MATHEMATICS CLUBS IN THE HIGH SCHOOL

A Thesis

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by

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CHAPTER I

INTRODUCTION

Introduction. Painful as it must seem to many, traditional philosophies of education existing in any country are difficult to change, in particular over relatively short periods of time. Thus, all the strenuous efforts toward achieving better preparation and a greater challenge for our more gifted students within the classes of our high schools are so contrary to our basic philosophy of equal opportunity for all, that significant progress of this worth-while goal is slow. Consequently help toward making some vitally needed improvement toward this end must be looked for outside the traditional classroom lectures.

Statement of problem. The purpose of this study is to determine the status of mathematics clubs in the general education programs of the public high schools of Kansas. An attempt has been made to get as accurate an analysis as possible of the mathematics clubs and what they contribute to the general education objectives of our youth. The study has been concerned as much with terminal programs as with the programs of students who plan to continue their work on the college level, or the university level. It is the outgrowth of individual and group studies that are both local and national in scope. An attempt has been made to arrive at a program designed to fulfill the needs of the high school students.

In the search for schools that had a mathematics club, it was found that there was a wide variation in the offerings in mathematics among the high schools of Kansas. This was in part due to the size of the school, and also because some schools still believe in the old mathematics, and have not revised their curriculum to fit the present day situations. Some schools included the mathematics club; others included a mathematics club combined with a science club; others did not offer a mathematics club in their school; and others were planning a club for the future.

Importance of study. A casual acquaintance with the literature on the subject and the schools with or without mathematics clubs, gives the impression that there is considerable disagreement among authorities on whether mathematics clubs should be organized in high schools, and whether they are useful to the mathematics students.

Because of the discrepancies that are apparent, a study of the problem seems to be urgently needed. Such a study, if successful, would be very useful to those concerned.

Data used in study. The data used in this study were collected by personal interviews with the instructors of mathematics, and by research. A questionnaire was sent out to noted mathematics instructors throughout the United States who could conceivably contribute to the program. A questionnaire was used as a guide for the interviews. It was designed to provide as complete information as possible. Some of the more important items were as follows: Size of school, presence or absence of a mathematics club, reason for no club, qualification of members, projects undertaken, courses taught in school, how the gifted pupils are kept interested in mathematics, and the purposes of the mathematics club. It was felt that by making personal interviews the information gained would be more reliable because the questions could be asked in such a way that each instructor would be interpreting the questions as intended by the interviewer. A copy of the questionnaire is included in the appendix of this study.

Difficulties encountered. The three items, "why don't you have a mathematics club, what courses are taught, and how are the gifted taken care of," were the most troublesome from the standpoint of the instructor's thinking. It was felt that the responses given to these items in an interview would be more reliable because they were more nearly a reflection of the instructor's thinking than would otherwise be the case. Another problem that arose was the fact that the questionnaire was designed to fit all schools. In trying to gather information on all classes of schools, it was found that some questions on the questionnaire could be answered differently by the large or small school.

__Organization of study.__ The thesis is divided into chapters, each of which is devoted to some aspect to the study which is of major interest. Chapter I is the introduction, in which the desirability of the thesis is indicated, Chapter II is the study of the objectives as stated by nationally known specialists in mathematics, Chapter III deals with the need for a mathematics club, Chapter IV consists of the programs that can be used for a mathematics club, Chapter V is the survey of schools in Kansas, and Chapter VI is the summary and conclusion, followed by Bibliography and appendices.

CHAPTER II

OBJECTIVES AS STATED BY NATIONAL SPECIALISTS

Many worth-while objectives have been contributed by individual writers on mathematics clubs. The literature is so voluminous, however, that it is beyond the scope of this study to make an exhaustive survey. Therefore, it is the purpose of this section to present a representative sampling of the contributions by these writers.

George R. Hunt, Mathematics Department, Odessa College, Odessa, Texas. Mr. Hunt presents the following as prime objectives of mathematics clubs in high schools:

1. To bring together for mutual acquaintance those students who have a special interest in mathematics.

2. To provide means for these students to become better acquainted with the literature in the field of mathematics, both ancient and modern.

3. To provide opportunity for these students to hear, meet and associate with professional people in the field of mathematics.

4. To provide opportunity for club members to meet with club members of other areas.

Organizations carrying out these objectives will develop in the students a feeling of unity and companionship in a field of activity. It will provide an opportunity to develop an appreciation of the role mathematics has played in the development of civilization.

All these objectives and accomplishments are met somewhat through ordinary classes if the teachers are interested and able to carry them out, but at best classes are limited in this respect. Clubs offer a greater opportunity to broaden the students perspective.

Mu Alpha Theta, the National High School and Junior College Mathematics Club, which was organized in February of 1957 is carrying out these objectives quite well. The organization has almost 400 chapters. It has a traveling library with about 500 volumes. It was circulated this past winter among about 73 chapters. The organization in California and in some other localities has maintained lecture circuits for the chapters. The lectures are presented by professors from universities and other career men in the field of mathematics.

There has been in the last few years a revival of interest in mathematics clubs, but in my opinion, they have not yet won the respect and encouragement from the schools that other activities enjoy.[1]

Dr. Albert E. Meder, Jr., Professor of Mathematics at Rutgers University, New Brunswick, New Jersey. Dr. Meder lists the following as objectives:

1. To stimulate interest in mathematics.
2. To encourage independent work by pupils.
3. To provide an opportunity for enrichment of course content (by discussion of topics omitted or treated only briefly in regular class work).
4. To provide opportunities to become acquainted with the applications of mathematics.
5. To provide opportunities to become acquainted with History of Mathematics.
6. To provide opportunities to become acquainted with mathematical games.[2]

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[1]George R. Hunt, of Odessa College, in private communication, July, 1959.

[2]Albert E. Meder, Jr., of Rutgers University, in private communication, January, 1960.

Virginia Lee Pratt, Instructor at Central High School, Omaha, Nebraska, has the following objectives of her mathematics club:

> As I see it such clubs are organized to provide added stimulus to scholarship in the subject and for enjoyment and further understanding of it. They offer an opportunity to enrich the mathematics program and to improve teacher pupil relationships. Healthy competition is possible without the pressure of grades. If handled properly a mathematics club can so recognize outstanding achievement in the subject that greater respect and prestige is gained for such. I think the greatest value is realized if time and facilities are provided for maximum student planning and participation, but the use of community leaders in the field as speakers has a place also.[3]

Irvin H. Brune, Professor of Mathematics, Iowa State Teachers College, Cedar Falls, Iowa. In stating conclusions he has observed from many years of teaching experience, Mr. Brune offered the following recommendations for the content and purpose of a mathematics club:

> The chief purpose of such a club, I think, should be to afford additional opportunities to pupils to investigate mathematics. Fascination with the subject is a pupil's main reason for studying it. He may be interested in its applications, of course, but liking the subjects motivates his study of it.
>
> This need not imply exalting the theme, "Mathematics is Fun." All too often we mislead pupils toward the belief that life is a sweet song to be sung whenever, and only whenever, we feel the urge. There are, on the other hand, elusive ideas, close reasonings, and stern disciplines to be pursued. At times even drudgery

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[3]Virginia Lee Pratt, of Central High School, in private communication, July, 1959.

appears. But, through it all, mathematics intrigues those who stay with it. A mathematics club, therefore, should bring genuine satisfactions. It should not be just another social club; pupils should not attend merely because refreshments highlight the meetings. Social pleasantries, of course, have their place, but a good mathematics club can prosper either with or without them.

With this in mind we should, I believe, seek to augment, supplement, and enhance the regular curricular offerings in mathematics. The club can hardly become the dependable vehicle for all that pleases people in mathematics. All too often pupils who should attend every meeting can not; other activities compete for the spare time on which the club subsists. But a good club can enlarge the learnings a good day-by-day program provides.

Accordingly, the mathematics club probably should provide pupils with opportunities to hear guests--lecturers, engineers, operations analysts, computers, scientists. Whether a professor from a nearby campus, a professional mathematician from a neighboring industry, or a pupil from one's own school, the speaker should show the members how mathematics brings satisfactions to people who do things. The high-school pupil who needs to read extra mathematics to prosecute his hobby in electronics; the thinker in industry who needs to tackle problems involving hundreds of variables; the teacher who traces the implications of a few unusual, but simple assumptions; all these support but even supplant, the work a good teacher does.

But pupils in the club should not be passive. They should work on projects of their own. As individuals and as members of teams they should investigate, experiment, and report. Even as listeners the members presumably are thinking things through. Doubtless other objectives also merit consideration. In the name of simplicity, though, and directness, the single aim of satisfaction via mathematics seems paramount.[4]

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[4]Irvin H. Brune, of Iowa State Teachers College, in private communication, August, 1959.

Nellie M. Kitchens, Governor of National High School and Junior College Mathematics Club, Mu Alpha Theta. Miss Kitchens states the following objectives for a mathematics club:

> To stimulate a deeper and more effective interest in mathematics and at the same time to promote enjoyment of mathematics.
>
> To give instruction in the deeper meanings and the historical importance and power of mathematics.
>
> To help those with great potential to become eager to make maximum growth.
>
> To provide more opportunities to develop the tremendous inventive genius that Americans have.
>
> To keep genius busy--Genius is 90% industry.
>
> To bring pleasure and honor to the school by having a group who will work to develop their potentialities in the field of mathematics.
>
> To help teachers to hold attention of the students. (We become tired, worn out, from the battle for attention--so we lose our potential.)
>
> To provide group cooperative activity--inspiring others.
>
> To give a rewarded feeling of accomplishment.
>
> To help the student understand mathematics as a continuing creative endeavor with an aesthetic value; making clear that it is a living subject.
>
> To supplement the content of regular courses that are often too thin for the ablest student.[5]

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[5]Nellie M. Kitchens, of Hickman High School, Columbia, Missouri, in private communication, September, 1959.

W. R. Krickenberger, Instructor, The Arsenal Technical High School, Indianapolis, Indiana. Mr. Krickenberger gives the following reasons for a mathematics club:

> I believe the objectives of a club should depend, to a certain degree upon the location of the high school and the general type of the schools' pupils. Without being specific, I would suggest the following objectives: (1) to arouse and maintain a greater interest in mathematics, (2) to learn more about the history of mathematics and the great mathematicians, (3) to create a closer fellowship among the mathematics students, (4) to influence more students in the applications of mathematics, (5) to undertake projects which will benefit the school as a whole.
>
> These are a few objectives which could be used in most schools. Of course these overlap each other in some cases. Any group of students with good leadership can be of great benefit to a school and community.
>
> I wish to give a proverb clothed in poor language but which I find very useful. It is: If you never do more than you are paid for, you will never be paid for more than you work for.[6]

Karl R. Douglas, Director of the Graduate School, Colorado University, Boulder, Colorado, states the following are the main objectives of a mathematics club:

> To create interest in the study of mathematics.
> To help understand uses of mathematics.
> To provide opportunity for socialization training.
> To help sponsor get to know students better individually.[7]

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[6] W. R. Krickenberger, of The Arsenal Technical High School, in personal communication, December, 1959.

[7] Karl R. Douglas, of Colorado University, in private communication, February, 1960.

Samuel I. Jones, Author of books on mathematical recreations, states the following as purposes of the mathematics club:

> The purpose of the mathematics club is manyfold, (1) to promote interest in the study of mathematics, (2) to bring together kindred spirits, bound by an appreciation of the beauties and significance of mathematics, (3) to give pupils glimpses of the future, which serve as incentives to continue the study, (4) to afford opportunity for discussing the many interesting features of the various mathematical subjects; (5) to furnish an outlet for their social instincts, (6) to illuminate the by-paths of mathematics; to study certain interesting matters connected with mathematics which do not find a place in the usual classroom, (7) to develop an appreciation for the truth and beauty in mathematics and our dependence upon it in practical life, and (8) to inspire the members--the future teachers--with the nobler phases of the subject enabling them to turn to inspire the coming generation.[8]

F. Lynwood Wren, Professor of Mathematics, George Peabody College, Nashville, Tennessee, states that the objectives of a mathematics club:

> I think that a High School Mathematics Club should attempt to provide opportunities for the more capable students to take excursions into the more fundamental aspects of mathematics. They should have strong encouragement to enter into individual research and independent investigations. There should be occasional relaxation projects which would offer opportunities to see some of the lighter side of mathematics.[9]

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[8]Samuel I. Jones, Mathematical Wrinkles (Nashville: Samuel I Jones Publisher, 1930), p. 309.

[9]F. Lynwood Wren, of George Peabody College, in private communication, July, 1960.

Dr. Harry Lewis, Chairman, Mathematics Department, East Side High School, Newark, New Jersey. Dr. Lewis gives the following views on mathematics clubs in the high schools:

> It is my belief that topics that are taught or touched upon in any area of the secondary school mathematics program should not be considered by the members of a mathematics club. Among these I would list such things as: (1) the teaching of the use of the slide rule, (2) permutations and combinations, (3) probability, (4) the use of the transit.
>
> On the other hand a mathematics club can be used to promote interest in the subject. I believe that meetings should be devoted to: (1) guest speakers such as: engineer, actuary, physicst, chemist, and a person familiar with computors, (2) visits to local planatariums, air craft plants, insurance companies, engineering schools, teacher's colleges, etc., (3) construction of mathematical models, such as: construction of curves by plotting real values of x against imaginary values of y; or generating curves through a series of straight lines; or construction of nomographs, (4) learning the operation of calculators, (5) reports on topics relating to the history of mathematics, (6) compiling a mathematics bulletin, magazine, or paper.
>
> You may have noticed that in the topics that I have included, I have carefully avoided to cite topics that find their way into the mathematics classroom.
>
> Frankly, I am thoroughly convinced that a mathematics club, as such, has little value in stimulating interest in the continued study of mathematics by the student. The real burden for motivating interest lies with the classroom teacher. He will either inspire a student to creativity in mathematics or make the field so distasteful that the weekly or bi-weekly mathematics club meetings could not possibly undo the harm.[10]

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[10] Harry Lewis, of East Side High School, in private communication, December, 1959.

