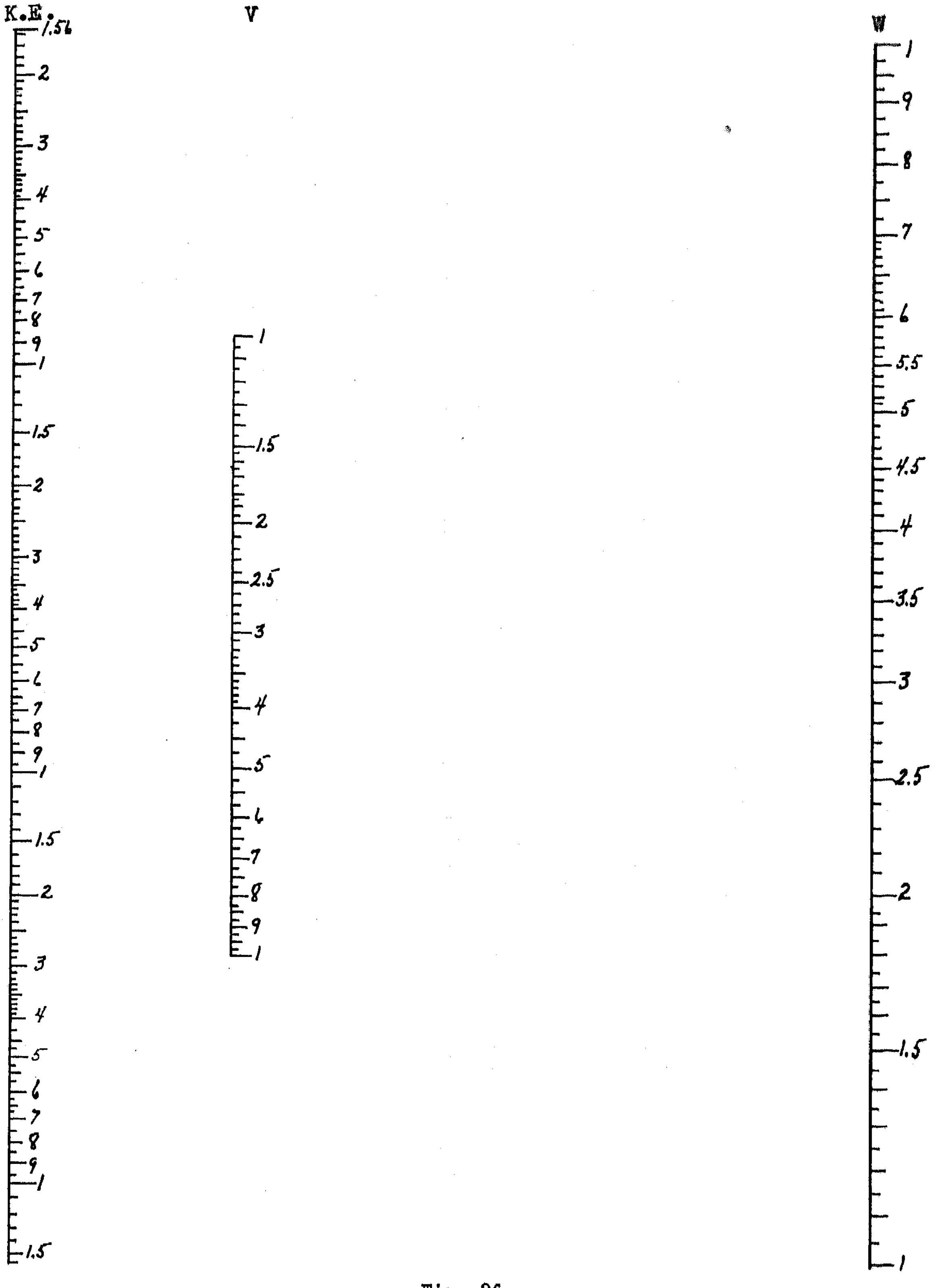


Fig. 26

K.E.= $WV^2/2g$ for g=32

K.E.= kinetic energy
V = velocity
W = weight

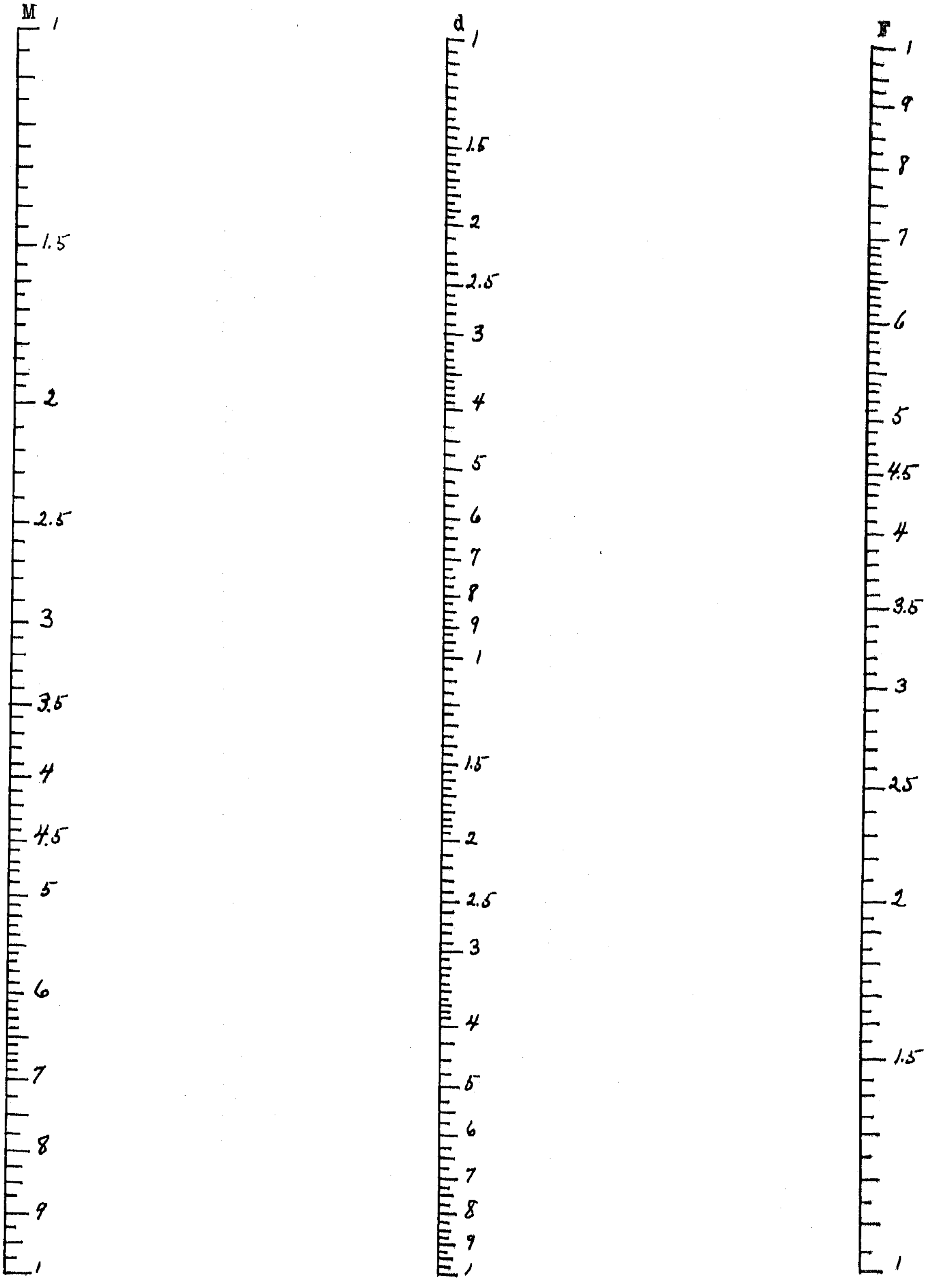


Fig. 27

M = F d

M = moment of force
F = force
d = distance from fulcrum

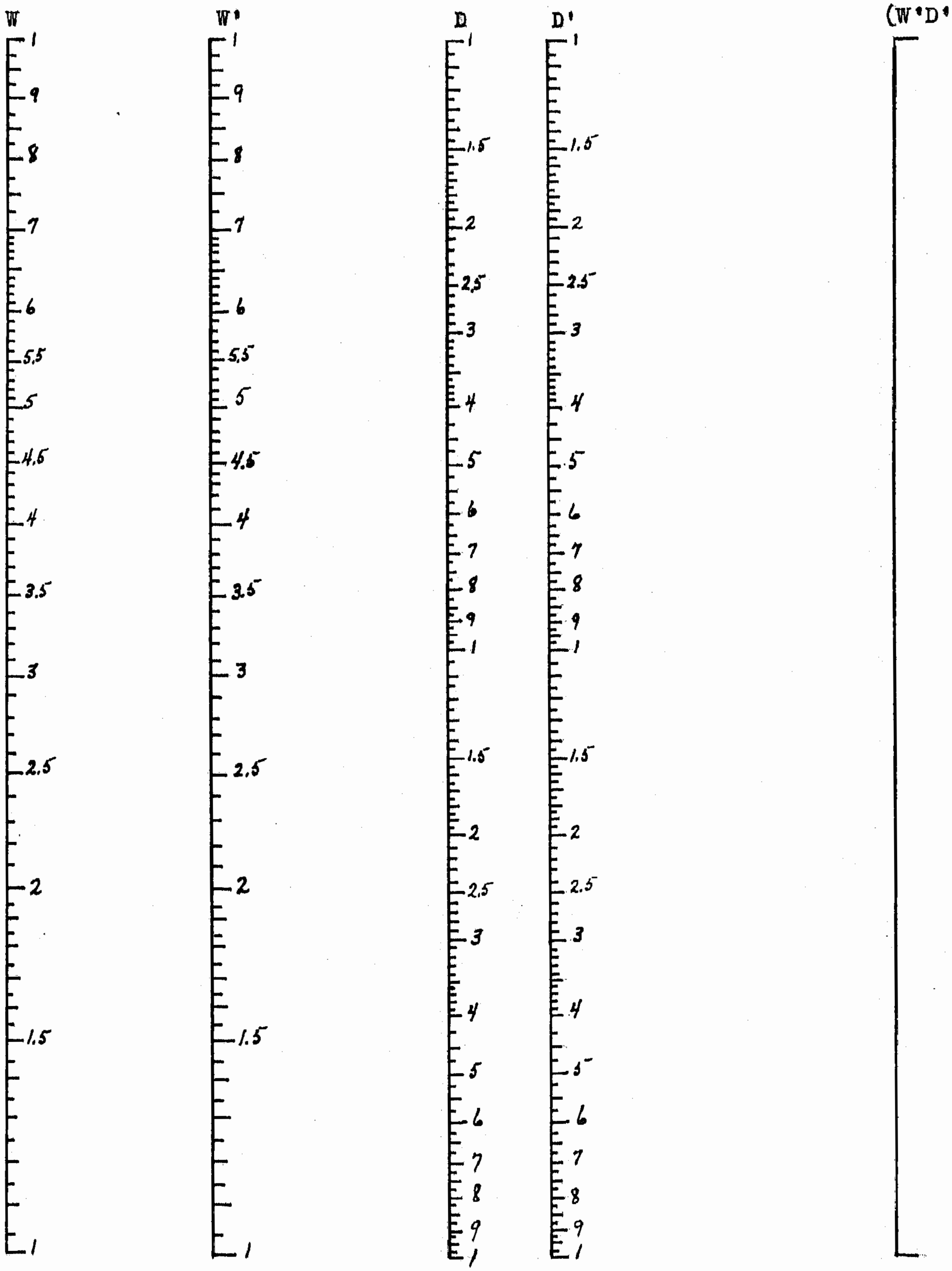


Fig. 28

W = W'D'/D

W = first weight
W'= second weight
D = first lever distance
D'= second lever distance

