AN EXPERIMENT IN THE TEACHING OF ALGEBRA I FROM THE CONTEMPORARY POINT OF VIEW

A THESIS

SUBMITTED TO THE DEPARTMENT OF
MATHEMATICS AND THE GRADUATE COUNCIL OF THE KANSAS STATE
TEACHERS COLLEGE OF EMPORIA IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE

by
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Mary Una Hamilton

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CHAPTER I

INTRODUCTION

Beginning algebra is perhaps the most critical point in the study of mathematics, since a feeling of success at this point is usually the decisive factor governing the student's continued interest in the subject.

I. THE PROBLEM

<u>Statement of the problem</u>. The purpose of this study is to develop a plan for applying the contemporary point of view to a course in beginning algebra while using a traditional textbook and to compare, both statistically and subjectively, the results obtained by following this plan with those obtained by adherence to the traditional point of view.

<u>Importance of the study</u>. The contemporary point of view is being used in many college mathematics courses. The student whose background does not contain suitable preparation has difficulty in adjusting to the more flexible patterns of thinking involved. Several topics not in the traditional course for the ninth grade need to be included, but it may be several years before these topics become incorporated in the majority of the accepted textbooks.

Recommendations have been made by the Commission on Mathematics,[1] and complete revisions of the course of study are being worked out by the School Mathematics Study Group, by the University of Illinois Committee on School Mathematics, and by other groups. In these the reorganization is too extensive for easy use by teachers who must use adopted textbooks of the traditional type. Books have been written on these newer topics, and lectures given, but in neither case is the teacher told much about how to use them in conjunction with the traditional material. In fact, when isolated, as in a lecture, these basic concepts sound trivial, for then their place in the total continuity of mathematics is not apparent.

Many excellent and experienced teachers, therefore, wonder whether these topics would really help students understand the subject matter better, and if so, whether the benefits would be sufficient to justify the expenditure of time and effort necessary. Others wonder how they can be connected with the usual material, while still others question whether anything can be done in that direction before such textbooks are available. In this experimental study an attempt has been made to introduce some of these

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[1]Commission on Mathematics, Program for College Preparatory Mathematics (New York: College Entrance Examination Board, 1959), pp. 36-37.

topics to students simply, to use these basic concepts in explaining some of the textbook materials, and to compare the results with similar results obtained from a control group.

II. DEFINITIONS OF TERMS USED

The traditional point of view. In the traditional approach, which has been used for several decades, emphasis has been placed almost exclusively on the rote learning of mechanical manipulations with the chief aim of acquiring skill in performing operations.[2]

The contemporary point of view. This approach places emphasis on the development of the basic unifying mathematical concepts along with the acquisition of adequate manipulative skills. Great emphasis is being placed on the study of mathematical structure and on the understanding of mathematical reasons for operations. According to the Commission on Mathematics, the characteristics of contemporary mathematics are:

> (1) A tremendous development quantitatively; (2) the introduction of new content; (3) the reorganization and extension of older content; and (4) renewed, increased, and conscious emphasis upon the view that mathematics is concerned with abstract patterns of thought.[3]

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[2] *Ibid.*, p. 2.

[3] *Ibid.*, pp. 2-3.

These characteristics are so comprehensive that the change in emphasis cannot be accomplished by merely replacing a few assignments by new materials. Instead, the fundamental concepts must permeate the entire content, clarifying, simplifying, and unifying topics that seem unrelated in the traditional treatment.[4]

Definitions of mathematical terms. Mathematical terms not used in the textbook, or defined differently in the textbook, are defined or explained as they occur in the units or other supplementary materials.

III. MECHANICS OF EXPERIMENT

Textbook used. The textbook used was *First Algebra* by Virgil S. Mallory.[5]

Experimental group. The experimental group began in September, 1959, as two sections composed of 55 sophomores and freshmen. In March, in a reorganization caused by the increasing population, two separate junior high schools were established. The class reported on in this study was a part of the original group, the 25 freshmen who moved to the

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[4]Commission on Mathematics, *Appendices* (New York: College Entrance Examination Board, 1959), p. 8.

[5]Virgil S. Mallory, *First Algebra* (revised; Chicago: Benj. H. Sanborn and Company, 1950).

Derby Junior High School. Except for the element of selection introduced by the fact that ninth grade students had a choice between General Mathematics and Algebra I, there was no grouping according to previous grades in mathematics or by I. Q. scores.

Control group. Because of the projected move, it was deemed impossible to hold another class together, other than the one experimental one, so the members of this control group were selected from the traditionally taught classes. Since the distribution of the I. Q. scores for the experimental group was found to possess marked skewness (Sk = 1.65, by the formula,[6] $Sk = \frac{(Q_3 - M_d) - (Md - Q_1)}{Q}$), each member of the control group was selected to match, as nearly as possible, a member of the experimental group in recorded I. Q. score and apparent industry. This set up a one-to-one correspondence between the students of the two groups which makes possible the direct comparison of corresponding scores and averages instead of the more subjective letter grades.

Plan for daily lesson. The following general plan was followed on most of the routine assignments for both groups:

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[6]Cecil B. Read, "Manual of Statistics" (Wichita, Kansas: University of Wichita, 1940), p. 53. (Mimeographed.)

A. Treatment of assignment

1. Enough review was given to recall the preceding assignment. This ranged from one sentence to 5-10 minutes of discussion or questions and answers.

2. A survey of work done was made by one of the following methods:

   (a) Students solved several problems on board, explained the first few (usually) in detail, and answered any questions asked about the others.

   (b) A short quiz was given, and the first papers were checked as received, in order to identify the main difficulties.

   (c) Students with difficulties were allowed to select problems to be solved on the board (by other students).

B. Assignment of next lesson (maximum time on homework: one hour)

1. For the experimental group the connection was shown with previous structure, basic laws, or future aims while solving one or more problems.

2. For the control group, one or more (usually more) of the same kind of problems were solved and attention was called to the danger points.

C. Supervised study period.

Plan for drill from other textbooks. The students were given a large number of problems of varying degrees of difficulty on which to spend the hour. They were allowed to work together. An answer sheet was available, work was not graded and was taken up only if students were found not working.

The purpose of these drill periods was to perfect acquired skills to allow for individual differences, and to give individual help. The good student gained speed and familiarity with the type of problem being done and clarified his thinking while explaining to others. The average student found a supply of problems he was able to do to help fix the processes in his mind. The poor student received individual help from other students and from the teacher to aid him in achieving minimal mastery.

Objective evaluation. At the end of the second semester all students in both groups were given the Breslich Algebra Survey Test[7] and the Colvin-Schrammel Algebra Test[8] and scores were compared statistically.

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[7]E. R. Breslich, Breslich Algebra Survey Test, Second Semester, Form A (Bloomington, Illinois: Public School Publishing Company, n. d.).

[8]E. S. Colvin and H. E. Schrammel, Colvin-Schrammel Algebra Test, Test II, Form A (Emporia, Kansas: Bureau of Educational Measurements, Kansas State Teachers College, 1937).

Subjective evaluation. Results were evaluated subjectively by the students' estimates of their liking for algebra and by the teacher's observation of attitudes and of indications of the students' understanding of concepts.

IV. ORGANIZATION OF REMAINDER OF THESIS

Chapter II contains the list of assignments for both groups. Chapters III through IX inclusive give materials that were prepared especially for the experimental group. The explanatory portions of these were condensed from tape recordings made during class sessions. Chapter X gives statistical data and Chapter XI gives conclusions with suggestions for further study. A bibliography follows the final chapter.

CHAPTER II

ASSIGNMENTS

Table I shows the assignments for each group. Unless otherwise stated, page numbers refer to First Algebra, by Virgil S. Mallory. Many drill exercises were taken from two other textbooks. Page numbers from these are preceded by (FBJ) or (SCS) indicating their source.[1] Materials for assignments marked with * are included in this thesis.

TABLE I

ASSIGNMENTS, 1959-60

No.	Experimental Group	Control Group
1	*Numbers and numerals (See Ch. III)	Pp. 4-5:1-26
2	(Cont.)	Pp. 6-7:1-7; (FBJ) Pp. 9-11:1-20
3	(Cont.)	Pp. 7-8:1-20
4	*Decimal and non-decimal numeration (See Ch. IV)	P. 8:1-8; P. 10:1-7
5	*Exercises I	P. 11:1-7
6	*Exercises II	P. 12:1-10

[1]Julius Freilich, Simon L. Berman, and Elsie Parker Johnson, Algebra for Problem Solving, Book I (Boston: Houghton Mifflin Company, 1952); Raleigh Schorling, John R. Clark, and Rolland R. Smith, First-Year Algebra (Yonkers-on-Hudson, New York: World Book Company, 1943).

TABLE I (CONTINUED)

No.	Experimental Group	Control Group
7	*Exercises III	P. 13:1-7
8	(Cont.)	Pp. 14-15:1-8
9	*Exercises IV; *Sets (See Ch. V)	Pp. 16-17:1-7
10	*Exercises V	Pp. 19:3-17; Pp. 19-21:1-10
11	*Exercises VI	Pp. 21-22:1-16
12	*Sets (cont.)	P. 27:1-20
13	*Exercises VII	Test on Chapter I
14	*Laws of operations (See Ch. VI)	P. 35:4-23
15	(Cont.)	P. 36:1-8
16	*Test I (See Ch. VII)	P. 38:4-15
17	Pp. 7-8:11-20; P. 10:3, 4	P. 40:1-8
18	P. 25:word list; P. 35:4-23 (odd) P. 36:1-8 (odd)	P. 41:8-20; P. 43:1-7
19	P. 38:4-18	P. 45:2-21
20	P. 40:1-8	Pp. 45-46:1-40
21	P. 41:2-21	P. 48:1-29 (odd); P. 48:30-34
22	Pp. 45-46:1-40	P. 48:1-40 (even)
23	*Exercises VIII	P. 50:1-5; P. 51:1-5
24	*Exercises IX	P. 52:1-5; P. 53:1, 3, 4, 6, 7

TABLE I (CONTINUED)

No.	Experimental Group	Control Group
25	P. 48 (odd)	P. 55:1-15
26	P. 48 (even)	P. 56:1-10
27	Test	P. 58:1-14
28	P. 50:1-5; P. 51:1-5	P. 61:1-20
29	P. 52:1-8	Test on Chapter II
30	Informal deduction[2]	P. 69:1-4; P. 72:1-11
31	(Cont.)	Pp. 74-75:1-20
32	(Cont.)	(FBJ) P. 66:1-8; Pp. 72-73:1-10; P. 76:1-12
33	(Cont.)	(FBJ) Pp. 80-81:1-18
34	(Cont.)	P. 76:1-10; P. 77:1-5
35	P. 53:1-10	Pp. 78-79:1-20
36	P. 11:3, 5, 7; P. 12:5, 7, 9; P. 13:1, 3, 5, 7	P. 82:3-22
37	P. 19:11-14; P. 19-21:1, 3, 5, 9, 11	P. 84:1-20
38	(FBJ) P. 66:1-8; Pp. 72-73: 1-10; P. 76:1-12	P. 86:3-18; P. 87:3-18
39	(FBJ) Pp. 80-81:1-18	P. 90:1-15
40	P. 55:6-15; P. 56:1-10	Pp. 91-92:1-10
41	P. 58:1-14; P. 59:1-12	Pp. 93-94:1-15

[2]Commission on Mathematics, Informal Deduction in Algebra: Properties of Odd and Even Numbers (New York: College Entrance Examination Board, 1959).

TABLE I (CONTINUED)

No.	Experimental Group	Control Group
42	Pp. 59-60:1-20	(FBJ) P. 159:1-28
43	P. 69:1-4; P. 72:1-11	Test over pp. 63-94
44	Pp. 74-75:1-20	P. 94:1-15
45	Pp. 78-79:1-20	P. 96:3-22
46	Pp. 80-82:3-22 (odd); P. 84:1-20 (odd)	(FBJ) Pp. 160-161:1-20 (SCS) P. 171:1-16
47	P. 86:3-18; P. 87:3-18	(SCS) P. 186:34-38; P. 202:83-90; (FBJ) P. 163:7-18
48	P. 90:1-15	P. 100:1-10
49	Pp. 91-92:1-10; P. 93:1-10	P. 101:1-10
50	(FBJ) P. 159:1-28	P. 103:1-10
51	Pp. 93-94:1-15; P. 94:1-15	P. 106:9-29 (odd); P. 106, B:1-9
52	P. 96:1-20	Test on Chapter III
53	(FBJ) Pp. 160-161:1-20 (SCS) P. 171:1-16	P. 115:1-4; P. 116:1-4
54	(SCS) P. 186:34-38; P. 202: 83-90; (FBJ) P. 163:7-18	P. 117:1-4
55	P. 100:1-10	P. 119:1-6
56	P. 101:1-10	P. 120:1-2; P. 121:1-8
57	P. 103:1-10	P. 123:1, 2
58	P. 112:1-20	P. 123:3, 4
59	Test on Chapter III	P. 125:1-5; P. 126:1-3; P. 127:1-2

TABLE I (CONTINUED)

No.	Experimental Group	Control Group
60	Sets of ordered pairs Sentences in two variables[3]	P. 130:1-4
61	Relations, Domain and range[4]	P. 133:1-12
62	Lattice graphs, *Exercises X	P. 135:1-8
63	*Exercises XI	P. 137:1-4
64	P. 133:1-12; P. 135:1-8	P. 139:1-4
65	P. 137:1-4; *Exercises XII	P. 139:5-8
66	P. 139:1-13	P. 139:9-13
67	*Exercises XIII	P. 142:1-8
68	P. 142:1-8	P. 142:9-17
69	P. 142:9-17	P. 144-145:5-9
70	P. 144-145:5-9	P. 146:5-10; Pp. 146-147:1-4
71	P. 146:5-10; Pp. 146-147: 1-4	P. 148, B:1-6; A:1-16
72	P. 148, B:1-6; A:1-16	Pp. 148-149, B:1-8
73	Pp. 148-149, B:1-8	Test on Chapter IV
74	Test on Chapter IV	Pp. 155-156:1-15
75	Pp. 155-156:1-20	P. 156, B:1-10
76	P. 156, B:1-10; A:21-24	P. 156, A:15-24

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[3]Commission on Mathematics, Appendices (New York: College Entrance Examination Board, 1959), pp. 11-13.

[4]Ibid., pp. 14-17.

TABLE I (CONTINUED)

No.	Experimental Group	Control Group
77	P. 157-158:1-10; P. 158:1-10	P. 157-158:1-10; P. 158:1-10
78	P. 161:1-10	(FBJ) Pp. 237-238:1-24; (FBJ) Pp. 243-244:1-20
79	*Exercises XIV	P. 161:1-10
80	Test, for Semester review	Test, for Sem. review
81	Pp. 162-163:1-10	Pp. 162-163:1-10
82	Pp. 164-165:1-10	Pp. 164-165:1-10
83	Review	Review
84	Semester Examination	Sem. Exam.
85	P. 167:2-6; P. 169:3-7	P. 167:2-6; P. 169:3-7
86	Pp. 171-172:1-5; P. 175:4-6; P. 176:6-7	Pp. 171-172:1-5; P. 175: 4-6; P. 176:6-7
87	P. 181:1-10	P. 181:1-10
88	P. 182:4-13	P. 182:4-13
89	P. 183:1-20	P. 183:1-20
90	P. 184:1-20	P. 184:1-20
91	Test on Chapter V	Test on Chapter V
92	P. 191:18-27; P. 193:16-30	P. 191:18-27; P. 193: 16-30
93	P. 195:15-24; P. 196:1-10	P. 195:15-24; P. 196: 1-10
94	Pp. 197-198:1-9	Pp. 197-198:1-9
95	Pp. 198-199:1, 3; Pp. 199-200: 1-7	Pp. 198-199:1,3; Pp. 199-200:1-7
96	P. 201:1-12 (odd); P. 202:1-14 (odd)	P. 201:1-12

TABLE I (CONTINUED)

No.	Experimental Group	Control Group
97	P. 203:1-20	P. 202:1-14
98	P. 209:1-36 (odd); P. 211:1-20 (odd)	P. 203:4-23
99	P. 211:21-25; P. 212:1-8	P. 209:1-36
100	P. 214:1-24	P. 211:1-20
101	(FBJ) P. 172:1-22	P. 211:21-25; P. 212:1-8
102	P. 230:2-31	P. 214:1-24
103	Pp. 231-232:1-50	(FBJ) P. 172:1-22
104	Pp. 233-234:1-25	Test on Chapter VI
105	(FBJ) P. 297:1-28; P. 299:1-28	P. 228:1-20; P. 230:1-20
106	(SCS) Pp. 331-332:1-66	Pp. 231-232:1-40
107	P. 234:26-50	Pp. 233-34:1-25
108	Pp. 234-235:1-20	P. 234:26-50
109	P. 236:1-48	Pp. 234-235:1-20
110	P. 238:1-30	(FBJ) P. 297:1-28; P. 299:1-28
111	P. 239:1-16	(SCS) Pp. 331-332:1-66
112	P. 241:5-24	P. 236:1-40
113	P. 242:6-25	P. 238:1-30
114	P. 245:1-20	P. 239:1-16
115	P. 246:5-24	P. 241:5-24

TABLE I (CONTINUED)

No.	Experimental Group	Control Group
116	P. 247:2-20	Test over pp. 228-241
117	P. 249:1-20	P. 242:6-25
118	Prime factorization;[5] *Exercises XV	P. 245:1-20
119	Polynomial factors[6]	P. 246:5-24
120	P. 252:5-24	P. 247:2-16
121	P. 253:1-20	P. 249:5-24
122	P. 254, B:1-20 (odd); C:1-20 (odd)	P. 252:5-24
123	P. 259:1-20	Pp. 253-254, A:5-14; B:7-11; C:12-16
124	P. 260:1-5; P. 261:5-9	P. 259:1-20
125	(FBJ) P. 303:1-30; Pp. 301-302:1-24	P. 260:1-5; P. 261:5-9
126	Test on Chapter VII	(FBJ) P. 303:1-30; Pp. 301-302:1-24
127	P. 277:15-24 (oral); 1-10	Test on Chapter VII
128	P. 278:11-26	Pp. 277-278:1-20
129	Pp. 279-280:11-30	Pp. 279-280:11-30
130	P. 282:8-27	P. 282:8-27

[5]School Mathematics Study Group, First Course in Algebra (Part 2) (Preliminary edition; New Haven, Connecticut: Yale University, 1959), pp. 264-268; pp. 273-277.

[6]Ibid., pp. 333-338.

TABLE I (CONTINUED)

No.	Experimental Group	Control Group
131	P. 283:5-14	P. 283:5-14
132	P. 285:7-26	P. 285:7-26
133	Lowest common denominator[7]	P. 286:1-9 (odd); Pp. 287-288:1-20
134	P. 286:1-9 (odd); Pp. 287-288:1-20	P. 291:5-24
135	P. 291:5-24	P. 293:11-30
136	P. 293:11-30	(SCS) P. 370:1-35; (FBJ) P. 349:1-16; P. 351:1-12
137	(SCS) P. 370:1-35; (FBJ) P. 349:1-16; P. 351:1-12	P. 296:1-10
138	P. 296:1-10	P. 297:1-10; P. 299:3-12
139	P. 297:1-10; P. 299:3-12	P. 300:1-20
140	P. 300:1-20	(SCS) Pp. 379-380:8-13; 16-63
141	(SCS) Pp. 379-380:8-13; 16-63	P. 307:1-10; 19-28
142	P. 307:1-10; 19-28	Test on Chapter VIII
143	Test on Chapter VIII	P. 320:1-20
144	P. 320:1-20	P. 322:1-10
145	P. 322:1-10	P. 326:4-18; Pp. 326-327: 1-5

[7] *Ibid.*, pp. 268-273.