Mary Caroline Hathon, Instructor, states the aims of a mathematics club:

> To open up possibilities in the field of mathematics for a worthy use of leisure time. To create an interest in the practical application of mathematics as the specialist uses it to-day. To acquaint the student with the scholarly work that has been done in the development of mathematics, and to stimulate admiration for the scientists in this field and their work.[11]

Dr. Harold Fawcett, Professor, Ohio State University, Columbus, Ohio. Dr. Fawcett reveals the following on mathematics clubs:

> It is my judgment that mathematics clubs provide a very real stimulus to those students who participate in them. In such a program it is possible to engage in many interesting activities which limited classroom time makes impossible. I know of actual cases where the program of a mathematics club has brought to life latent mathematical potentialities in a number of students.[12]

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[11] Mary Caroline Hathon, "A Mathematics Club," The Mathematics Teacher, XX (January, 1927), 39.

[12] Harold Fawcett, of Ohio State University, in private communication, August, 1960.

CHAPTER III

THE NEED OF A MATHEMATICS CLUB

Probably every teacher of secondary mathematics feels that one of his important problems is that of arousing and holding the interest of his pupils in mathematical work. The gifted child presents a definite challenge to any teacher who is concerned about the welfare of each individual pupil. In the mathematics classes there is a great need to give more attention to the gifted pupil, to see that his curiosity and his interests are stimulated and directed, and to help him choose the work for which his talents best fit him and which will enable him to render the greatest service to society. In the elective work of the third and fourth years, one may expect to find some degree of interest on the part of the pupil, but in algebra or geometry, one frequently encounters the pupil who has "no head for mathematics," who announces cheerfully, and honestly believes, that he has always found the subject difficult, and seems reconciled to the fact that he is invulnerable to any kind of mathematical instruction.

In many high schools the courses of study are so over crowded with formal work therefore much exercise in topics of interest to the pupils is prevented. It is largely because of the failure of teachers to provide opportunity for exploring and expanding these mysterious byways that many of the high school pupils regard mathematics as a necessary evil, or as an affliction to be escaped as soon as the minimum requirement has been met. To solve this problem the teacher must make mathematics interesting to the pupil.

Little has been done in many of our schools to make the pupil see the pleasure and cultural value of mathematics. Mathematical recreations and diversions are a legitimate and profitable expenditure of time if, without diluting the instruction, they create such an interest that the hard work, which is connected with any course that is worth-while, does not seem like drudgery, for while valuable discipline may be developed by doing unpleasant tasks, the pupil who attacks his problems in a cheerful and interested frame of mind has a great advantage in accomplishing work and overcoming obstacles.

In addition to the problem or arousing the full or indifferent pupil from his lethargy, there is the difficulty to keeping the brighter and more original pupils working at concert pitch, so while new interest is being created, care must be taken to preserve that which already existed. While more intensive work on the subject in hand may be assigned for extra credit to these more ambitious pupils and other devices may be used to retain their interest, still it is a lamentable fact that the amount of uniformity necessary in classroom work often makes it difficult to bring out the capacity of the individual pupil. Yet teachers owe it to the superior pupil to hold his interest, to stimulate his enthusiasm, and to open up to him new fields of thought. Inspiring the student to the development of these objects may be accomplished in class, but every enthusiastic teacher of mathematics will feel that his opportunities for doing so are much too limited.

Too often the pupil feels that the class and the teacher are too busy with the work at hand to be side-tracked with some question not primarily on the subject. And too seldom does a pupil take the initiative to approach the teacher with his questions after class, so they remain unanswered. Herein lies an important challenge to the mathematics teachers to find a time and a way to stimulate this, building interest and curiosity rather than let them die a gradual but certain death.

Here a mathematics club can be of invaluable assistance in motivating the pupil's curiosity and imagination in mathematics. The mathematics club is at least a partial solution of this difficulty. In any such a club it is possible to supplement the work of the classroom by offering opportunities that the class does not afford, to add to the pupil's background of mathematical knowledge, to inspire pupils to original work, and to lead them to realize that the subject is rich in interest and expansion. Boys and girls who become so interested in the classroom work that they voluntarily remain after school for an hour or more to discuss subjects which have been suggested, and who are constantly reaching out for more information, will be glad of an opportunity to meet regularly for more or less informal discussion, and the work of such a club reacts favorable on the attitude toward mathematics throughout the school.

This is one organization where time can be allotted to create in the mind of each club member a sense of appreciation for engineering and scientific applications of mathematical facts; for re-emphasizing fundamental concepts that up to this time have had little meaning to the individual; and for instilling a feeling that will impel him to investigate unique applications whenever or wherever they arise. It can bring together the elements of physics, chemistry, and mathematics, and to demonstrate forcefully the interrelationship of these subjects.

Types of a mathematics club. Every mathematics club should be organized to assist its members in learning mathematics. The mathematics club has become the proving ground for the discovery and cultivation of skills and talents based on mathematics; a place to prepare for careers and hobbies in mathematics.

Probably the most familiar to teachers is the popular type of mathematics club wherein all pupils who so desire are welcomed as members. There isn't any limit to size or qualification of members. Due to the heterogeneity of talent which is embraced by such a plan, the group activities are in most cases limited to such things as mathematical recreations, puzzles, plays, simple application, and some simple projects. The club has the advantage of appealing to large numbers, but also has the disadvantage of prohibiting discussions of an advanced nature, therefore, the brighter students are again neglected.

Another type of mathematics club is the one that limits its membership, but permits almost complete freedom of activity. A mathematics club of this type is now being put into operation in the Kingman High School, and although sponsored by staff members, it is student controlled. The club constitution limits membership to a small number of students. Admission requirements are that a student must have taken a year of mathematics and that he plans to take another. The student who belongs to the club also gives programs to other classes. Therefore, the knowledge attained in the club is also spread throughout the school. The student receives recognition and works that much harder to receive more recognition. These students are also used to present different topics on mathematics to the social clubs of the community. This has proved a very valuable public relations instrument between school and community.

Organization of a mathematics club. The teacher who has not had experience with mathematics clubs, but wishes to organize one may hesitate because he is uncertain as to how to go about it or will it be accepted. The essential steps used to organize a club are: First, talk with the principal, discuss the benefits that the club could offer, and ask for his approval. Be sure to have everything all planned and an outline of the plans before going to the principal. Second, talk with other mathematics teachers of the school and enlist their approval, ideas, and help. Third, talk with a few good pupils who are interested in mathematics either in class groups or individually, and tell them briefly what a club could accomplish. Discuss the matter more than once so that all are clear as to the work to be done and the responsibilities to be carried as well as the benefits to be expected of a club. Be sure of the active interest of a number of good pupils before taking further steps. Fourth, have other teachers present the matter in their classes that have eligible pupils and analize the results. Have them talk with recommended pupils individually or in groups. Fifth, call an organization meeting, inviting only those who are eligible according to the standards the sponsor, principal, students, and other mathematics teachers have decided should be held. Sixth, start the program with aggressive initiative, so that it will not falter along the first few meetings and then die out because of lack of interest. The first few meetings are the most crucial.

At the first meeting outline the general plan of organization that the sponsor, other teachers, and pupils have decided would be best, and once more speak of the work to be done by the club in the future. It would be well to hand out a list of subjects that would constitute suitable material for club programs, so that pupils get a clear realization of the possible interest and profit to be expected from participation. The mathematics club should try to bridge the gap between textbook study and actual application. The club should try and avoid dry old programs, and recognize the necessity for making provisions for the systematic conduct of club business and program meetings.

In consideration of the matter and in discussions with the principal and teachers, a conclusion should have been reached as to what responsibilities the pupils are to carry and what responsibilities are to be carried by the sponsor. In case the decision has been reached for the sponsor to carry major responsibility for all details, then the pupils should be told of this. If the decision has been reached that pupils are to write and adopt a constitution and bylaws and elect their own officers, then the next step is to have the pupils elect a president pro tem, and appoint a committee to take steps to provide the instruments for organization. This should be enough of a program for the first meeting.

The temporary president and the committee work to write the instruments of the club. The sponsor will, of course, work with these pupils at every step of their activity, but this is the only time the sponsor will be of great help to the pupils. In the future the pupils will do most of the work and depend upon the sponsor only for limited help.

After the constitution and bylaws have been written, a second meeting of the pupils is called by the temporary president, and the chairman of the writing committee reads the proposed constitution. There will follow the usual discussion and debates for or against the provisions of the constitution. After the constitution has been revised and adopted, and election of officers is held, the club is under way.

Basic principles of club organization. In the organization of the mathematics club, the following principles were used and a very successful club has resulted.

The club should be based on definite and worthy objectives, which have room for expansion. No school club should ever be allowed to exist if its aims and purposes are not definite. Furthermore, these purposes should be worthy and plainly stated in words so everyone in the club can understand them.

The purposes and activities of the mathematics club should be those of its pupil members, who take an active part in the club. Very frequently the club attempts to do the work of adults by imitating adult organizations. This is a student organization and should operate on the student level. Sometimes the teacher so dominates it that it cannot do otherwise. The club should exist for the education of the pupil members and not for the teacher sponsor, and consequently its program must be interesting, appropriate, and valuable to its pupil members.

Wherever possible mathematics club activities should grow out of curricular activities, or activities associated with the curricular activities. The main material of education in the school is its so-called curricular work, and therefore, the club has values in motivating and enriching this work. The club should expand on the curricular work which is taught in the classroom or associated work. Applications of the class work are excellent.

Proper balance should be preserved between the mathematics club activities and the regular activities of the school. The mathematics club program, while important, is not the only important part of the school program and should receive only a proper share of school time and attention. Therefore, meetings can be arranged after school. In following this principle the sponsor will eliminate the friction that could exist between teachers and clubs.

The club program must fit the local situation to succeed. The program which fits one school does not necessarily fit another school. No club methods or program should be taken from another school and adopted wholly. Differences in size and type of school, in background and club experience of pupils and of teachers, in ability and general attitude of the staff, and in many other items must be considered.

Mathematics club membership should be voluntary. No pupil should feel that he is forced to join the club. Such forcing will only kill interest and spontaneity because it would dilute existing interest within the club. No pupil should be allowed to waste his time by loafing either within a club or outside of it. The program should be based upon interest of its programs and not compulsion. A few interested students is more worth-while than a group of uninterested but bright students.

Provision should be made for recognizing and capitalizing individual abilities. In any school will be found a variety of abilities, capacities, and interests, and for their proper capitalization there must be a complementary variety of educational possibilities. Club membership should be kept small in order that variety in possibility and variety in participation may be obtained. This variety should be based upon pupil interests rather than upon the interests represented by the faculty.

In the mathematics club there should be no excessive dues, fees, assessments, or similar restrictions on membership. One aim of many clubs is exclusiveness and a usual method of attaining this aim is the establishment of high entrance fees or yearly dues. If dues are necessary they should be very small, so small that they will never bar pupils from membership in the club. It is best not to have dues. Any expenses that are encountered by the club should be taken out of the departments fund, or donations can be accepted. Both of these methods have been used very successfully. Other requirements, such as high average in school work should not be allowed except, of course, in the case of the Honor Club. All pupils who are interested in mathematics and are willing to learn and work should be allowed to join. Of course, they should be taking mathematics and be among the better students, but not necessarily the gifted students.

The mathematics club should be limited in size. A large club defeats its own purpose because it offers too few opportunities for direct participation and becomes a convenient place for loafers. No one can say exactly just what the limit of club membership should be, because it should depend upon its activities, frequency of meetings, and experience and general ability of the sponsor. In most cases it should be small enough to permit all members to participate freely in its activities during the school year.

All members of the club should participate in its activities. It should be a club rule that any one who benefits by what other members of the club do, should be expected to contribute his share to the clubs life and activities through its program. No pupil should be forced to participate against his will any more than he should be forced to join a club against his will. On the other hand a loafer should not be tolerated. If a pupil refuses to cooperate, he should be dropped from the club membership. The club should have pride in its membership and therefore loafers cannot be tolerated.

The mathematics club should not be considered vocational in purpose. The activities of most school clubs have important vocational aspects, and these should be suggested in appropriate meetings of the club. However, the main purpose of the club should be that of widening, deepening, and broadening the interests of the members, and not the giving of complete information about the vocations represented. This should be pushed into the background and only drawn out on occasion through club programs.

Whenever possible the mathematics club should be scheduled on regular school time. If a club program has important educational values for the pupil member on his own time, it is valuable enough to be included in his regular school schedule, or right before or after school. Scheduling clubs on school time will mean that both the teacher and the pupil will have better attitudes towards them; it will dignify the activity; and it will require high returns for the time invested. Other values are that it will insure full attendance of its membership; provide for regularity and consistency; and lift the program to the level of importance of the so-called regular work of the school. It should be placed on the school calendar for the entire year, so faculty and students can plan ahead.

Whenever possible, club meetings should be held on school premises. This is not always possible, or even desirable, in the case of trips and visits to places other than the school. In general the club meetings should be held in the school building where they can easily be controlled; where responsibility is not divided; and where the school and its interests can be definitely safeguarded.

Club sponsors should be carefully chosen and assigned. Probably nothing will kill a club more quickly than an uninterested, unsympathetic or unprepared sponsor. Mere membership in a department is not sufficient warrant for appointment to the club. A careful consideration of interests, preparation, capabilities, personal qualifications, and pupil's like and dislikes should be considered.

The sponsor should be a counselor and not a dictator. In spite of the fact that the sponsor is the most potent force in the functioning of a good club, it is a mistake to have the club run by the sponsor. The main function of the sponsor should be that of counselor, a supplier of judgement and experience which the pupil's lack. He should be an enthusiastic member of the club, but at the same time a reserved member. The club should be a cooperative enterprise, with the teacher acting as chief stimulator and at the same time as chief balance wheel. If any criticisms are made concerning programs, they should be given in private to the individual pupils or pupil concerned.

The faculty and community should be thoroughly educated in club ideals, objectives, and activities. The whole-hearted support of the club program by the entire faculty and community is essential, for nothing can be more disastrous than an unsympathetic attitude which finds expression either in open ridicule and direct opposition or in half-hearted support and mere toleration. Those clubs which are most successful and have the most enthusiasm are the ones which have school and community backing and which have interested and capable people in the community, backing and lending their assistance to the students.

There should be no physical initiation of new members into the club. The initiation should consist of an educational program by the new members. With this type of program, the students will realize that the club is a mature self-educating organization, that will not tolerate foolishness or immaturity.

Model constitution. The following constitution was written for the Kingman High School Mathematics Club, and it can be used for a model to develop other constitutions for other schools.

Preamble: For the purpose of associating ourselves in an ordered manner for the cultivation of our common interests in mathematics, we the students of ______ High School, adopt this constitution as the basis of the activities of a mathematics club.