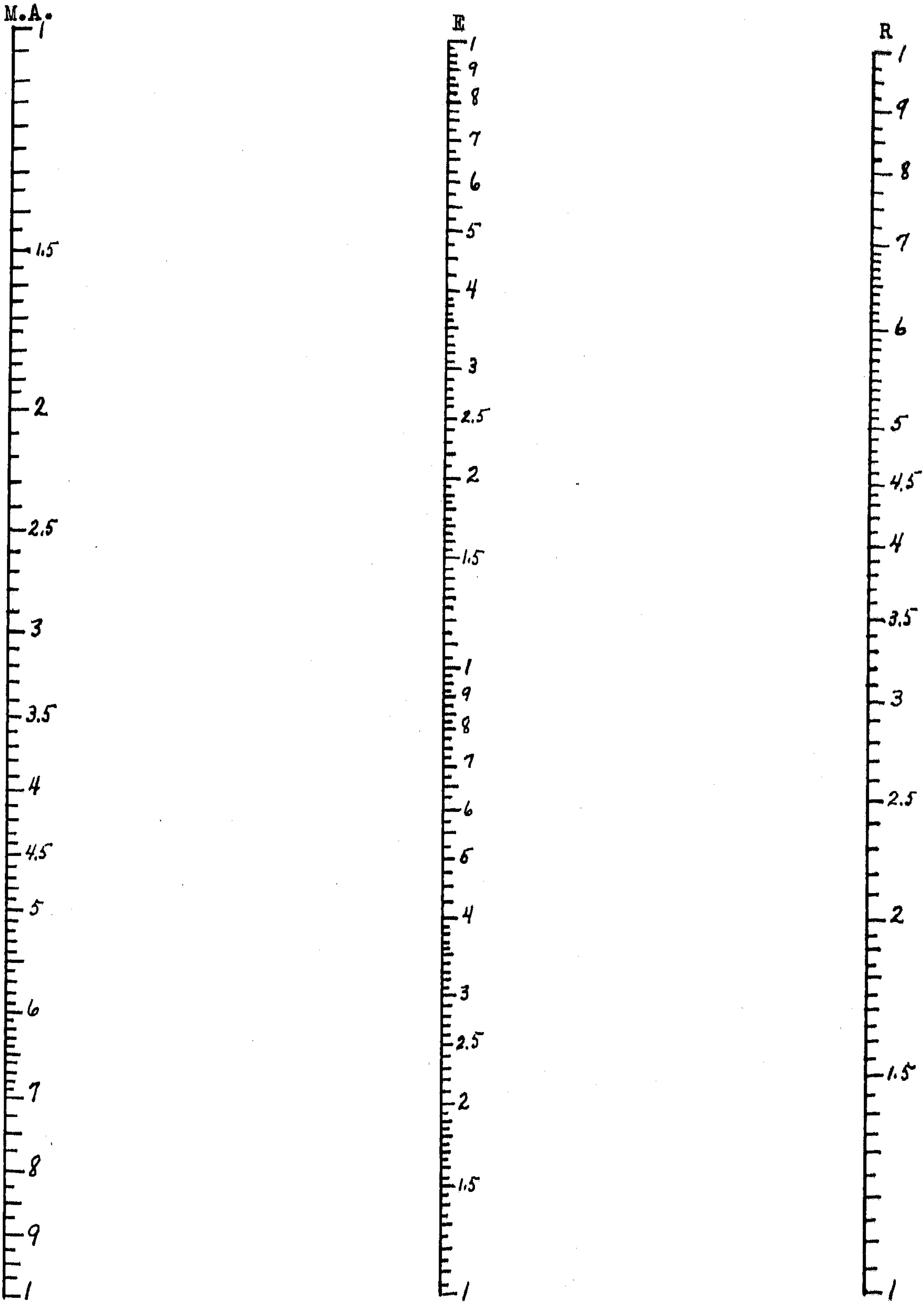


Fig. 29

M.A.= R/E

M.A.= mechanical advantage
R = resistance
E = effort

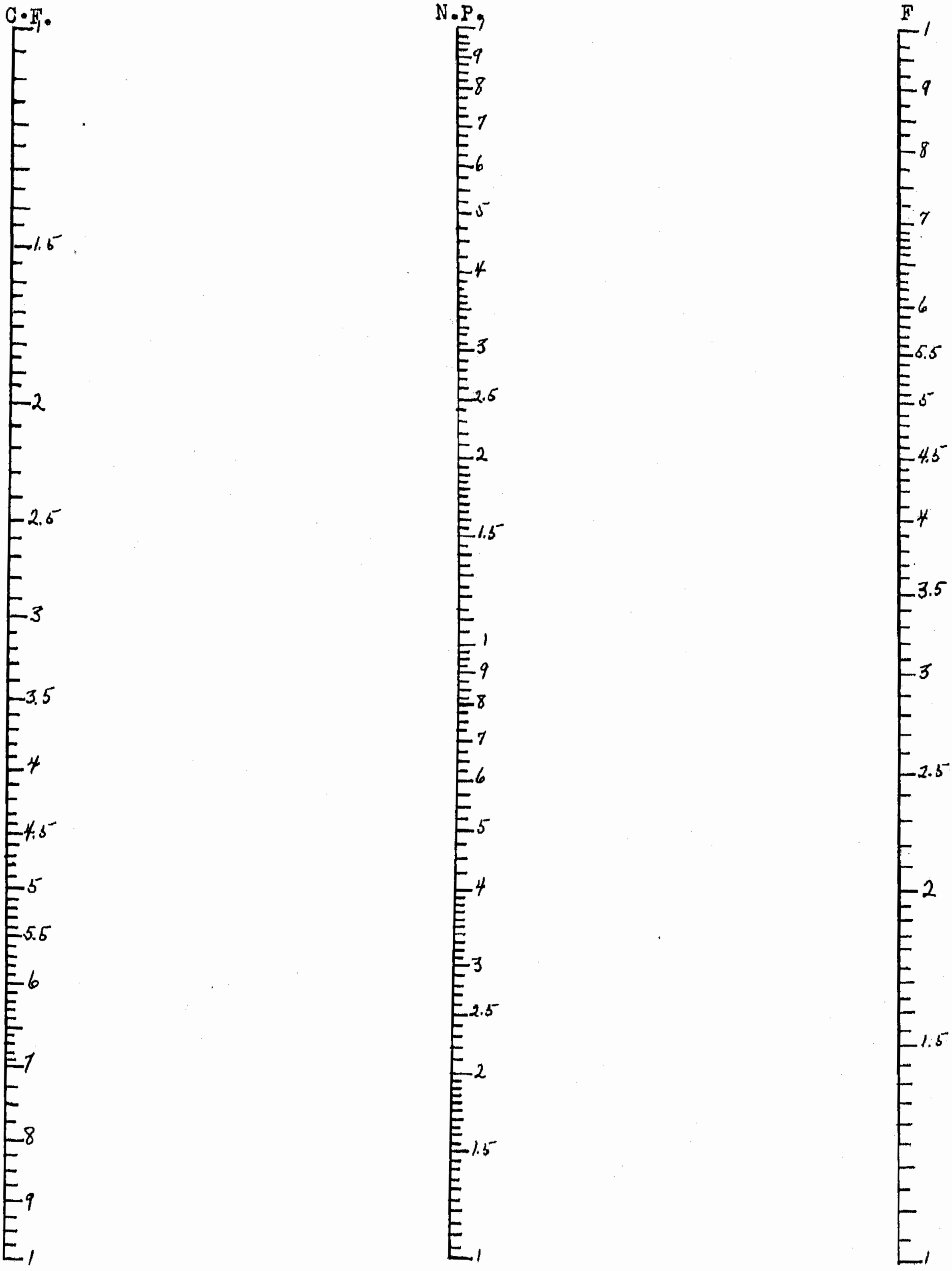


Fig. 30

C.F.= F/N.P.