TABLE I (CONTINUED)

No.	Experimental Group	Control Group
146	P. 326:1-18; Pp. 328-329: 1-10	PP. 328-329:1-10
147	P. 374:1-40	Pp. 331-332:1, 2, 3, 6, 7; P. 334:1-8; P. 337:1-8
148	P. 378:1-8; Pp. 378-379: 1-10	P. 374:1-40
149	P. 380:1-20	P. 378:1-5; Pp. 378-379: 1-5
150	P. 381:1-20	P. 380:1-20
151	P. 382:1-20 (odd); Pp. 383-384:1-20 (odd)	P. 381:1-20
152	P. 388:1-20 (odd); P. 390:1-20 (odd)	P. 382:1-20
153	Test on Chapters IX and XI	Pp. 383-384:1-20
154	P. 399:1-10; P. 401:1-20 (odd)	P. 385:1-20 (odd); P. 388:1-20 (odd)
155	P. 402:1, 3, 5, 7; P. 404:1-20	Test on Chapters IX and XI
156	P. 406:1-12; P. 407:1-8	P. 399:1-10 (odd); P. 401:1-20 (odd)
157	P. 409:1-10	(FBJ) P. 306:1-30
158	(FBJ) P. 306:1-30	P. 402:1, 3, 5, 7; P. 404:1-20
159	P. 409:11-20	P. 406:1-12; P. 407:1-8
160	P. 410:5-14	P. 409:1-10
161	P. 412:3-12	P. 409:11-20 (odd); P. 410:5-14 (odd)

TABLE I (CONTINUED)

No.	Experimental Group	Control Group
162	P. 413:1-20	(SCS) Pp. 446-448:1-52
163	(SCS) Pp. 446-448:1-52	P. 412:3-12
164	Test on Chapter XII	Test on Chapter XII
165	P. 415:2-6	P. 413:1-20
166	Quadratic inequalities[8] *Exercises XVI	P. 415:2-6
167	Review	Review
168	Colvin-Schrammel Algebra Test	Colvin-Schrammel Algebra Test
169	Breslich Algebra Survey Test	Breslich Algebra Survey Test

[8]F. Dawson Trine, "An Introduction to Algebra with Inequalities," The Mathematics Teacher, LIII (January, 1960), pp. 44-45.

CHAPTER III

NUMBERS AND NUMERALS

This unit was presented in class in the following manner:

"What algebra does with numbers. Algebra is a course in mathematics. In it we shall take the number system used in arithmetic and learn more about its basic ideas. We shall extend that system to include kinds of numbers you have not yet studied. We shall make generalizations concerning the basic ideas, or concepts, of mathematics. Instead of learning how one problem is solved, we shall classify problems according to kind and learn methods that can be used in solving all problems of each of those kinds. We shall learn laws that apply to all numbers in the system. We shall condense this information by translating it into the language of mathematics so that concepts and relationships can be seen more quickly. We shall add to our knowledge of numbers step by step to build a solid foundation for further mathematical and scientific study.

"This foundation can be built in more than one way and mathematics can be looked at from many points of view. We are now in a period of time when this is being done, so if you are to be prepared for whatever mathematics you may take later, it is necessary that we add many topics and explanations as we go along. Please understand that this is not meant to take the place of reading the textbook. Also, whenever we discuss points of view that differ with statements in the textbook, we will try to understand why.

"Meaning of "number" and "numeral." We need to consider how our number system developed. We might pause to investigate the word number. Just what is a number? If we write a "2" on the board, is it a number? Then if we write "John" on the board, is he actually up there? The number is a mental concept, its name is a numeral, and the mark we make on the board or on paper is a symbol used to represent it. In ordinary conversation we use the word "number" to include all of them, but we need to be able to distinguish between them whenever it is necessary.

"Ways of answering the question, "How many?". Undoubtedly, mental concepts of number began before any of our written history. Man asked himself the question, "How many?", frequently. How many enemies were there? How many supplies did his family need? How many days did a journey take? How many sheep went out to graze in the morning and how many came back at night? He needed numbers to be able to plan, to record as an aid to his memory, and to communicate ideas of number to others.

"Here are ways he used to express the idea:

A. Use of whole object and part of object

B. One-to-one correspondence

C. Standard sized set for counting large numbers of objects

D. Place value notation systems[1]

"Method A is still used when the fisherman brings in a string of fish or a hunter mounts the horns of an animal he kills.

"One-to-one correspondence is used by the small child who holds up three fingers to tell how old he is or by the teacher who checks attendance by a seating chart or by the student who makes dots on the board while he is adding. Many jokes and puzzles hinge on one-to-one correspondence. This is done in the story of the innkeeper with nine empty rooms, in a bit of old nineteenth century verse quoted by Martin Gardner.[2]

"A group of ten travelers asked for single rooms and each refused to share a room with any of the others. The host temporarily put two men in room A, the third in B,

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[1]Lester E. Laird, "Elementary Mathematics from a Slightly Advanced Viewpoint," Reports, Workshop in High School Mathematics, Fifth Year (Emporia, Kansas: Kansas State Teachers College, 1959), p. 15. (Mimeographed.)

[2]Martin Gardner, "Mathematical Games," The Scientific American, Vol. 198 (January, 1958), p. 92.

the fourth in C, the fifth in D, the sixth in E, the seventh in F, the eighth and ninth in G and H, and then ran back to room A and got one of the two he had there, the tenth and last and put him in room I. How did he do it?

"We are led to believe he put these men in one-to-one correspondence, but we have a feeling something is wrong, for ten objects cannot be put in one-to-one correspondence with only nine. To clear up the confusion we need to write down the arrangement.

```
  A    B   C   D   E   F   G   H   I
 ↗↖    ↕   ↕   ↕   ↕   ↕   ↕   ↕
 1 2   3   4   5   6   7   8   9
```

No matter which man he takes from A and puts in I, the tenth man still doesn't have a room.

"The standard sized set was used by the Egyptian and the Roman additive systems. Let's turn to p. 173 while we discuss this method of expressing numbers. To express a number such as 279 it was necessary for them to include enough symbols to add to that amount. This seems to us to be quite awkward, but the real trouble comes when we try to multiply in one of these systems. Let's try to multiply 23 by 7 in Roman Numerals.

```
            XXIII
              VII
         ---------
      XX      III
      XX     III
   LL    VVV
   ------------
     CLXI
```

"Our place value system. Our system of numeration is usually called the Hindu-Arabic, since it came to us from India by way of Arabia. Early calculating had been done with an abacus or with a counting board, but the place value system with zero for an empty space made it possible to calculate with numerals.[3]

"In a place value system each symbol has its own value multiplied by a value due to the position in which it stands. Our system is based on ten, so we call it a decimal system. The value of the first place on the right is one. The value of the next place to the left of that one is ten, the next is 10 x 10, the next is 10 x 10 x 10, and so on. Thus 256 means 2 x 10 x 10 + 5 x 10 + 6.

"Denominations in our system of numeration. Our language has names for denominations with up to sixty-three zeros. In order, they are thousand, million, billion, trillion, quadrillion, quintillion, sextillion, septillion, octillion, nonillion, decillion, undecillion, duodecillion, tredecillion, quattuordecillion, quindecillion, sexdecillion, septendecillion, octodecillion, novemdecillion, and vigintillion.[4] These names are interesting but you need not

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[3]Lancelot Hogben, The Wonderful World of Mathematics (Garden City, New York: Garden City Books, 1955), p. 45.

[4]"Numeration," Webster's New Collegiate Dictionary (Second edition; Springfield, Massachusetts: G. and C. Merriam Company, 1953), p. 577.

remember them. They were devised at a time when people had little use for such large numbers, so very few learned them past trillion. As more emphasis was placed on how to do arithmetic instead of why, they dropped out of our textbooks. Now when we speak of large quantities, such as astronomical distances, we say, "a billion billion."

"Whatever names we may use for our numbers, the important thing to remember is that there are infinitely many counting numbers. No matter how far we count, there will always be a next one, or successor; there is no last one.[5]"

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[5]School Mathematics Study Group, First Course in Algebra (Part 1) (Preliminary edition; New Haven, Connecticut: Yale University, 1959), p. 2.

CHAPTER IV

DECIMAL AND NON-DECIMAL NUMERATION[1]

This unit was presented in class in the following manner:

"How the odometer counts. A good way to get a realization of the meaning of each place in a numeral of several digits is to study the odometer in an automobile. How does it work? When it comes from the factory the odometer shows all zeros. The first wheel registers tenths of a mile up to nine tenths. At the end of another tenth of a mile the numeral 1 shows on the second or unit's wheel and the first wheel shows 0, meaning one mile and no tenths. The process repeats, adding a mile each time, till the unit's wheel registers nine miles. After another mile the odometer reads 000100, meaning one group of ten miles and no units and no tenths. When we have ten groups of ten miles, it reads 001000, meaning one group of ten times ten, no group of ten, no units and no tenths, or

$$1 \times 10 \times 10 + 0 \times 10 + 0 + \frac{0}{10} = 100 \text{ miles.}$$ [2]

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[1] School Mathematics Study Group, Experimental Units for Grades Seven and Eight (New Haven, Connecticut: Yale University, 1959), Unit II.

[2] Irving Adler, Mathematics: The Story of Numbers, Symbols and Space (In The Golden Library of Knowledge. New York: Golden Press, 1958), pp. 44-45.

"Counting by different groups. Ten may have become the base of our number system because we have ten fingers, but it is not the only base used for counting. The digital computers use the base two system, with some use of base eight numerals.

"Let's take 27 objects and count them in different ways. If we count in groups of 10,

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0,

we get 2 x 10 + 7, written 27. But suppose we count the same objects in groups of 8.

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

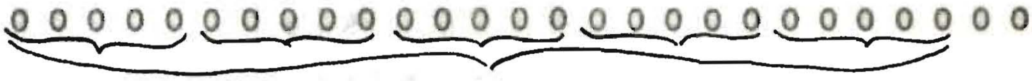
We have 3 groups of 8 with 3 more objects, written 33. We cannot call this numeral "thirty-three", because that name means 3 groups of 10 with 3 more. We have no single word for it so we must read, "three-three", and add the words "base eight" if there is any danger of being misunderstood. For the same reason, we would write

$$27_{(10)} = 33_{(8)}$$

Now let's count the same objects in groups of five.

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

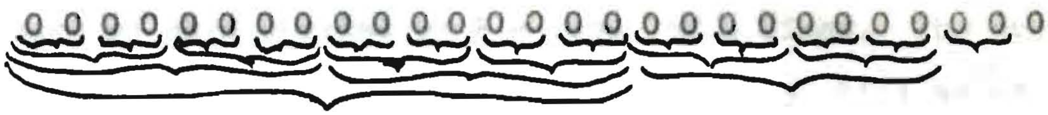
This gives us 5 groups of 5 with 2 more, but our digits in base five can go only as high as 4, because when we get a group of five in any place we carry one to the next place.



Now we have $1(5)^2 + 0(5) + 2$, or 102_{5}

$$27_{10} = 33_{8} = 102_{5}$$

Now let's count the same objects by groups of 2.



This we would write 11011_{2}. Then we have

$$27_{10} = 33_{8} = 102_{5} = 11011_{2}.$$

These are all the same number of objects. We have written different names for that number.

"Changing from base ten to other bases. Now suppose we have 52 objects to express in base two. It would be inconvenient to draw a large number of objects every time we want to change bases, so we divide the number into groups of two by "notch arithmetic".[3] Let's see how it works with 27_{10}, a numeral we have already changed to base two.

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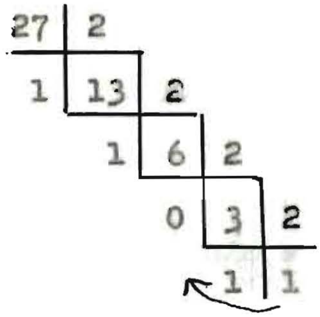
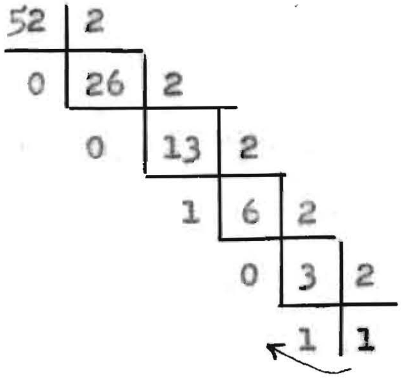
[3]Aaron Bakst, Mathematical Puzzles and Pastimes (Princeton, New Jersey: D. Van Nostrand Company, Inc., 1954), pp. 23-28.



The result is read from the bottom toward the left and top. The last "1" we wrote says we have 1 group of 2 x 2 x 2 x 2, because we have divided by 2 a total of 4 times. The next says we have 1 group of 2 x 2 x 2, and so on, till we come to the first "1" we wrote which says we had one for units place. We can check our result.

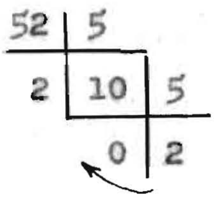
$1(2)^4 + 1(2)^3 + 0(2)^2 + 1(2) + 1 = 16 + 8 + 0 + 2 + 1 = 27_{10}$

Now let's apply notch arithmetic to 52.



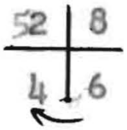
Check: $1(2)^5 + 1(2)^4 + 0(2)^3 + 1(2)^2 + 0(2) + 0 = 32 + 16 + 0 + 0 = 52_{10}$

In base 5,



Check: $2(5)^2 + 0(5) + 2 = 52_{10}$

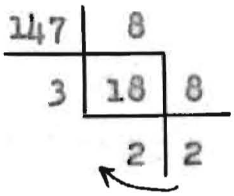
In base 8,



Check: $6(8) + 4 = 52_{10}$

Then, $52_{10} = 64_{8} = 202_{5} = 110100_{2}$.

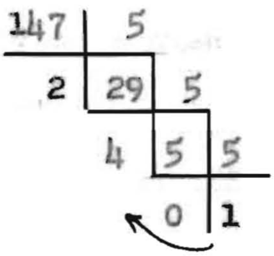
Let's express 147_{10} in base 8.



Check: $2(8)^2 + 2(8) + 3$

$= 128 + 16 + 3 = 147_{10}$

In base 5,



Check: $1(5)^3 + 0(5)^2 + 4(5) + 2$

$= 125 + 0 + 20 + 2 = 147_{10}$

Then, $147_{10} = 223_{8} = 1042_{5}$.

Exercises I

1. Make a table showing how numerals would be written in each base (10, 8, 5, 2) to count to twenty.
2. Change each of these numerals to numerals in the other three bases we are studying. (If the number is not written in base 10 numerals, change to base 10 before changing to the other two.)

(a) $147_{(10)}$	(e) $.5_{(10)}$	(i) $13_{(5)}$
(b) $25_{(10)}$	(f) $121_{(8)}$	(j) $10110_{(2)}$
(c) $10_{(10)}$	(g) $42_{(8)}$	(k) $10111101_{(2)}$
(d) $17_{(10)}$	(h) $132_{(5)}$	(l) $1001_{(2)}$
		(m) $10_{(2)}$

"Notation of exponents. This notation, $(5)^2$, that we have been using instead of 5 x 5, makes some definitions necessary.

"An exponent is a small numeral above and to the right of a quantity to show how many times it is taken as a factor.

"A factor is any one of the quantities multiplied together to form a product.

"Three dots, ···, used after an expression mean "and so on."

"If we place a numeral before the parenthesis, as $3(5)^2$, we mean 3 x 5 x 5. We also speak of $(5)^2$ as "the second power of five" or "five raised to the second power."

"We are always looking for patterns in mathematical expressions to help us organize, generalize, and remember. In $1562 = 1(10)^3 + 5(10)^2 + 6(10) + 2$, it seems we have multiplied each digit except the last two by a power of ten, but in the case of 6(10) we could say "six times ten taken once" and in the case of the unit's digit, we could say "two times ten taken not at all." That would give us $6(10)^1$ and $2(10)^0$ for the two last parts. Notice this makes $(10)^0 = 1$. Then when we write $1562 = 1(10)^3 + 5(10)^2 + 6(10)^1 + 2(10)^0$, our pattern is complete and consistent; all parts of it agree. This is very satisfying to the mathematician and makes his generalizations more useful.

"General place value pattern. Now we could say that in the same general manner, we could express a number by numerals written in any base we might choose.

TABLE II

PLACE VALUES FOR VARIOUS BASES

Base		Place value					
10	...	$(10)^5$	$(10)^4$	$(10)^3$	$(10)^2$	$(10)^1$	$(10)^0$
9	...	$(9)^5$	$(9)^4$	$(9)^3$	$(9)^2$	$(9)^1$	$(9)^0$
8	...	$(8)^5$	$(8)^4$	$(8)^3$	$(8)^2$	$(8)^1$	$(8)^0$
7	...	$(7)^5$	$(7)^4$	$(7)^3$	$(7)^2$	$(7)^1$	$(7)^0$
6	...	$(6)^5$	$(6)^4$	$(6)^3$	$(6)^2$	$(6)^1$	$(6)^0$
5	...	$(5)^5$	$(5)^4$	$(5)^3$	$(5)^2$	$(5)^1$	$(5)^0$
4	...	$(4)^5$	$(4)^4$	$(4)^3$	$(4)^2$	$(4)^1$	$(4)^0$
3	...	$(3)^5$	$(3)^4$	$(3)^3$	$(3)^2$	$(3)^1$	$(3)^0$
2	...	$(2)^5$	$(2)^4$	$(2)^3$	$(2)^2$	$(2)^1$	$(2)^0$

"This makes a nice pattern. It says that this is the way we always do this kind of problem. We are not limited to the first nine digits, either. The same method has been used to express numbers in base 12 by using single symbols, x for ten and e for eleven.

"Addition and subtraction. If we can use these systems of numeration for counting, we should also be able to add and subtract with them. Let's try a problem first in base 8, remembering that 7 is our largest digit in that base.

$3572_{(8)}$	Check:	$3(8)^3 + 5(8)^2 + 7(8) + 2 = 1914_{(10)}$
$1245_{(8)}$		$1(8)^3 + 2(8)^2 + 4(8) + 5 = 677_{(10)}$
$5037_{(8)}$		$5(8)^3 + 0(8)^2 + 3(8) + 7 = 2591_{(10)}$

When we add 2 and 5 we get 7, which gives us no trouble, but when we add 4 and 7 in the second column, we get one 8 and 3 more, so we write the 3 and carry 1. In the third column 5 and 2 plus the one we had to carry gives one 8 and no more. We write 0 and carry 1. In the last column 1 and 3 plus the 1 that was carried gives 5. Each time you add, if your sum is less than 8, write it. If your sum is 8 or more, determine how many groups of 8 you have and how many more. Write the surplus and carry the number of groups to the next column.

$$\begin{array}{r} 7545_{8} \\ 1267_{8} \\ \hline 11034_{8} \end{array} \qquad \begin{array}{r} 1324_{5} \\ 4233_{5} \\ \hline 11112_{5} \end{array} \qquad \begin{array}{r} 1334_{5} \\ 2313_{5} \\ \hline 4202_{5} \end{array}$$

"Now let's try subtraction. How much is $5764_{8} - 2167_{8}$?

$$\begin{array}{r} 5764 \\ 2167 \\ \hline 3575_{8} \end{array}$$

Since 7 is larger than 4, we must take 1 group of 8 from the top 6 in the next column. Now we have one 8 and 4 more. Seven from this leaves 5. In the next column, since we have used one group of 8, the top digit is now 5 and again the bottom digit is larger and we must take one group from the next column before we can subtract. Six from (8 and 5) leaves 7, 1 from 6 gives 5, and finally, 2 from 5 gives 3.

Exercises II

1. Add and check:

   (a) 5267_{8} and 3257_{8}

   (b) 423_{5} and 214_{5}

   (c) 10101_{2} and 111001_{2}

2. Subtract and check:

   (a) 3257_{8} from 5267_{8}

   (b) 214_{5} from 423_{5}

   (c) 10101_{2} from 111001_{2}

"Multiplication and division. Now that we know how to add and subtract, let's try multiplication and division. A multiplication table isn't absolutely necessary but will be helpful, so we shall make one for base 8. What is the first product that we may expect to be different from our base 10 table? The product, 2 x 4, which is one 8 and no more, written 10. Another will be 2 x 5, which is one 8 and 2 more, written 12.