Name and Purpose: The name of this club shall be ______ Mathematics Club. The purpose shall be to broaden and deepen the interest of its members in mathematics, to increase knowledge of the subject, to pass on to others an appreciation of the values and beauties of the subject, to learn to perfect our skills in mathematics, to understand the importance of mathematics in our lives, and to explore the possibility that mathematics can be fun.

Membership: No one shall become a member of this club unless he agrees to be bound by the provisions of this constitution. This Mathematics Club shall consist of those students of ______ High School who have successfully passed one course in mathematics, who are at the time of application for membership enrolled in some course in mathematics and are doing satisfactory work in that course, or who have successfully completed one year of mathematics beyond the first year.

Membership Limit: Membership in this club at any one time shall not be limited to any fixed number of persons, but rather shall be limited to the situation. Application for membership will be made in writing stating the reason why one wants to be a member of the mathematics club. Prior to organization, the sponsor, other mathematics teachers, and pupils will select members. After organization, election to membership shall be by a majority vote of the members present. Any elected members who fail to attend three consecutive meetings of the club without acceptable excuse, shall automatically cease to be a member. The sponsor and club officers will comprise the official reviewing board to decide on dismissals. At the beginning of the school year, the membership roll shall be revised to include only active members who are in good standing. Any member in good standing shall be eligible to serve on any committee and to be elected to office.

Sponsor: The club sponsor shall be a teacher of _______ High School who is designated by the principal or by the faculty to take that responsibility. He shall be eligible to attend all meetings of the club, to attend meetings of all committees, and to attend meetings of the officers. He shall advise on all matters pertaining to club activities and to veto any action proposed or taken by any committee, the officers, or the club. He shall be privileged to refuse to approve for membership any student who is a candidate, and to refuse to approve for election any student who is nominated for office.

Officers: The officers of the mathematics club shall be president, vice president, secretary-treasurer, librarian, and reporter.

Duties of Officers: It shall be the duty of the president to appoint all committees and to advise them in their work, to preside at every meeting of the club, and in general to be responsible for the welfare of the club.

It shall be the duty of the vice president to preside at any meeting of the mathematics club during the absence of the president and to take other responsibilities of any executive nature when so requested by the president, sponsor or club.

It shall be the duty of the secretary-treasurer to keep minutes of the meetings and to keep accurate records of membership, dues received, if any, and motions made and submitted to vote. He shall take care of all correspondence.

The librarian and library committee shall be responsible for the storing and care of the supplies, books, papers, and equipment of the club.

The reporter shall handle publicity of the club for the club newspaper, school paper, and the community paper.

Election of Officers: Officers shall be elected annually by a majority vote of members at a regular meeting during the last month of the school year. Regular meetings will be held on alternate Mondays after school for the purpose of business, projects, experimental work, films, or guest speakers. Regular meetings may be cancelled at the discretion of the sponsor when the school schedule or other circumstances demand a change. Special meetings may be called when needed.

Amendments: This constitution may be amended by two-thirds vote of members present at the time an amendment is properly presented for action.

CHAPTER IV

PROGRAM MATERIALS

The mathematics club is dependent upon its programs for its success. Therefore in this chapter, some programs have been worked out and also a list of topics that would make good programs is included.

Many recreational values are achieved through mathematics clubs in secondary schools. For many years mathematics has had the reputation of being the hardest subject in the curriculum and most students dread taking mathematics. Also years ago people believed that the harder and more abstract the study, the greater was the resulting mental discipline. These ideas are gradually leaving the schools. Mathematics employs a symbolic language and therefore requires logical reasoning to understand it.

To prove that mathematics is really down to earth let us analyze briefly its relation to the seven cardinal principles.

1. Health: Mathematics is not directly connected with the teaching of health except as the general idea of quantity and the measurement of foods and the proper proportions of necessary elements enter into the health teaching.

2. Command of the Fundamental Processes: The elements of arithmetic are, of course, included in this objective.

3. <u>Worthy Home Membership</u>: For meeting this objective one naturally thinks of the social studies, literature, music, art, and home economics. But I suggest that the knowledge of the making of budgets and of keeping household accounts would keep many a home from being broken.

4. <u>Vocation</u>: Mathematics of various sorts has long been recognized as an essential part in the training for many vocations.

5. <u>Civic Education</u>: This objective deals with the social and personal sides of life, particularly the developments of right civic attitudes and the spirit of cooperation. For this training, mathematics is not so essential, however important it may be in carrying on the business of government.

6. <u>Ethical Character</u>: How can the teaching of mathematics or the work of a mathematics club have as a by-product the development of ethical character? A study of nature reveals the most marvelous mathematical relationships. The very planets in the heavens are arranged with their distances apart following a numerical series with astonishing exactness. The buds on the stems occur in strange mathematical orders. Snow crystals illustrate static symmetry. The human body, the butterfly, the iris and all growing plants are based on the more subtle dynamic symmetry. These ratios are the basic of the beauty in nature and in art. Great spiritual uplift comes from the mere contemplation of these mathematical relationships in nature. A deep respect for such creations begets an abiding faith in the creator.

7. <u>Worthy Use of Leisure</u>: The development of the doctrine of interest and the study of the learning process brought out the fact that children learn more readily the thing in which they are interested and in which they are doing something. In play or recreation there are always both interest and participation. Therefore play or recreation has come to have its rightful place in the classroom.

Mathematics has such a wealth of materials and the time given in the school program is so limited, that the only way is to organize a mathematics club. In a club there is real serious work done. But since it is self imposed, it is recreational. A club develops

> initiative, interest to a big degree, and appreciations that are carried over into the classroom. Leisure time if used worthily means growth and re-creation in body, mind, and spirit.[1]

Newspaper. Probably the most important project of a mathematics club is a club paper. News about the mathematics club and articles originating from its discussions should be given prominence, but also other mathematics department news is included. The club should depend quite largely on advertising its meetings and programs through the paper. For example, announcements that the laws of chance will be studied at the next meeting if everyone brings six pennies for experimental purposes, or that mathematical magic will be demonstrated, are plugged in enthusiastic headlines that do bring out the members in full force. Problems, puzzles, and tricks that are put in the paper become the center of argument at meetings, and program material. Keen interest in the program results because the subject has already been thought over for days by the members and because each member has a copy of the problem that was in the paper under discussion, which makes the task of the leader of the program much simpler.

News from other mathematics club and similar activities in other schools are included whenever possible.

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[1]Marie Gugle, "Recreational Values Achieved Through Mathematics Clubs in Secondary Schools," Mathematics Teacher, Vol. 19, pp. 214-218, April, 1926.

Sometimes contests are conducted between schools through the paper.

The paper should contain poetry and composition on mathematical subjects of a high order of merit, as well as descriptions of puzzles and tricks, and in the last issue a summary of the years events is noted.

There can be a puzzle column which is always a major feature of interest. Usually a contest of one type or another is run in connection with the column. The column conductor is allowed to use his discretion as to methods of maintaining interest, a small prize can be given for the best solution. Pencils stamped in gold with the words of the paper's name on it have proved effective and popular prizes, and very economical.

The paper can be sold through the mathematics classes and in the school lobby for 10 cent a copy or can be given away. If it is sold, profits can be used to pay for the mathematics clubs expenses.

A person may ask himself if a mathematics club paper is worth-while. If he will think of the following reasons he will be convinced that a paper is worth-while.

There is a sense of responsibility in completing assignments on time, and fine examples of cooperation, loyalty, knowledge, and service at the cost of hours of personal sacrifice have been given many times by the students engaged in the work. Initiative and ingenuity in finding subjects, looking up material, and adapting a program or article for the paper are required in unusual measure of those whose responsibility includes historical and feature articles. The paper should stimulate and have a direct effect on class work. The articles and programs in the paper should stimulate students to expand upon the work and find more curious corners of mathematics.

The only purpose of the mathematics paper should be to spread among students the idea that mathematics is a very vital subject, that it is full of fun and thrills in its own right, that it has been created through the efforts of a long line of interesting personalities, a line reaching from the dawn of civilization down to the present moment, and that mathematics is vitally tied up with many phases of modern life, industry, and science.

The mathematics paper should be organized as any other paper. All the members of the club should have specific duties to carry out and should fulfill these duties if the paper is to be a success.

Christmas decorating. The mathematics club can be very popular at Christmas time by decorating with mathematical figures. If a Christmas dinner is served a big rectangular table decorated with tall straight red cylinders set in the center of little holly circles can be set up. The place cards can be angles, cone shape or squares, and circles. The food can be blocks of ice cream, delicious sandwiches and frosted cookies cut in squares, triangles, circles, rectangles, trapezoids, semicircles, octagons, and stars, and then finish with big cylinders of chocolate milk or other refreshments with ice cubes.

In the auditorium or in the lobby of the school a geometrical Christmas tree could be set up. The base can be a cylinder, the stems a very narrow rectangle, and the main part of the tree could have several tiers of graduated trapezoids surmounted at the top by a triangle. On the tree can hang colored or glittering circles, rectangles, squares, triangles, semicircles, parallel lines, perpendicular lines, hexagons, stars, pyramids, cones, cubes, octagons, and other n sided figures.

The class room can be decorated by painting geometric designs on the windows and on the boards. Geometric figures, constructed of D-sticks, can hang from the ceiling of the room on pieces of string.

Mathematical films. The following is a list of mathematical films, with their description, rental price, and rental company, that could be used as a program for the mathematics club.

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Mathematical films. The following is a list of mathematical films, with their description, rental price, and rental company, that could be used as a program for the mathematics club.

INDIRECT MEASUREMENT: 20 min. $3.00 for five days. Need for indirect measurement techniques is established by showing specific situations where direct measurements cannot be made. This sound film then demonstrates three methods of indirect measurement--congruent triangles, similar triangles, and trigonometry. Rental company: Bureau of Audio-Visual Instruction, University Extension Division, University of Nebraska, Lincoln 8, Nebraska.

QUADRILATERALS: 10 min. $1.75 for five days. It is the purpose of this film to illustrate and explain the chief properties of the important quadrilaterals, such as: parallelogram, rectangle, rhombus, square, trapezoid, and trapezium. Rental company: Bureau of Audio-Visuals Instruction, University Extension Division, University of Nebraska, Lincoln 8, Nebraska.

ARCHIMEDES' PRINCIPLE: 6 min. $1.25 for five days. Recreates Archimedes' experiments under conditions that show the development of his principle. Demonstrates experimentally the measurement of the buoyant force of fluid upon an immersed solid. Rental company: Bureau of Audio-Visual Instruction, Extension Division, University of Colorado, Boulder, Colorado.

LANGUAGE OF GRAPHS, THE: 13 min. $2.00 for 5 days. The use of graphs to sum up a situation. Boys and girls considering the financial and circulation problems of their school newspaper and employ bar, line, circle, and equation graphs, to picture relationship and make comparisons. Rental company: Bureau of Audio-Visual Instruction, Extension Division, University of Colorado, Boulder, Colorado.

LANGUAGE OF MATHEMATICS: 10 min. $1.25 for five days. The practical application of mathematical terms showing how mathematics is fundamental to our society and to every phase of modern living. Students see "in action" the precise and meaningful symbols of mathematics. They learn how this unique language helps them to state and solve problems more rapidly and accurately. Rental company: Bureau of Audio-Visual Instruction, Extension Division, University of Colorado, Boulder, Colorado.

ORIGIN OF MATHEMATICS: 10 min. $1.25 for 5 days. A background film for understanding the history of numbers, measurement and calculation. The methods of the cave dwellers, Egyptians, Babylonians, Greeks, Romans, and Arabs are illustrated. Rental company: Bureau of Audio-Visual Instruction, Extension Division, University of Colorado, Boulder, Colorado.

GEOMETRY AND YOU: 10 min. $1.25 for five days. In constructing a model porch Jim and Bob use protractor and ruler to apply their study of such figures as rectangles, triangles, and circles, and such principles as congruence, similarity, and symmetry. Rental company: Bureau of Audio-Visual Instruction, Extension Division, University of Colorado, Boulder, Colorado.

ALGEBRA IN EVERYDAY LIFE: 10 min. $1.25 for 5 days. Shows how algebra is used in everyday life as well as in specialized fields. Emphasizes the basic algebraic steps of observation, translation, manipulation, and computation. Rental company: Bureau of Audio-Visual Instruction, Extension Division, University of Colorado, Boulder, Colorado.

LOCUS: 12 min. $1.25 for 5 days. Visualizes and explains by a combination of photography, drawings, and the spoken word the concept of locus. Rental company: Bureau of Audio-Visual Instruction, Extension Division, University of Colorado, Boulder, Colorado.

MEANING OF PI: 10 min. $1.25 for 5 days. In step-by-step procedure the numerical value of pi is arrived at and sequences show the use of circles in art, industry, and commerce. In a closing historical sequence the discovery of pi is described. Rental company: Bureau of Audio-Visual Instruction, Extension Division, University of Colorado, Boulder, Colorado.

MEASUREMENT: 10 min. $1.25 for 5 days. A boy of twelve encounters everyday situations involving seven kinds of measurement--linear, square, cubic, weight, liquid, temperature, and time. Rental company: Bureau of Audio-Visual Instruction, Extension Division, University of Colorado, Boulder, Colorado.

PARALLEL LINES: 10 min. $1.25 for 5 days. Explains the concept of parallel lines, illustrates the prevalence of parallel lines in industry and architecture, and gives specific instances of the application of laws of parallel lines. Rental company: Bureau of Audio-Visual Instruction, Extension Division, University of Colorado, Boulder, Colorado.

EXPLORING THE UNIVERSE: 11 min. $1.25 for 5 days. Demonstrates the principles and construction of telescopes. Shows by animation the binaries, trinaries, the variables and why they vary, galaxies and galactic rotation. Shows also what will happen to the Big Dipper in 100,000 years and the theory of the expanding universe. Rental company: Bureau of Audio-Visual Instruction, Extension Division, University of Colorado, Boulder, Colorado.

METRIC SYSTEM: 11 min. $2.00 for 3 days. The history of the metric system and its uses today. Comparison of English and Metric units, advantages of computations in the metric system. Rental company: Audio-Visual Education Center, University of Michigan, Ann Arbor, Michigan.

VECTORS: 12 min. $2.00 for 3 days. Explanation, plotting, and solution of vectors. Rental company: Audio-Visual Education Center, University of Michigan, Ann Arbor, Michigan.