C.F.= coefficient of friction
N.P.= Normal pressure
F = force

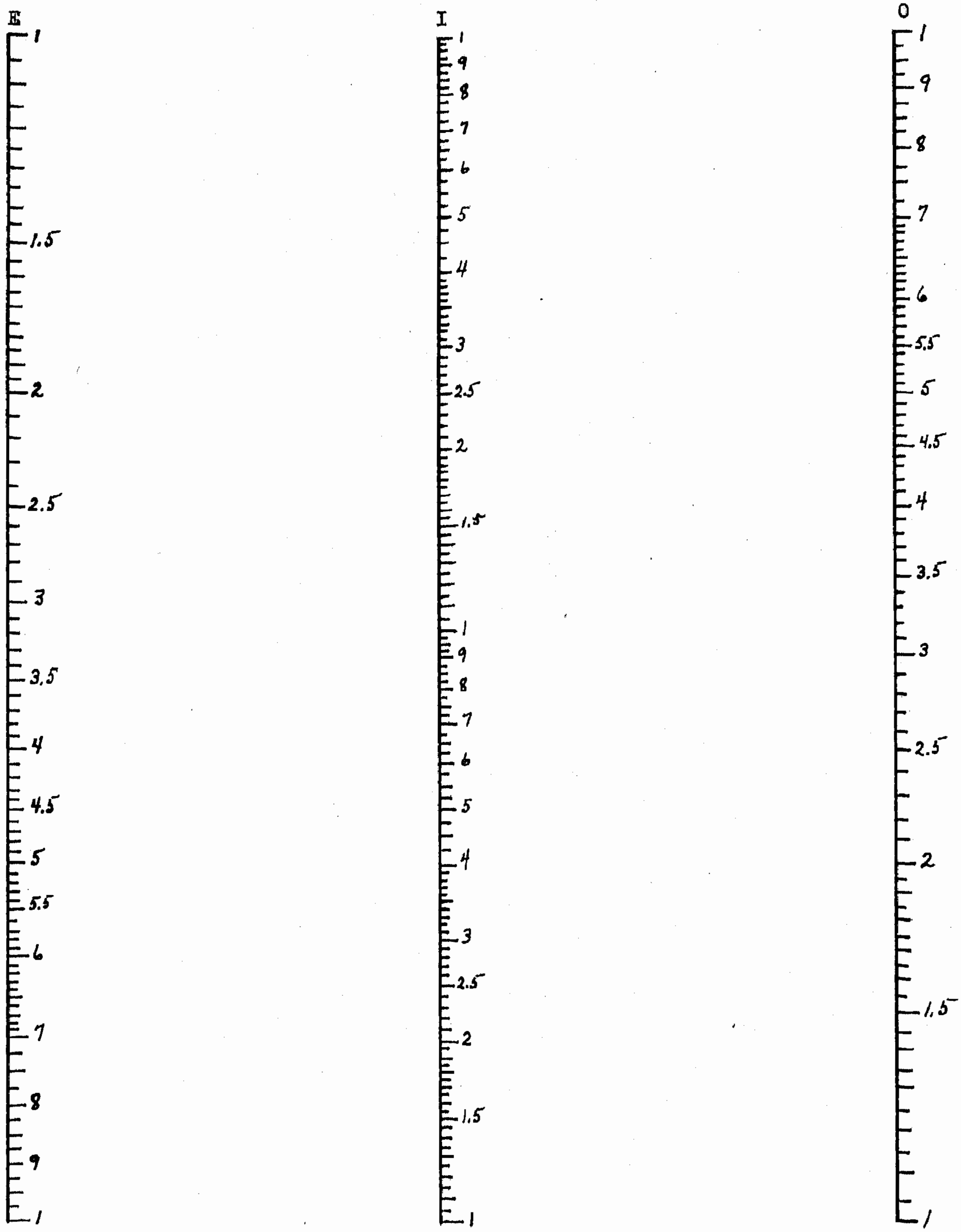


Fig. 31

$E = O/I$

E = efficiency of a machine
O = output
I = input

CHAPTER III

NOMOGRAMS
COVERING THE DIVISION OF
HEAT

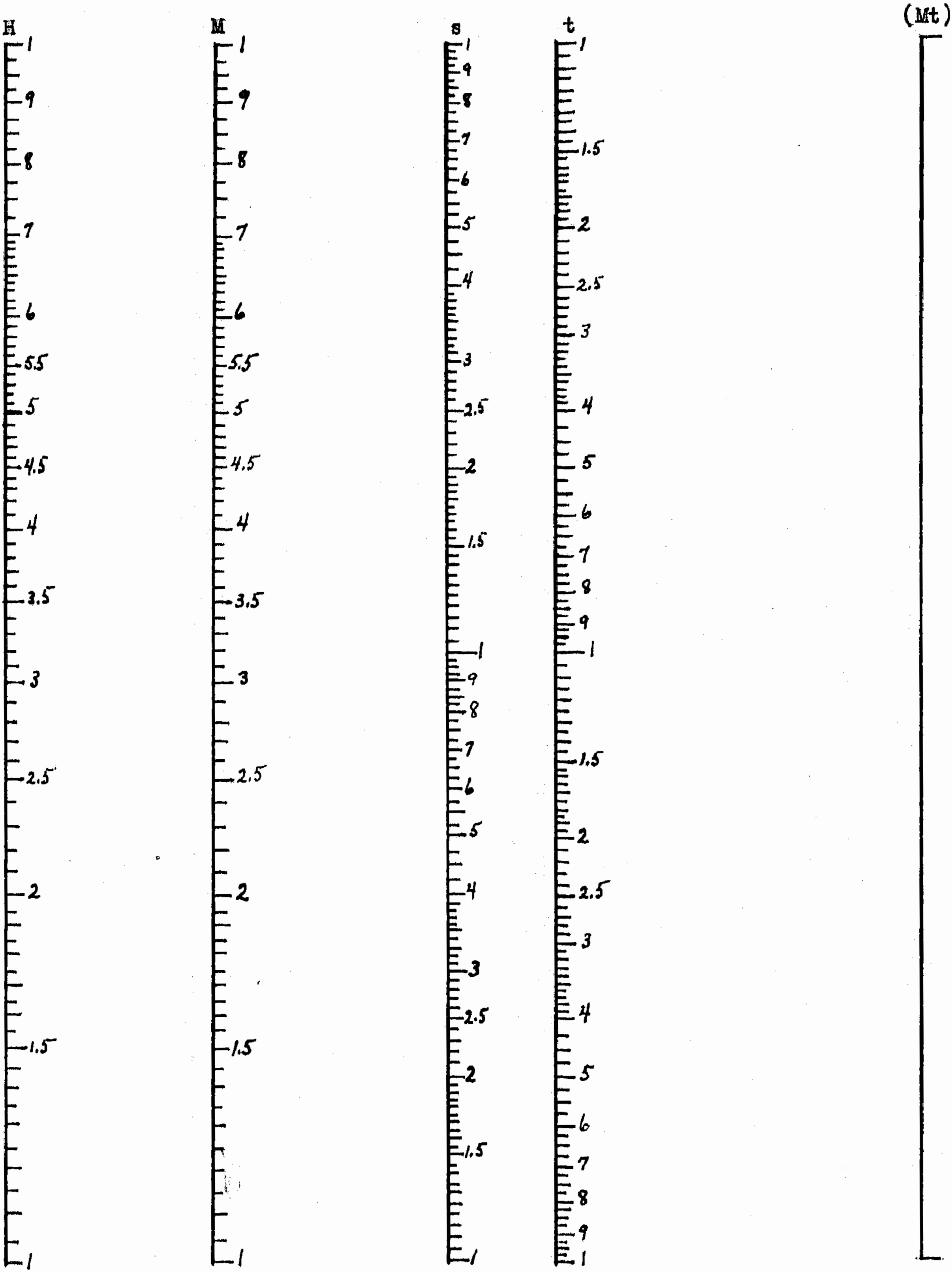


Fig. 32

$$H = M\ t\ s$$

H = heat units
M = mass
t = change in temperature
s = specific heat

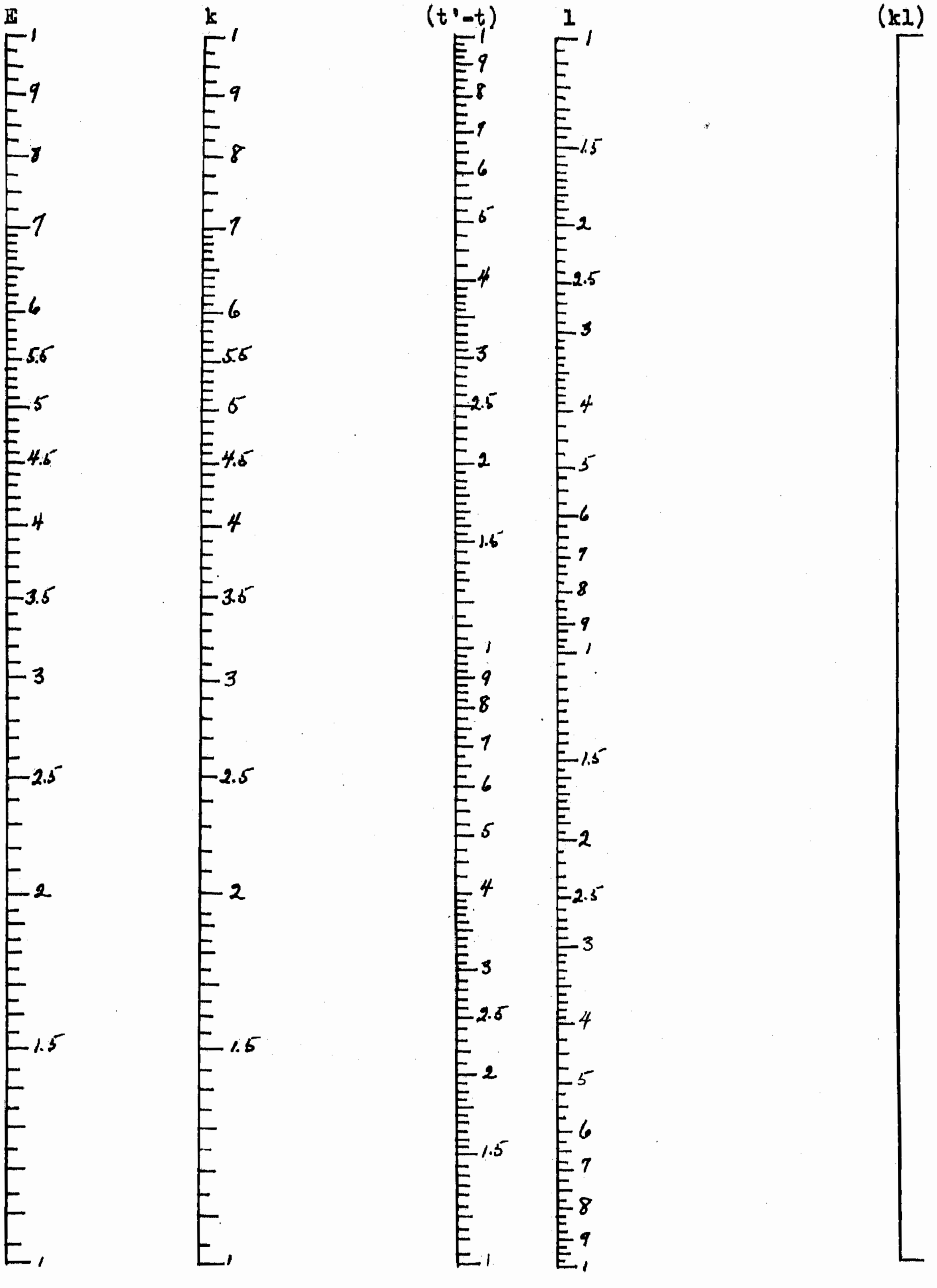


Fig. 33

$$E = k\,l\,(t'-t)$$

E = expansion
k = coefficient of linear expansion
l = original length
(t'-t) = change in temperature

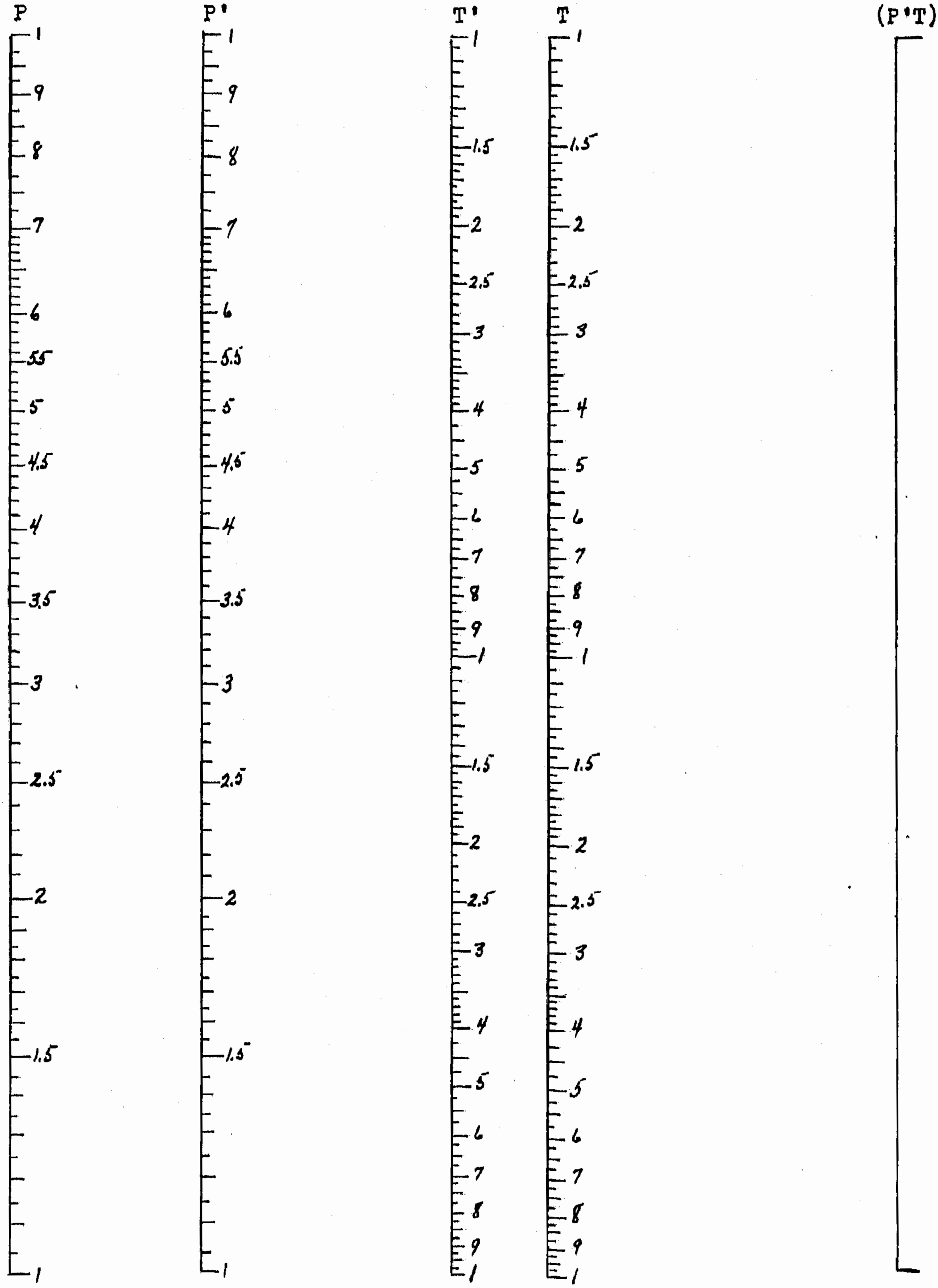


Fig. 34

$$P = P'T/T'$$

P = first pressure of a gas
P'= second pressure of a gas
T = first temperature (absolute sclae)
T'= second temperature (absolute scale)