TABLE III

MULTIPLICATION TABLE, BASE EIGHT

x	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	10	12	14	16
3	0	3	6	11	14	17	22	25
4	0	4	10	14	20	24	30	34
5	0	5	12	17	24	31	36	43
6	0	6	14	22	30	36	44	52
7	0	7	16	25	34	43	52	61

"Let's multiply 372_{8} by 27_{8}.

By our table, seven times two is one-six, so we write six and carry one. Seven times seven is six-one, and one carried makes six-two, so we write two and carry six. Seven times three is two-five, and six carried makes three-three, which we write 33. Next multiplying by 2 groups of 8, we have 2 x 2 groups of 8, which is 4 groups of 8 and should, therefore, be written in the second column (or place) under the 2 groups of 8 of our previous partial product. Using words again, two times seven gives one-six, so we write six and carry one. Two times three gives six, and one carried makes seven. Now, let's add, remembering our base. In first place, 6, and in second place, 6. In third place, 6 + 3 gives one 8 and 1 more, or one-one, so we write 1 and carry 1. Finally, 7 + 3 + 1 (carried) gives one 8 and 3 more, or 13.

```
    372
     27
   ----
   3326
   764
  -----
  13166 (8)
```

"Now we may use division to check this.

In division we must estimate the digits in the quotient in the same way we do in base ten division. According to our table, two is contained 5 times in one-three, for 5 x 2 is one-two, but we also must multiply 7 by that digit. If we take

```
        372 (8)
    ________
 27 ) 13166
      105
      ---
       246
       241
       ---
         56
         56
         --
```

27 x 5, we get 163, which would make our partial product too large to subtract. If we use four, 4 x 27 = 145, again too large. So we take 3 for the first digit in our quotient. Subtracting (base 8) we have two-four. Bringing down the next digit, we have two-four-six. Again we must estimate. Two-seven is less than three-oh, but not much, so we estimate by looking in the table to see what number multiplied by three gives a product close to two-four. We find 3 x 6 = 22 and 3 x 7 = 25. Since the latter is very little larger than two-four, we try 7 in our quotient. Our final partial dividend is five-six. In this 27 is contained 2 times.

"Of course, another way to check either multiplication or division would be to change all numbers to base ten and perform the same operation.

"Now that we can multiply and divide, let's solve the fraction problem in Exercises I, 2(e) by division.

```
      .4             .222...          .1
  2 ) 1.00 (8)   2 ) 1.00 (5)   10 ) 1.0 (2)
      1 0             4               1 0
      ---            ---              ---
        0             10
                       4
                      ---
                      10
```

Exercises III

1. (a) Make a multiplication table for base five.

   (b) Do the same for base two.

2. Multiply and give answer in same base, and check by changing to base ten:

   (a) $13_{8} \times 26_{8}$

   (b) $124_{5} \times 314_{5}$

   (c) $10111_{2} \times 1001_{2}$

3. Divide and give answer in same base, and check by changing to base ten:

   (a) $2676_{8} \quad 25_{8}$

   (b) $413_{5} \quad 121_{5}$

   (c) $11011_{2} \quad 1001_{2}$

Exercises IV

(Short Quiz)

1. Write 13_{10} in base 8, in base 5, and in base 2.
2. Write 23_{5} in base 10.
3. Multiply 14_{5} by 3_{5}.

CHAPTER V

SETS[1]

This unit was presented in class in the following manner:

"Notation. A set is a collection such that we can always tell whether or not something is a member of the set. We speak of a "set of dishes," a "set of dominoes" or say, "She and I do not run around in the same set," and everyone knows what we mean. There are two ways of indicating a set.

1. Listing: $S = \{0, 1, 2, \cdots 9\}$.
2. Describing: $S = \{\text{Whole numbers less than } 10\}$.

"A member of a set is called an element of the set. The Greek letter epsilon, $\in$, is used to mean "is an element of." The expression, $2 \in S$ is read, "two is an element of the set S."

"A subset is a set of elements within another set. In the set $S = \{0, 1, 2, \cdots 9\}$, we could consider a subset made up of the numbers in S that are divisible by three, $B = \{3, 6, 9\}$.

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[1]This presentation is drawn largely from class lectures by Dr. Oscar Peterson in Ma 470, "High School Mathematics for Teachers" at Kansas State Teachers College, Emporia, Kansas, Summer, 1959.

"A <u>null-set</u> is an empty set, written $\emptyset$ and pronounced "fee" by mathematicians.

"Whether we list or describe a set depends on the number of elements in the set. Take the set of odd numbers less than 10. This is a <u>finite</u> set, which means it has a last member. A small finite set is usually listed, but may be described if we prefer. If we want to indicate an infinite set, say all the counting numbers, we must describe, S = {the counting numbers}. But consider the counting numbers between 100 and 1000. They could be listed but it would not be practical to list so many, so we describe them by B = {the counting numbers between 100 and 1000}. There is a shorter way of saying this in symbols, $B = \{x / 100 < x < 1000, x \text{ is a counting number}\}$, which means "B is equal to the set of all elements x such that x is greater than 100 and is less than 1000 and x is a counting number." Prime numbers between two and ten would be listed, $A = \{3, 5, 7\}$. A <u>prime</u> number is one that cannot be the product of two other numbers, can only be divided by itself or one.

Exercises V

Designate the following collections in the notation of sets:

1. The boys in this class who go out for football.
2. The months of the year containing the letter "r".

Exercises V (Continued)

3. The T. V. stations in this locality.
4. The programs that may be viewed from eight to nine o'clock on Monday evenings.
5. The boys in this class who are also in the Clothing I class.
6. The numbered highways that pass through the county seat of this county.
7. The states of the United States that touch the Pacific Ocean.
8. The states of the United States that touch the border of Mexico.
9. The national parks in our state.
10. The presidents of the United States who have been assasinated.
11. The main branches of the U. S. armed forces.
12. The odd numbers less than ten.
13. The one-digit counting numbers.
14. The prime numbers less than ten.
15. The odd numbers greater than ten.
16. All the counting numbers.
17. The movable inanimate objects in this room.
18. The girls in this class who are in the third hour English class.

"Open sentences and sets. A sentence that contains a placeholder is an open sentence. A placeholder is any symbol that holds a place. If we write, "_____ has a book in class today," we have an open sentence. We can replace the blank with the name of anyone in this class. If he has a book, the statement is true; if he has none the statement is false. In either case, we have replaced the placeholder with a member of the set, A = {members of this class}. We call this set, then, the replacement set (R. S.), the set from which we can choose elements to use instead of the placeholder. The solution set (S. S.), is made up of all the elements that we could use in the open sentence and get a true statement. If everyone here has a book, the solution set is the same as the replacement set, but if only John, Pat, and Barbara have books today, then S. S. = {John, Pat, Barbara}. For the open sentence, "______ is a boy in this class who goes out for football,"

R. S. = {boys in this class}

S. S. = {Frank, Gene, Goff, Joe, John, Lyle}

Exercises VI

For each open sentence give a replacement set and find the solution set.

1. The capitol of Kansas is ______.

2. ______ is a state adjoining Kansas.

Exercises VI (Continued)

3. The numeral ____ is a digit.

4. A ______ is a plane figure having four sides.

5. ______ is a state bordering on the Gulf of Mexico.

6. _____ is a whole number greater than four and less than ten.

7. The base of our number system is _____.

8. _____ is the base of a number system we have used in some exercises.

9. One of the seasons of the year is _____.

10. A _____ is used by students to fasten sheets of paper together.

11. Students use a _____ to write on paper.

12. A student rides to our school in a _____.

13. _____ is considered a major sport in our school.

14. The name of our school in Derby is _____.

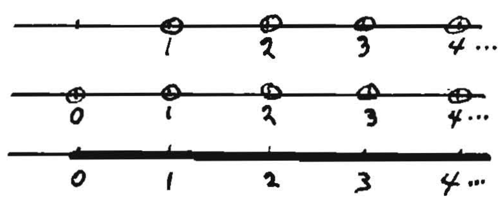
15. The largest city in Kansas is _____.

"Sets of numbers. In algebra we are more interested in sets of numbers for replacement sets. All the sets of numbers you have studied can be represented on a number line by graphs.

N = {natural numbers}

W = {whole numbers}

A = {numbers of arithmetic}



Each set is an extension of the earlier ones and includes them. Then we can call set N a subset of set A because it is made up of part of the elements of the larger set. We use $N \subset A$ to indicate that N is a subset of A.

"It is not necessary for placeholders to be blanks. The set of placeholders includes ___, □, ?, ○, x, y, ···. When we use a letter to hold a place we call the letter a variable. If we have the open sentence ___ + 3 = 7 and our R. S. = {natural numbers}, we must remember that we cannot use zero or any fractions in this sentence because they are not in the replacement set we are using, but we can choose any of the natural numbers. If we choose 9, 9 + 3 = 7 is false. We usually write $9 + 3 \neq 7$, $\neq$ meaning "is not equal to."

If we choose 5, $5 + 3 \neq 7$

But if we choose 4, $4 + 3 = 7$

Therefore, S. S. = {4}.

Exercises VII

In the following exercises, R. S. = {natural numbers}. Find the solution set for each open sentence.

1. ___ + 9 = 14
2. 5 + ___ = 14
3. 2 + ? = 5
4. 3 + ___ = 15
5. 8 + □ = 6
6. $2\frac{1}{2}$ + ? = 5

Exercises VII (Continued)

7. $3 + x = 7$

8. $9 - x = 1$

9. $11 - y = 1\frac{1}{2}$

10. $8 + 3x = 14$

11. $2x + 3x = 15$

12. $2x + 5x = 7x$

13. $a + a = 2a$

14. $6a - 5a = a$

15. $x + 3 = 5$

16. $\frac{x}{2} = 5$

17. $2x = 8$

18. $56 = 7x$

19. $\frac{a}{3} = 2$

20. $\frac{y}{3} = 8$

21. $x - 3 = 5$

22. $y - 8 = 16$

23. $5 + 4x = 25$

24. $\frac{3x}{2} = 15$

25. $5x = x + 4$

26. $\frac{4x}{3} - 2 = 6$

27. $3x - 1 = 2x + 4$

28. $3x + 4 = 16$

29. $5x - 2 = 23$

30. $17y - 5y = 12y$

31. $8x = 13$

32. $2\frac{1}{2}x = 7\frac{1}{2}$

33. $2x = 0$

34. $\frac{y}{3} = 18$

35. $x - 2\frac{1}{2} = 5\frac{1}{2}$

CHAPTER VI

LAWS OF OPERATIONS

This unit was presented in class in the following manner:

"You have heard addition, subtraction, multiplication and division called the fundamental operations of arithmetic. The principal operations of algebra are addition and multiplication. We shall study a set of numbers that will enable us to express a subtraction problem as an addition problem having the same answer, and division as multiplication. Therefore, we need only study the laws of operations for addition and multiplication.

"The commutative law of addition. If we say 2 + 3 = 3 + 2, we are talking about something that is true of these two numbers, but we are not saying anything about any other numbers. Even if we did many more problems of the same kind, we would still be talking about only those numbers we were using. But if a and b are variables replaceable by any element of a set of numbers, when we say a + b = b + a, we are making a generalization. We are saying this is true for all elements of that set, that whenever we add any of the numbers of that set, the order of the numbers does not affect the result. We call this the commutative law of addition and it holds true for all the sets of numbers you have had. On page eight of your textbook, this is called "the law of order in addition."

"The commutative law of multiplication. The statement of the similar truth for multiplication, a x b = b x a is called the commutative law of multiplication. It is usually written ab = ba, because it is commonly understood that ab means a x b just as 2a means 2 x a.

"The associative law of addition. No matter how we group the numbers we are adding, the result will be the same. We call this the associative law of addition and write it in symbols (a + b) + c = a + (b + c).

"The associative law of multiplication. It is also true in multiplication that changes in the grouping of the numbers have no effect on the result. This is the associative law of multiplication and is written (ab)c = a(bc).

"The distributive law. When the sum of two numbers is to be multiplied by a third number, the two numbers may be added first and then multiplied by the third number, or they may each be multiplied separately by the third number and the products added. This is the distributive law, written, a(b + c) = ab + ac. In words it is long and confusing; in symbols we can take it all in at a glance.[1]

"This law frequently lets us choose the easier of two ways of solving a problem. Suppose we want to find the perimeter of a rectangle when we know l and w. We can use $p = 2l + 2w$ or $p = 2(l + w)$. If $l = 6.24$ and $w = 3.76$, for example, we could save work if we noticed that $l + w = 10$ and then multiplied by 2.

"Identity elements. We also know from arithmetic that $5 + 0 = 5$ and $3 + 0 = 3$. When we generalize this we say $a + 0 = a$. We are saying when we add zero to any member of the set we will get the same member. We call zero the identity element for addition. What is the identity element for multiplication? The element that you can multiply any member of the set by and get the same number. In general terms, $a \times 1 = a$.

"Substitution principle. If $a = b$, that is, if a and b are two numerals for the same number, either can be replaced by the other in any operation.[2]

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[1]Commission on Mathematics, Appendices (New York: College Entrance Examination Board, 1959), pp. 6-7.

[2]Moses Richardson, Fundamentals of Mathematics (Revised edition; New York: The Macmillan Company, 1958), p. 45.

"How to make verbal definitions. A good definition of a mathematical word or expression should have at least the following characteristics:

1. It should be a complete sentence that makes us able to recognize the thing defined and to distinguish it from other things.
2. It should not include other forms of the word being defined.
3. It should give the special meaning the word has in mathematics.
4. It should be understandable by those who are going to use it.

"Usually in defining a word that is followed by "is", we should name the smallest subset to which the thing belongs and then tell how it is different from other members of the set, for example: "Mathematics is a science in which we study operations dealing with quantities and in which we study methods for finding other quantities from those we know."[3]

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[3]This definition was worked out by the students in the experimental class.

CHAPTER VII

TEST I AND EXERCISES VIII-XVI

Test I, Assignment 16

1. Express $37_{(10)}$ in base 8; in base 5; in base 2.

2. Make multiplication table for base 5.

3. Add $2576_{(8)}$ and $3745_{(8)}$

4. Subtract $3542_{(8)}$ from $7641_{(8)}$

5. Multiply $2341_{(5)}$ by $34_{(5)}$

6. Define:

   (a) Set

   (b) Exponent

   (c) Variable

   (d) Numeral

   (e) Mathematics

In problems 7-9, R. S. = {natural numbers}.

7. $3x + 4 = 16$

8. $2\frac{1}{2} + x = 5$

9. $3a + a = 4a$

10. State the following laws in symbols:

   (a) Commutative law of addition

   (b) Commutative law of multiplication

   (c) Associative law of addition

Test I (Continued)

(d) Associative law of multiplication

(e) Distributive law

Exercises VIII, Assignment 23

Find the solution set for each of these inequalities, using R. S. = {natural numbers}.

$<$ means "less than".

$>$ means "greater than".

$\leq$ means "less than or equal to".

$\geq$ means "greater than or equal to".

Equivalent inequalities can be found by the methods used to find equivalent equations.

Example: $2x < 6$

$x < 3$

S. S. = $\{x / x < 3$ and is a natural number$\}$.

1. $2x > 6$
2. $2x \geq 6$
3. $2x < 6$
4. $2x \leq 6$
5. $5x > 15$
6. $5x < 15$
7. $10x \leq 50$
8. $7x < 40$
9. $3y \leq 15$
10. $8x > 23$
11. $\frac{x}{2} > 5$
12. $\frac{x}{3} < 10$
13. $\frac{x}{7} \leq 21$
14. $\frac{x}{2.5} > 12$
15. $\frac{x}{.25} \geq 4$
16. $\frac{y}{17} > 2$
17. $\frac{y}{10} \leq 3$
18. $\frac{r}{5} < 6$
19. $\frac{p}{12} \leq 5$
20. $\frac{x}{4} < 6$
21. $x + 4 \geq 10$
22. $x + 2\frac{1}{2} \leq 7\frac{1}{2}$
23. $r + 5 > 10$
24. $s - .25 < 9.25$

Exercises VIII (Continued)

25. $t - 6 \leq 15$

26. $w - 3 \geq 12$

27. $x + 7 < 14$

28. $y + 5 > 5$

29. $p - 8 < 29$

30. $q - 9 > 13$

31. $r + 7 \geq 24$

32. $s - 10 < 5$

33. $Q - 6 < 40$

34. $p + 2 \leq 10$

35. $w + 4 > 18$

36. $x - 3 > 24$

37. $y - 8 > 16$

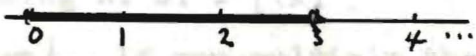
38. $x + 7 \leq 24$

39. $r - 5\frac{1}{2} < 12\frac{1}{2}$

Exercises IX, Assignment 24

Find the solution set for each of the following open sentences, using R. S. = {numbers of arithmetic}, and illustrating on a number line if possible.

Example: $2x < 6$



S. S. = $\{x \mid x < 3$ and is a number of arithmetic$\}$.

1. $2x < 6$

2. $4x > 4$

3. $x + 2 < 7$

4. $\frac{x}{3} \leq 5$

5. $x - 5 > 4$

6. $\frac{x}{2} \geq 3$

7. $x + 8 \leq 6$

8. $x - 14 > 1$

9. $\frac{x}{5} > 4$

10. $\frac{x}{1.5} < 3$

11. $x + 1\frac{1}{2} > 3\frac{1}{2}$

12. $x + 1\frac{1}{2} \geq 3$

13. $x - 4 \geq 0$

14. $\frac{x}{2} \geq 0$

15. $x + 1 < 3$

16. $x + .25 \leq 5$

17. $5x \leq 10$

18. $2.5x > 5$

19. $7x \leq 14$

Exercises X, Assignment 62

Construct lattice graphs, using R. S. = $\{(x,y) \mid -4 \le x \le 4,$ $-3 \le y \le 3$ and x, y, integers$\}$.

1. $y = x$
2. $y = -x$
3. $y = x - 1$
4. $y = x + 1$
5. $x - y = 2$
6. $y = 2x$
7. $y = 3x$
8. $y = \frac{x}{4}$
9. $y = x + 2$
10. $y = 2x + 1$
11. $y = 2x - 3$
12. $x + \frac{y}{2} = 1$
13. $\frac{x}{2} + y = 0$
14. $x + y = 0$
15. $x - y = 0$
16. $y = x - 3$
17. $2y = -x$
18. $x + y = x + 1$

Exercises XI, Assignment 63

Construct lattice graphs, using R. S. = $\{(x, y) \mid -4 \le x \le 4,$ $-3 \le y \le 3$ and x, y, integers$\}$. If you multiply the members of an inequality by a negative number, the inequality will be reversed.

Example: $x - y > 3$

$-y > 3 - x$

$y < -3 + x$

1. $y > x$
2. $y < x$
3. $y > x - 1$
4. $y > x + 1$
5. $x + y \ge 0$
6. $-x + y \ge 0$
7. $x - y < 2$
8. $x - y \ge 2$
9. $y < 3x$
10. $y > 2x$
11. $y < x - 1$
12. $y < x + 1$
13. $y \le x + 2$
14. $\frac{y}{2} > x$
15. $\frac{y}{2} \le x - 1$
16. $x + y > 1$
17. $x + y \le 1$
18. $x - y > y - x$

Exercises XII

Graph the following, using R. S. = {real numbers}:

1. $x = y$
2. $x + y = 5$
3. $x - y = 5$
4. $x + 2y = 6$
5. $x - 2y = 4$
6. $y + 4x = 2$
7. $2y + 3x = 6$
8. $x = 3y$
9. $y = 3x$
10. $3x - 2y = 4$
11. $x - y = 1$
12. $x = -y$
13. $2x - 3y = 6$
14. $y + 2x = 2$
15. $y - x = x - y$
16. $y - 4 = x$

Exercises XIII, Assignment 67

Graph the following inequalities, using R. S. = {real numbers}:

1. $x + y > 5$
2. $x - y < 5$
3. $x > y$
4. $y > x$
5. $x - y \leq 2$
6. $3x + 2y \geq 6$
7. $y \geq 2x + 4$
8. $y \leq x + 3$
9. $x + 4 > y$
10. $3y < 2x + 1$

Exercises XIV, Assignment 79

Graph the following pairs of inequalities, using R. S. = {real numbers}:

1. $x + y > 5$
   $x - y < 3$
2. $y \leq x$
   $y \geq 2x - 4$
3. $y > x - 1$
   $y > -x - 2$
4. $y \geq x$
   $y + x \geq 2$
5. $y > 2x$
   $y < -x$
6. $y < -\frac{x}{3} + 2$
   $y < x + 4$

Exercises XIV (Continued)

7. $y \leq 3x$
$5y - 2x \leq 10$

8. $y > -3x$
$\frac{-y}{2} + x \geq -1$

9. $y < -x$
$y + \frac{x}{4} < 1$

10. $\frac{1}{2}y + \frac{1}{7}x \leq 1$
$3y + 2x \geq 3$

Exercises XV,[1] Assignment 118

Factor, if possible, the following quadratic polynomials over the integers, making use of prime factorization of coefficients and explaining such use in each.