LINES AND ANGLES: 11 min. $2.00 for 3 days. Erection of a perpendicular; its relationship with the ordinary plumb bob, level and square; how angles are formed and measured; relationship of angles to each other and to complete circle. Rental company: Audio-Visual Education Center, University of Michigan, Ann Arbor, Michigan.

HOW MANY STARS: 11 min. $2.00 for 3 days. Galileo's work with the telescope; discoveries which led to explanations of our own galaxy; mans' study of distant universes; modern concepts of solar system; its place in the galaxy; meaning of a light year. Rental company: Audio-Visual Education Center, University of Michigan, Ann Arbor, Michigan.

STAR IDENTIFICATION: 18 min. $2.75 for 3 days. Apparent movement of stars across the sky. Locates and identifies 23 basic navigation stars. Rental company: Audio-Visual Education Center, University of Michigan, Ann Arbor, Michigan.

SOLAR SYSTEM: 11 min. $2.00 for 3 days. Names of planets; relative size; distance from the sun; forces at work in the solar system; orbits of the planets; differences between planets and stars; facts about gravitational attraction, light and heat. Rental company: Audio-Visual Education Center, University of Michigan, Ann Arbor, Michigan.

SLIDE RULE (The C and D Scales): 21 min. Black and white. $2.75 for 5 days. Explains in detail the C and D scales of the slide rule, the parts and markings of the rule, and how to use these scales for multiplication, division, and combinations of these two operations. Rental company: Audio-Visual Center, Indiana University, Bloomington, Indiana.

SLIDE RULE (Proportion, Percentage, Squares, and Square root): 21 min. $2.75 for 5 days. Shows how to use the B and C scales of the slide rule to calculate proportions and percentages, how to calculate squares and square roots, and how to determine the placing of decimals after the square root has been extracted. Rental company: Audio-Visual Center, Indiana University, Bloomington, Indiana.

STORY OF OUR NUMBER SYSTEM: 11 min. Color. $3.25 for 5 days. Presents the historical development of our number system including the earliest records of Babylonian, Mayan, and Roman recording and counting systems. Shows how impractical the Roman numeral system is in figuring problems. Traces the early use of the abacus, the introduction of the Zero, and the gradual use of the Arabic system which closely resembles the early Hindu-Arabic number symbols. Attributes the use of this system as one of the reasons for the rapid development of modern science. Rental company: Audio-Visual Center, Indiana University, Bloomington, Indiana.

THE EARLIEST NUMBERS: 30 min. Black and white, sound. $4.75 for 5 days. Shows man's first effort to count with symbols and demonstrates how Egyptian and Babylonian mathematics have contributed to our present number system. Stresses this contribution in terms of the essential elements of a modern numeration system: base, place, symbols, zero, decimal point. Presents the characteristics of the Egyptian number system. Discusses an even earlier tally stick of paleolithic man. Analyzes the Babylonian system of numeration. Traces our present use of "minutes" and "seconds" from Babylonian fractions. Through models, demonstrates and explains certain physical methods of writing and reconing with numbers. Devices shown are the English tally stick, quipu, abacus, and counting board. From these, such words and ideas are identified as "stock," "bank," "carry," and "borrow." Rental company: Audio-Visual Center, Indiana University, Bloomington, Indiana.

BASE AND PLACE: 30 min. Black and white, sound. $4.75 for 5 days. Presents the characteristics, history, and applications of the binary system. Through this system, emphasizes the basic principles of base and place in our system of numeration. Shows how numbers are represented in the binary system, its relationship to electronic digital computers, and how business applies the binary system through the use of keysort cards. Mentions specific applications and sketches the historical contribution of Leibniz and Harriot to the binary system. Demonstrates the importance of base and place in our number system. Shows such functions as the role of ten and checking the transposition of digits through division by nine. Rental company: Audio-Visual Center, Indiana University, Bloomington, Indiana.

BIG NUMBERS: 30 min. Black and white, sound. $4.75 for 5 days. Demonstrates how scientists and mathematicians write and use very large and very small numbers. Stresses the meaning and importance of exponents and powers and shows some of their uses. Traces the interest in large numbers of the Greeks, Hindus, and some New York City kindergarten children. Presents and demonstrates exponents and scientific notation as an easy way of writing and computing with large numbers. Explains negative exponents and demonstrates how very small numbers are written and used. Illustrates such other uses of exponents and powers as computing compound interest, calculating the healing time of a wound, determining belt friction, and assessing the value of a fleet of trucks. "Perfect numbers" are explained as an example of man's continual fascination with numbers. Euclid's formula for finding such numbers is introduced and demonstrated. Rental company: Audio-Visual Center, Indiana University, Bloomington, Indiana.

FUNDAMENTAL OPERATIONS: 30 min. Black and white, sound. $4.75 for 5 days. Analyzes modular arithmetic as an aid in understanding the principles of our own, or rational, arithmetic. Through arithmetic modulo five shows the closure, commutative, and identity element principles of rational arithmetic. Identifies the additional associative and distributive arithmetic principles using our own arithmetic. Demonstrates that the two fundamental operations are addition and multiplication; that subtraction and division are the inverse of these operations, and that an addition and multiplication table, limited in size by place values, are required to perform fundamental operations. Also shows the distinctive properties of arithmetic modulo six. Rental company: Audio-Visual Center, Indiana University, Bloomington, Indiana.

SHORT CUTS: 30 min. Black and white, sound. $4.75 for 5 days. Explains and demonstrates logarithms, the slide rule, and other methods for simplifying computation. Through the use of models and charts, presents finger multiplication, the lightning or cross method of multiplication, and Napier's "bones." Explains the development and application of logarithms. Shows how a log table is constructed and used. Relates this to a model of a slide rule and demonstrates its operation and uses. Indicates the many other uses of logarithms in representing important relationships in such areas as electricity and chemistry. Rental company: Audio-Visual Center, Indiana University, Bloomington, Indiana.

FRACTIONS: 30 min. Black and white, sound. $4.75 for 5 days. Presents the history and some of the characteristics of fractions. Indicates the derivation of the words "fraction," "numerator," and "denominator." Sketches the basic principles governing the use of fractions. Shows that a fraction such as 3/5 has five meanings: 3/5 of 1; 1/5 of 3; 3:5; 3÷5; (3,5). Traces the way in which our present representation of fractions has come to us from Babylonia, Egypt, Greece, Rome, India, and Belgium. Compares the properties of rational, decimal and duodecimal fractions. Discusses some of the arguments advanced by advocates of the duodecimal system. Rental company: Audio-Visual Center, Indiana University, Bloomington, Indiana.

NEW NUMBERS: 30 min. Black and white, sound. $4.75 for 5 days. Explains new and important number concepts in modern mathematics. Indicates the pattern of how new numbers arise. Presents some of the history, characteristics, and uses of negative, irrational, transfinite, and complex numbers as well as quaternions. Rental company: Audio-Visual Center, Indiana University, Bloomington, Indiana.

PATTERNS IN MATHEMATICS: 14 min. Black and white. $2.00 a week. Dr. Meder explains the statement of the

Commission on Mathematics, "Algebra must be treated as a study of mathematical structure, rather than only as the development of manipulative skills." An emphasis on this structure, or pattern, is the basis of all curriculum revision recommended by the Commission, and Dr. Meder explains clearly and with varied examples why mathematics should be considered a study of pattern, rather than a collection of isolated facts and tricks. With problems taken from arithmetic, algebra, geometry, and trigonometry, he shows how concepts of pattern can make mathematics teaching clearer and more meaningful. Rental company: Audio-Visual Center, K.S.T.C., Emporia, Kansas.

NUMBER FIELDS: 17 min. Black and white. $2.00 a week. Here, Dr. Meder takes up the specific problems of the student who solves an algebra problem without understanding why his answer works. Dr. Meder shows the weakness of manipulative solutions, and demonstrates how the teacher's understanding of the properties of a number field enables the teacher to make algebra more meaningful for students. With the use of animated diagrams, Dr. Meder discusses the system of integers, real numbers, rational numbers, and complex numbers, from the point of view of number fields and explains the requirements and properties of a number field. Rental company: Audio-Visual Center, K.S.T.C., Emporia, Kansas.

IRRATIONAL NUMBERS: 23 min. Black and white. $2.00 a week. Scenes of an actual classroom in action, with interpretative comments from Dr. Meder, vividly demonstrate the Commission's suggestion for teaching irrational numbers through the study of decimal fractions. The limited understanding of irrational numbers in past teaching is eliminated by this new method which leads to a positive demonstration of what an irrational number is. The teaching method for communicating these new ideas to others is clearly shown in these classroom scenes. Rental company: Audio-Visual Center, K.S.T.C., Emporia, Kansas.

CONCEPT OF FUNCTION: 16 min. Black and white. $2.00 a week. A new approach to the concept of function is given here, one which makes the teaching easier for the teacher and the concept clearer for the student. The former ambiguity in the teaching of this concept is eliminated by new and sharper definitions of such expressions as: F(x), range, domain and rule. Present confusion between single valued and multiple valued functions is completely eliminated by this new method of looking at the concept of function. Rental company: Audio-Visual Center, K.S.T.C., Emporia, Kansas.

SENTENCES AND SOLUTION SETS: 21 min. Black and white. $2.00 a week. Through actual classroom scenes, a ninth grade algebra teacher is shown teaching the concept of set. Dr. Meder explains and illustrates with the classroom scenes the way to introduce sets, variables, and open sentences to students. He gives examples that show how the concept of set makes the teaching of inequalities, as well as equations, simpler and clearer. With a solid understanding of these basic concepts, even the average student will progress. Rental company: Audio-Visual Center, K.S.T.C., Emporia, Kansas.

HOW MAN LEARNED TO COUNT. 30 min. Black and white. $7.50 per day. A trip back through history to see how arithmetic developed from the cave man to the present. Bil Baird and his puppets, "Snarky and Gargle," demonstrate the counting systems of Egypt, Rome, Phoenicia, Carthage, and India and how they contributed to our modern number system. Rental company: Association Films, Inc., 561 Hillgrove Ave., La Grange, Ill.

QUICKER THAN YOU THINK: 30 min. Black and white. $7.50 per day. Bil and the marionettes tell the story of the evolution of primitive counting systems into the creation of modern electronic calculator, based on the binary system. They emphasize that although it works at the speed of light, man must still plan its work. Rental company: Association Films, Inc., 561 Hillgrove Ave., La Grange, Ill.

MYSTERIOUS "X": 30 min. Black and white. $7.50 per day. Algebra is the key to the study of most mathematics used in science and business. Bil, Snarky and Gargle demonstrate everyday applications of algebra and show how the great scientists use its terms to explain the phenomena of nature. Rental company: Association Films, Inc., 561 Hillgrove Ave., La Grange, Ill.

WHAT'S THE ANGLE: 30 min. Black and white. $7.50 per day. The Baird marionettes use modern-day illustrations and go back to ancient times to explain the fundamental concepts of plane and solid geometry. They show how this science was used and developed by the

Egyptains, Chinese and Greeks and its use today in architecture, construction, surveying and navigation. Rental company: Association Films, Inc., 561 Hillgrove Ave., La Grange, Ill.

IT'S ALL ARRANGED: 30 min. Black and white. $7.50 per day. By using three familiar situations, Bil Baird demonstrates to his marionette helpers, how to figure combinations mathematically. The film dramatically points out that by rearranging a given number of units a large variety of different combinations are possible. Rental company: Association Films, Inc., 561 Hillgrove Ave., La Grange, Ill.

HOW'S CHANCES? 30 min. Black and white. $7.50 per day. Explains how to arrive at a probable total by taking a representative sampling of the whole and projecting the percentages. Demonstrates how mathematics is used in business, industry, and government to determine vital advance predictions of probable totals. Rental company: Association Films, Inc., 561 Hillgrove Ave., La Grange, Ill.

SINE LANGUAGE: 30 min. Black and white. $7.50 per day. Bil, Snarky and Gargle throw some light on the study of trigonometry. They show some of the everyday uses of trigonometry and its importance in navigation, land surveying and the description of periodic phenomena such as musical notes and electrical currents. Rental company: Association Films, Inc., 561 Hillgrove Ave., La Grange, Ill.

STRETCHING THE IMAGINATION: 30 min. Black and white. $7.50 per day. Bil Baird and the puppets use such commonplace items as a fun house mirror and figures drawn on a rubber sheet to demonstrate that while form may change, the essential substance remains the same. Practical applications of topology are shown including its applications in planning power lines and laying out efficient routes. Rental company: Association Films, Inc., 561 Hillgrove Ave., La Grange, Ill..

CAREERS IN MATHEMATICS: 30 min. Black and white. $7.50 per day. In this final film in the series, Bil, Snarky and Gargle recap the highlights of the previous eight films. They talk with Dr. Howard Fehr and other prominent figures of the mathematics field who tell how rich and rewarding a math career can be. Rental company: Association Films, Inc., 561 Hillgrove Ave., La Grange, Ill.

Compound statements. In the following two paragraphs the simple and compound statements are defined by John Kemeny.

> A statement is a verbal or written assertion. In the English language such assertions are made by means of declarative sentences. For example, "It is snowing" and "I made a mistake in signing up for this course" are statements.
>
> The two statements quoted above are simple statements. A combination of two or more simple statements is a compound statement. For example, "It is snowing, and I wish that I were out of doors, but I made the mistake of signing up for this course," is a compound statement.[2]

The fundamental property of any statement is that it is either true or false (and that it cannot be both true and false).

Take two simple statements, "It is raining" and "It is cold." Let p stand for "it is raining," and q stand for "it is cold." To make a compound statement a person would say, "It is raining and cold." To symbolize this statement the result would be $p \wedge q$. The symbol "$\wedge$" being read as "and." Notice that both statements must be true for p and q to be true.

To make a statement where one or the other of the simple statements is true it could be said "It is raining or cold." To symbolize this statement use $p \vee q$, where "$\vee$" is read as "or."

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[2]John G. Kemeny, J. Laurie Snell, Gerald L. Thompson, Introduction to Finite Mathematics (Englewood Cliffs: The Prentice-Hall Company, 1957), p. 1.

Suppose one of the simple statements would change, for example, "It is not raining." Symbolically we would write $\sim p$, where "$\sim$" is read "the negation."

The truth value of a statement is determined by the truth value of its components. The truth values are found by constructing truth tables. The first truth tables will be for $p \wedge q$ and $p \vee q$.