CHAPTER IV

NOMOGRAMS

COVERING THE DIVISION OF

ELECTRICITY

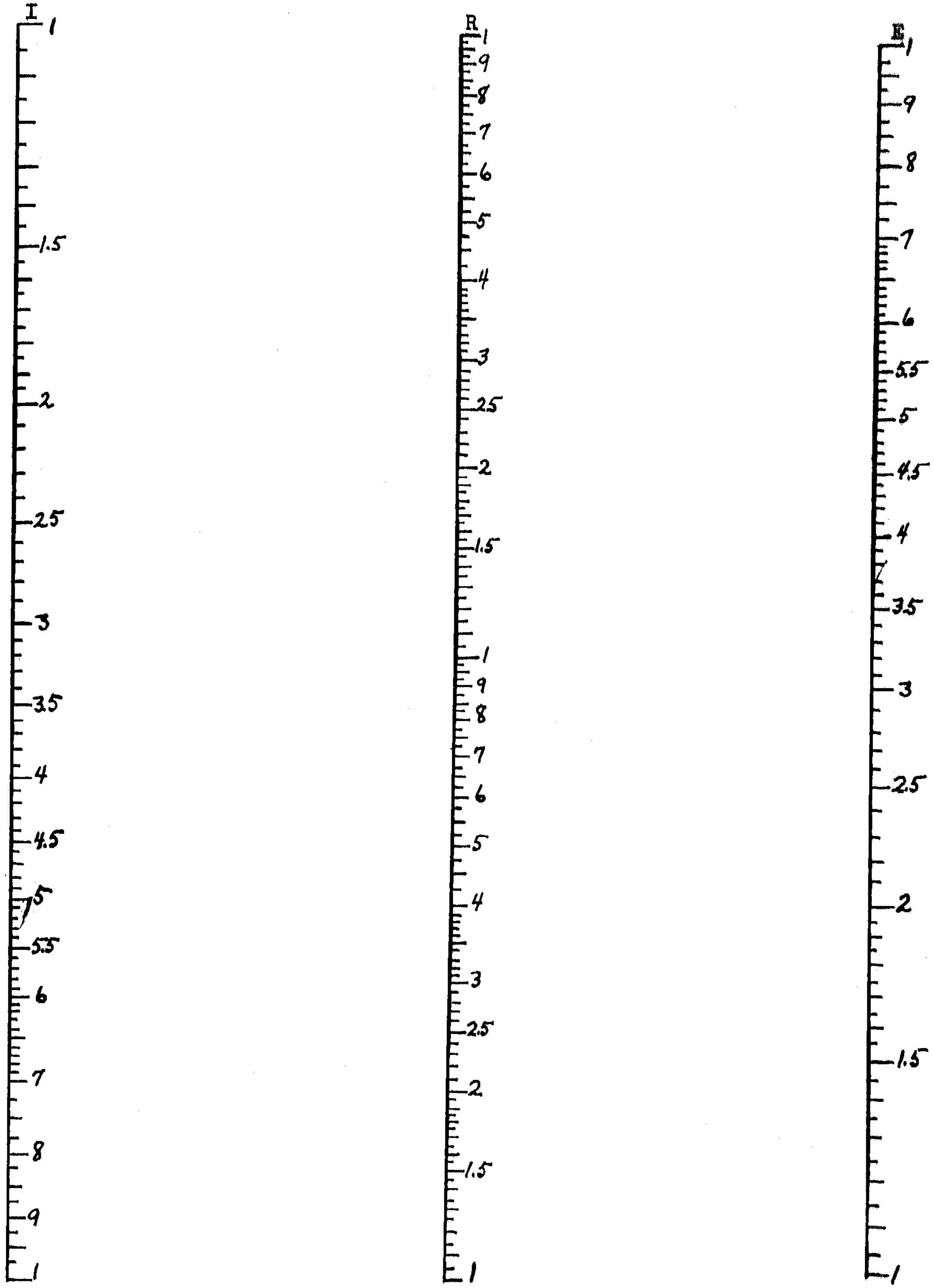


Fig. 35

I = E/R

I = amperes
R = ohms
E = volts

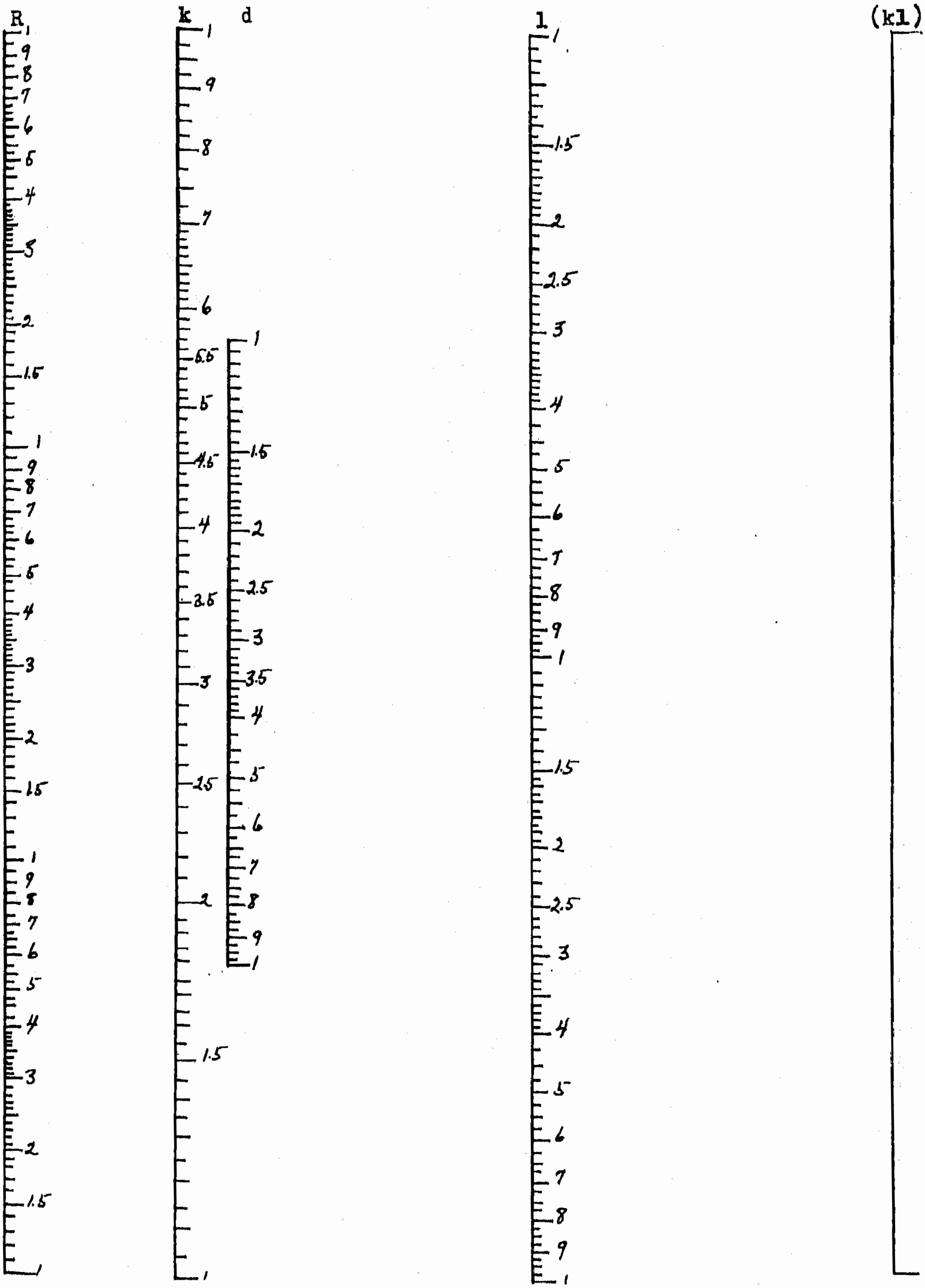


Fig. 36

$$R = kl/d^2$$

R = resistance in a conductor
k = specific resistance
d = diameter in mills
l = length

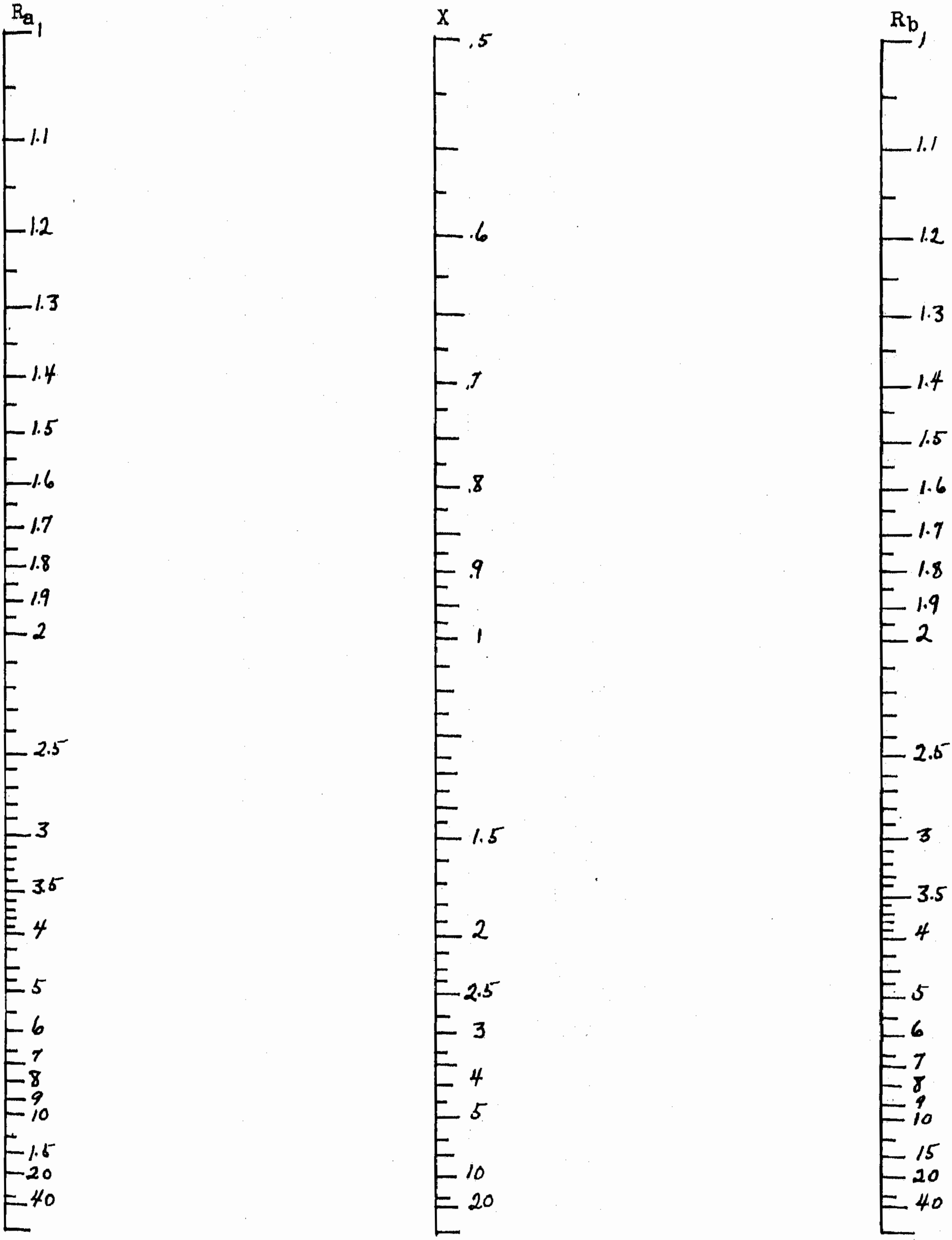


Fig. 37

$$1/X = 1/R_a + 1/R_b$$

X = total resistance for a parallel circuit
R_a= resistance in first branch
R_b= resistance in second branch

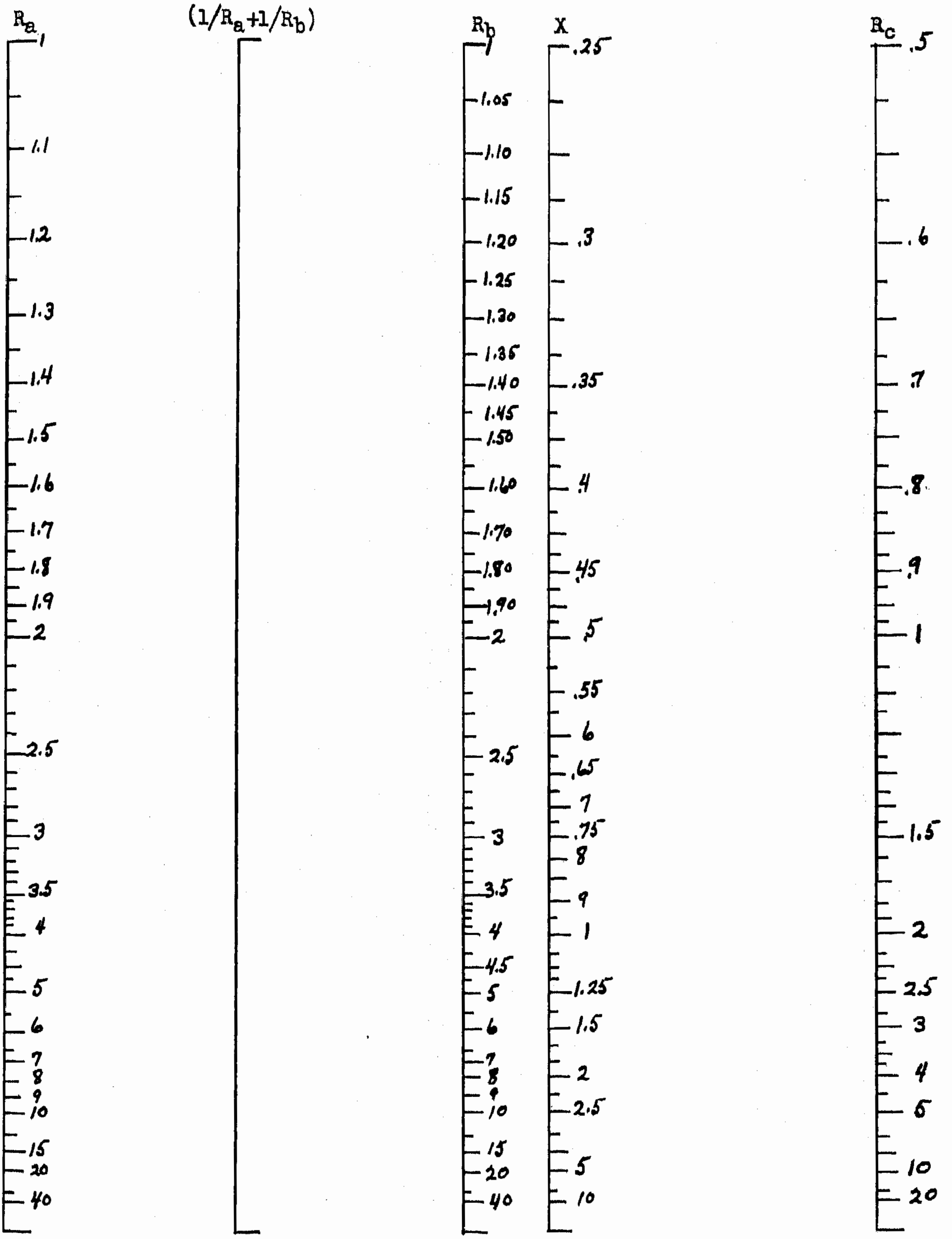


Fig. 38

$1/X = 1/R_a + 1/R_b + 1/R_c$

X = total resistance in ohms
R_a= resistance in first branch
R_b= resistance in second branch
R_c= resistance in third branch