1. $x^2 - 34x - 72$
2. $x^2 - 18x + 72$
3. $x^2 + 17x + 72$
4. $x^2 + 38x + 72$
5. $x^2 + 20x + 72$
6. $x^2 + 15x + 36$
7. $x^2 + 21x + 108$
8. $x^2 + 24x + 108$
9. $x^2 - 39x + 108$
10. $x^2 - 33x - 108$
11. $x^2 + 31x + 108$
12. $x^2 + 25x + 108$

Exercises XVI, Assignment 166

Find the solution set for each of the following open sentences, using R. S. = {real numbers}. Illustrate if possible, by graphing on a number line. Also (on coordinate axes) give rough sketch of graph of each quadratic function.

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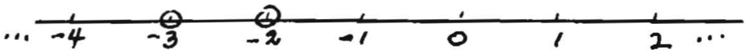
[1]Oscar J. Peterson, "Unpublished Notes," March, 1960.

Exercises XVI (Continued)

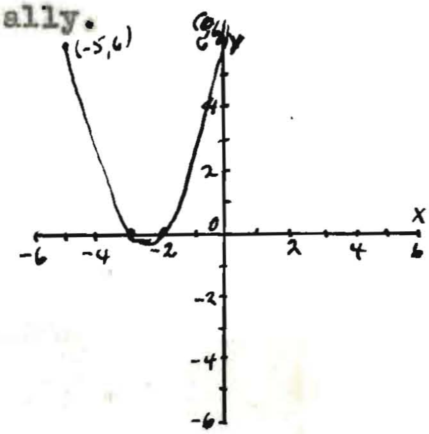
Example: 1. (a) $x^2 + 5x + 6 = 0$

$(x + 2)(x + 3) = 0$

S. S. = $\{-2, -3\}$.

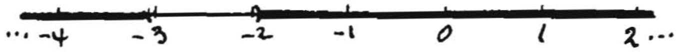


For the sketch of the graph of the function, $y = x^2 + 5x + 6$, if 0, -2, -3, and -5 are chosen for values of x, the corresponding values for y can be found mentally.



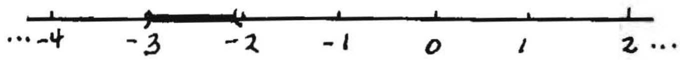
(b) $x^2 + 5x + 6 > 0$

S.S. = $\{x \mid x < -3 \text{ or } x > -2\}$.



(c) $x^2 + 5x + 6 < 0$

S.S. = $\{x \mid -3 < x < -2\}$.



2. (a) $x^2 + x - 6 = 0$
   (b) $x^2 + x - 6 > 0$
   (c) $x^2 + x - 6 < 0$

3. (a) $x^2 - x - 6 = 0$
   (b) $x^2 - x - 6 > 0$
   (c) $x^2 - x - 6 < 0$

4. (a) $x^2 + 3x + 2 = 0$
   (b) $x^2 + 3x + 2 > 0$
   (c) $x^2 + 3x + 2 < 0$

5. (a) $x^2 + x - 2 = 0$
   (b) $x^2 + x - 2 > 0$
   (c) $x^2 + x - 2 < 0$

6. (a) $x^2 - x - 2 = 0$
   (b) $x^2 - x - 2 > 0$
   (c) $x^2 - x - 2 < 0$

7. (a) $x^2 - 3x + 2 = 0$
   (b) $x^2 - 3x + 2 > 0$
   (c) $x^2 - 3x + 2 < 0$

Exercises XVI (Continued)

8. (a) $x^2 + 2x + 1 = 0$
   (b) $x^2 + 2x + 1 > 0$
   (c) $x^2 + 2x + 1 < 0$

9. (a) $2 - x - x^2 = 0$
   (b) $2 - x - x^2 > 0$
   (c) $2 - x - x^2 < 0$

10. (a) $6 + x - x^2 = 0$
    (b) $6 + x - x^2 > 0$
    (c) $6 + x - x^2 < 0$

CHAPTER VIII

SUPPLEMENTARY EXPLANATIONS ON TOPICS IN TEXTBOOK

This material was presented in class in connection with the correspondingly numbered sections in *First Algebra* by Virgil S. Mallory in the following manner:

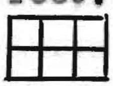
5. "A formula is a general rule in the form of an equation using two or more variables. To use it for a single numerical solution, we must be given the replacements for all the variables but one. When solving problems with formulas, use the following form:

(1) Formula, (2) Values given, (3) Drawing

(4) Replacement line, (5) Any algebraic steps necessary

(6) Solution

A drawing should be made whenever one is possible and should be labeled with the letters of the formula, not with the values given.

7. "You cannot multiply feet by feet and get square feet. The foot is a measure of length only; it has no thickness. The unit of area corresponding to this unit of length is the square foot. If a rectangle is two feet wide and three feet long,  , its area is made up of two rows of three square feet each. Because we multiply numbers and then apply the result to the practical problem in hand, we

indicate denominate units for the values given and for the solution, but they should not be used in any of the algebraic equations.

11. "Here ax = b is a general form. Our R. S. = {numbers of arithmetic}. In the equations we have had, a and b have been so small that our solution set has been obvious. If the numbers a and b are larger or contain difficult fractions, the solution set may not be obvious. We are hunting for a pattern or method for finding an equivalent equation whose solution set will be obvious. Two equations are equivalent if they have the same solution set.

"In 2x = 6, if we say x has been multiplied by 2 and the product is 6, we can undo what has been done by dividing 6 by 2, which gives 3.

"Another point of view is that 2x and 6 are both names for the same number, so if we want x, which is half of 2x, we take half of 6. $x = \frac{6}{2}$, x = 3, S. S. = {3}.

"In both instances we divided by the number that had been used to multiply the variable, so we find our pattern is $x = \frac{b}{a}$. Notice we do not need the division sign in this expression, for $\frac{1}{2}$ of $6 = \frac{1}{2} \times 6 = 6 \div 2 = \frac{6}{2}$. These are all numerals for the same number so we use the one that takes the least writing. We have a special term for generalizing here, also. To divide by a, we multiply by its <u>inverse</u>, $\frac{1}{a}$.

"We might restate the division axiom: Dividing both sides of an equation by the same number will give an equivalent equation.

13. "Here the general form for finding a more simple equivalent equation is x = ab. And our restatement of the multiplication axiom is: Multiplying both sides of an equation by the same number will give an equivalent equation.

15. "The general form for finding a more simple equivalent equation is x = b - a. Our restatement of the subtraction axiom is: Subtracting the same number from both sides of an equation will give an equivalent equation.

17. "The pattern we are seeking here is x = b + a. We restate the addition axiom: Adding the same number to both sides of an equation will give an equivalent equation.

27. "If we take the set S = {1, 2, 3, ···} and add any two members of the set, we get another member of the set. This property is called closure and we say the set is closed under addition. The set of natural numbers is closed under multiplication but not under division and subtraction. Fractions were invented so that division would always be possible. But in the set of numbers of arithmetic, subtraction was not always possible so negative (or opposite) numbers were invented about the middle of the sixteenth century.[1]

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[1]Joseph E. Hoffman, The History of Mathematics (New York: Philosophical Library, 1957), p. 82.

28. "We represent the opposite of a number by a point the same distance away from zero on the left half of the number line.

The sum of a number and its opposite (or additive <u>inverse</u>) is equal to zero.[2] In general terms, we write $a + (-a) = 0$. Negative numbers should be read "negative one", "negative two", and so on, whenever there is any danger of confusing them with subtrahends.

"We see we have doubled our number system by extending it to include negative numbers. We call this larger set, made up of the numbers of arithmetic and their opposites, the set of <u>real numbers</u>. The natural numbers and their opposites with zero form the set of <u>integers</u>. We can now define another set, the set of <u>rational numbers</u>. A <u>rational number</u> is any number that can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. This set includes fractions and whole numbers, negative and positive. All the real numbers can be represented by points on a number line. Remember $I \subset R$, the set of integers is a subset of the reals. The set of rationals is a subset of the reals. All the sets of numbers we have studied are subsets of this

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[2]Commission on Mathematics, <u>Appendices</u> (New York: College Entrance Examination Board, 1959), p. 7.

larger set, the real numbers, but this is not the last extension that has been made.

32. "The following table is a summary of algebraic addition:

TABLE IV

ALGEBRAIC ADDITION

Signs of terms	Find	Sign of answer
like	sum of absolute values	same as that of terms
unlike	difference of absolute values	sign of larger term

33. "To subtract a real number, add its opposite (by the rules of algebraic addition). In other words, we change a subtraction problem to an equivalent addition problem.

Thus: $(-2) - (-5) = (-2) + (+5) = +3$.

Or we could write $(-2) - (-5) = -2 + 5 = +3$, dropping the parentheses and the sign of the operation, since there is a conventional agreement that we are to add signed numbers written horizontally unless there are signs giving other instructions.

36. "Here is a method of developing the rules for the multiplication of real numbers.[3]

From arithmetic, $(+2)(+3) = +6$

and $2(0) = 0$. Substituting

$(+3) + (-3)$ for zero, $2\ (+3) + (-3) = 0$.

---

[3]Dr. Henry Van Engen, of the University of Wisconsin, in a lecture at Kansas State Teachers College, Emporia, Kansas, summer, 1959.

By the distributive law, $(+2)(+3) + (+2)(-3) = 0$, but $(+2)(+3) = 6$, so $(+2)(-3)$ will have to be equal to the additive inverse of $+6$,

$$\therefore (+2)(-3) = -6.$$

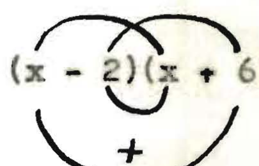
Again, since $a(0) = 0$, $(-2) \quad (+3) + (-3) \quad = 0$.
By distributive law, $(-2)(+3) + (-2)(-3) = 0$, but $(-2)(+3) = -6$, $\quad \therefore (-2)(-3) = +6$.

58. "The x-distance is called the <u>abscissa</u>; the y-distance is called the <u>ordinate</u>.

61. "Let us consider the pair of equations, $x - y = 1$ and $x + y = 5$. If the set $S = \{(5, 4), (4, 3), (3, 2), \cdots\}$ is the set of points on the graph of $x - y = 1$ and the set $R = \{(1, 4), (2, 3), (3, 4), (4, 1), \cdots\}$ is the set of points on the graph of $x + y = 5$, then $S \cap R$ (which is read "S intersect R") $= \{(3,2)\}$ is the intersection of the two graphs and the solution set of the problem. The intersection of the two sets is made up of the elements that are common to both sets.

65. "In the example shown, $x = 5 - y$ gives us two names for the same number, then x in the second equation may be replaced by $5 - y$ to give us an equivalent equation containing only one variable.

93. "'The Binomial Face' is an aid to memory, a mnemonic. Or we can use the



word "foil" for a mnemonic to be certain we get all the partial products. When we have $(x - 2)(x + 6)$, f reminds us to multiply together the first terms of the two binomials to find the first term of our product, x^2; o reminds us to multiply the two outside terms to find one of the cross products, $6x$; i reminds us to multiply the two inside terms to find the other cross product, $-2x$; and l reminds us to multiply the last terms of the two binomials to find the final term of the product, -12. We then have $x^2 + 6x - 2x - 12$, which equals $x^2 + 4x - 12$.

116. "When you express a fraction in another form, say $\frac{1}{3}$ in ninths, be careful to multiply by unity, thus:

$$\frac{1}{3} \cdot \frac{3}{3} = \frac{3}{9}.$$

This method leads to greater accuracy and clearly shows that we are not changing the value of the fraction."

CHAPTER IX

KEY TO EXERCISES AND TEST I

Exercises I-VII are given in Chapters IV and V.

Exercises I

1.

Verbal	Base 10	Base 8	Base 5	Base 2
one	1	1	1	1
two	2	2	2	10
three	3	3	3	11
four	4	4	4	100
five	5	5	10	101
six	6	6	11	110
seven	7	7	12	111
eight	8	10	13	1000
nine	9	11	14	1001
ten	10	12	20	1010
eleven	11	13	21	1011
twelve	12	14	22	1100
thirteen	13	15	23	1101
fourteen	14	16	24	1110
fifteen	15	17	30	1111
sixteen	16	20	31	10000
seventeen	17	21	32	10001
eighteen	18	22	33	10010
nineteen	19	23	34	10011
twenty	20	24	40	10100

2.

Base 10	Base 8	Base 5	Base 2
147	223	1042	10010011
25	31	100	11001
10	12	20	1010
17	21	32	10001
0.5	0.4	0.222...	.1
81	121	311	1010001
34	42	114	100010
42	52	132	101010
8	10	13	1000
22	26	42	10110
189	275	1224	10111101
9	11	14	1001
2	2	2	10

Exercises II

1. (a) $10546_{(8)}$
   (b) $1142_{(5)}$
   (c) $1001110_{(2)}$

2. (a) $2010_{(8)}$
   (b) $204_{(5)}$
   (c) $100100_{(2)}$

Exercises III

1. (a)

x	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

(b)

x	0	1
0	0	0
1	0	1

2. (a) 362_8 Ans.

Multiplication:

$13 = 1(8) + 3 = 11_{10}$

$\underline{26} = 2(8) + 6 = 22_{10}$

102

$\underline{26}$

$362_8 = 3(8)^2 + 6(8) + 2$

$= 192 + 48 + 2 = 242_{10}$

Check:

11

$\underline{22}$

22

$\underline{22}$

242_{10}

(b) 101101_5 Ans.

Multiplication:

$124 = 1(5)^2 + 2(5) + 4$

$\underline{314} = 3(5)^2 + 1(5) + 4$

1111

124

$\underline{432}$

$101101_5 = 1(5)^5 + 0(5)^4 + 1(5)^3 + 1(5)^2 + 0(5) + 1$

$= 3125 + 0 + 125 + 25 + 0 + 1$

$= 3276_{10}$

Check:

39

$\underline{84}$

156

$\underline{312}$

3276_{10}

Exercises III (Continued)

(c) 11001111_{2} Ans.

Multiplication: Check:

$$10111 = 1(2)^4 + 0(2)^3 + 1(2)^2 + 1(2) + 1 = 23$$

$$1001 = 1(2)^3 + 0(2)^2 + 0(2) + 1 = 9$$

```
     10111          23
      1001           9
     10111        207(10)
  10111
  11001111
```

$$11001111_{2} = 1(2)^7 + 1(2)^6 + 0(2)^5 + 0(2)^4 + 1(2)^3 + 1(2)^2 + 1(2) + 1 = 207_{10}$$

3\. (a) 106_8 Ans.

Division: Check:

```
       106(8)             70(10)
  25 ) 2676          21 ) 1470
       25                 147
        176                 0
        176
```

$25_{8} = 2(8) + 5 = 21_{10}$

$2676_{8} = 2(8)^3 + 6(8)^2 + 7(8) + 6 = 1470_{10}$

$106_{8} = 1(8)^2 + 0(8) + 6 = 70_{10}$

Exercises III (Continued)

(b) 3_{5} Ans.

Division:

$$121 \overline{)\,413} = 3_{5}$$
$$\underline{413}$$

$121_{5} = 1(5)^2 + 2(5) + 1 = 36_{10}$

$413_{5} = 4(5)^2 + 1(5) + 3 = 108_{10}$

$3_{5} = 3_{10}$

Check:

$$36 \overline{)\,108} = 3_{10}$$

(c) 11_{2} Ans.

Division:

$$1001 \overline{)\,11011} = 11_{2}$$
$$\underline{1001}$$
$$1001$$
$$\underline{1001}$$

$11011_{2} = 1(2)^4 + 1(2)^3 + 0(2)^2 + 1(2) + 1 = 27_{10}$

$1001_{2} = 1(2)^3 + 0(2)^2 + 0(2) + 1 = 9_{10}$

$11_{2} = 1(2) + 1 = 3_{10}$

Check:

$$9 \overline{)\,27} = 3_{10}$$

Exercises IV

1. (a) 15_{8} (b) 23_{5} (c) 1101_{2}
2. 13_{10}
3. 102_{5}

$$14 = 1(5) + 4 = 9_{10}$$
$$\underline{3} = 3 = \underline{3_{10}}$$
$$102_{5} = 1(5)^2 + 0(5) + 2 = 27_{10}$$

Exercises V

1. {Frank, Gene, Goff, Joe, John, Lyle}
2. {January, February, March, April, September, October, November, December}
3. {KARD, KAKE, KTVH}
4. {Peter Gunn, Joseph Cotton, Goodyear Theater}
5. $\emptyset$
6. {81, 42, 54, 96, 15}
7. {California, Oregon, Washington, Alaska, Hawaii}
8. {Texas, New Mexico, California, Arizona}
9. $\emptyset$
10. {McKinley, Lincoln, Garfield}
11. {Navy, Army, Air Force, Marines}
12. {1, 3, 5, 7, 9}
13. {1, 2, 3, ··· 9}
14. {1, 2, 3, 5, 7}
15. $\{x \mid x > 10, \text{ and odd}\}$
16. {counting numbers} or {natural numbers}
17. {chairs, desk, chalk, erasers, yardstick}
18. {Mary, Pat, Barbara}

Exercises VI

1. R. S. = {state capitols}; S. S. = {Topeka}
2. R. S. = {states of United States}; S. S. = {Oklahoma, Missouri, Nebraska, Colorado}

Exercises VI (Continued)

3. R. S. = {all numerals}; S. S. = {0, 1, 2, ··· 9}
4. R. S. = {all plane figures}; S. S. = {square, rectangle, parallelogram, rhomboid, rhombus, trapezoid, quadrilateral}
5. R. S. = {states of United States}; S. S. = {Texas, Louisiana, Mississippi, Alabama, Florida}
6. R. S. = {whole numbers}; S. S. = {5, 6, 7, 8, 9}
7. R. S. = {bases of number systems}; S. S. = {10}
8. R. S. = {bases of number systems}; S. S. = {8, 5, 2, 10}
9. R. S. = {spring, summer, fall, winter}; S. S. = {spring, summer, fall, winter}
10. R. S. = {all fasteners}; S. S. = {pin, clip, staple, tape}
11. R. S. = {writing tools}; S. S. = {pen, pencil, ballpoint, typewriter}
12. R. S. = {vehicles}; S. S. = {car, bus}
13. R. S. = {all sports}; S. S. = {football, basketball, track}
14. R. S. = {names of schools in Derby}; S. S. = {Derby Junior High School}
15. R. S. = {cities in Kansas}; S. S. = {Wichita}

Exercises VII

1. {5}
2. {9}
3. {3}
4. {12}
5. ∅
6. ∅

Exercises VII (Continued)

7. {4}
8. {8}
9. ∅
10. {2}
11. {3}
12. {natural numbers}
13. {natural numbers}
14. {natural numbers}
15. {2}
16. {10}
17. {4}
18. {8}
19. {6}
20. {24}
21. {8}
22. {24}
23. {5}
24. {10}
25. {1}
26. {6}
27. {5}
28. {4}
29. {5}
30. {natural numbers}
31. ∅
32. {3}
33. ∅
34. {54}
35. {8}