In the compound statement $p \wedge q$, p could be either true or false and so could q. Therefore there are four pairs of truth values. It must be found in each case if $p \wedge q$ is true. If p and q are both true, then $p \wedge q$ is true, if $p \wedge q$ are false or either is false the statement is false.

p	q	$p \wedge q$
T	T	T
T	F	F
F	F	F
F	T	F

In the compound statement $p \vee q$, if either of the simple statements is true, then the compound statement is true. If both are false then the compound statement is false. Now if both simple statements are true and it is possible for both to be true at the same time, then the compound statement is true. If it is impossible for both simple statements to be true at the same time, then the compound statement is false. This symbol "$\veebar$" is used for one another but not both.

p	q	$p \veebar q$
T	T	F
T	F	T
F	T	T
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

In the statement $\sim p$, the "not" means the negation of p.

p	$\sim p$
T	F
F	T

The statement, "If the weather changes, then I will go swimming," is an assertion containing a condition. The statement is of the form "if p then q," and to symbolize this an arrow "$\rightarrow$" is used. If p and q are both true, then $p \rightarrow q$ is true, and if p is true and q false, then $p \rightarrow q$ is false. It is known that a statement is either true or false, so an arbitrary decision is made and it is said that $p \rightarrow q$ is true whenever p is false regardless of the truth value of q.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The last symbol used is the double arrow "$\leftrightarrow$." This asserts that if p is true, then q is true, and if p is false then q is false. The statement $p \leftrightarrow q$ may be read "p if and only if q."

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

To summerize the above material we have:

symbol	name	translated as
$\wedge$	conjunction	"and"
$\vee$	Disjunction (inclusive)	"or"
$\underline{\vee}$	Disjunction (exclusive)	"or"
$\sim$	Negation	"not"
$\rightarrow$	Conditional	"if then"
$\leftrightarrow$	Biconditional	"if and only if"

When using two statements four possibilities will result, when there are three statements there will be a total of eight possible triples of truth values.

Construct truth tables for:

1. $(p \wedge r) \vee (p \rightarrow q)$
2. $[p \rightarrow (q \wedge r)] \leftrightarrow (q \vee \sim r)$
3. $(q \underline{\vee} p) \rightarrow p$
4. $\sim [(\sim q \vee \sim p) \vee (r \vee p)]$

Truth tables can be used in checking the validity of an argument. By an argument is meant the assertion that a statement (the conclusion) follows from other statements (the premises). An argument is valid if and only if the conjunction of the premises implies the conclusion and where all are true.

$$\begin{array}{l} p \rightarrow q \\ \underline{p \quad\;\;} \\ \therefore q \end{array}$$

p	q	[(p→q)	∧	p]	→	q
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	F	F	T	T
F	F	T	F	F	T	F

This is valid because all are true.

Truth tables can be used in working with sets and electrical circuits.

<u>Mathematical fallacies</u>. The following are seven mathematical fallacies that could be used in a mathematics club, as presented by W. W. Rouse Ball.

One of the oldest fallacies is as follows:

Suppose that $a = b$, then $ab = a^2$,

$$ab - b^2 = a^2 - b^2,$$

$$b(a - b) = (a + b)(a - b),$$

$$b = a + b,$$

$$b = 2b,$$

$$1 = 2.$$

Another instance of a fallacy is as follows:

Let a and b be two unequal numbers, and let c be their arithmetic mean, hence $a + b = 2c$,

$$(a + b)(a - b) = 2c(a - b),$$

$$a^2 - b^2 = 2ca - 2bc,$$

$$a^2 - 2ac = b^2 - 2bc,$$

$$a^2 - 2ac + c^2 = b^2 - 2bc + c^2,$$

$$(a - c)^2 = (b - c)^2,$$

$$a = b$$

Another example may be stated as follows:

We have $(- 1)^2 = 1$,

Take logarithms of both sides,

$$2 \log (-1) = \log 1 = 0,$$

$$\log (-1) = 0,$$

$$-1 = 10^0,$$

$$-1 = 1.$$

The same argument may be expressed thus: Let x be a quantity which satisfies the equation $e^x = -1$,

Square both sides $e^{2x} = 1$,

$$2x = 0,$$

$$x = 0,$$

$$e^x = e^0$$

$$e^x = -1,$$

$$e^0 = 1,$$

$$-1 = 1.$$

The fifth mathematical fallacy is as follows:

We know that $\log(1+x) = x - 1/2\,x^2 + 1/3\,x^3 - \ldots,$

If $x = 1$, the resulting series is convergent;

hence we have $\log 2 = 1-1/2+1/3-1/4+1/5-1/6+1/7-\ldots,$

therefore $2\log 2 = 2-1+2/3-1/2+2/5-1/3+2/7-\ldots,$

Taking those terms together which have a common denominator,

we obtain

$$
\begin{aligned}
2\log 2 &= 1+1/3-1/2+1/5-1/7+1/4-1/9+\ldots,\\
&= 1-1/2+1/3-1/4+1/5-\ldots,\\
&= \log 2,\\
2 &= 1.
\end{aligned}
$$

The following fallacy is using square roots. We can write the identity $\sqrt{-1} = \sqrt{-1}$ in the form

$$\sqrt{\frac{-1}{1}} = \sqrt{\frac{1}{-1}},$$

$$\frac{\sqrt{-1}}{\sqrt{1}} = \frac{\sqrt{1}}{\sqrt{-1}},$$

$$(\sqrt{-1})^2 = (\sqrt{1})^2$$

$$-1 = 1.$$

Another mathematical fallacy using square roots:

Again we have $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab},$

$$\sqrt{-1} \times \sqrt{-1} = \sqrt{(-1)(-1)},$$

$$(\sqrt{-1})^2 = \sqrt{1},$$

$$-1 = 1.$$ [3]

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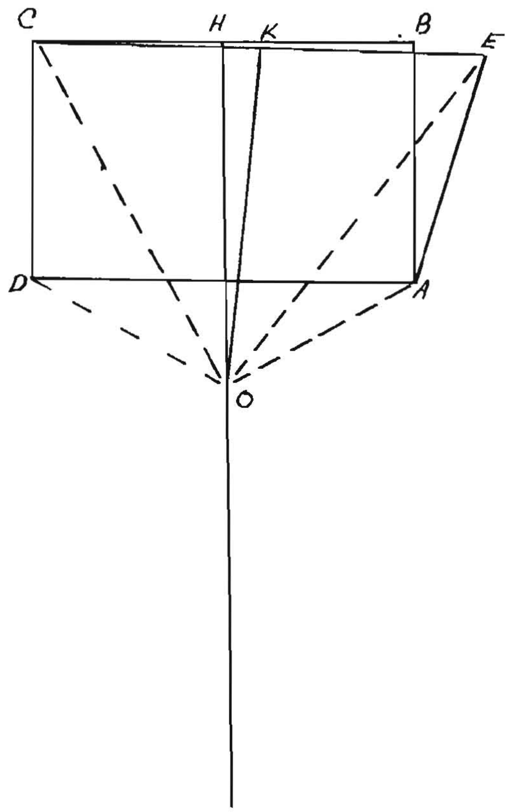
[3] W. W. Rouse Ball, Mathematical Recreations and Essays (New York: The Macmillan Company, 1905), pp. 24-26.

Geometrical fallacies. To prove that a right angle is equal to an angle which is greater than a right angle:

Let ABCD be a rectangle. From A draw a line AE outside the rectangle, equal to AB or DC and making an acute angle with AB, as indicated in the diagram.



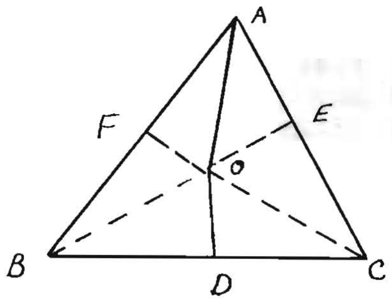
Bisect CB in H, and through H draw HO at right angles to CB. Bisect CE in K, and through K draw KO at right angles to CE. Since CB and CE are not parallel the lines HO and KO will meet at O. Join OA, OE, OC, and OD.

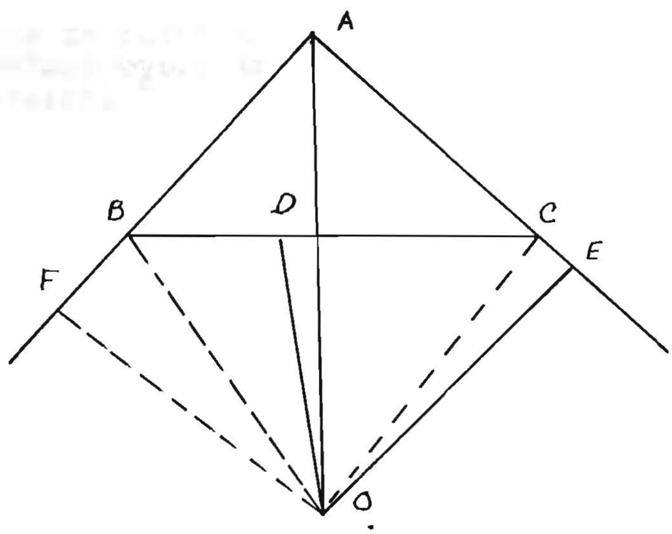
The triangles ODC and OAE are = in all respects. For, since KO bisects CE and is perpendicular to it, we have OC = OE. Similarly, since HO bisects CB and DA and is perpendicular to them, we have OD = OA. Also, by construction, DC = AE. Therefore the three sides of the triangle ODC are equal respectively to the three sides of the triangle OAE. Hence the triangles are ≅ and therefore the angle ODC is equal to the angle OAE.

Again, since HO bisects DA and is perpendicular to it, we have the angle ODA equal to the angle OAD. Hence the angle ADC (which is the difference of ODC and ODA) is equal to the angle DAE, (which is the difference of OAE and OAD). But ADC is a right angle, and DAE is necessarily greater than a right angle. Thus the result is impossible.[4]

To prove that every triangle is an isosceles triangle, is presented in this geometrical fallacy which follows.

Let ABC be any triangle. Bisect BC in D, and through D draw DO perpendicular to BC. Bisect the angle BAC by AO. First: If DO and AO do not meet, then they are parallel. Therefore AO is at right angles to BC. Therefore AB = AC. Second: If DO and AO meet, let them meet in O. Draw OE perpendicular to AC. Draw OF perpendicular to AB. Join OB, OC.





Let us begin by taking the case where O is inside the triangle, in which case E falls on AC and F on BC.

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[4]*Ibid.*, pp. 42-43.

The triangles AOF and AOE are ≅ since the side AO is common, angle OAF = angle OAE, and angle OFA = angle OAE. Hence AF = AE. Also, the triangles BOF and COE are ≅. For since OD bisects BC at right angles, we have OB = OC; also, since the triangles AOF and AOE are ≅, we have OF = OE; lastly, the angles at F and E are right angles. Therefore the triangles BOF and COE are ≅. Hence FB = EC. Therefore AF + FB = AE + ED, that is AF = AC.

The same demonstration will cover the case where DO and AO meet at D, as also the case where they meet outside BC but so near it that E and F fall on AC and AB and not on AC and AB produced.

Next take the case where DO and AO meet outside the triangle, and E and F fall on AC and AB produced. Draw OF perpendicular to AB produced. Join OB, OC.

Following the same argument as before, from the equality of the triangles AOF and AOE, we obtain AF = AE; and, from the equality of the triangles BOF and COE, we obtain FB = EC. Therefore AF - FB = AE - EC, that is, AB = AC.

Thus in all cases, whether or not DO and AO meet, and whether they meet inside or outside the triangle, we have AB = AC; and therefore every triangle is isosceles, a result which is impossible.[5]

To prove that, if two opposite sides of a quadrilateral are equal, the other two sides must be parallel, is one of the most interesting of geometrical fallacies.

Let ABCD be a quadrilateral such that AB is equal to DC. Bisect AD in M, and through M draw MO at right angles to AD. Bisect BC in N, and draw NO at right angles to BC. If MO and NO are parallel, then AD and BC (which are at right angles to them) are also parallel.

If MO and NO are not parallel, let them meet in O; then O must be either inside the quadrilateral as in

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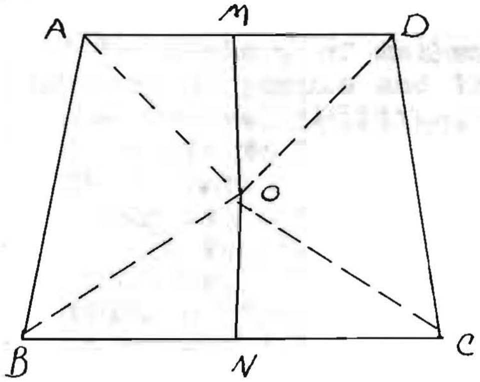
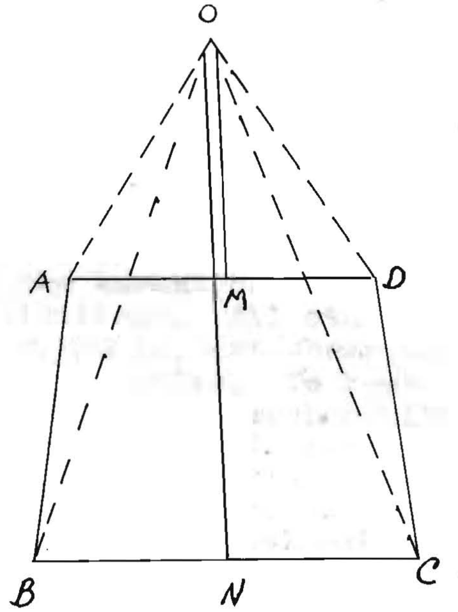
[5]*Ibid.*, pp. 44-45.

the left-hand diagram or outside the quadrilateral as in the right-hand diagram. Join OA, OB, OC, OD.





Since OM bisects AD and is perpendicular to it, we have OA = OD, and the angle OAM equal to the angle ODM. Similarly OB = OC, and the angle OBN equal to angle OCN. Also by hypothesis AB = DC, hence the triangles OAB and ODC are equal in all respects, and therefore the angle AOB is equal to the angle DOC.

Hence in the left-hand diagram the sum of the angles AOM, AOB is equal to the sum of the angles DOM, DOC; and in the right-hand diagram the difference of the angles AOM, AOB is equal to the difference of the angles DOM, DOC; and therefore in both cases the angle MOB is equal to the angle MOC, i.e., OM (or OM produced) bisects the angle BOC. But the angle NOB is equal to the angle NOC, i.e., ON bisects the angle BOC; hence OM and ON coincide in direction. Therefore AD and BC, which are perpendicular to this direction, must be parallel. This result is not universally true, and the above demonstration contains a flaw.[6]

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[6]*Ibid.*, pp. 46-47.