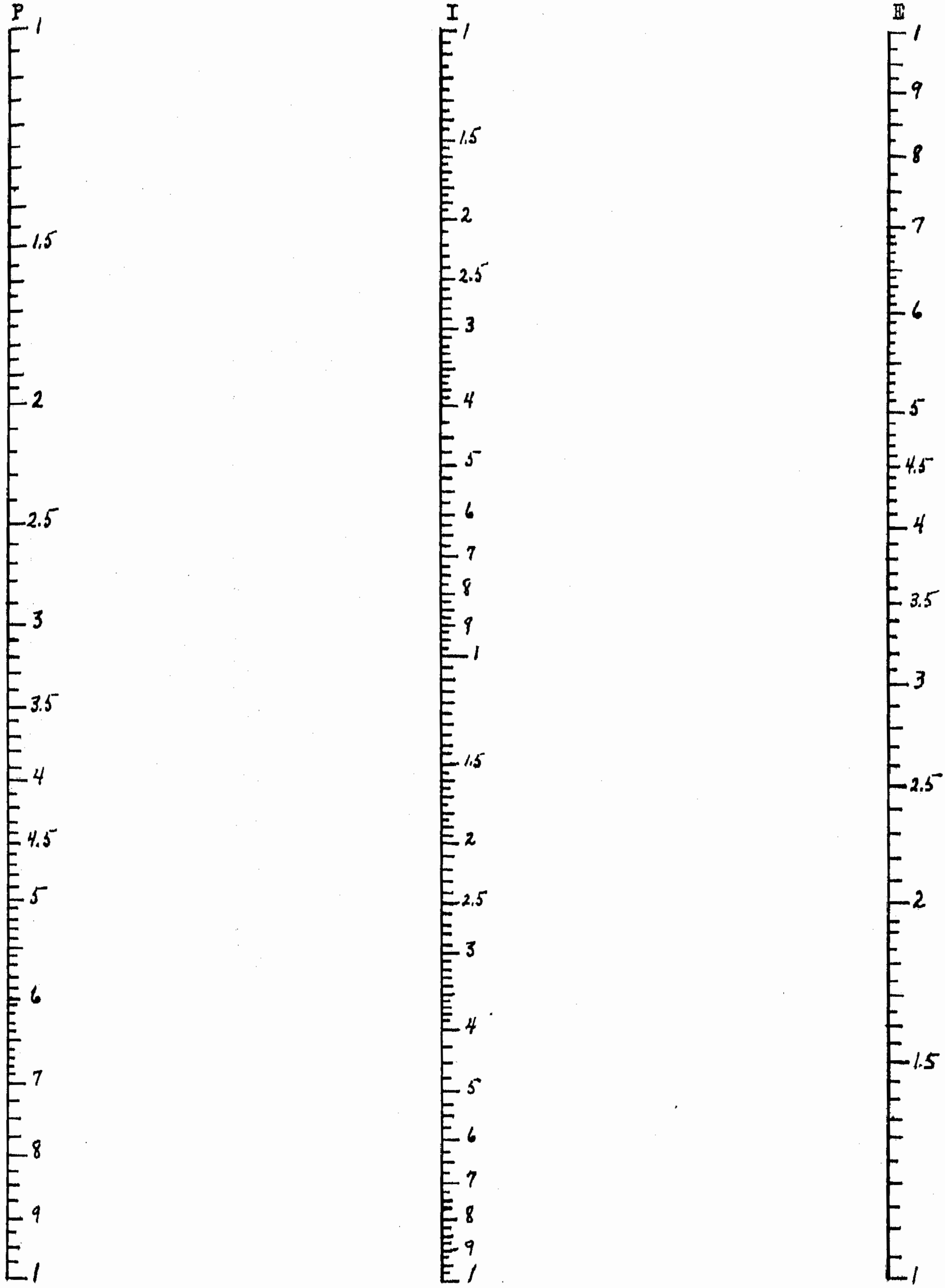


Fig. 39

P = E I

P = electric power in watts
E = volts
I = amperes

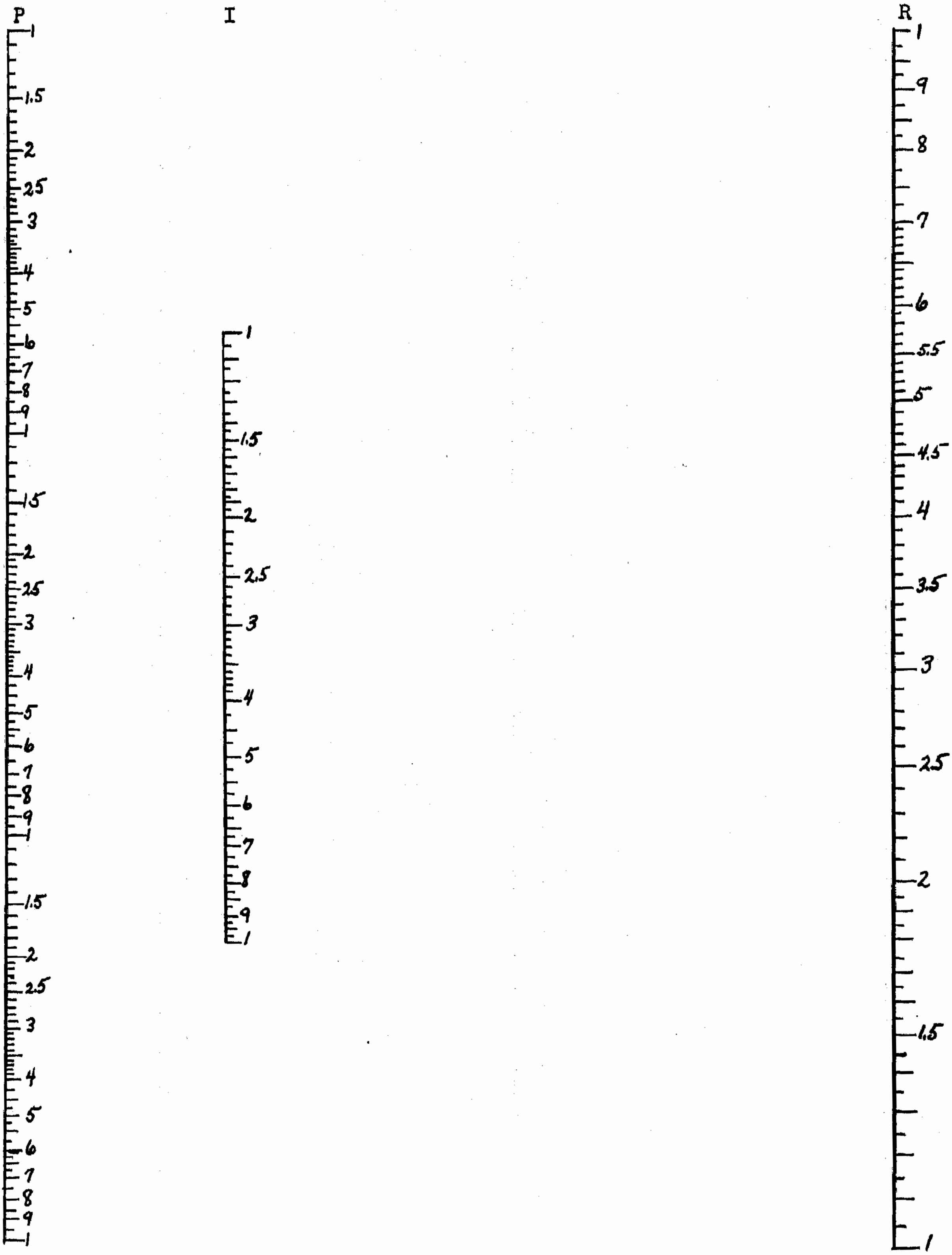


Fig. 40

$$P = I^2 R$$

P = electric power in watts
I = amperes
R = ohms

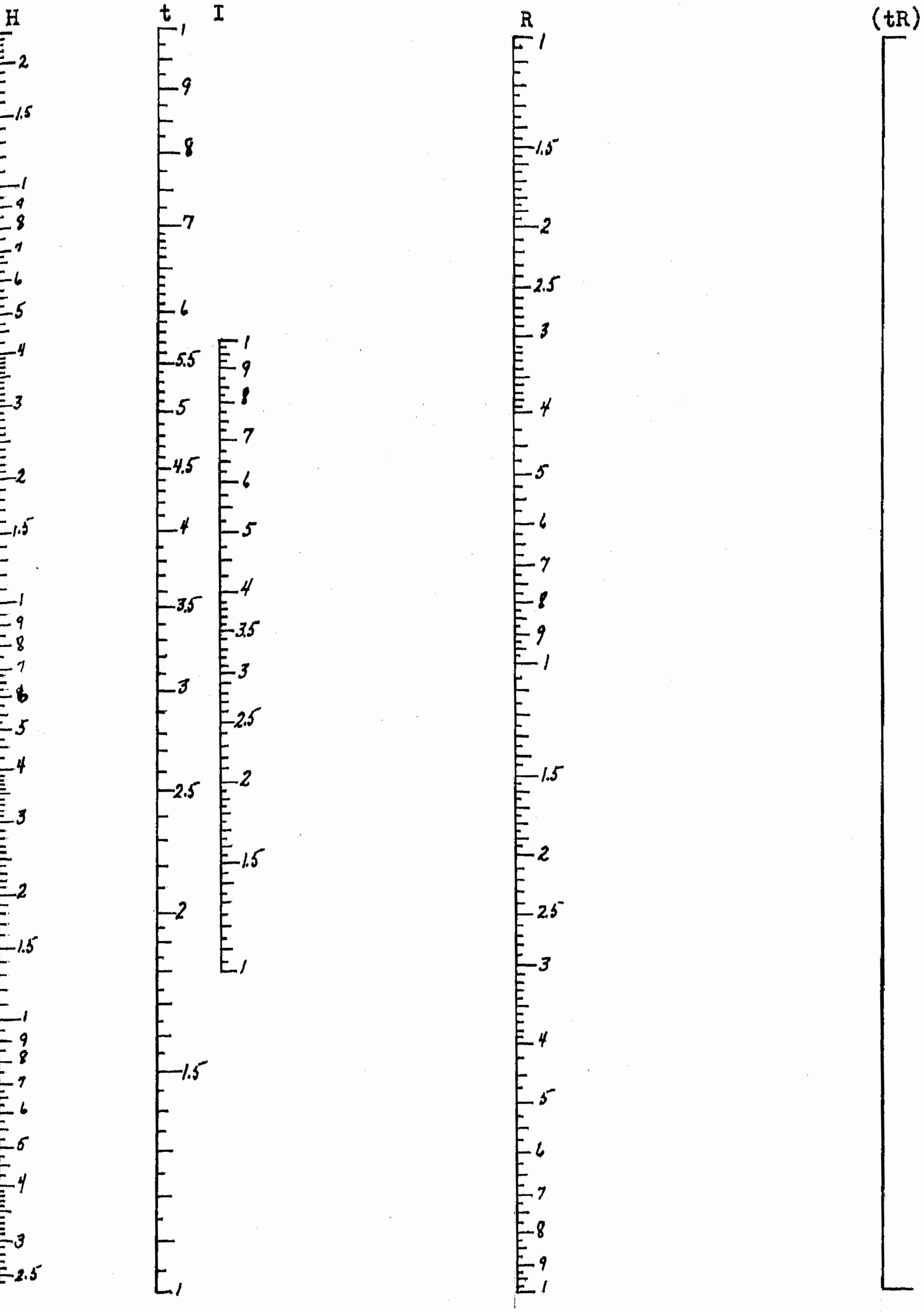


Fig. 41

$$H = .24\, t\, I^2\, R$$

H = heat in calories
t = time in seconds
I = current in amperes
R = resistance in ohms