Test I

1. 45_{8} ; 122_{5} ; 100101_{2}
2.

x	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

3. 6543_{8}
4. 4077_{8}
5. 202244_{5}
7. {4}
8. ∅

Test I (Continued)

9. {natural numbers}

10. (a) $a + b = b + a$

(b) $ab = ba$

(c) $a + (b + c) = (a + b) + c$

(d) $a(bc) = (ab)c$

(e) $a(b + c) = ab + ac$

Exercises VIII

1. $\{4, 5, 6, \cdots\}$
2. $\{3, 4, 5, \cdots\}$
3. $\{1, 2\}$
4. $\{1, 2, 3\}$
5. $\{4, 5, 6, \cdots\}$
6. $\{1, 2\}$
7. $\{1, 2, 3, 4, 5\}$
8. $\{1, 2, 3, 4, 5\}$
9. $\{1, 2, 3, 4, 5\}$
10. $\{3, 4, 5, \cdots\}$
11. $\{11, 12, 13, \cdots\}$
12. $\{x \mid x < 30$, x is natural number$\}$
13. $\{x \mid x \leq 147$, x is natural number$\}$
14. $\{31, 32, 33, \cdots\}$
15. $\{1, 2, 3, \cdots\}$

Exercises VIII (Continued)

16. $\{35, 36, 37, \cdots\}$

17. $\{y \mid y \leq 30$, y is a natural number$\}$

18. $\{r \mid r < 30$, r is a natural number$\}$

19. $\{p \mid p \leq 60$, p is a natural number$\}$

20. $\{x \mid x < 24$, x is a natural number$\}$

21. $\{6, 7, 8, \cdots\}$

22. $\{1, 2, 3, 4, 5\}$

23. $\{6, 7, 8, \cdots\}$

24. $\{1, 2, 3, \cdots, 9\}$

25. $\{t \mid 6 < t \leq 21$, t is a natural number$\}$

26. $\{15, 16, 17, \cdots\}$

27. $\{1, 2, 3, 4, 5, 6\}$

28. $\{1, 2, 3, \cdots\}$

29. $\{p \mid 8 < p < 37$, p is a natural number$\}$

30. $\{23, 24, 25, \cdots\}$

31. $\{17, 18, 19, \cdots\}$

32. $\{10, 11, 12, 13, 14\}$

33. $\{Q \mid 6 < Q < 46$, Q is a natural number$\}$

34. $\{1, 2, 3, \cdots 8\}$

35. $\{15, 16, 17, \cdots\}$

36. $\{28, 29, 30, \cdots\}$

37. $\{25, 26, 27, \cdots\}$

38. $\{x \mid x \leq 17$, x is a natural number$\}$

39. $\{6, 7, 8, \cdots 17\}$

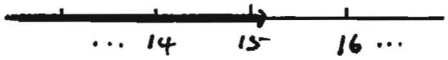
Exercises IX

1. {x | x < 3, x is a no. of arith.}

2. {x | x > 1, x is a no. of arith.}

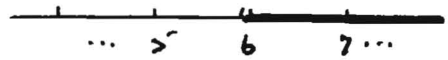
3. {x | x < 5, x is a no. of arith.}

4. {x | x ≤ 15, x is a no. of arith.}



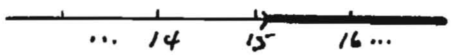
5. {x | x > 9, x is a no. of arith.}

6. {x | x ≥ 6, x is a no. of arith.}

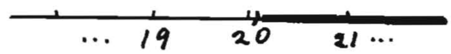


7. ϕ

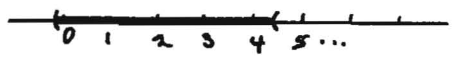
8. {x | x > 15, x is a no. of arith.}



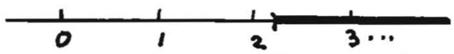
9. {x | x > 20, x is a no. of arith.}



10. {x | x < 45, x is a no. of arith.}

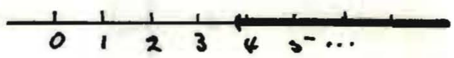


11. {x | x > 2, x is a no. of arith.}



12. {x | x ≥ $1\frac{1}{2}$, x is a no. of arith.}

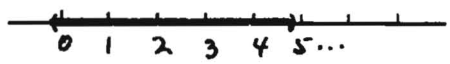
13. {x | x ≥ 4, x is a no. of arith.}



14. {x | x is a no. of arith.}

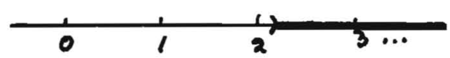
15. {x | x < 2, x is a no. of arith.}

16. {x | x ≤ 4.75, x is a no. of arith.}

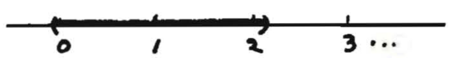


17. {x | x ≤ 2, x is a no. of arith.}

18. {x | x > 2, x is a no. of arith.}

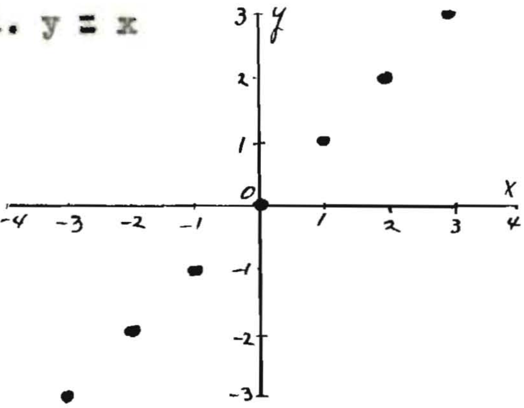


19. {x | x ≤ 2, x is a no. of arith.}

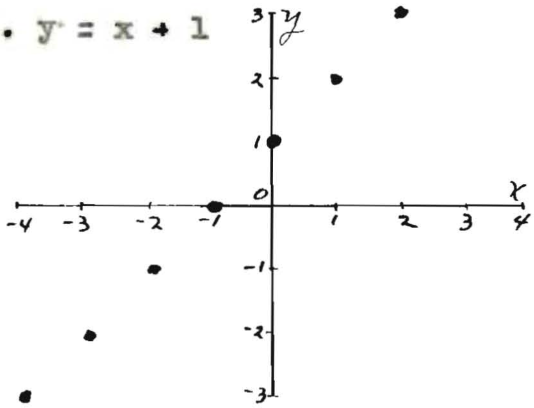


Exercises X

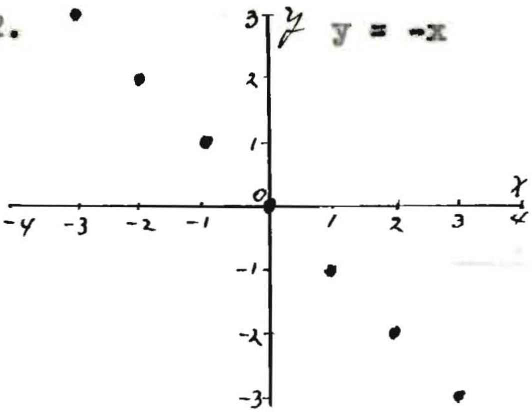
1. y = x



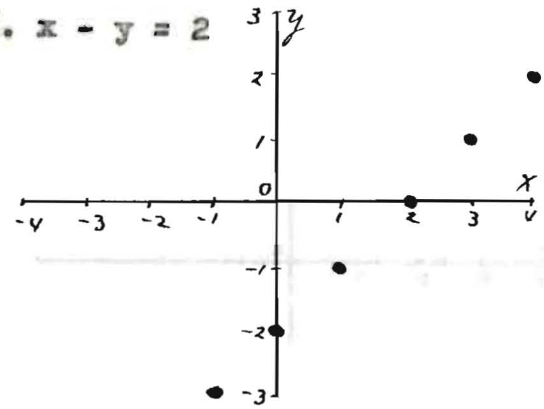
4. y = x + 1



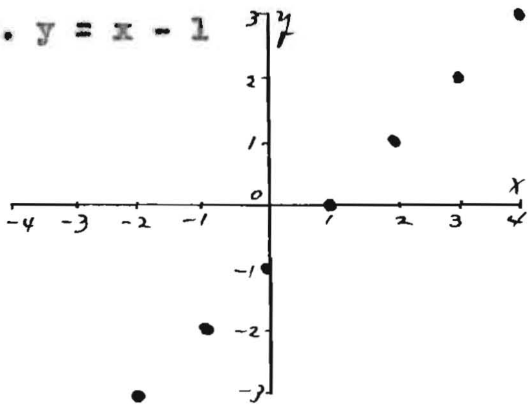
2. y = -x



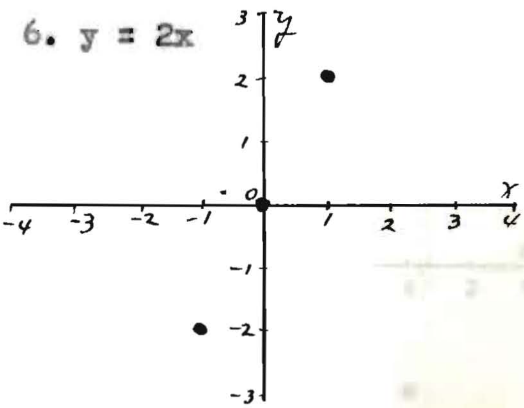
5. x - y = 2



3. y = x - 1

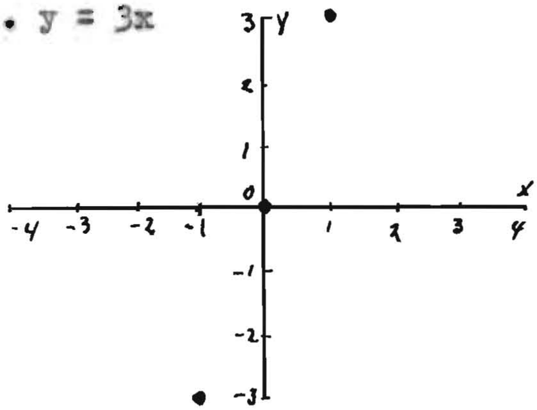


6. y = 2x

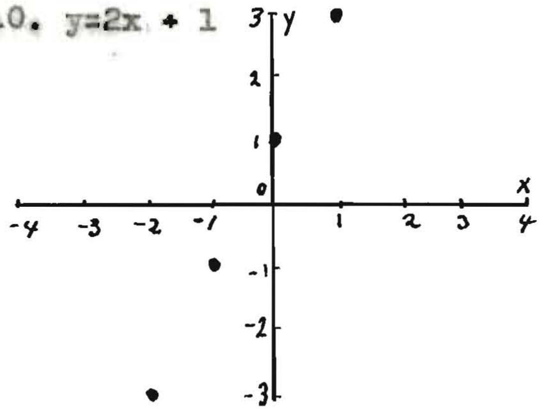


Exercises X (Continued

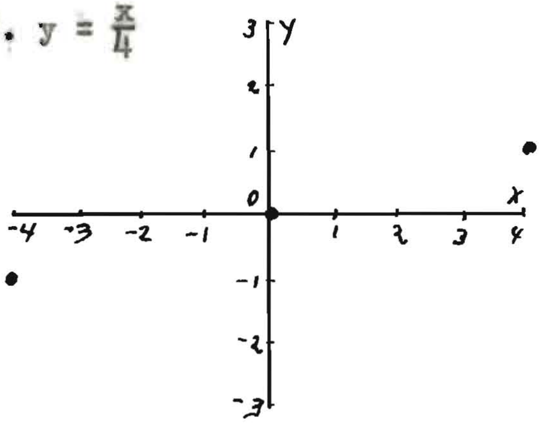
7. $y = 3x$



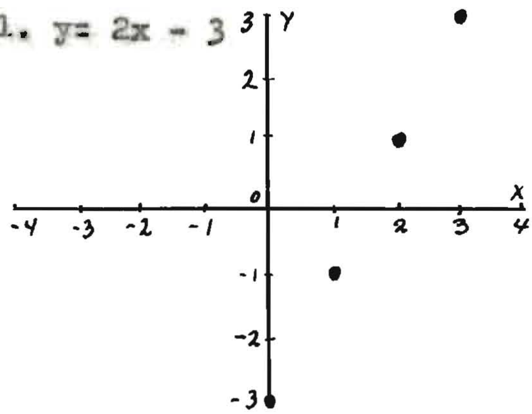
10. $y = 2x + 1$



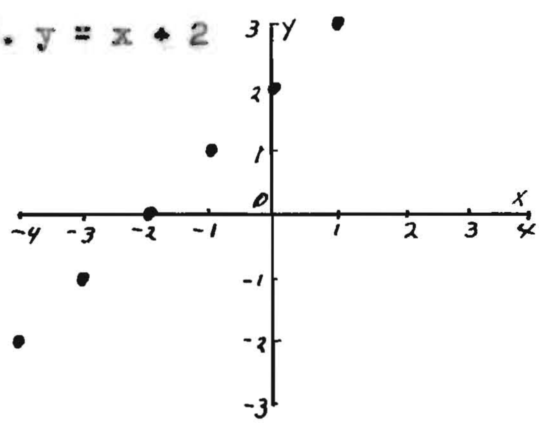
8. $y = \frac{x}{4}$



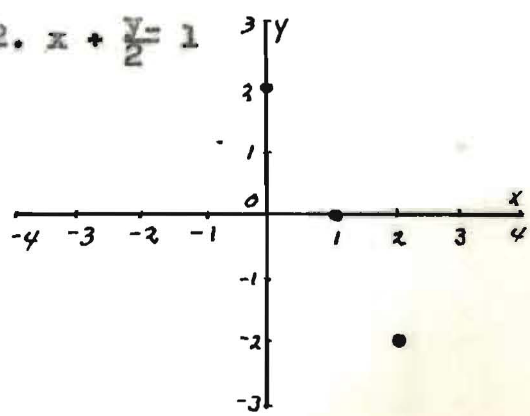
11. $y = 2x - 3$



9. $y = x + 2$

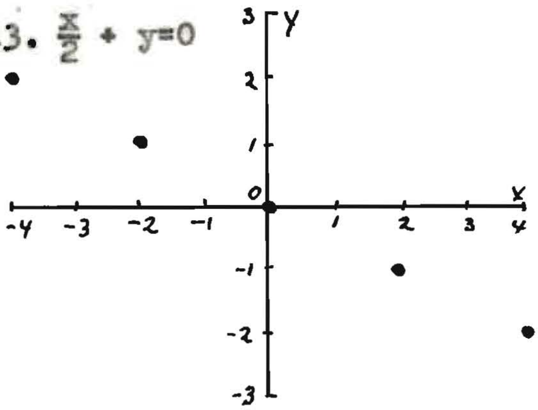


12. $x + \frac{y}{2} = 1$

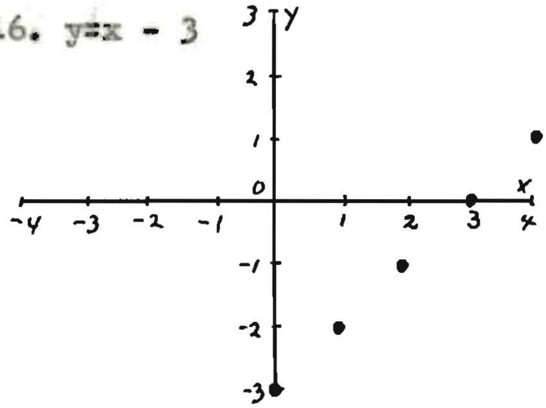


Exercises X (Continued)