<u>Men of mathematics chart</u>. To study mathematics a person must also study it's exciting history. Without studying some history of mathematics people cannot appreciate mathematics in its full glory.

Julia B. Shelton of Eastern Montana College of Education, Billings, Montana, says:

> The history of mathematics is the kernel of the history of people and their civilizations. All other histories--of politics, literature, music, art--bear on and relate to the development of mathematics. To keep this history secret is to rob mathematics of much of its glamour and human appeal and to prevent a full grasp of and perspective on the interrelationships of the facets of culture. The lives of the men of mathematics are stories of struggles and victories, of incomprehensible genius, unmatched in any field.[7]

The following chart of the history of mathematics would make an excellent program for a mathematics club. The members could investigate a small group of mathematicians at a time and then report their findings to the whole group. In this manner their knowledge of the men of mathematics would be greatly increased.

600 BC Thales--Demonstrative Geometry
540 BC Pythagoras--Postulational thinking
Geometry
500 BC Sulvasutras--Religious mathematical writer
450 BC Zeno--Paradoxes of motion
440 BC Anaxagoras--Geometry
430 BC Antiphon--Gr. Method of exhaustion
425 BC Theodorus of Cyrene--Irrationality

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[7]Julia B. Shelton, "A History of Mathematics Chart," <u>The Mathematics Teacher</u>, LII, November, 1959, p. 563.

410 BC Democritus--Method of indivisibles
Atomic Theory
400 BC Archytas--Proportion
380 BC Plato--Foundations of Mathematics
370 BC Eudoxus--Method of Exhaustion
350 BC Menaechums--Conic Sections
Xenocrates--History of Geometry
340 BC Aristotle--Applications of Mathematics
Logic
300 BC Euclid--Elements of Geometry
Perfect numbers
287 BC Aristarchus--Copernican system
230 BC Eratosthenes--Prime number sieve
Size of earth
225 BC Apollonius--Conic Sections
Archimedes--Method of Equilibrium
Infinite series
Mechanics
Hydrostatics
180 BC Hypsicles--Astronomy, number theory
Nicomedes--Conchoid
Diocles--Cissoid
140 BC Hipparchus--Star catalogue
50?AD Heron--Machines
Surveying
Diophantus--Syncopation of Greek Algebra
Number Theory
100 AD Menelaus--Spherical Trigonometry
Nicomachus--Number Theory
125 AD Theon of Smyrna--Theory of numbers
History of Pythagoras
150 AD Ptolemy--Planetary theory
Star catalogue
Trigonometry
Table of chords
275 AD Diophantus--Algebra, theory of numbers
300 AD Pappus--Mathematical collection
Centroid theorems
320 AD Iamblichus--Number theory
390 AD Theon--Edited Euclid's elements
410 AD Hypatia--1st woman mathematician
470 AD Proclus--Valuable commentary
505 AD Varahamihira--Hindu Astronomer
Table of sines
510 AD Boethius--Geometry, theory of numbers
Aryabhata the Elder--General mathematics
pi=3.1416
628 AD Brahmagupta--Cyclic Quadrilaterals

710 AD Bede--Calendar
Finger Reckoning
820 AD Al-Khowarizmi--Algebra
Hrabanus Maurus--Computus
840 AD Honein ibn Ishaq--Greek mathematics
850 AD Mahavira--Arithmetic, algebra, mensuration
860 AD Alchindi--Astronomy, optics, proportion
Almahani--Trigonometry, cubic equation
920 AD Al-Battani--Moslem astronomer
Law of cosines in spherical trig.
980 AD Abul-Wefa--Trig. Tables
Geometry
1000 AD Alhazen--Optics
1100 AD Omar Khayyam--Cubic Equations
Calendar
1140 AD Abraham ben Ezra--Magic squares, calendar
1150 AD Bhaskara--Indeterminate equation
1202 AD Fibonacci--Liber Abaci. Geometry
1250 AD Sacrobosco--Numerals and the sphere
Roger Bacon--Astronomy, general mathematics
1345 AD Richard Suiceth--Coordinates
1360 AD Nicole Oresme--Exponents, proportion,
coordinates
1470 AD Johann Muller--Translator of Math.
1494 AD Pacioli--Double-entry bookkeeping
1500 AD Leonardo Da Vinci--Geometry
Art
Mechanics
1506 AD Del Ferro--Cubic equations
1510 AD Albrecht Durer--Projective Geometry
1522 AD Tonstall--First arithmetic printed in England
Rudolff--Algebra, decimals
1530 AD Copernicus--Planetary theory
Trigonometry
1542 AD Robert Recorde--equality sign
1545 AD Tartaglia--Cubic equations
Artillery prob.
Cardano--cubic equations
Ferrari--quartic equations
1553 AD Stifel--Number mysticism
1572 AD Bombelli--Algebra
1580 AD Vieta--Plane and Sph. triangles-6
Trig. functions
Theory of equations
1590 AD Stevin--Decimal fractions
Cataldi--Continued fractions

1600 AD Galileo--Falling bodies
Pendulum
projectiles
telescopes
Harriot--Algebra symbolism
Burgi--Logarithms
1610 AD Kepler--Laws of planetary motion
Principle of continuity
1612 AD Bachet de Meziriac--On Diophantus,
recreations
1614 AD Napier--Trig
Logarithms
Computing rods
1624 AD Briggs--Logs to base 10
1630 AD Cavalieri--Method of indivisibles
Logarithms
Oughtred--Algebra
Symbolism
Slide Rule
1st table of natural logs
1635 AD Fermat--Analytic Geometry
Theory of numbers
Probability
1637 AD Descartes--Analytic Geometry
1640 AD Desargues--Projective geometry
Roberval--Geometry
1650 AD N. Mercator--Mercators series
Astronomy
Cosmography
B. Pascal--Conics
Cycloids
Probability
Computing
Pascal triangle
Wallis--Precalculus
Imaginary numbers
1670 AD Christopher Wren--Astronomy
Architecture
J. Gregory--Infinite Series
Barrow--Differentiation
Muygens--Probability
1672 AD Mohr--Constructions with compasses only
1680 AD Isaac Newton--Fluxions
Gravitation
Hydrostatics
Dynamics

1682 AD Leibniz--Symbolic logic
Calculus
Determinants
Computing machines
1690 AD Halley--Astronomy
Life Insurance tables
J. Bernoulli--Probability
Marquis de l'Hospital--Applied calculus
1720 AD Taylor--Infinite series
De Moivre--Probability
Actuarial Math.
Complex numbers
1733 AD Saccheri--Non-Euclidean geometry prelude
1730 AD Matsunaga--pi to 50 figures
1740 AD Maclaurin--Fluxions
Curves
1750 AD Euler--Writer of Mathematics
Notations
Formulas
Montucla--History of Mathematics
1760 AD D'Alembert--Differential equations
Astronomy
Physics
John Landen--Elliptic integrals
1770 AD Lambert--Pi irrational
Hyperbolic functions
Map projections
1780 AD Lagrange--Calculus
Mechanics
Theory of numbers
1794 AD Monge--Descriptive Geometry
1800 AD Carnot--Modern geometry
Delambre--Astronomy, geodesy
Lacroix--Analysis
Mascheroni--Geometry of the compasses
Pfaff--Astronomy, analysis
1805 AD Legendre--Elements of Geometry
Number theory
Laplace--Differential Equations
Celestial Mechanics
Probability
1806 AD Brianchon--Duality
Argand--Imaginary numbers
1820 AD Gauss--Non-Euclidean Geometry
Fundamental Theorem of Algebra
Theory of numbers
Fourier--Series expansions

1824 AD Abel--Algebraic solution of general quintic
impossible elliptic functions
1825 AD J. Bolyai--Non-Euclidean Geometry
Lobachevsky--Non-Euclidean Geometry
1830 AD Poncelet--Projective geometry
Cauchy--Rigorization of Analysis
Definitions
Galois--Groups
Theory of Equations
Babbage--Calculating machine
1840 AD Steiner--Modern Geometry
Lame--Elasticity, surfaces
1850 AD Cayley--Matrices
Determinants
Hyperspace
Invariant theory
Liouville--Transcendental numbers
De Morgan--History of mathematics, logic
George Boole--Logic, differential equations
Gudermann--Hyperbolic functions
Quetelet--Statistics
Wronski--Philosophy of mathematics
1853 AD Hamilton--Quaternions
1854 AD Riemann--Riemannian Geometry
Riemann Surfaces
Grassmann--Ausdehnungslehre
1875 AD Brocard--Modern geometry of triangle
1880 AD Georg Cantor--Irrational numbers
Transcendental numbers
Transfinite numbers
1881 AD Gibbs--Vector Analysis
1888 AD Lemoine--Geometrography
1890 AD Weierstrass--Arithmetization of Analysis
1891 AD Heaviside--Vector analysis
1905 AD Einstein--Theory of Relativity
Atom bomb

The following group of men are listed by the dates of their birth and death.

Beltrami, Eugenio (1835-1900)--Non-Euclidean and infinitesimal Geometry

Bertrand, Joseph (1822-1900)--Analysis, probabilities, history of science, mathematical physics

Ball, Sir Robert Stawell (1840-1913)--Theory of screws, dynamics, mathematical astronomy

Bachmann, Paul (1837-1920)--Theory of numbers

Baire, Rene (1874-1932)--Theory of functions, irrationals, continuity, theory of aggregates
Appell, Paul (1855-1930)--Analysis, Mechanics, theory of functions
Hermite, Charles (1822-1901)--Algebra and analysis, theory of functions, elliptic functions
Tait, Peter Guthrie (1831-1901)--Mechanics, mathematical physics, quaternions
Fuchs, Lazarus (1833-1902)--Linear differential equations
Cremona, Luigi (1830-1903)--Synthetic geometry
Stokes, Sir George Gabriel (1819-1903)--Mathematical physics
Boltzmann, Ludwig (1844-1906)--Mathematical physics, principles of mechanics
Kummer, Ernst Eduard (1810-1893)--Theory of numbers, ideal numbers
Chebyshev, Pafnutii L'vovich (1821-1894)--Theory of numbers
Minkowski, Hermann (1864-1909)--Relativity
Newcomb, Simon (1835-1909)--Celestial mechanics
Meray, Charles (1835-1911)--Theory of irrationals
Fiedler, Otto Wilhelm (1832-1912)--Cyclography, descriptive geometry and its relationship to synthetic geometry
Gordan, Paul (1837-1912)--Theory of invariants
Lemonie, Emile (1840-1912)--New triangle geometry, geometrography
Poincare, Henri (1854-1912)--Theory of functions, Fuchsian and Abelian functions, differential equations
Couturat, Louis (1868-1914)--Mathematical logic
Hill, George William (1838-1914)--Celestial mechanics, lunar theory
Dedekind, Richard (1831-1916)--Theory of irrationals
Darboux, Gaston (1842-1917)--The General theory of surfaces
Sylow, Ludvig (1832-1918)--Substitution groups
Ramanujan, Srinivasa (1887-1920)--Theory of numbers
Schwarz, Hermann Amandus (1843-1921)--Conform representation of surfaces, minimal surfaces
Jordan, Camille (1838-1922)--General analysis, substitutions and algebraic equations
Frege, Gottlob (1848-1925)--Mathematical logic, foundations of arithmetic
Edgeworth, Francis Ysidro (1845-1926)--Probabilities, econometry

Fredholm, Eric Ivar (1866-1927)--Integral equations
Mittag--Leffler, Gosta (1846-1927)--Theory of functions
Painleve, Paul (1863-1933)--Differential equations
Noether, Emmy (1882-1935)--Theory of ideals, non-commutative algebras
Von Neumann, John (1903-1957)--High Speed electronic computers, Theory of games, Mathematical Foundations of quantum Mechanics
Hilbert, David (1862-1953)--Foundations of Geometry, Algebraic number fields, Integral equations

Notch arithmetic or the binary system. The following program of Notch Arithmetic was taken from the work of Aaron Bakst.[8]

In our every day arithmetic numbers are written according to a definite system. Each digit in a number occupies a position or place which is assigned a definite value known as a place value. For example, the number 537 is written with three digits 5, 3, and 7. The place value of the digit "7" is one. The place of the digit "3" is ten. The place value of the digit "5" is one hundred. In other words, every place immediately to the left of a given place has a value ten times larger. This system of numeration is known as the decimal system of numeration (or as the ten-system). Note that in this system of numeration ten symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) are employed for writing numbers.

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[8] Aaron Bakst, Mathematical Puzzles and Pastimes (New York: The D. Van Nostrand Company, 1954), pp. 22-37.

In mathematics, when writing the product of a number multiplied by itself, a simplified procedure is employed. The product of the same number is written as a power of this number with an indicator showing how many times the number was multiplied by itself. This indicator is written to the right and slightly above the number. Thus, 100 is written as 10^2 (although, this writing of the "1" by general agreement is omitted), 1,000 is written as 10^3. Thus "537" may be represented as (5 x 100) + (3 x 10) + (7). Also, 6,824 may be written as (6×10^3) + (8×10^2) + (2 x 10) + (4). This idea of writing a number so that each digit is assigned a place value according to the position which it occupies is not the specific property of the decimal system of numeration only. If, instead of ten symbols or digits, a person has at his disposal only two symbols or two digits, that is, 1 and 0 (or "the Notch" and the symbol zero), arithmetic becomes very simple. In this system of numeration the symbol 10 plays the same role as it does in the decimal system of numeration. But in "notch arithmetic" the value of the symbol 10 is two, since the number two does not have a special symbol for itself, whereas in the decimal system 10 has the value ten. The "notch mathematician" must have made a great discovery when he found that in place of "many" he could use the word "two."