CHAPTER V

NOMOGRAMS

COVERING THE DIVISION OF

SOUND

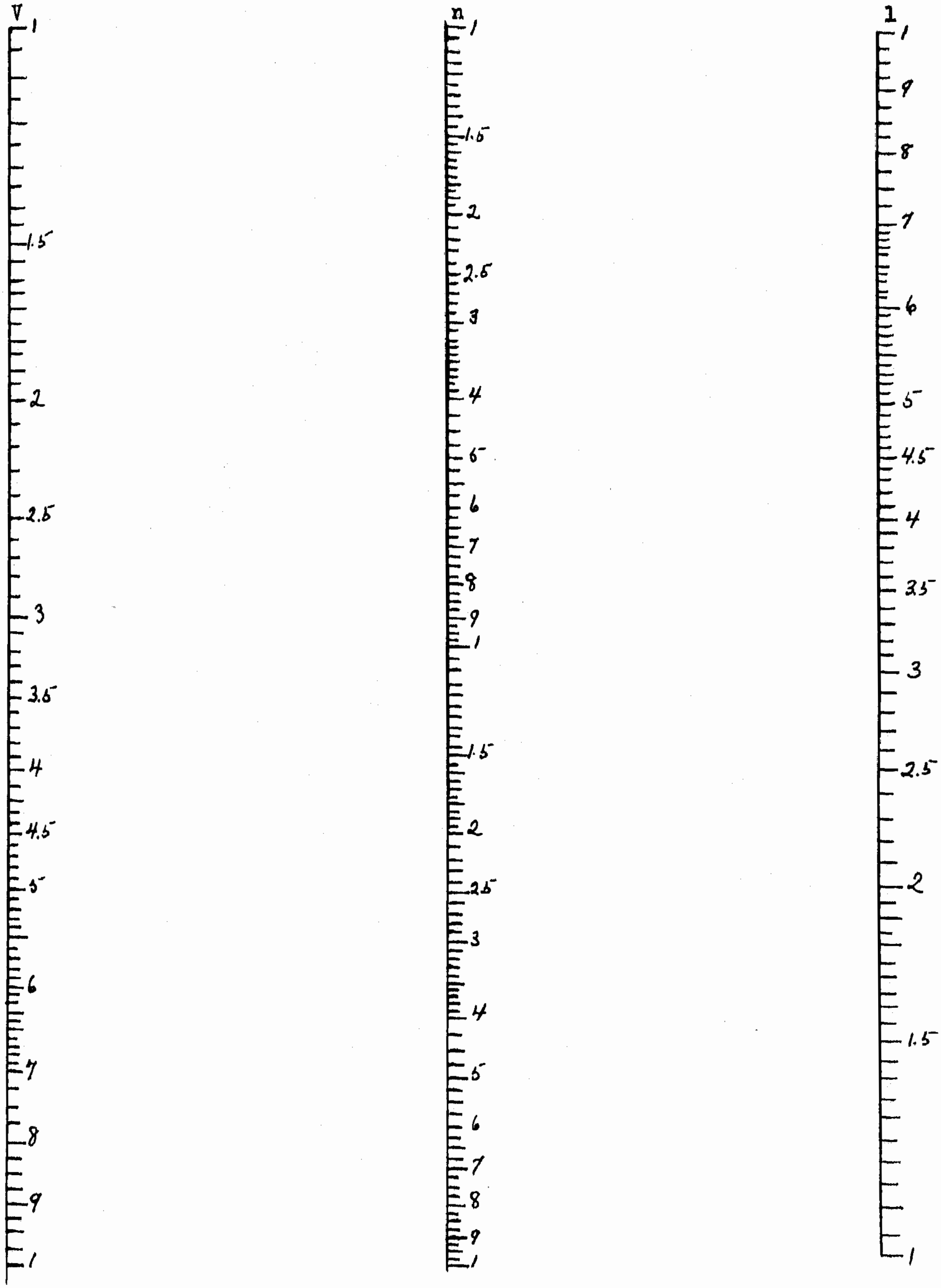


Fig. 42

V = nl

V = velocity
n = frequency
l = length

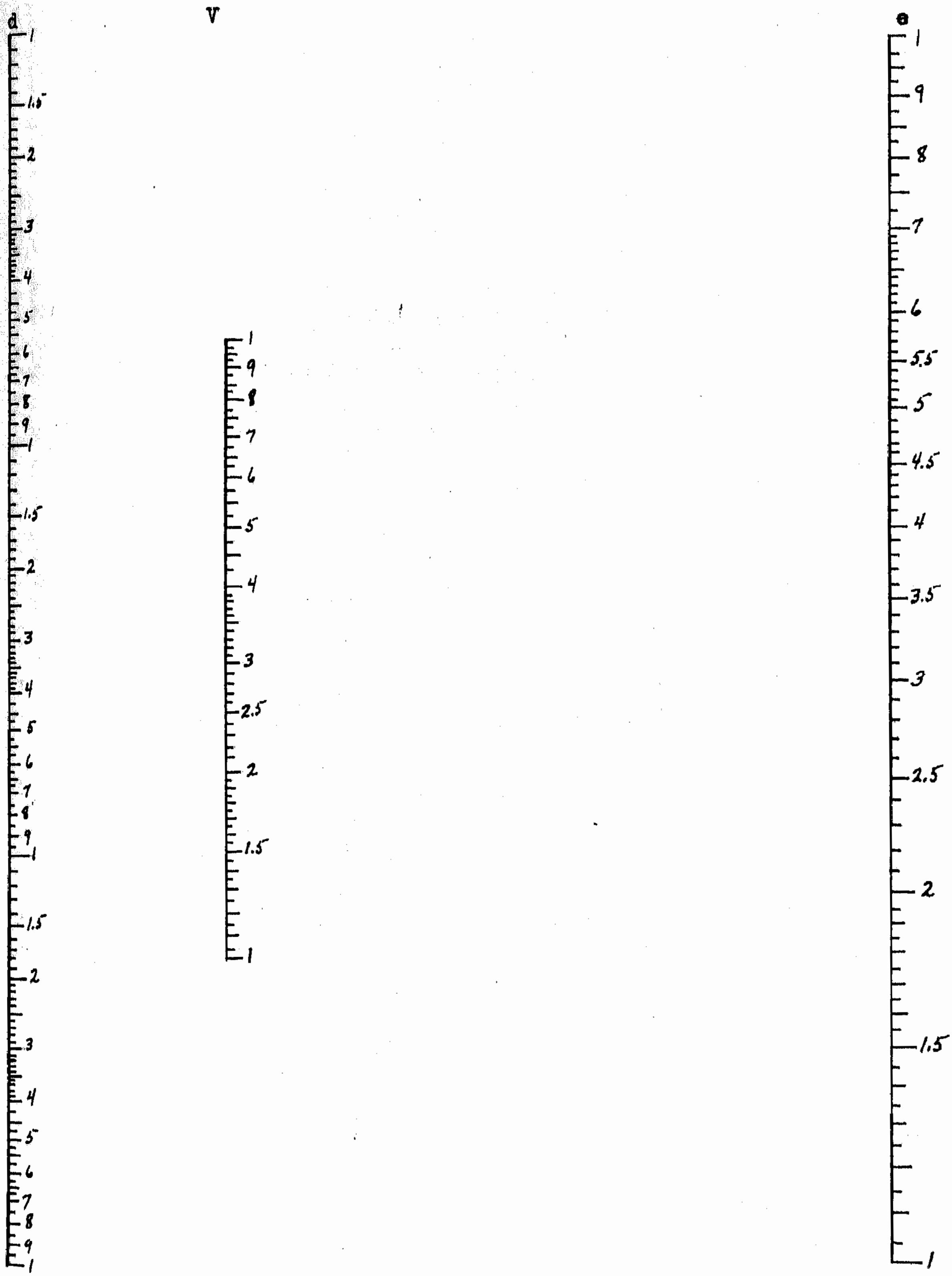


Fig. 43

$$V = \sqrt{e/d}$$

V = velocity of sound
e = elasticity of the medium
d = density of the medium

CHAPTER VI

NOMOGRAMS
COVERING THE DIVISION OF
LIGHT

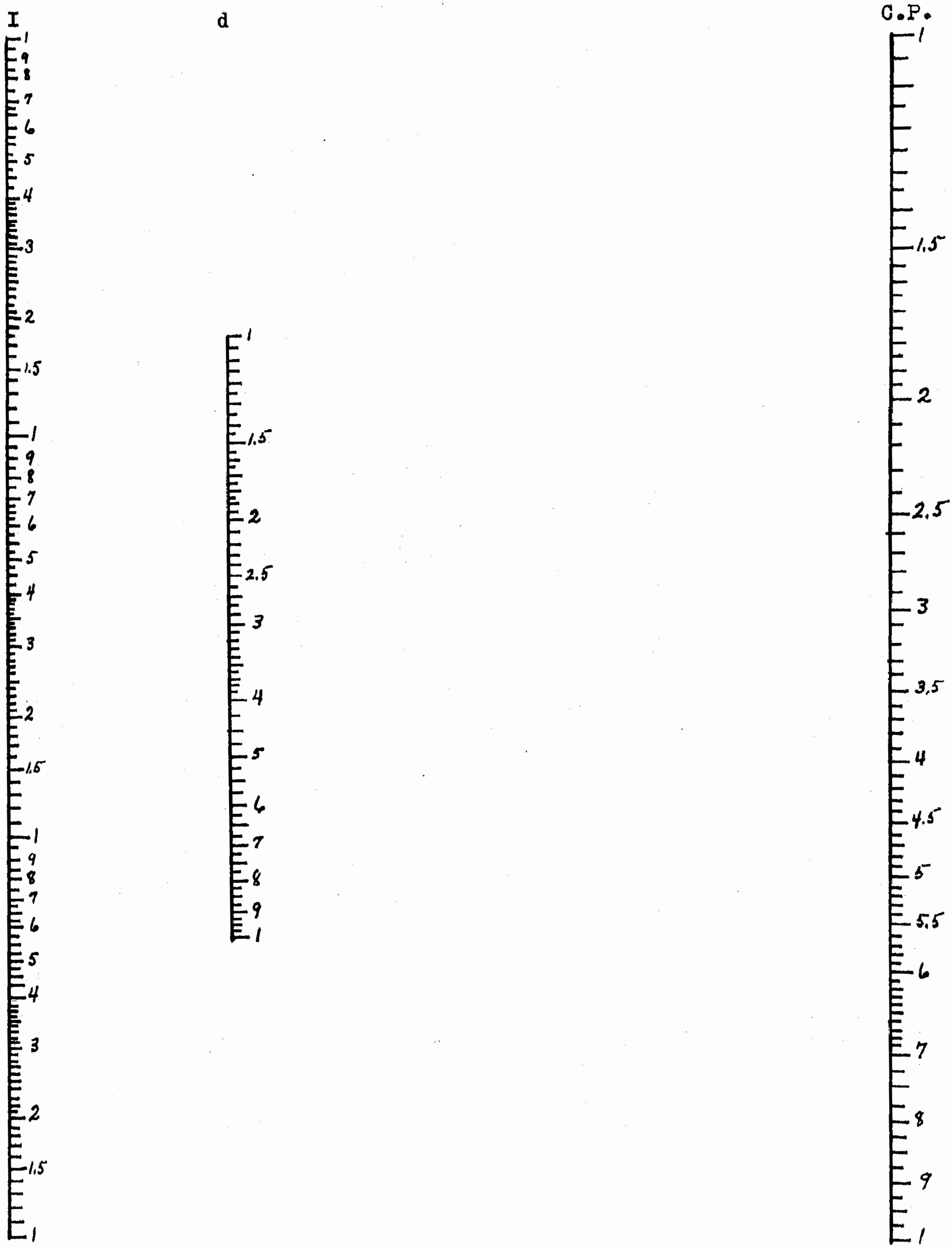


Fig. 44

$$I = C.P./d^2$$

I = intensity of illumination in foot candles
C.P. = candle power
d = distance in feet

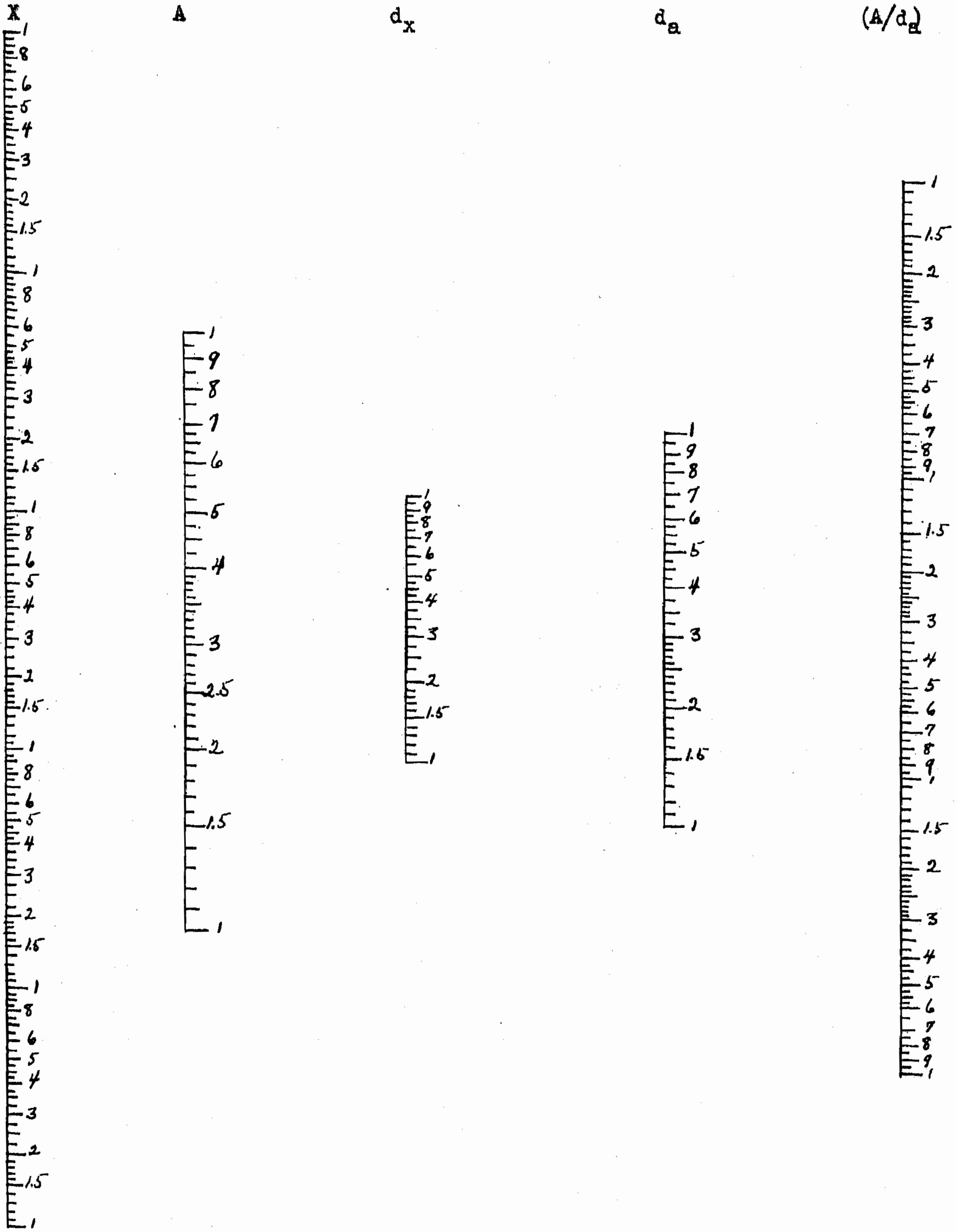


Fig. 45

$$X = A\, d_x^2 / d_a^2$$

X = candle power of unknown lamp
A = candle power of known lamp
d_x= distance of unknown lamp
d_a= distance of known lamp