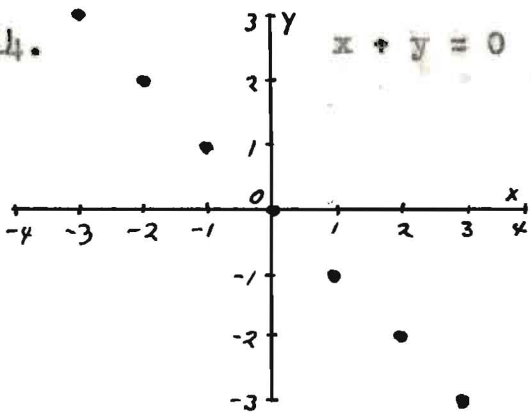
13. $\frac{x}{2} + y = 0$



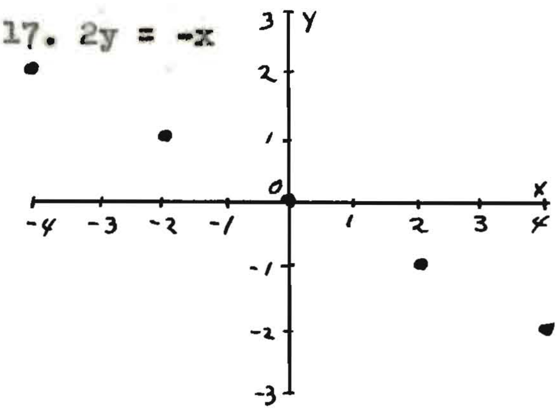
16. $y = x - 3$



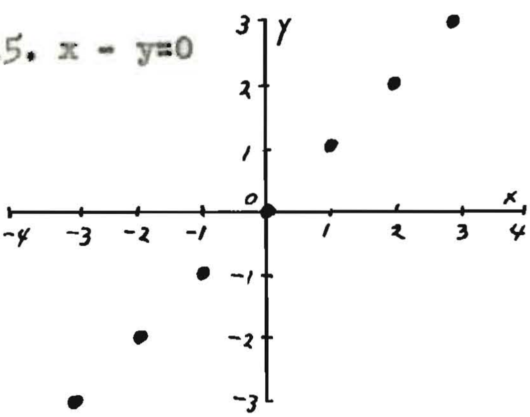
14. $x + y = 0$



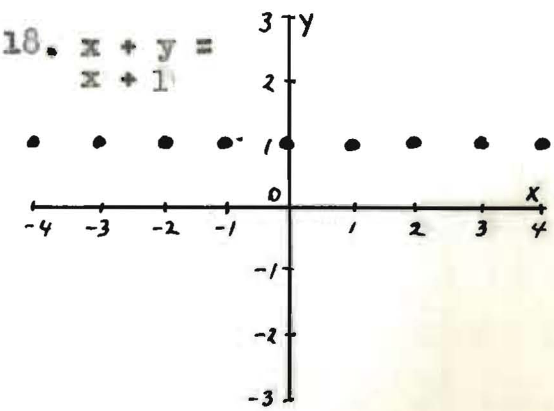
17. $2y = -x$



15. $x - y = 0$

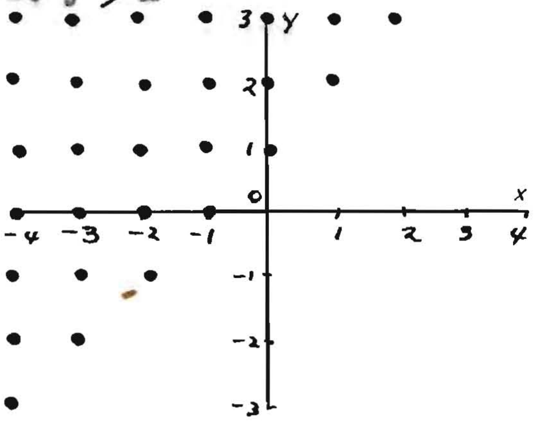


18. $\frac{x + y}{x + 1} =$

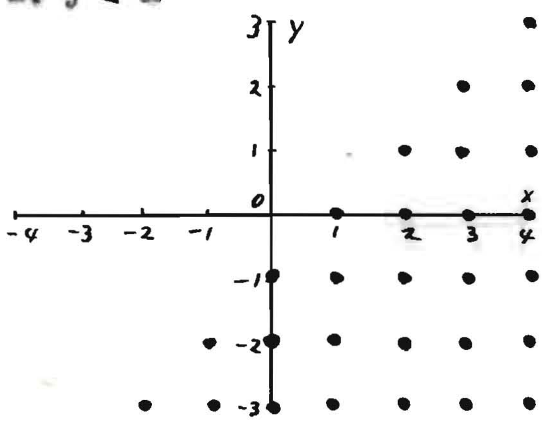


Exercises XI

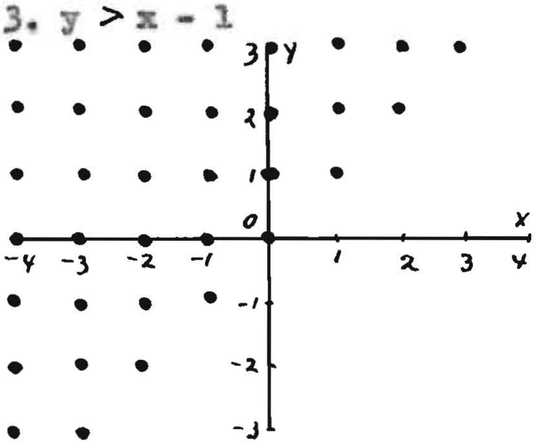
1. y > x



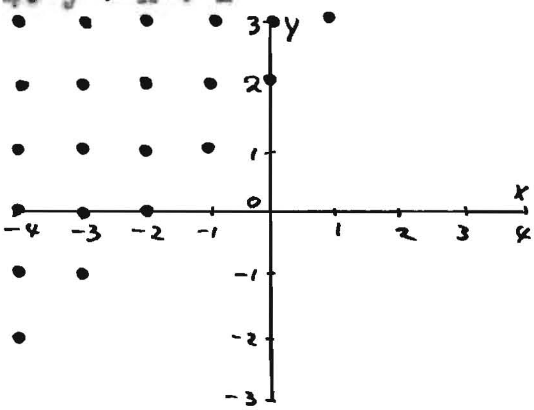
2. y < x



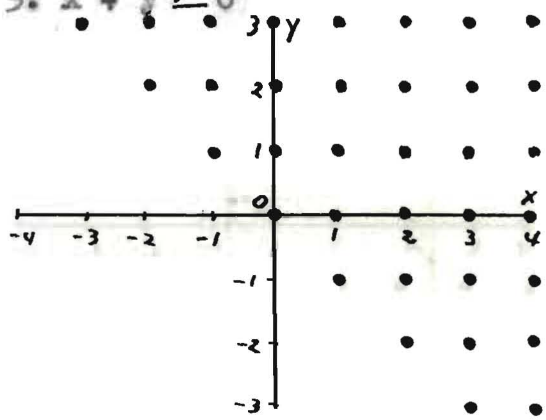
3. y > x - 1



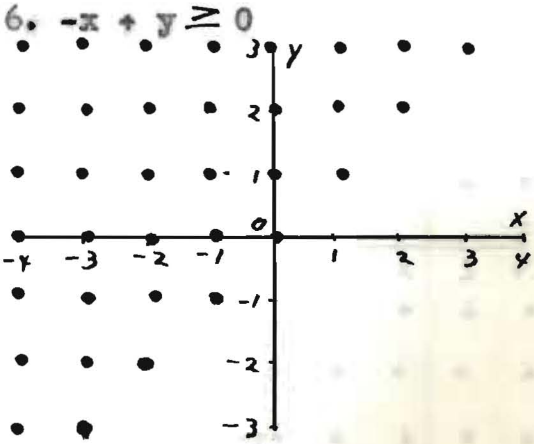
4. y > x + 1



5. x + y ≥ 0

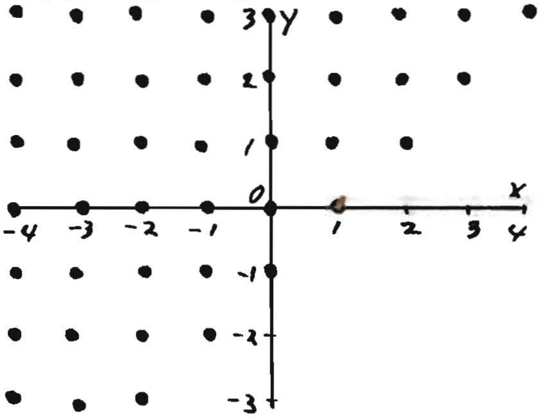


6. -x + y ≥ 0

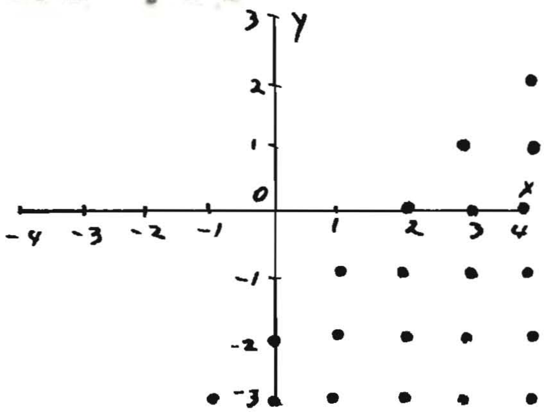


Exercises XI (Continued)

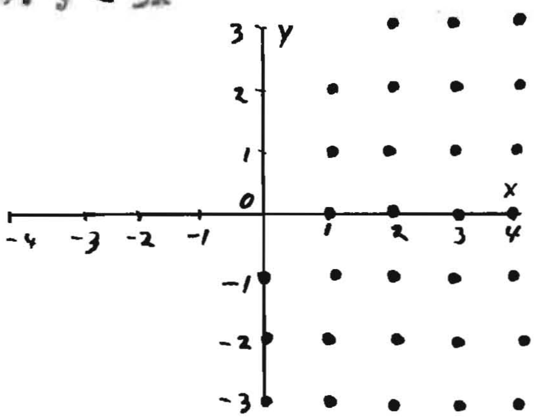
7. $x - y < 2$



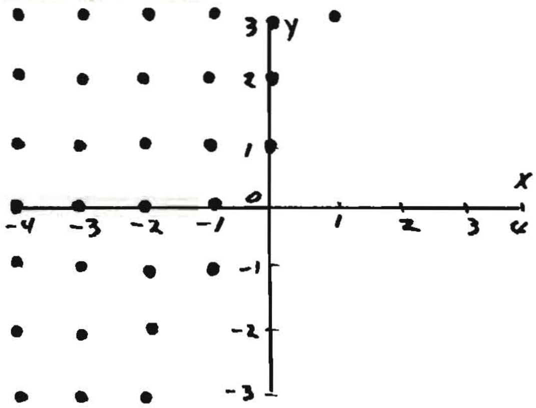
8. $x - y \geq 2$



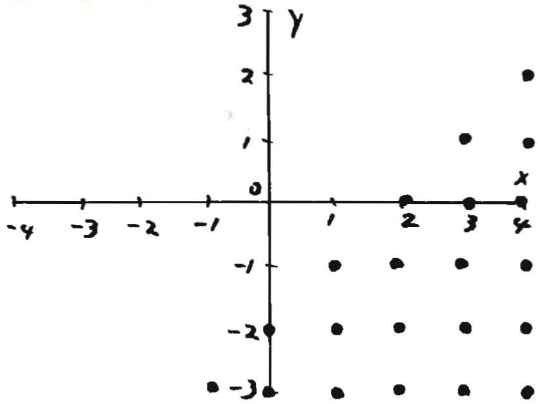
9. $y < 3x$



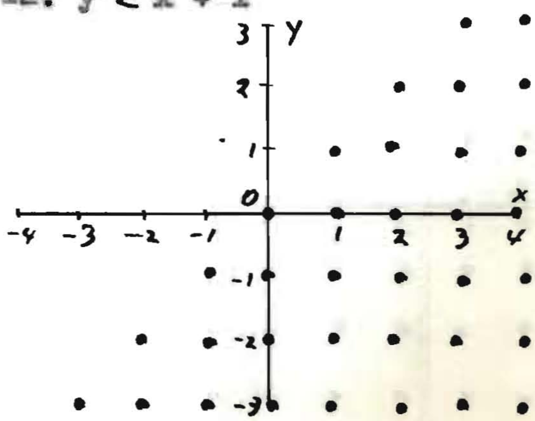
10. $y > 2x$



11. $y < x - 1$

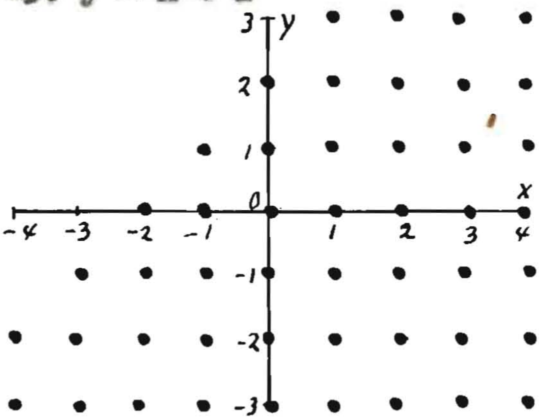


12. $y < x + 1$

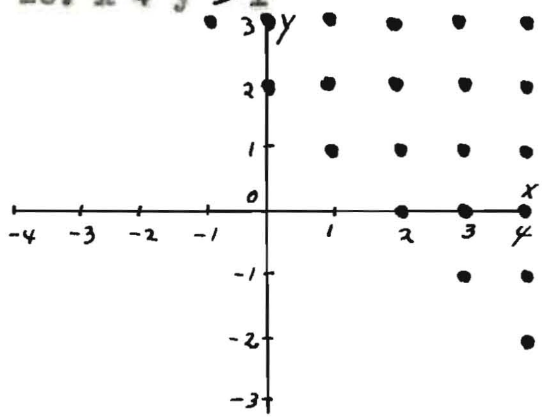


Exercises XI (Continued)

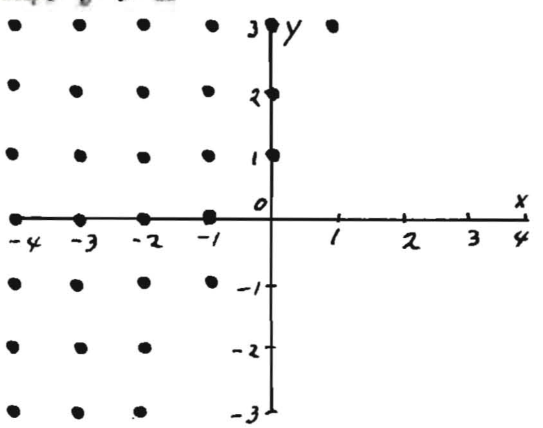
13. $y \leq x + 2$



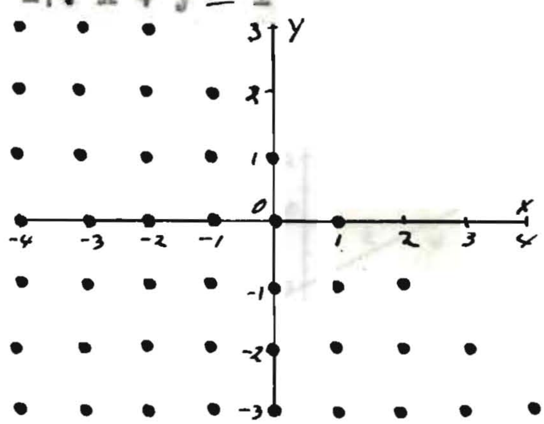
16. $x + y > 1$



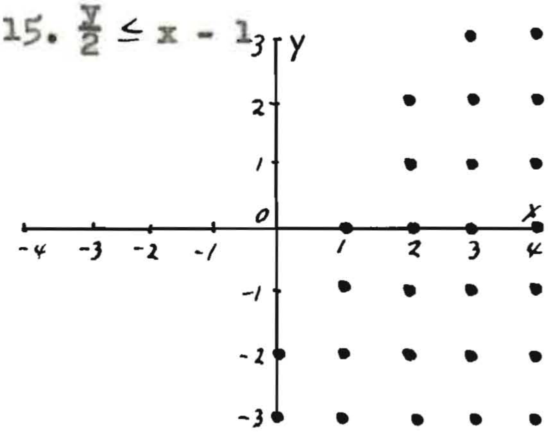
14. $y > x$



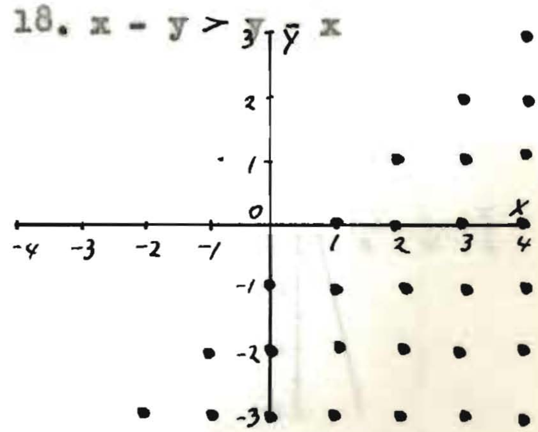
17. $x + y \leq 1$



15. $\frac{y}{2} \leq x - 1$

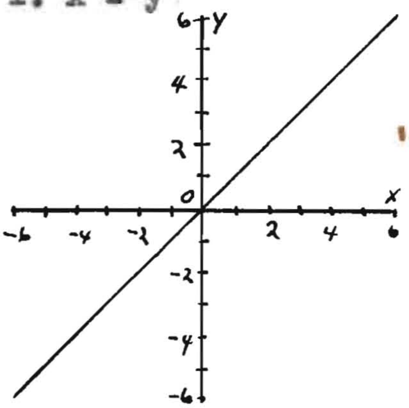


18. $x - y > y - x$

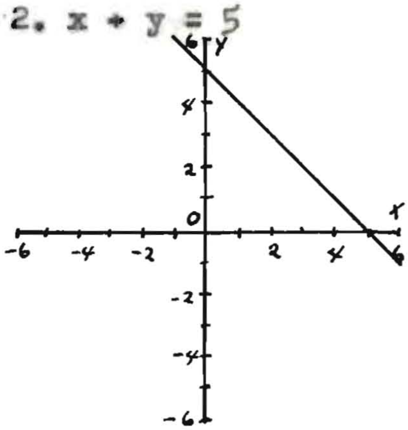


Exercises XII

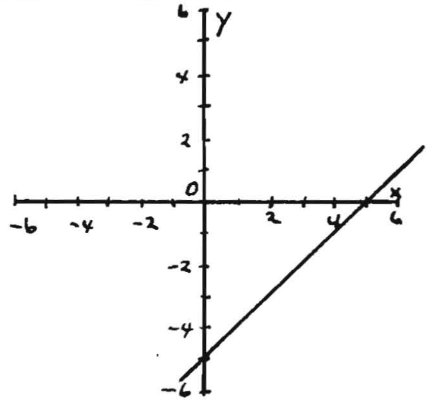
1. x = y



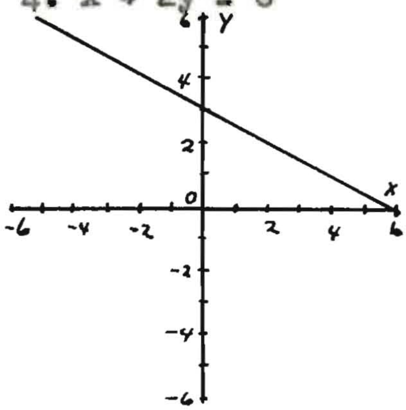
2. x + y = 5



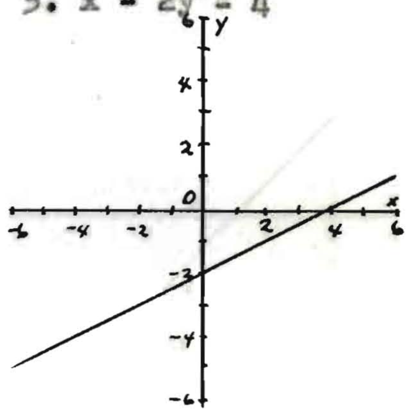
3. x - y = 5



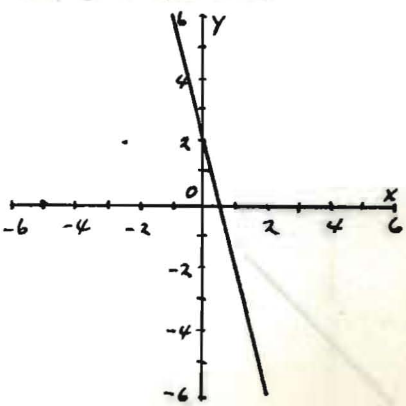
4. x + 2y = 6



5. x - 2y = 4

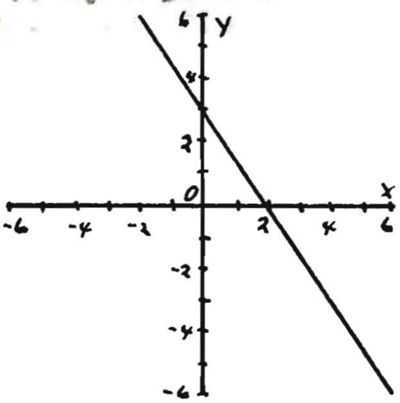


6. y + 4x = 2

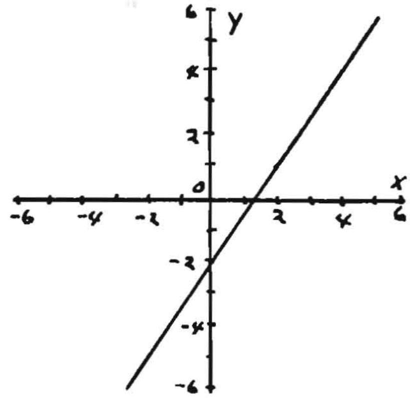


Exercises XII (Continued)

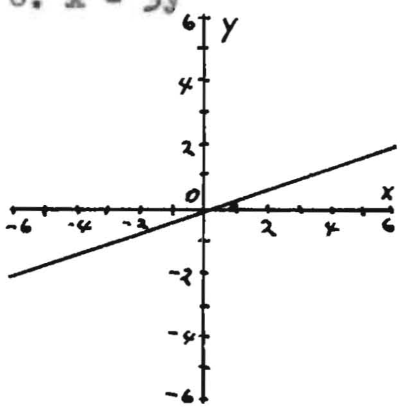
7. $2y + 3x = 6$



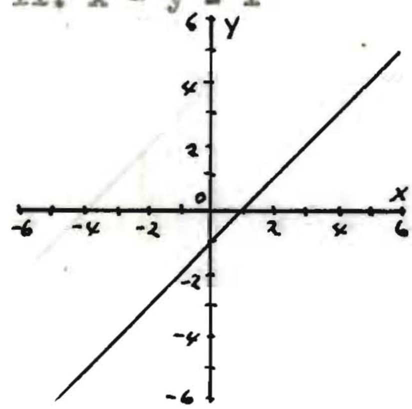
10. $3x - 2y = 4$



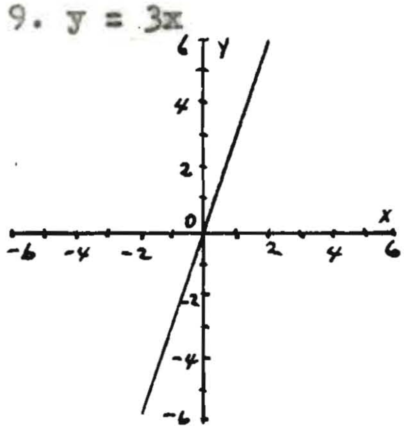
8. $x = 3y$



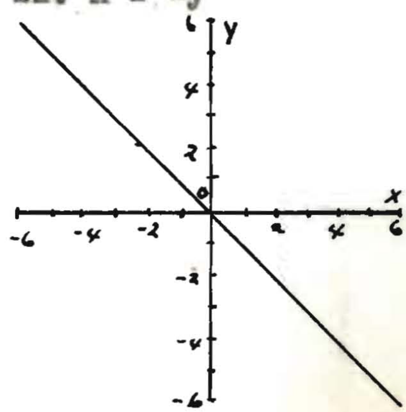
11. $x - y = 1$



9. $y = 3x$

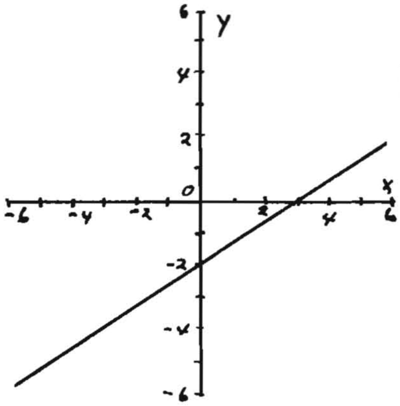


12. $x = -y$

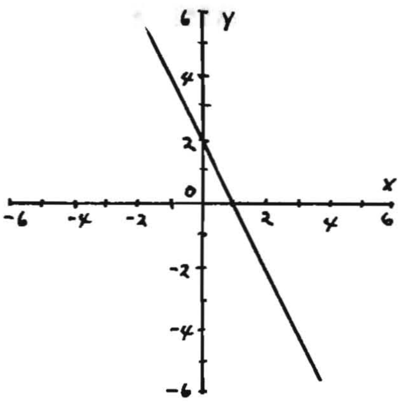


Exercises XII (Continued)