Today "notch arithmetic" is given a special name. It is called the binary system of numeration or the two-system of numeration. In this arithmetic, each place value immediately to the left of a given place is twice the value of the given place. In the decimal system of numeration, the symbol "10" has the value "ten." In the two-system of numeration the symbol "10" has the value "two." Thus, when written in the two-system of numeration, the symbol 11 has the value of (1 x 2) + 1 or "3" (three). When written in the two-system of numeration, 1,111 is represented as $(1 \times 2^3) + (1 \times 2^2) + (1 \times 2) + (1) = 8 + 4 + 2 + 1 = 15.$

Numbers which are written in the decimal system of numeration may be translated into the two-system of numeration by computing the number of "twos" in the number written in the decimal system of numeration. This computation is performed by means of "long division." Suppose that the number 15,683 which is written in the decimal system of numeration is used. Now perform the consecutive divisions of this number by 2 as follows:

15,683 divided by 2 = 7,841 + remainder 1
7,841 divided by 2 = 3,920 + remainder 1
3,920 divided by 2 = 1,960 + remainder 0
1,960 divided by 2 = 980 + remainder 0
980 divided by 2 = 490 + remainder 0
490 divided by 2 = 245 + remainder 0
245 divided by 2 = 122 + remainder 1
122 divided by 2 = 61 + remainder 0
61 divided by 2 = 30 + remainder 1
30 divided by 2 = 15 + remainder 0

15 divided by 2 = 7 + remainder 1
7 divided by 2 = 3 + remainder 1
3 divided by 2 = 1 + remainder 1

The divisions may also be written as:

```
15,683 | 2
     1 | 7,841 | 2
             1 | 3,920 | 2
                     0 | 1,960 | 2
                             0 | 980 | 2
                                   0 | 490 | 2
                                         0 | 245 | 2
                                               1 | 122 | 2
                                                     0 | 61 | 2
                                                          1 | 30 | 2
                                                               0 | 15 | 2
                                                                    1 | 7 | 2
                                                                        1 | 3 | 2
                                                                            1 | 1
```

The last quotient becomes the digit on the extreme left of the translation. The number 15,683 in the two-system of numeration is then 11,110,101,000,011.

The translation of 73 into the two-system of numeration is obtained as follows:

```
73 | 2
 1 | 36 | 2
      0 | 18 | 2
           0 | 9 | 2
               1 | 4 | 2
                   0 | 2 | 2
                       0 | 1
```

When 73 is translated into the two-system of numeration, it becomes 1,001,001.

A number which is written in the two-system of numeration may be translated into the ten-system (the decimal system) of numeration, provided it is kept in mind that the place values in the two-system are all powers of 2. Thus the number 1,111,011,111 is translated as follows:

$(1 \times 2^9) + (1 \times 2^8) + (1 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2) + 1$ or

$512 + 256 + 128 + 64 + 16 + 8 + 4 + 2 + 1 = 991.$

The two-system of numeration is so versatile that by its application some very interesting tricks may be performed. The applications of this system of numeration to practical problems are also numerous. The writing of coded messages was once based in principle on this system of numeration.

The two-system of numeration equivalents of a series of ten-system numbers are as follows:

Ten-system	Two-system
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1,000
9	1,001
10	1,010
11	1,011
12	1,100
13	1,101
14	1,110
15	1,111
16	10,000
17	10,001
18	10,010
19	10,011
20	10,100
21	10,101
22	10,110
23	10,111
24	11,000
25	11,011
26	11,010
27	11,011
28	11,100
29	11,101
30	11,110
31	11,111
32	100,000
33	100,001
34	100,010
35	100,011
36	100,100
37	100,101
38	100,110
39	100,111
40	101,000
41	101,001
42	101,010
43	101,011
44	101,100
45	101,101
46	101,110

Ten-system	Two-system
47	101,111
48	110,000
49	110,001
50	110,010
51	110,011
52	110,100
53	110,101
54	110,110
55	110,111
56	111,000
57	111,001
58	111,010
59	111,011
60	111,100
61	111,101
62	111,110
63	111,111
64	1,000,000
65	1,000,001
66	1,000,010
67	1,000,011
68	1,000,100
69	1,000,101
70	1,000,110
71	1,000,111
72	1,001,000
73	1,001,001
74	1,001,010
75	1,001,011
76	1,001,100
77	1,001,101
78	1,001,110
79	1,001,111
80	1,010,000
81	1,010,001
82	1,010,010
83	1,010,011
84	1,010,100
85	1,010,101
86	1,010,110
87	1,010,111
88	1,011,000
89	1,011,001
90	1,011,010
91	1,011,011
92	1,011,100

Ten-system	Two-system
93	1,011,101
94	1,011,110
95	1,011,111
96	1,100,000
97	1,100,001
98	1,100,010
99	1,100,011
100	1,100,100

Using the foregoing table of numbers, one can construct seven tables of numbers.

The first table of numbers (all written in the ten-system of numeration) will consist of numbers whose equivalents in the two-system of numeration have the digit "1" on the extreme right (that is, the equivalents of the numbers 1, 11, 101, 1,011, 100,011, etc.). The second table will contain all the numbers in the ten-system of numeration which have the digit "1" in the second place from the right (that is, the equivalents of the numbers 10, 110, 11,010, etc.). In like manner, Tables 3 through 7 will contain all the numbers in the ten-system of numeration whose equivalents in the two-system of numeration have the digit "1" in the third, fourth, fifth, sixth, and seventh places from the right, respectively.

The tables follow:

Table 1

1	21	41	61	81
3	23	43	63	83
5	25	45	65	85
7	27	47	67	87
9	29	49	69	89
11	31	51	71	91
13	33	53	73	93
15	35	55	75	95
17	37	57	77	97
19	39	59	79	99

Table 2

2	22	42	62	82
3	23	43	63	83
6	26	46	66	86
7	27	47	67	87
10	30	50	70	90
11	31	51	71	91
14	34	54	74	94
15	35	55	75	95
18	38	58	78	98
19	39	59	79	99

Table 3

4	22	44	62	79
5	23	45	63	84
6	28	46	68	85
7	29	47	69	86
12	30	52	70	87
13	31	53	71	92
14	36	54	76	93
15	37	55	77	94
20	38	60	78	95
21	39	61		100

Table 4

8	26	44	62	88
9	27	45	63	89
10	28	46	72	90
11	29	47	73	91
12	30	56	74	92
13	31	57	75	93
14	40	58	76	94
15	41	59	77	95
24	42	60	78	
25	43	61	79	

Table 5

16	26	52	62	88
17	27	53	63	89
18	28	54	80	90
19	29	55	81	91
20	30	56	82	92
21	31	57	83	93
22	48	58	84	94
23	49	59	85	95
24	50	60	86	
25	51	61	87	

Table 6

32	42	52	62
33	43	53	63
34	44	54	96
35	45	55	97
36	46	56	98
37	47	57	99
38	48	58	100
39	49	59	
40	50	60	
41	51	61	

Table 7

64	74	84	94
65	75	85	95
66	76	86	96
67	77	87	97
68	78	88	98
69	79	89	99
70	80	90	100
71	81	91	
72	82	92	
73	83	93	

These tables may be transcribed on cards, one table to a card, and used for the following tricks.

Ask anyone to select any number between 1 and 100. Then ask him to tell the numbers of the tables in which the selected number appears.

If he chose the number 59, he will say his number appears in Tables 1, 2, 4, 5, and 6. Then add the first number in each of the respective tables (in this case--1, 2, 8, 16, and 32), and the sum of these numbers will be the number chosen.

A persons age can be told if that person tells the numbers of the tables in which is age appears.

If some young lady reveals that her age appears in the Tables 2, 3, and 5, tell her that she has seen 22 springs. To arrive at the age add the first numbers on each of the cards she names.

To mystify everybody further, don't even consult the tables. If someone gives a number of a table in which his selected number appears, raise 2 to the power of that number which is 1 less than the number of the table. For example, if the number of the table is 6, then 2 must be raised to the power 5, and this gives 32. Now the first number of table 6 is known. In similar manner one can determine the first number of any other table and, following the procedure given above, a person can tell what the selected number is.

Remember, a number raised to the zero power is one, so that the first number of Table 1 will always be one.

The foregoing tables may be enlarged.

If in addition to the notch (which for our purposes may be replaced by the symbol "1") use is also made of the symbol "0," an introduction to a system of numeration in arithmetic of the notch is made. This is the two-system of numeration which was described earlier. The arithmetic operations of the two-system of numeration are simple. The facts one needs to remember may be condensed into three statements:

$1 + 1 = 10, \qquad 1 \times 1 = 1, \qquad \text{and } 1 \times 0 = 0.$

Addition in the two-system of numeration is performed as follows:

```
   11 010 100 110 011
    1 110 101 011 101
  -------------------
  101 001 010 010 000
```

Since 1 + 1 = 10, whenever this addition combination occurs, write "0" in that place and "carry" 1 into the next place to the left.

Multiplication in the two-system of numeration is performed as follows:

```
                    110 010 100 101 001
                        101 001 111 001
                    -------------------
                    110 010 100 101 001
                110 010 100 101 001
              1 100 101 001 010 01
             11 001 010 010 100 1
            110 010 100 101 001
        110 010 100 101 001
     11 001 010 010 100 1
    -----------------------------------
    100 001 000 110 110 101 001 100 001
```

Subtraction in the two-system is performed as follows:

```
••   •  •   •••  •
11 011 001 100 000 111
 1 110 010 011 110 101
 ---------------------
 1 100 111 000 010 010
```

The "dots" above the numerals indicate "borrowing," a procedure which is also employed in the arithmetic of the decimal system.

Division in the two-system of numeration is performed as follows:

```
11 100 111 010 101    1001
10 01                 ---------
-----                 110101000
 1  01 1
 1  00 1
 -------
     1 011
     1 001
     -----
        10 01
        10 01
        -----
            0 101
```

The quotient is 110 101 000, and the remainder is 101.

The division may be carried on, and this will yield a decimal (in the sense of the two-system of numeration) fraction. Thus,

```
101           1001
1001          ---------
----          0.1000111
   10001
    1001
    ----
    10000
     1001
     ----
      1110
      1001
      ----
       101
```

The sequence of digits 101 indicates that a repeating fraction occurs. (We will call the fractions in the two-system of numeration two-mals.) 0.1000111 1000111 1000111 ....

The two-mal fraction 0.1 is equivalent to 1/2 in the decimal system of numeration. The two-mal fraction 0.01 is equivalent to 1/4, and the two-mal fraction 0.001 is equivalent to 1/8 in the decimal system of numeration. Each and every position to the right is equal in value to one-half of the value of the place to its left, whether these positions are indicated by a "two-mal point" or otherwise.

The "two-mal fraction" 0.1000111 has the value of $1/2 + 1/32 + 1/64 + 1/128 = 71/128$. The fraction 101/1001 has the value of 5/9 which is larger than the value of 0.1000111 by 1/1152.[9]

Binomial theorem. The following proofs of the binomial theorem were taken from the works of Samuel Goldberg.[10]

If n is any positive integer and x and y are any numbers, then $(x+y)^n = \binom{n}{0}x^n + \binom{n}{1} x^{n-1}y + \binom{n}{2} x^{n-2}y^2 + \binom{n}{3} x^{n-3}y^3 + \binom{n}{4} x^{n-4}y^4 \ldots + \binom{n}{n} y^n$ or, more concisely, $(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r}y^r$. Proof: In computing $(x+y)^n$, a choice is made between the letter x or the letter y from each of the n factors in $(x+y)^n = (x+y)(x+y)\ldots(x+y)$ and multiply these n choices. If this is done for all possible

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[9] Aaron Bakst, Mathematical Puzzles and Pastimes (New York: The D. Van Nostrand Company, 1954), pp. 22-32, 35-37.

[10] Samuel Goldberg, An Introduction Probability (Englewood Cliffs: The Prentice-Hall Company, 1960), pp. 149-152.

choices of x's and y's and the results are added the answer will be $(x+y)^n$. For example, the product x^n is arrived at, by choosing x from each factor, the product $x^{n-1}y$ whenever x is chosen from all but one factors, etc.

For a given integer r ($0 \leq r \leq n$), the product $x^{n-r}y^r$ is obtained whenever one chooses exactly r y's (and therefore n-r x's). To determine the choice uniquely, one has only to decide from which r of the n factors to select y's. Hence there are $\binom{n}{r}$ choices, each leading to the product $x^{n-r}y^r$ and so the term $\binom{n}{r}x^{n-r}y^r$ appears in the expansion of $(x+y)^n$. Since r is any integer from 0 to n inclusive, the theorem is proved. Because the numbers $\binom{n}{r}$ appear as coefficients in the expansion of a power of a binomial, they are called binomial coefficients.

Example: Expand $(x+y)^4$

$$(x+y)^4 = \binom{4}{0}x^4 + \binom{4}{1}x^3y + \binom{4}{2}x^2y^2 + \binom{4}{3}xy^3 + \binom{4}{4}y^4$$

$$(x+y)^4 = \sum_{r=0}^{4} \binom{4}{r} x^{4-r}y^r$$

For any positive integers r and n with $r < n$,

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

Proof: The $\binom{n}{r}$ r-subsets of a set with n elements can be divided into those that include a given element, and those that do not. The number of r-subsets including the given element is $\binom{n-1}{r-1}$ since fixing one element leaves us free to select r-1 others from the remaining n-1. The number of r-subsets that do not include the given element is $\binom{n-1}{r}$, since r is chosen from n-1 elements. Hence all r-subsets have been accounted for.

Pascal's triangle. The most convenient device for calculating and displaying the binomial coefficients is known as Pascal's Triangle.[11] The first few rows are illustrated below. The row for n = 0 lists the one coefficient in the expansion of $(x+y)^0$, and the row for n = 1 lists the two coefficients in the expansion of $(x+y)^1$, the row for n = 2 lists the three coefficients in the expansion of $(x+y)^2$, and the row for n = 3 lists the four coefficients in the expansion of $(x+y)^3$, and so on. Since every set with n elements has exactly one null subset and exactly one n-subset (namely, itself), we find 1's under the column headed r = 0 and also among the hypotenuse of the triangular array.

n \ r	0	1	2	3	4	5	6	7	8	9	10	11	12
0	1												
1	1	1											
2	1	2	1										
3	1	3	3	1									
4	1	4	6	4	1								
5	1	5	10	10	5	1							
6	1	6	15	20	15	6	1						
7	1	7	21	35	35	21	7	1					
8	1	8	28	56	70	56	28	8	1				
9	1	9	36	84	126	126	84	36	9	1			
10	1	10	45	120	210	252	210	120	45	10	1		
11	1	11	55	165	330	462	462	330	165	55	11	1	
12	1	12	66	220	495	792	924	792	495	220	66	12	1

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[11] Ibid.