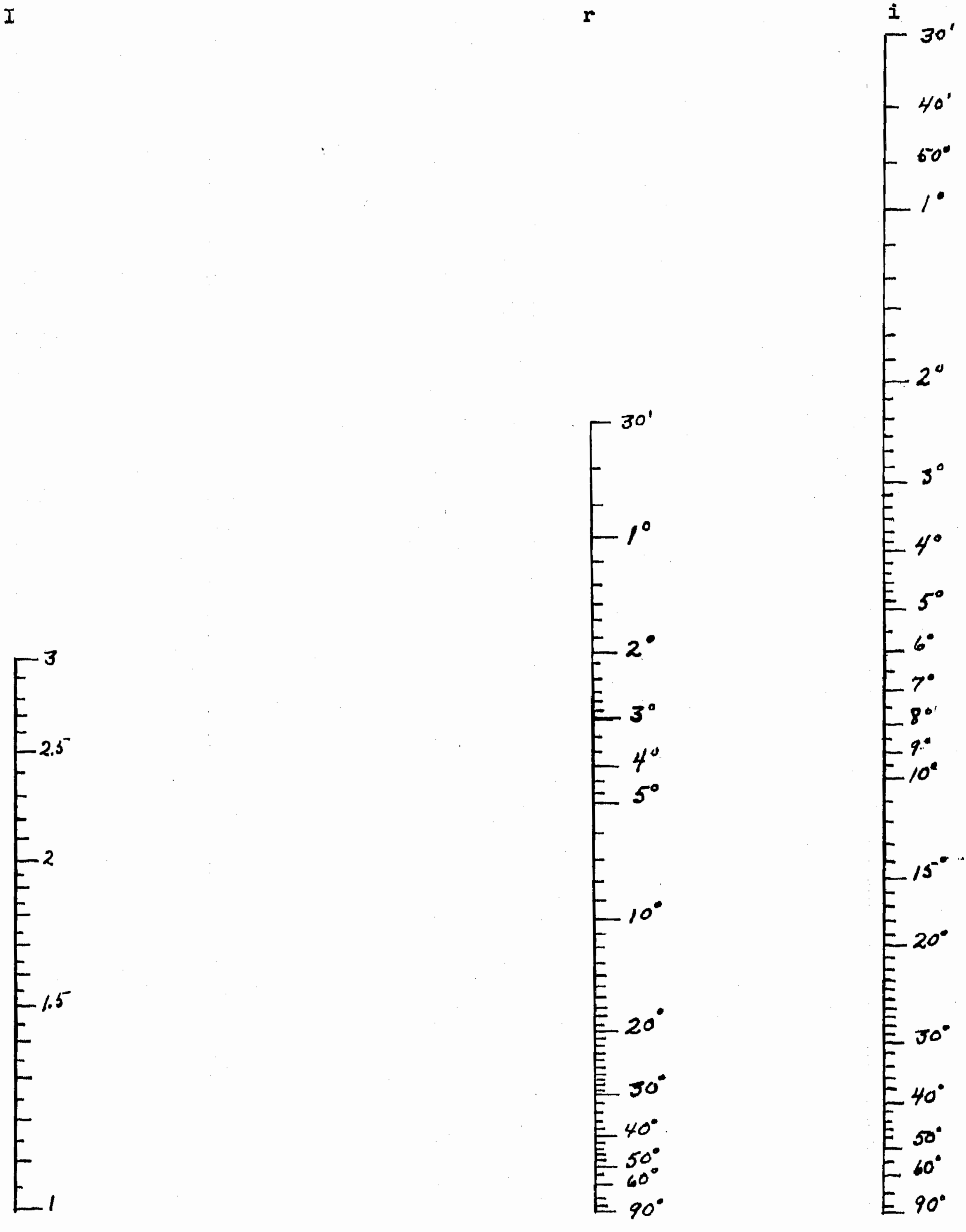


Fig. 46

I = sin i/sin r

I = index of refraction
i = angle of incidence
r = angle of refraction

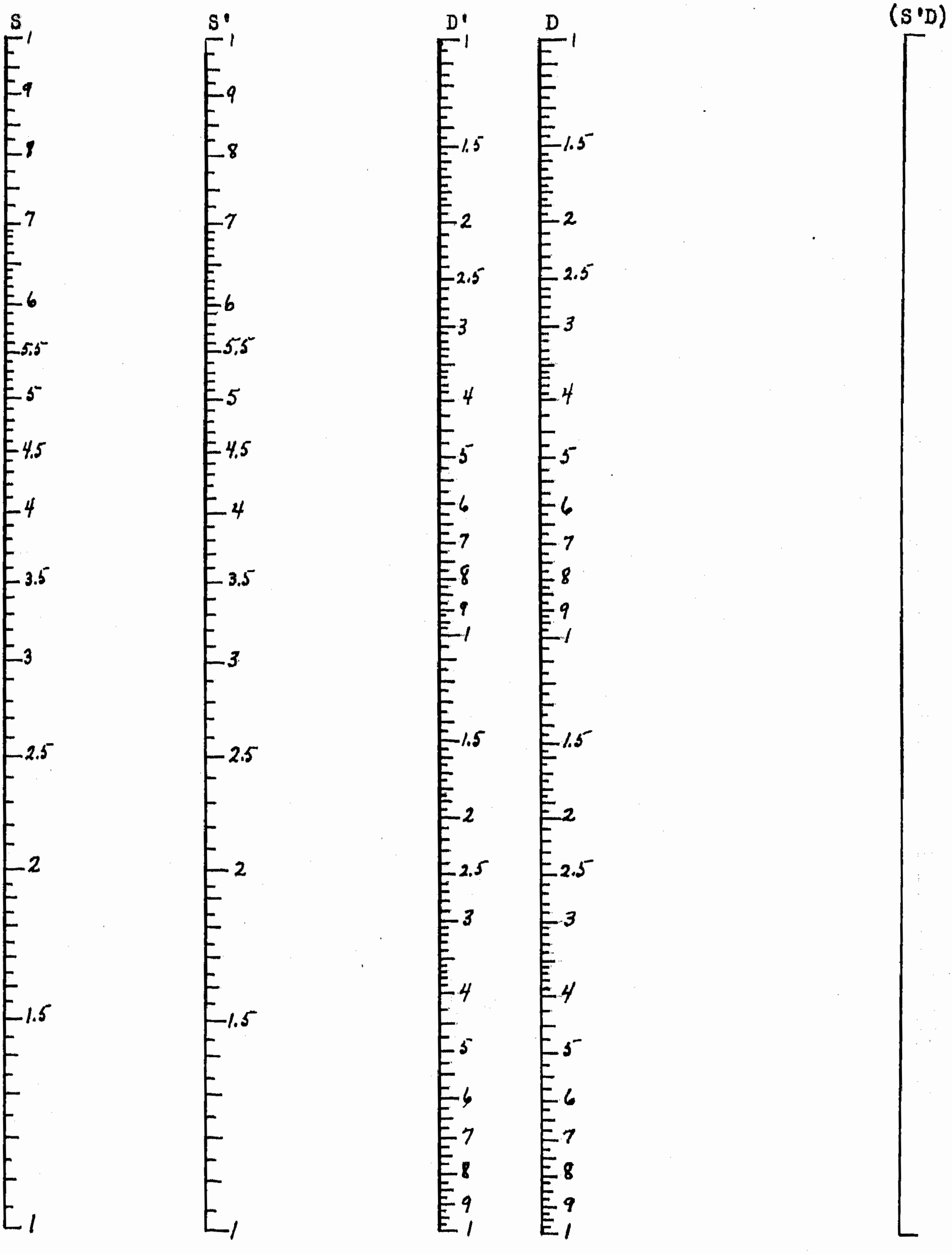


Fig. 47

S = S' D/D'

S = size of object
S'= size of image
D = object distance
D'= image distance

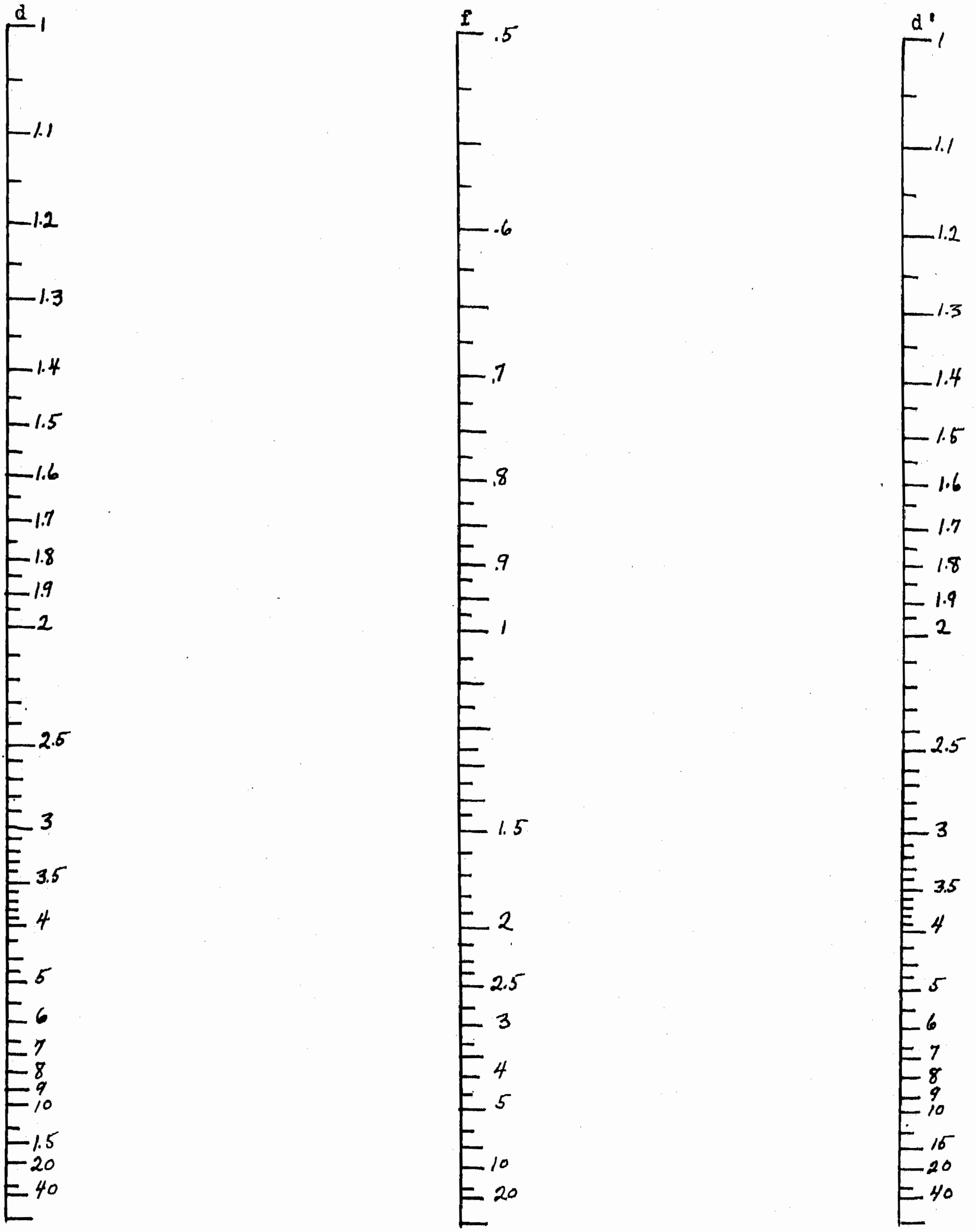


Fig. 48

$1/f = 1/d + 1/d'$

f = focal length
d = object distance
d'= image distance

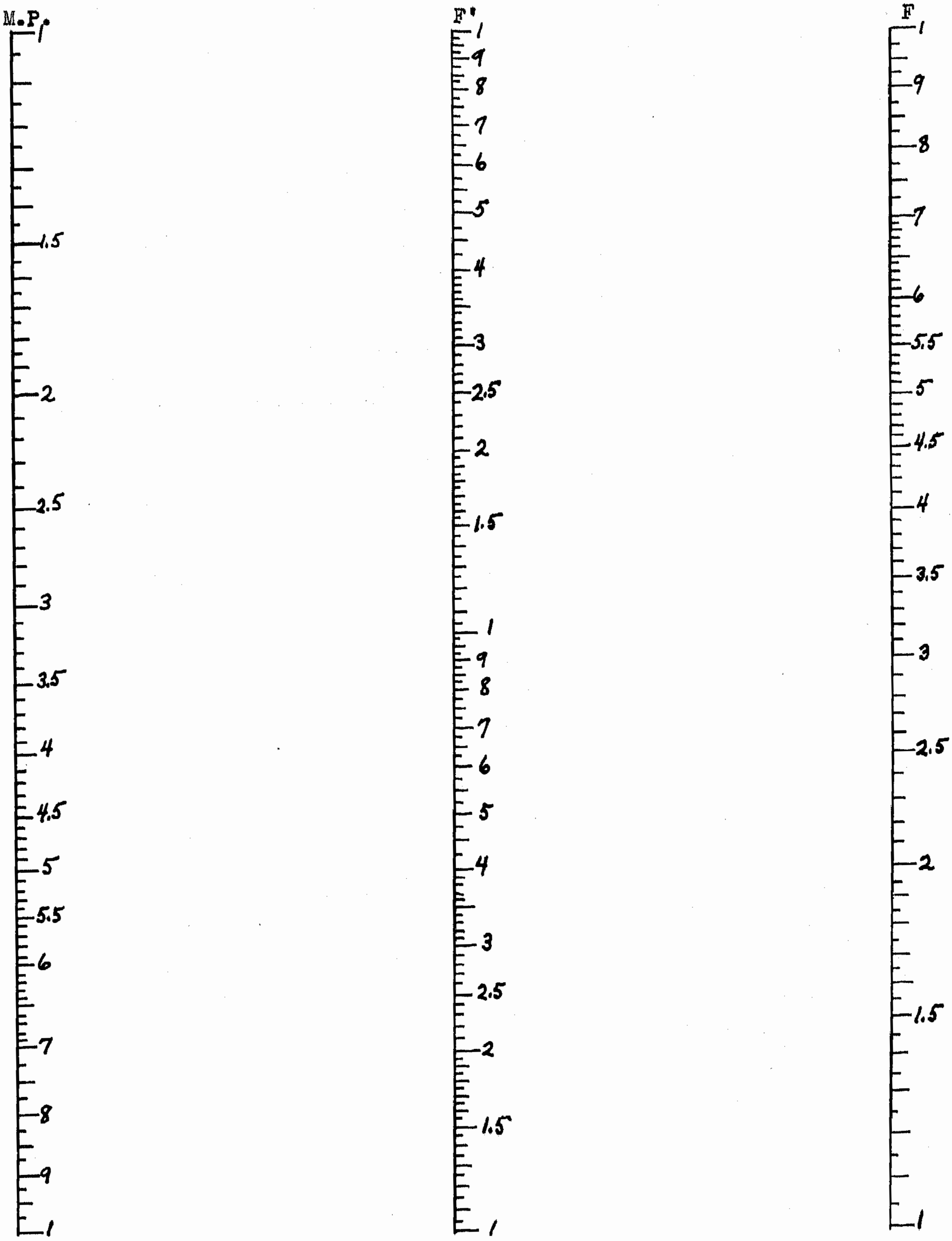


Fig. 49

M.P.= F/F'

M.P.= magnifying power
F = focal length of object lens
F'= focal length of eyepiece

CHAPTER VII

NOMOGRAMS

COVERING A FEW OF THE GENERAL

FORMULAS

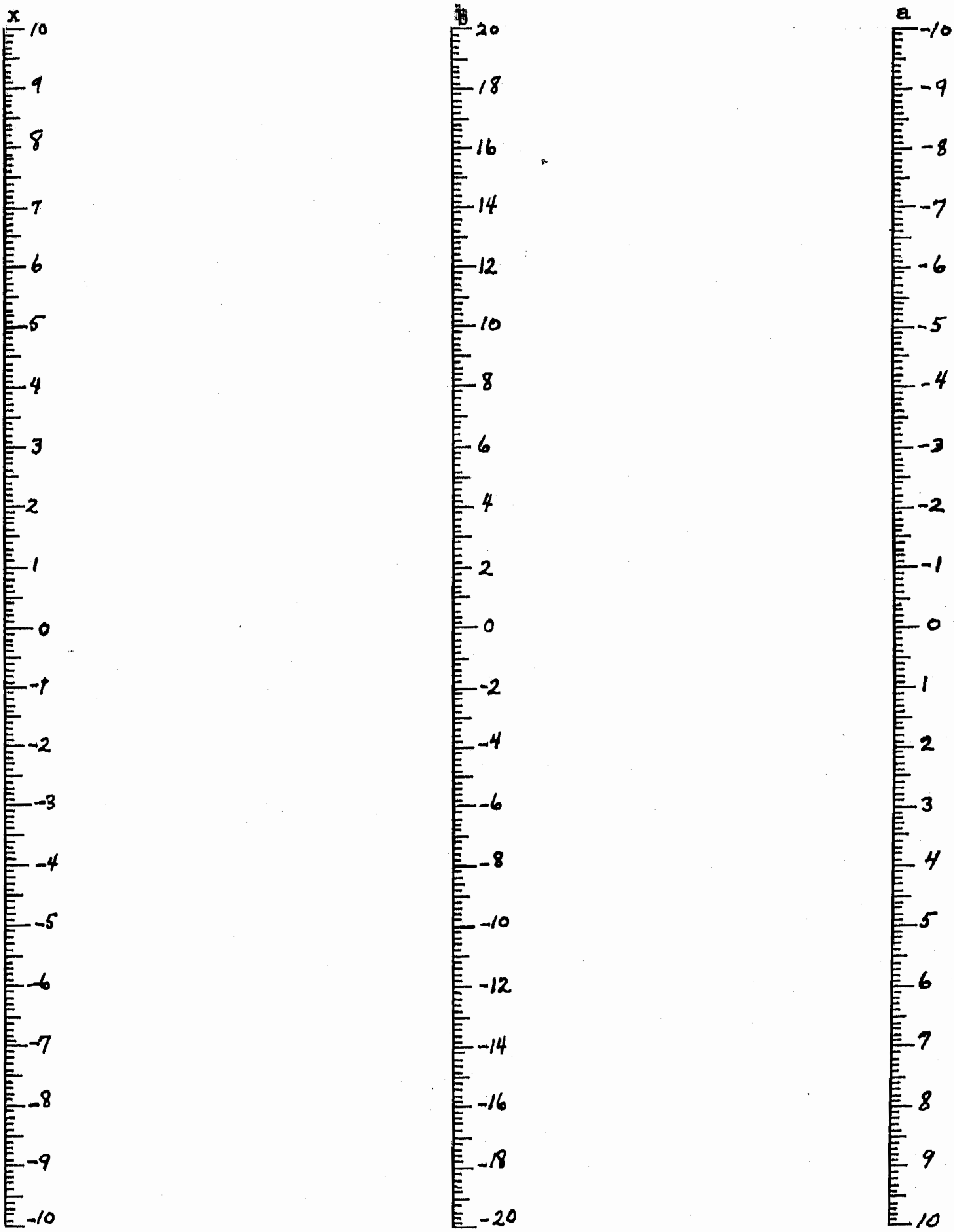

x
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
b
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
a
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10