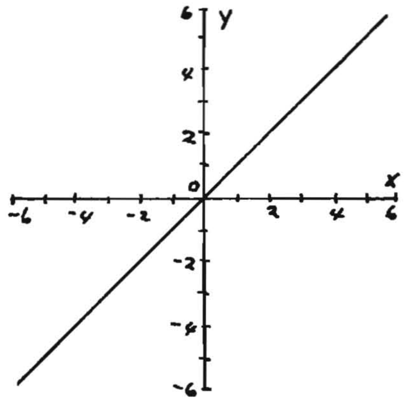
13. $2x - 3y = 6$



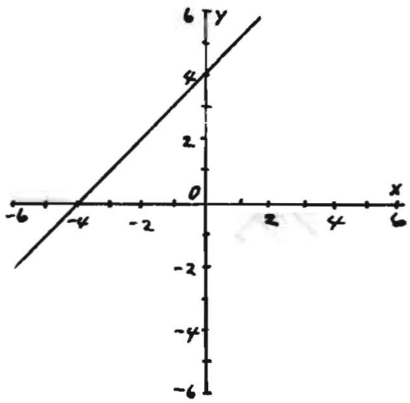
14. $y + 2x = 2$



15. $y - x = x - y$

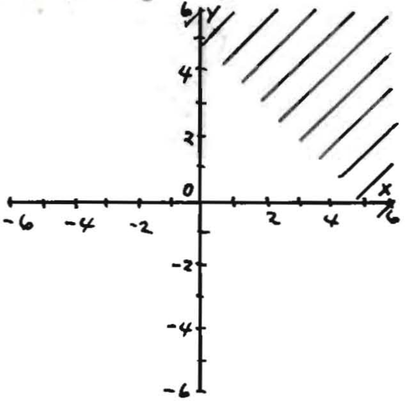


16. $y - 4 = x$

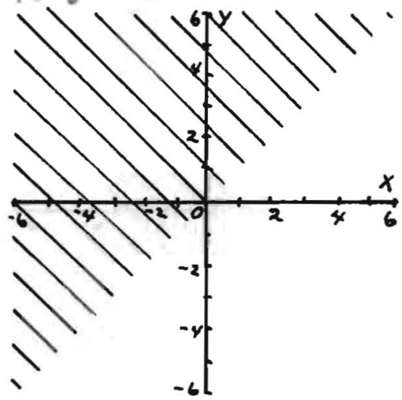


Exercises XIII

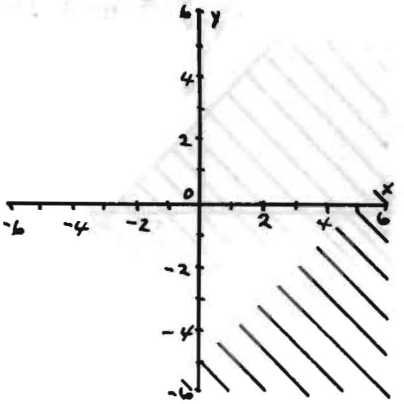
1. x + y > 5



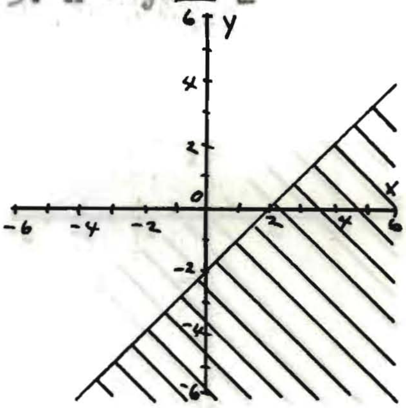
4. y > x



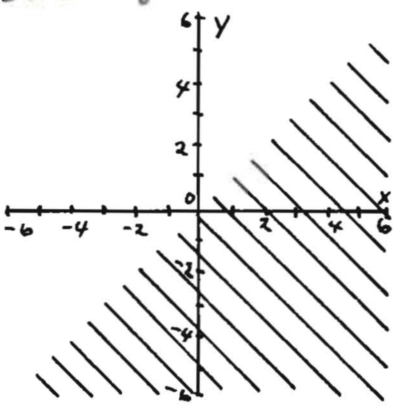
2. x - y < 5



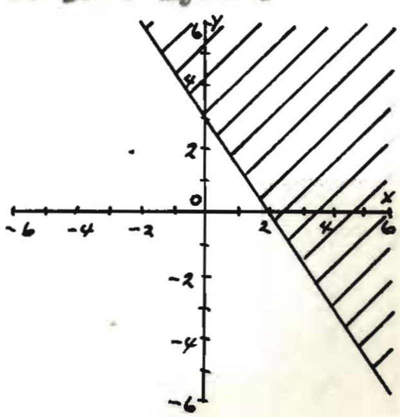
5. x - y ≥ 2



3. x > y

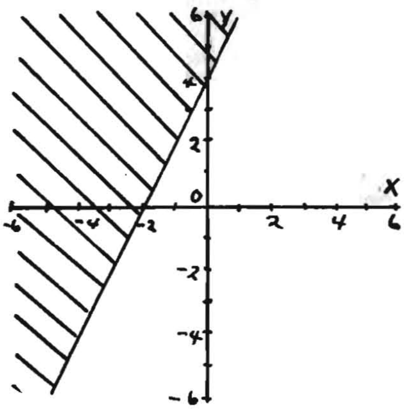


6. 3x + 2y ≥ 6

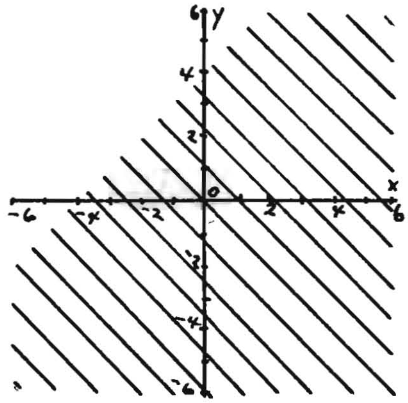


Exercises XIII (Continued)

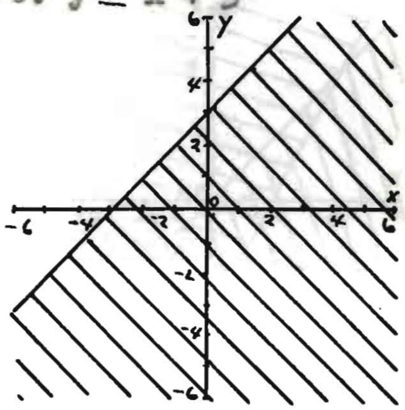
7. $y \geq 2x + 4$



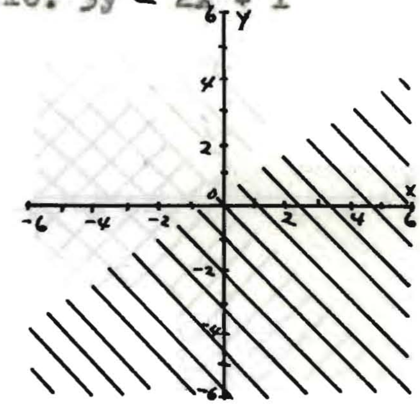
9. $x + 4 > y$



8. $y \leq x + 3$

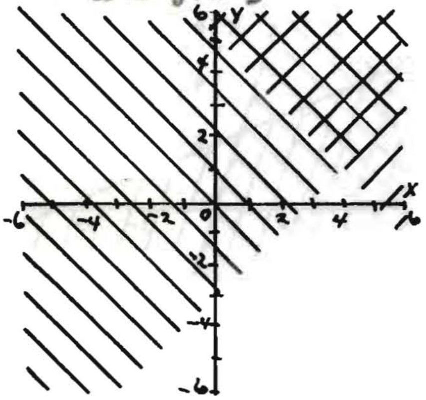


10. $3y < 2x + 1$

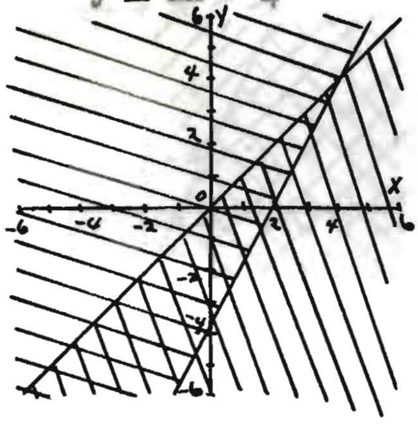


Exercises XIV

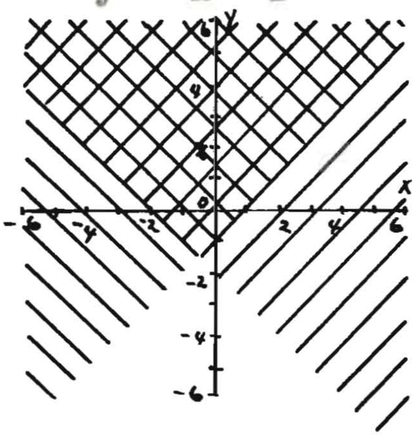
1. $x + y > 5$
   $x - y < 3$



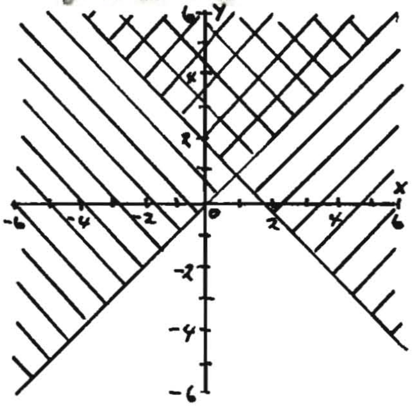
2. $y \leq x$
   $y \geq 2x - 4$



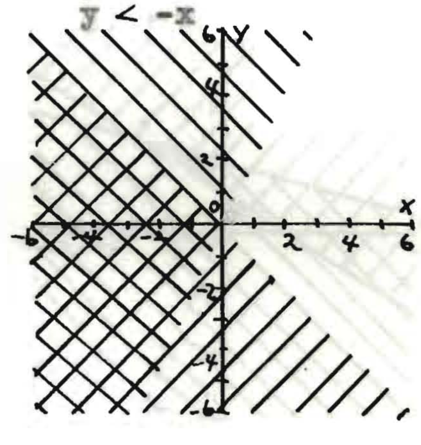
3. $y > x - 1$
   $y > -x - 2$



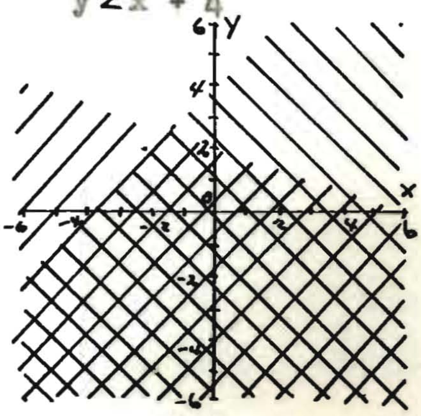
4. $y \geq x$
   $y + x \geq 2$



5. $y > 2x$
   $y < -x$

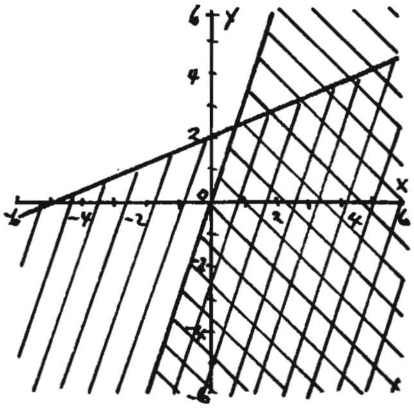


6. $y < \frac{x}{3} + 2$
   $y < x + 4$

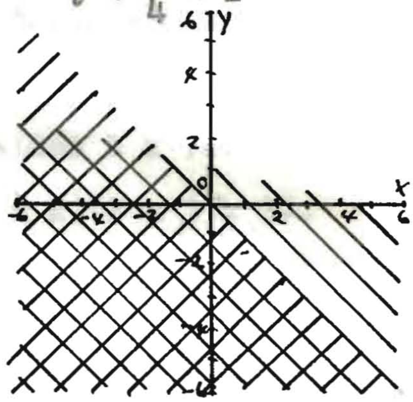


Exercises XIV (Continued)

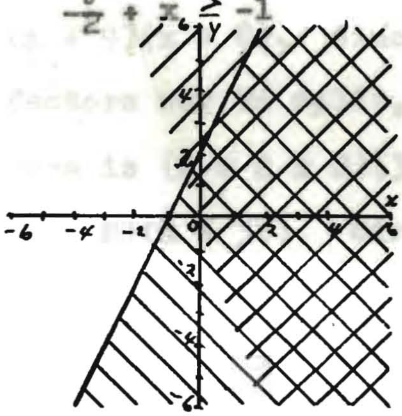
7. $y \le 3x$
   $5y - 2x \le 10$



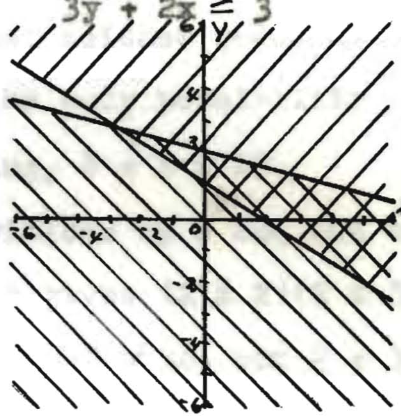
9. $y < -x$
   $y + \frac{x}{4} < 1$



8. $y > -3x$
   $\frac{-y}{2} + x \ge -1$



10. $\frac{1}{2}y + \frac{1}{7}x \le 1$
    $3y + 2x \ge 3$



Exercises XV

1. $(x - 36)(x + 2)$. The numerical term, $72 = (2 \times 2 \times 2)(3 \times 3)$. Since 34 is divisible by 2 but not by 3, the "2's" must be split and the "3's" must not. That gives two possibilities (besides 1×72) to consider, $(2)(2 \times 2 \times 3 \times 3)$ and $(2 \times 2)(2 \times 3 \times 3)$. Adding, $-36 + 2 = -34$.
2. $(x - 6)(x - 12)$. Since 18 is divisible by both 2 and 3, both groups must be split, giving $(2 \times 3)(2 \times 2 \times 3)$. Adding, $-6 - 12 = -18$.
3. $(x + 9)(x + 8)$. Since 17 is prime, neither group of factors may be split, therefore the only possibility here is $(2 \times 2 \times 2)(3 \times 3)$. Adding, $8 + 9 = 17$.
4. $(x + 2)(x + 36)$. Since 38 is divisible by 2 but not by 3, only the "2's" are split. This gives $(2 \times 2)(2 \times 3 \times 3)$ or $(2)(2 \times 2 \times 3 \times 3)$. Adding, $4 + 18 \neq 38$, but $2 + 36 = 38$.
5. Not factorable over the integers. Since 20 is divisible by 2 but not by 3, only the "2's" are split. That gives the same grouping as in No. 4, but neither of those groups add to 20, and $1 + 72 \neq 20$.
6. $(x + 12)(x + 3)$. Factoring, $36 = (2 \times 2)(3 \times 3)$. Since 15 is divisible by 3 but not by 2, the "3's" are split but not the "2's", giving $(2 \times 2 \times 3)(3)$. Adding, $12 + 3 = 15$.

Exercises XV (Continued)

7. $(x + 9)(x + 12)$. Factoring, $108 = (2 \times 2)(3 \times 3 \times 3)$. Since 3 is a factor of 21, but 2 is not, the "3's" must be split and the "2's" must stay together. Then 108 will either be factored $(2 \times 2 \times 3)(3 \times 3)$ or $(2 \times 2 \times 3 \times 3)(3)$. We find that $12 + 9 = 21$.

8. $(x + 6)(x + 18)$. Since 2 and 3 are both factors of 24, both groups of factors must be split, which makes $(2 \times 3)(2 \times 3 \times 3)$ the only combination to consider here. Adding, $6 + 18 = 24$.

9. $(x - 36)(x - 3)$. Since 3 is a factor of 39, but 2 is not, the "3's" must be split and the "2's" not split. Possibilities to consider are $(2 \times 2 \times 3 \times 3)(3)$ and $(2 \times 2 \times 3)(3 \times 3)$. Adding, $-36 - 3 = -39$.

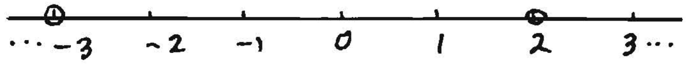
10. $(x - 36)(x + 3)$. The same reasoning applies here as in No. 9. Adding, $-36 + 3 = -33$.

11. $(x + 27)(x + 4)$. Since 31 is prime, neither group of factors may be split, so $(3 \times 3 \times 3)(2 \times 2)$ is the grouping to consider. Adding, $27 + 4 = 31$.

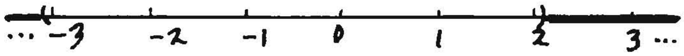
12. Not factorable over the integers. Since 25 is not divisible by 2 nor by 3, neither group of factors may be split. The only possible groupings to consider are $(3 \times 3 \times 3)(2 \times 2)$ and 1×108. But $27 + 4 \neq 25$, and $1 + 108 \neq 25$.