Fig. 50

$x = a + b$

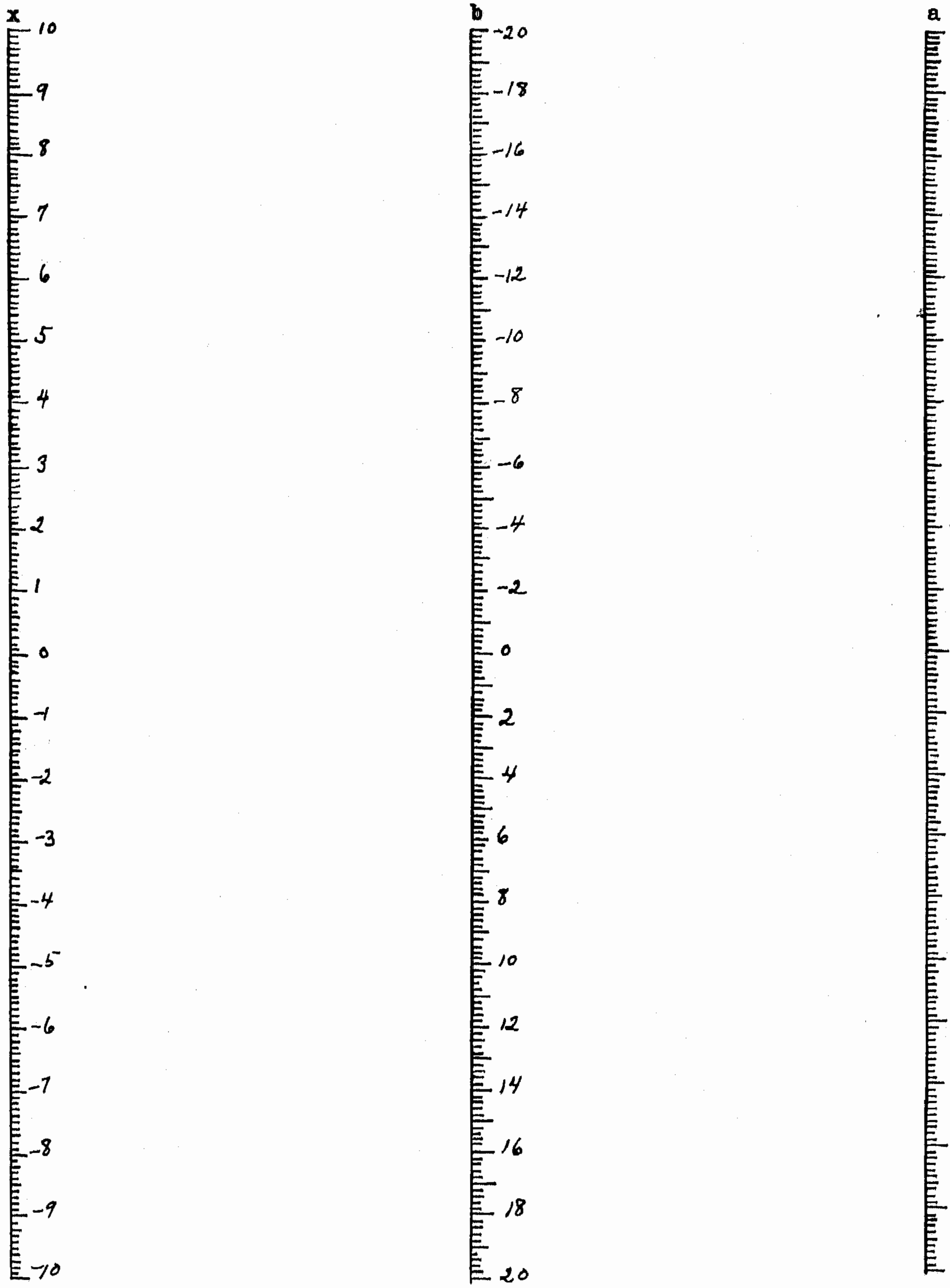

x
10 9 8 7 6 5 4 3 2 1 0 -1 -2 -3 -4 -5 -6 -7 -8 -9 -10
b
-20 -18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18 20
a
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

Fig. 51

$$x = a - b$$

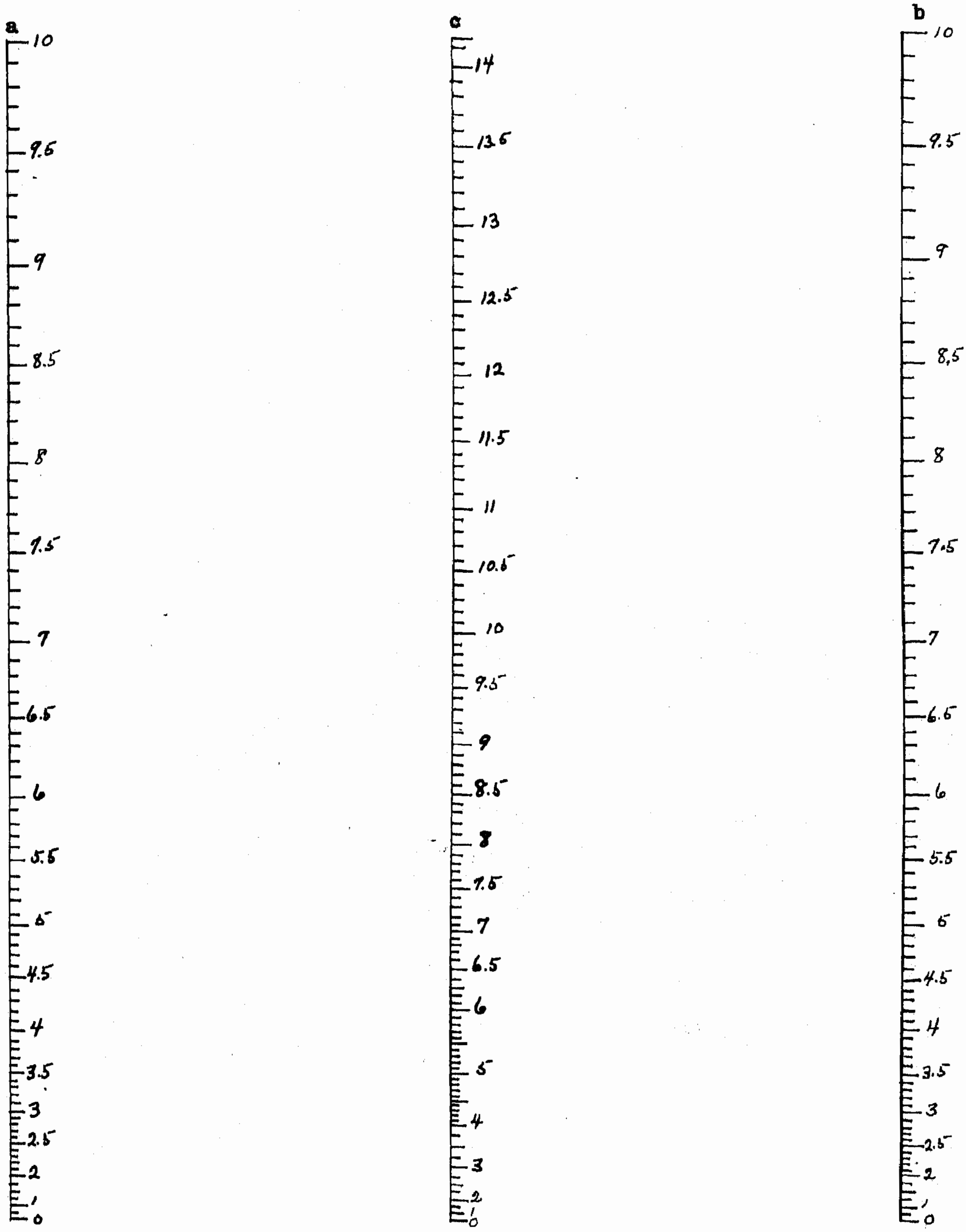
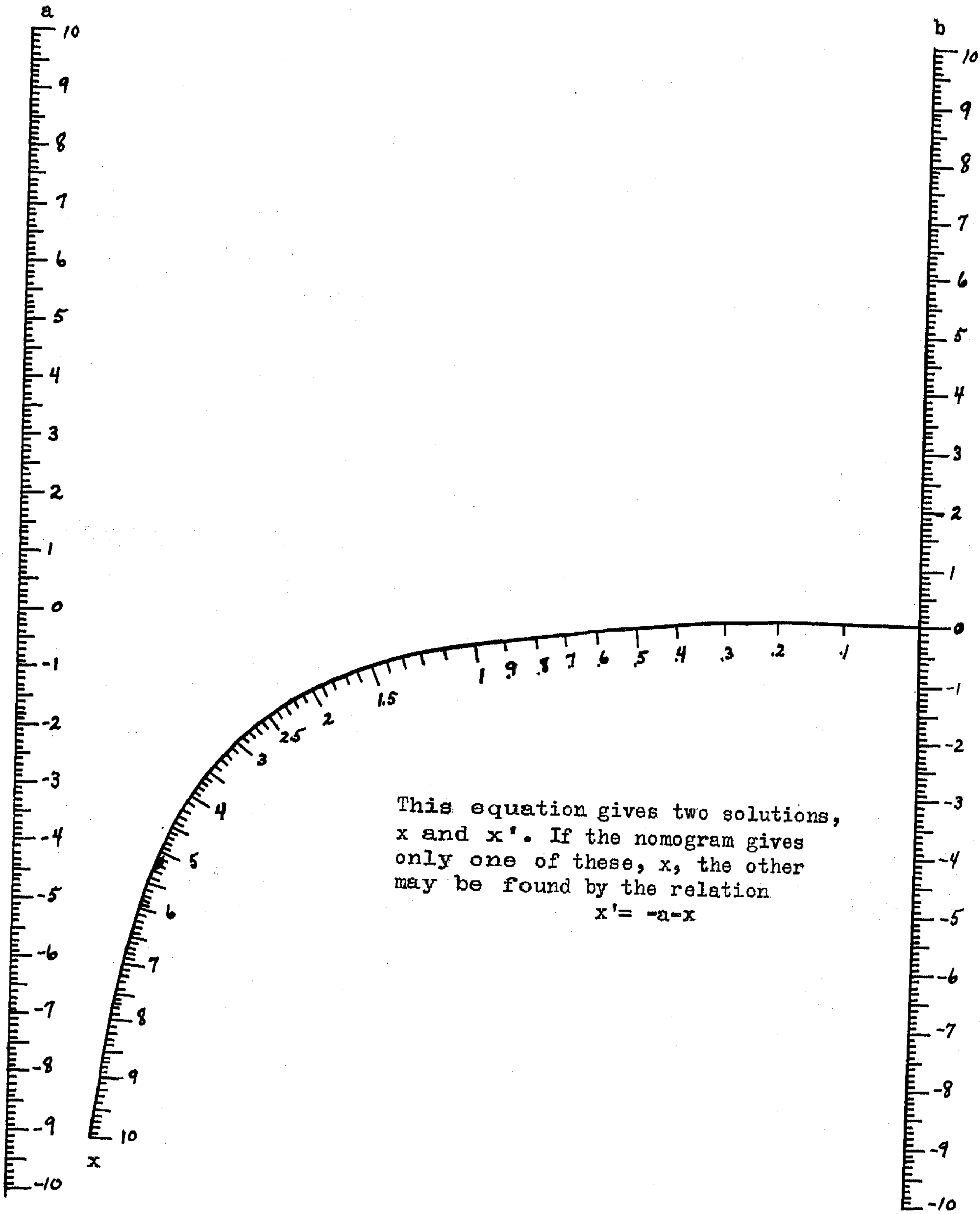


Fig. 52

$$c^2 = a^2 + b^2$$



This equation gives two solutions, x and x'. If the nomogram gives only one of these, x, the other may be found by the relation

$$x' = -a - x$$

Fig. 53

$$x^2 + ax + b = 0$$

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