Exercises XVI

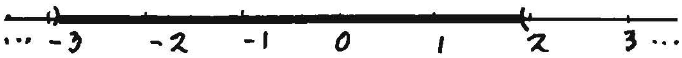
2. (a) $\{-3, 2\}$

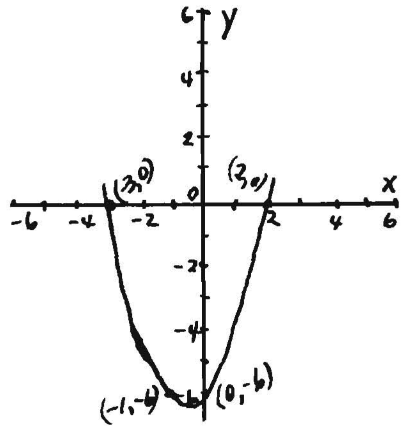


(b) $\{x \mid x < -3 \text{ or } x > 2\}$

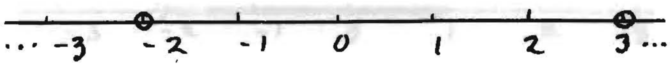


(c) $\{x \mid -3 < x < 2\}$

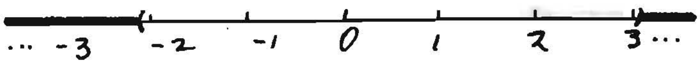




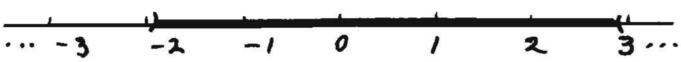
3. (a) $\{-2, 3\}$

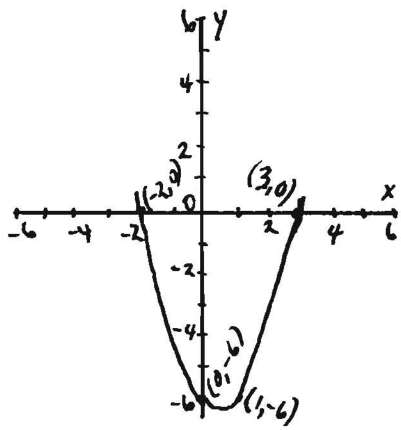


(b) $\{x \mid x < -2 \text{ or } x > 3\}$

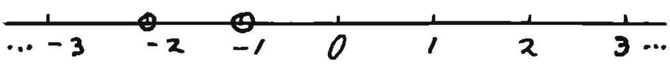


(c) $\{x \mid -2 < x < 3\}$

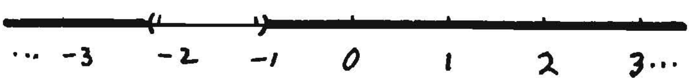


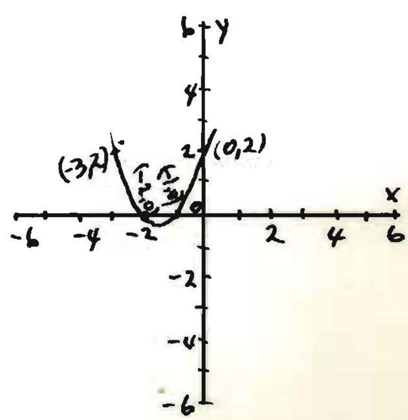


4. (a) $\{-2, -1\}$



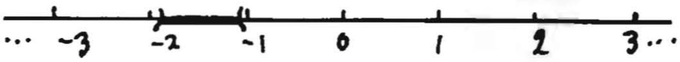
(b) $\{x \mid x < -2 \text{ or } x > -1\}$



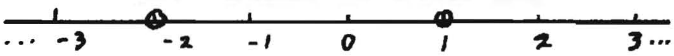


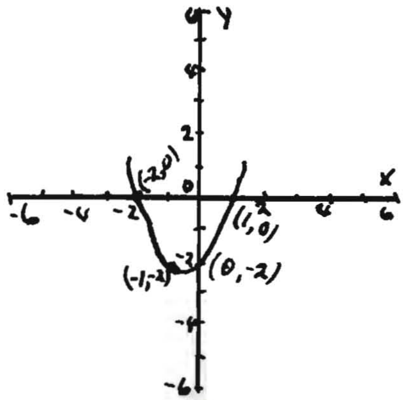
Exercises XVI (Continued)

4\. (c) $\{x \mid -2 < x < -1\}$

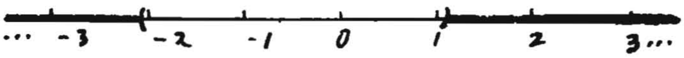


5\. (a) $\{-2, 1\}$



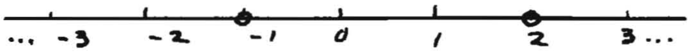


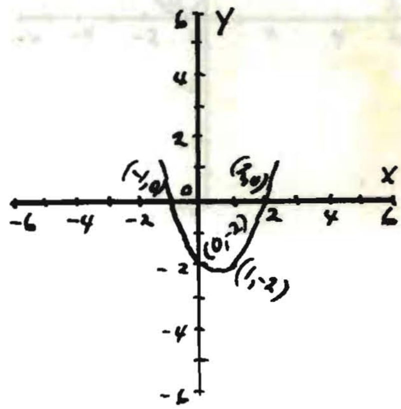
(b) $\{x \mid x < -2 \text{ or } x > 1\}$



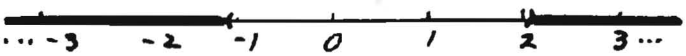
(c) $\{x \mid -2 < x < 1\}$

6\. (a) $\{-1, 2\}$



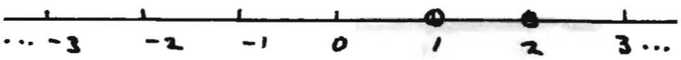


(b) $\{x \mid x < -1 \text{ or } x > 2\}$



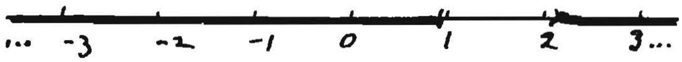
(c) $\{x \mid -1 < x < 2\}$

7\. (a) $\{1, 2\}$

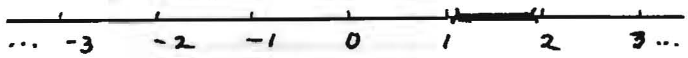


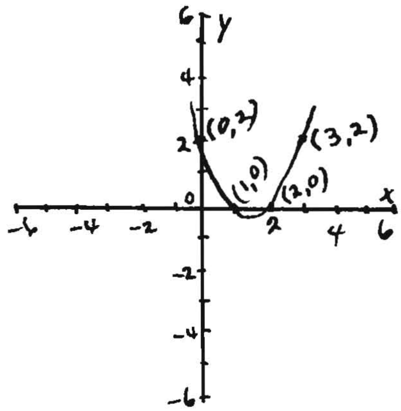
Exercises XVI (Continued)

7. (b) $\{x \mid x < 1 \text{ or } x > 2\}$

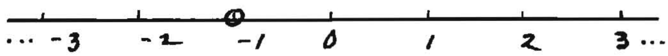


(c) $\{x \mid 1 < x < 2\}$



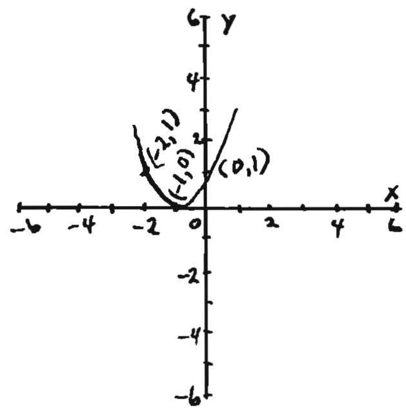


8. (a) $\{-1\}$

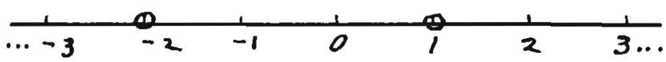


(b) $\{x \mid x < -1 \text{ or } x > -1\}$

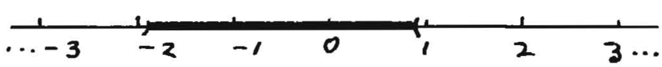
(c) ϕ



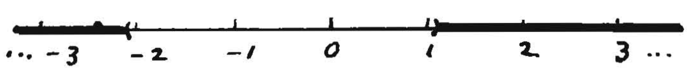
9. (a) $\{-2, 1\}$

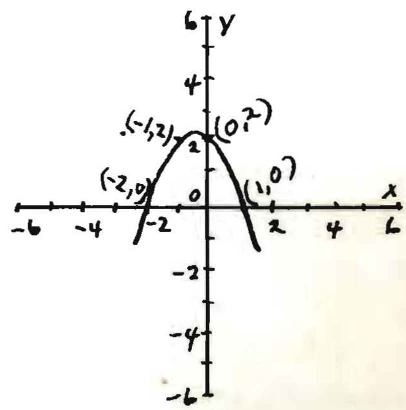


(b) $\{x \mid -2 < x < 1\}$



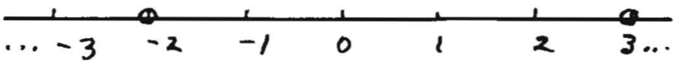
(c) $\{x \mid x < -2 \text{ or } x > 1\}$





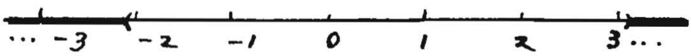
Exercises XVI (Continued)

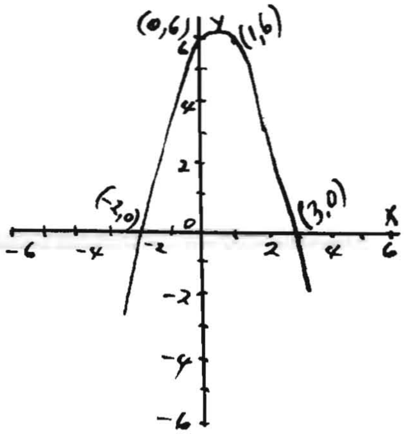
10. (a) $\{-2, 3\}$



(b) $\{x \mid -2 < x < 3\}$

(c) $\{x \mid x < -2 \text{ or } x > 3\}$





CHAPTER X

STATISTICAL DATA

TABLE V

SCORES ON COLVIN-SCHRAMMEL ALGEBRA TEST, TEST II, FORM A, AND I. Q. SCORES[1] FOR BOTH GROUPS

Experimental Group				Control Group			
Student Number	I.Q.	Test Score	Test Grade	Student Number	I.Q.	Test Score	Test Grade
1	94	26	D	26	94	32	C
2	101	32	C	27	102	35	C
3	122	36	C	28	123	39	C
4	123	30	C	29	121	45	C
5	79	39	C	30	84	38	C
6	99	33	C	31	98	31	C
7	123	45	C	32	125	40	C
8	91	42	C	33	94	32	C
9	95	34	C	34	95	25	D
10	96	33	C	35	98	41	C
11	97	33	C	36	97	24	D
12	122	45	C	37	120	45	C
13	94	48	B	38	88	35	C
14	97	38	C	39	95	24	D
15	106	36	C	40	103	38	C

[1]George K. Bennett, Harold G. Seashore and Alexander G. Wesman, Differential Aptitude Tests (New York: Psychological Corporation, 1947).

TABLE V (CONTINUED)

Experimental Group				Control Group			
Student Number	I.Q.	Test Score	Test Grade	Student Number	I.Q.	Test Score	Test Grade
16	116	57	A	41	119	50	B
17	106	35	C	42	104	42	C
18	115	35	C	43	111	34	C
19	128	57	A	44	128	46	C
20	136	40	C	45	139	63	A
21	88	39	C	46	92	30	C
22	97	40	C	47	97	31	C
23	99	32	C	48	95	37	C
24	98	40	C	49	100	47	B
25	103	51	B	50	103	50	B
Ave.	105	39.5		Ave.	105	38.0	
M_d	99	38.0		M_d	100	38.0	

Published standards: median, 37; standard deviation, 11.9; standard error of measurement, 4.3

NOTE: This table should be read as follows: Student No. 1, in the Experimental Group, who had an I. Q. score of 94, made a test score of 26, which was rated a D. Student No. 26, in the Control Group, who was paired with him, and who also had an I. Q. score of 94, made a test score of 32, which was rated a C.

TABLE VI

SCORES OF EXPERIMENTAL GROUP ON BRESLICH ALGEBRA SURVEY TEST, SECOND SEMESTER, FORM A

Student Number	Part I	II	III	IV	V	VI	Total Score	Test Grade
1	8	6	11	2	6	20	53	C
2	12½	6	15	6	3	25	67½	C
3	8	9	11	8	12	25	73	C
4	13½	8	17	4	9	31	82½	B
5	5	6	2	2	0	20	35	D
6	6	5	4	0	0	21	36	D
7	5½	8	9	4	6	18	50½	C
8	5	4	6	6	9	20	50	C
9	10	4	4	4	3	11	36	D
10	15	8	8	2	6	21	60	C
11	12	6	5	2	3	11	39	D
12	12½	10	10	8	9	9	58½	C
13	16	11	11	6	9	19	72	C
14	12	7	6	4	3	10	42	D
15	9½	8	9	4	9	21	60½	C
16	13	12	11	6	9	28	79	B
17	10	8	10	6	6	25	65	C
18	12	7	10	8	18	20	75	B
19	14	10	16	8	18	31	98	A

TABLE VI (CONTINUED)

Student Number	Part I	II	III	IV	V	VI	Total Score	Test Grade
20	15	13	28	10	12	30	108	A
21	7	7	14	2	3	12	45	D
22	10	9	13	2	3	16	53	C
23	11	8	9	4	3	23	58	C
24	12	11	11	6	6	28	74	C
25	16	13	23	8	12	30	102	A
Ave.	10.8	8.2	10.9	4.9	7.1	21.0	62.9	

Published median: 60.4. Median: 60.0

NOTE: This table should be read as follows: Student No. 1, of the Experimental Group, made a score of 8 on Part I of the Breslich Algebra Survey Test, 6 on Part II, and so on, making a total score of 53, which was rated a C.

Table VII should be similarly read.

TABLE VII

SCORES OF CONTROL GROUP ON BRESLICH ALGEBRA SURVEY TEST, SECOND SEMESTER, FORM A

Student Number	Part I	Part II	Part III	Part IV	Part V	Part VI	Total Score	Test Grade
26	9	8	10	6	6	15	54	C
27	6	6	6	2	9	9	38	D
28	14	13	12	4	9	20	72	C
29	11½	10	15	2	15	24	77½	B
30	6	7	8	0	9	8	38	D
31	5½	5	10	2	3	10	35½	D
32	10	9	8	8	9	27	71	C
33	5	7	10	4	6	19	51	C
34	7	3	2	0	0	5	17	F
35	9	10	6	4	3	18	50	C
36	7	7	2	0	3	8	27	F
37	11	12	14	4	6	25	72	C
38	10	11	14	6	6	17	64	C
39	5½	3	2	0	0	3	13½	F
40	8	8	15	4	6	19	60	C
41	8	10	24	6	9	23	80	B
42	9	12	20	8	9	15	73	C
43	8½	10	21	6	9	21	75½	B
44	16	10	18	6	18	28	96	A
45	14	12	21	12	21	30	110	A

TABLE VII (CONTINUED)

Student Number	Part						Total Score	Test Grade
	I	II	III	IV	V	VI		
46	3	5	9	6	9	21	53	C
47	3½	11	10	2	6	18	50½	C
48	7½	8	8	2	6	22	53½	C
49	14	13	7	6	6	17	63	C
50	16	13	19	10	24	28	110	A
Ave.	8.9	8.9	11.6	4.5	8.3	18.0	60.2	

Published median: 60.4. Median: 63.0

CHAPTER XI

CONCLUSIONS

Objective evaluation. The test score averages and medians for the two groups were remarkably close, as is shown in the following table:

TABLE VIII

MEDIANS AND AVERAGES OF TOTAL TEST SCORES

Test	Published Median	Experimental Group Average	Experimental Group Median	Control Group Average	Control Group Median
Colvin-Schrammel	37.0	39.5	38.0	38.0	38.0
Breslich	60.4	62.9	60.0	60.2	63.0

To determine whether there was any significant difference in the performance of the two groups the corresponding averages of the scores on each test were compared statistically.[1] The standard deviation, σ, of each set of scores given in Tables V, VI, and VII in Chapter X was found by the formula,

$$\sigma = i\sqrt{\frac{\sum_{j=1}^{n} f_j d_j^2}{N} - c^2},$$

[1]Cecil B. Read, Manual of Statistics (Wichita, Kansas: University of Wichita, 1940), pp. 116-125. (Mimeographed.); and Albert Waugh, Elements of Statistical Method (New York; McGraw-Hill Book Company, Inc., 1943), pp. 250-252.

where i = the size of class interval,

d_j = the deviations from an arbitrary mean,

f_j = the frequencies,

N = the number of cases,

and c = the correction $\frac{\sum_{j=1}^{n} f_j d_j}{N}$ needed when using an arbitrary starting point for computing deviations.

The standard error of the mean, σ_M, (the amount it might vary due to sampling) was found by the formula,

$$\sigma_M = \frac{\sigma}{\sqrt{N}}.$$

The final comparison between corresponding averages, M_1 and M_2, was made by finding the ratio of their difference to the standard error of the difference, $\sigma_{M_1 - M_2}$, by the formula,

$$\frac{M_1 - M_2}{\sigma_{M_1 - M_2}} = \frac{M_1 - M_2}{\sqrt{(\sigma_{M_1})^2 + (\sigma_{M_2})^2}}.$$

Results are shown in Table IX.

TABLE IX

STATISTICAL COMPARISON OF TEST AVERAGES

Average,	M	σ	σ_M	$\sqrt{(\sigma_{M_1})^2+(\sigma_{M_2})^2}$	$\frac{M_1-M_2}{\sqrt{(\sigma_{M_1})^2+(\sigma_{M_2})^2}}$
Colvin-Schrammel					
Experimental Group	39.5	7.90	1.58	2.52	$\frac{1.5}{2.52}$ = .60
Control Group	38.0	9.85	1.97		
Experimental Group	39.5	7.90	1.58	4.58	$\frac{2.5}{4.58}$ = .54
*Standard for Test	37.0	11.9	4.3		
Control Group	38.0	9.85	1.95	4.72	$\frac{1.0}{4.72}$ = .21
*Standard for Test	37.0	11.9	4.3		
Breslich, Total Scores					
Experimental Group	62.9	19.99	3.99	6.51	$\frac{2.7}{6.51}$ = .41
Control Group	60.2	25.70	5.14		
Part I					
Experimental Group	10.8	3.25	.65	.96	$\frac{1.8}{.96}$ =1.87
Control Group	8.9	3.55	.71		
Part II					
Experimental Group	8.2	2.68	.54	.60	$\frac{.7}{.60}$ =1.17
Control Group	8.9	1.37	.27		
Part III					
Experimental Group	10.9	5.85	1.17	1.62	$\frac{.7}{1.62}$ = .43
Control Group	11.6	5.65	1.13		

TABLE IX (CONTINUED)

Average*	M	σ	σ_M	$\sqrt{(\sigma_{M_1})^2+(\sigma_{M_2})^2}$	$\frac{M_1 - M_2}{\sqrt{(\sigma_{M_1})^2+(\sigma_{M_2})^2}}$
Part IV					
Experimental Group	4.9	2.86	.57	.79	$\frac{.4}{.79}$ = .51
Control Group	4.5	2.76	.55		
Part V					
Experimental Group	7.1	4.71	.94	1.20	$\frac{1.20}{1.20}$ =1.00
Control Group	8.3	3.99	1.20		
Part VI					
Experimental Group	21.0	6.70	1.34	1.97	$\frac{3.0}{1.97}$ =1.52
Control Group	18.0	7.20	1.44		

*Since 11,246 cases were used in standardizing the test, it was assumed that the distribution was normal, and therefore, that the arithmetic mean was approximately the same as the median.

The range of three standard errors includes all but one-fourth of one per cent of all the possible cases, therefore, any ratio greater than three is considered as indicating that the difference in the means could not have happened by chance variation due to sampling. None of the comparisons in Table IX gave a ratio greater than three, so none of the differences can be considered statistically significant.

Indications. In the experimental group more than one-fifth of the total teaching time was spent on supplementary materials. Since that resulted in no significant difference in performance on a test covering the traditional materials, the group gained by whatever amount they learned from the supplementary materials.

Subjective evaluation. The responses of the students to the question, "How do you like Algebra I?" are tabulated in Table X.

TABLE X

STUDENTS' LIKING FOR ALGEBRA I

Group	Disliked very much	Liked less than ave.	Ave.	Liked more than ave.	Liked very much
Experimental	0	3	2	12	8
Control	2	3	5	9	6

A better attitude was evidenced in the experimental group by an almost total lack of complaints. They wanted to do more graphs than time permitted. When solving quadratic equations or pairs of linear equations, they found the total solution. They seemed to have a much better than usual understanding of the dependence of one variable upon another.

One unexpected benefit was a transfer into the field of English. For years attempts have been made to get grammatically correct definitions in mathematics without much success. Therefore, it was especially gratifying to find that a mathematical reason for good grammar helped improve definitions in the English class.

<u>Suggestions for further study</u>. Division using numerals in bases other than ten proved to be too difficult for a large portion of the group. Since these same students had trouble dividing in base ten, there is a definite need for a good presentation in that area. Supplementary material on approximation and on irrationals needs to be included, also.

It should be interesting to make a follow-up study of the success in geometry of students who have been taught algebra by this approach.

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