There is a simple relation among numbers in the triangle. If we start at any number not on the hypotenuse and move to the right one number and then drop down to the row below we note that the sum of the first two numbers is precisely the number in the row below. For example, starting at the left in the row for $n = 4$, the numbers in the row for $n = 5$ are obtained. $1 + 4 = 5$, $4 + 6 = 10$, $6 + 4 = 10$, $4 + 1 = 5$.

The table shows that the above observation is generally true and can thus be used to extend the table one row at a time and thereby to compute binomial coefficients $\binom{n}{r}$ for larger and larger values of n.

<u>The multinomial theorem</u>. The following work on this theorem was taken from the works of Samuel Goldberg.[12] Now let n be any positive integer and $x_1, x_2, x_3, \ldots x_k$ any k numbers. Then $(x_1 + x_2 + x_3 + \ldots + x_k)^n =$

$$\sum \binom{n}{n_1, n_2, \ldots, n_k} x_1^{n_1} x_2^{n_2} \ldots x_k^{n_k}$$ where the sum is

taken over all nonnegative integers $n_1, n_2, n_3, \ldots n_k$ such that $n_1 + n_2 + n_3 + \ldots n_k = n$. Proof: Having n factors to multiply, but now each is the multinomial $(x_1 + x_2 + \ldots + x_k)$ instead of the binomial $(x+y)$ a person chooses from each

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[12] Samuel Goldberg, <u>An Introduction Probability</u> (Englewood Cliffs: The Prentice-Hall Company, 1960), pp. 154-5.

factor x_1, or x_2, ... or x_k, multiply n choices and add these products for all possible choices. For example, one gets the product x_1^n, by choosing x_1 from each factor, the product $x_1^{n-2}x_2x_k$ by choosing x_1 from n-2 of the factors, x_2 from one factor and x_k from another factor, etc.

Now for given nonnegative integers $n_1, n_2, \ldots, n_k$ (whose sum is n) the product $x_1^{n_1} x_2^{n_2} \ldots x_k^{n_k}$ is attained whenever one chooses exactly $n_1 x_1$'s, $n_2 x_2$'s ... $n_k x_k$'s from the n available factors. There are as many ways of making such a choice as there are ways of placing the n factors into k cells, the first cell containing the n_1 factors from which x_1 is chosen, the second cell containing the n_2 factors from which x_2 is chosen, etc. By the number of ways of placing n distinct objects into k cells so that n_i objects are in cell i for i = 1, 2, ..., k ($n_1 + n_2 + \ldots n_k = n$) $\binom{n}{n_1, n_2, \ldots, n_k} = \frac{n!}{n_1!\, n_2! \ldots n_k!}$ therefore there are $\binom{n}{n_1, n_2, \ldots n_k}$ choices, each leading to the product $x_1^{n_1} x_2^{n_2} \ldots x_k^{n_k}$. Thus the multinomial theorem is attained. The numbers $\binom{n}{n_1, n_2, \ldots n_k}$ are called multinomial coefficients. Example: To expand $(p + q + r)^3$.

$(p + q + r)^3 = \sum \binom{3}{n_1, n_2, n_3} p^{n_1} q^{n_2} r^{n_3}$ where the sum is taken over all nonnegative integers n_1, n_2, n_3 such that

$n_1 + n_2 + n_3 = 3$. Hence $(p + q + r)^3 = \binom{3}{3,0,0} p^3 + \binom{3}{0,3,0} q^3 + \binom{3}{0,0,3} r^3 + \binom{3}{2,1,0} p^2q + \binom{3}{1,2,0} pq^2 + \binom{3}{2,0,1} p^2r + \binom{3}{1,0,2} pr^2 + \binom{3}{0,2,1} q^2r + \binom{3}{0,1,2} qr^2 + \binom{3}{1,1,1} pqr = p^3 + q^3 + r^3 + 3p^2q + 3pq^2 + 3p^2r + 3pr^2 + 3qr + 3qr^2 + 6pqr$.

<u>Mathematical instruments</u>. The use of instruments in the classroom provides the opportunity for the direct application of many principles which are too often discussed as mathematics for mathematics sake. Many pupils respond with increased interest to the applicative method of presentation and the use of the practical and concrete.

The following is an outline that could be given in a mathematics club meeting.

INSTRUMENTS FOR MATHEMATICS

I. Measuring Instruments

1. Ruler
   a. History
   b. Counting and measuring
   c. Possible error and precision of measurement

2. Protractor
   a. History
   b. Measurement of an angle
   c. Constructing an angle of a given size
   d. Types
   e. Choosing a protractor

3. Vernier
   a. History
   b. Construction
   c. Reading the vernier
   d. The linear vernier
   e. The circular vernier

II. Calculating Instruments

1. The Slide Rule
   a. History
   b. Construction
   c. Scales

2. The abacus
   a. History
   b. Types
   c. Construction
   d. Using

3. The Calculator
   a. History
   b. Construction
   c. Types
   d. Uses

III. Miscellaneous Instruments

1. The Pantograph
   a. History
   b. Theory
   c. Construction
   d. Types
   e. Uses

2. Parallel Rulers
   a. History
   b. Theory
   c. Use

3. The Center Square
   a. Theory
   b. Construction
   c. Use

4. Proportional Dividers
   a. History
   b. Theory

c. Scale
d. Types

5. The Steel Square
a. History
b. Scale
c. Use

IV. Field Instruments

1. The Transit
a. History
b. Construction
c. Types
d. Uses

2. The Sextant
a. History
b. Construction
c. Use
d. Types

3. The Hypsometer-Clinometer
a. History
b. Construction
c. Use

4. The Angle Mirror
a. History
b. Construction
c. Use

5. Tapes
a. Types
b. History
c. Use

6. Jacob's Staff
a. History
b. Use

7. Plane Table
a. History
b. Types
c. Use

8. Alidade
a. History

b. Types
c. Uses

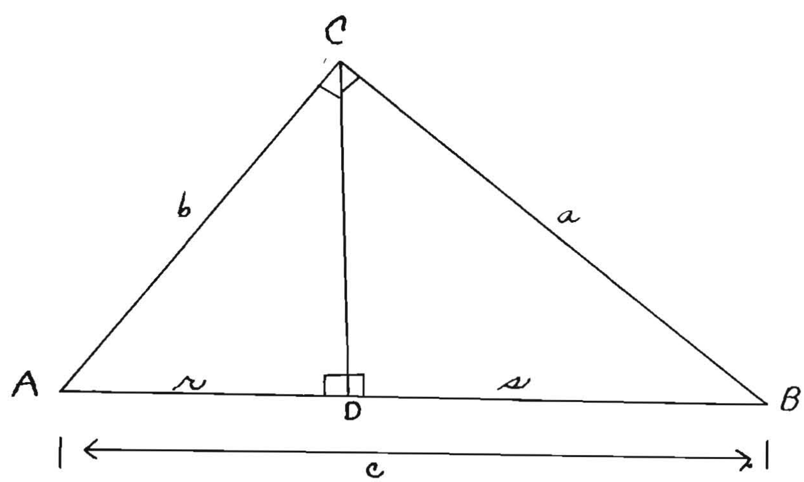
9. The Astrolabe
a. History
b. Use

<u>Pythagorean theorem</u>. The Pythagorean Theorem, in Geometry, states that in a right triangle the square of the hypotenuse equals the sum of the squares of the other two sides.

$$c^2 = a^2 + b^2$$

In the formula, c is the length of the hypotenuse and a and b are the length of the other two sides.

In Geometry, the Pythagorean theorem has many proofs. Following are a few of these proofs, which will make a very interesting and educational program for a mathematics club.



Given: $\triangle ABC$ with $\angle ACB$ a rt. $\angle$.

Prove: $c^2 = a^2 + b^2$

Proof: 1. Draw CD ⊥ AB

2. ΔABC, ΔBCD, and ΔADC similar

3. $\frac{c}{a} = \frac{a}{s}$ and $\frac{c}{b} = \frac{b}{r}$

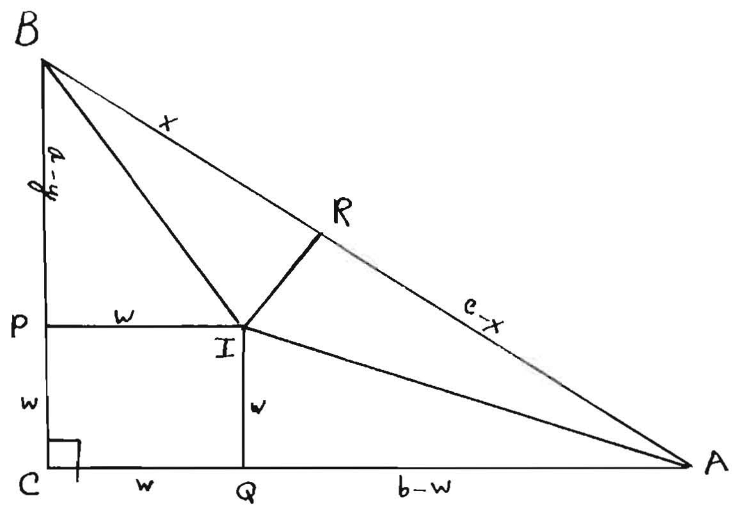
4. $a^2 = cs$ and $b^2 = cr$

5. $a^2 = b^2 = cs + cr$

6. $a^2 + b^2 = c(s + r)$

7. $s + r = c$

8. $\therefore a^2 + b^2 = c^2$



Given: Rt. ΔABC with rt. ∠ at C

I as center of inscribed circle

PQR points of contact of this circle with the sides of Δ

Prove: $c^2 = a^2 + b^2$

Proof: IR = IP = IQ = W = y

Area of ΔABC = area ΔAIB + area AIQ + area of trapezoid BIQC

$\frac{1}{2}ab = \frac{1}{2}cw + \frac{1}{2}w(b - w) + \frac{1}{2}w(a + w)$

$ab = cw + cwb - w^2 + w + w^2$

$x = a - y$

$w - y$

$c - x = b - w$

$x + w + c - x = a - y + y + b - w$

$2w = a + b - c$

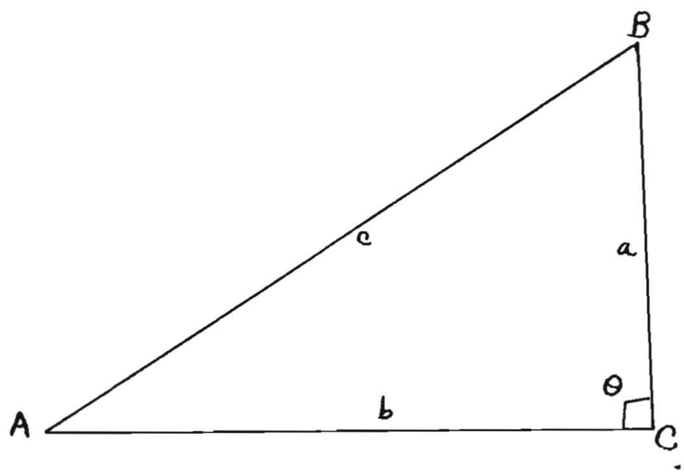
$w = \frac{1}{2}(a + b - c)$

$ab = \frac{c}{2}(a + b - c) + \frac{b}{2}(a + b - c) + \frac{a}{2}(a + b - c)$

$2ab = ac + bc - c^2 + ab + b^2 - bc + a^2 + ab - ac$

$2ab = a^2 + b^2 - c^2 + 2ab$

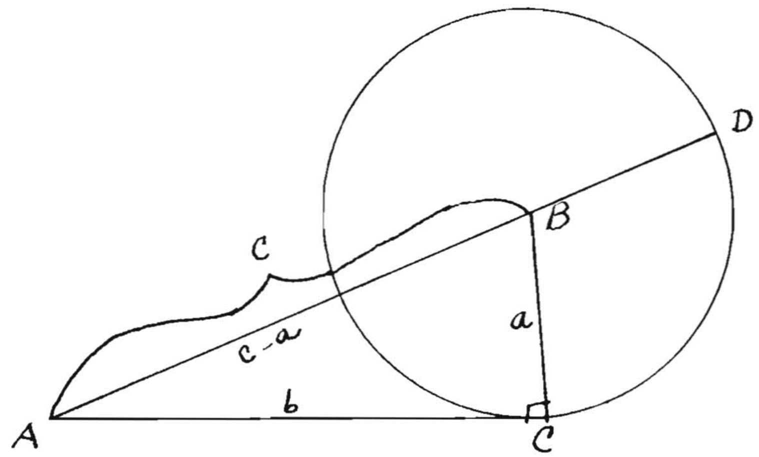
$c^2 = a^2 + b^2$



Given: Rt. Δ ABC

Prove: $c^2 = a^2 + b^2$

Proof: By use of Law of Cosines $c^2 = a^2 + b^2 - 2ab\cos\theta$



Given: $\triangle CBA$ with rt. $\angle$ ACB and sides a, b, c

Prove: $c^2 - a^2 = b^2$

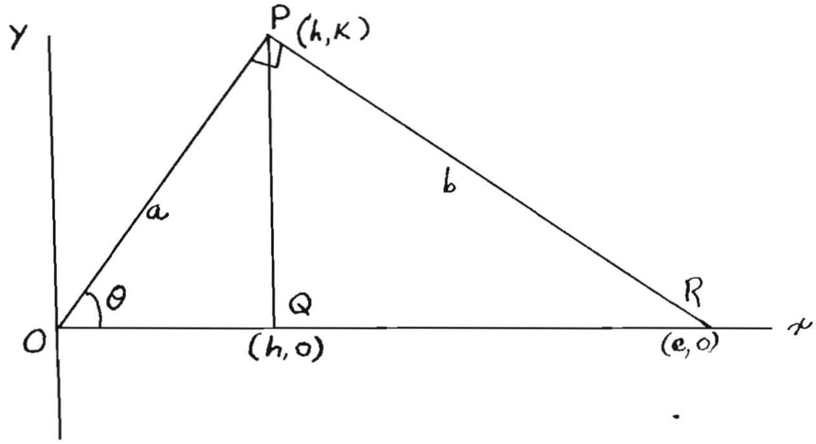
Proof: Construct a circle with radius a and center B

Extend AB to point D on the circle

$$\frac{c + a}{b} = \frac{b}{c - a}$$

$$(c + a)(c - a) = b^2$$

$$c^2 - a^2 = b^2$$



Let $\angle$ POR be θ

Since $\angle$ OPR is a rt. $\angle$ $\tan\theta = \frac{b}{a}$

Equation of line OP; $y = (b/a)x$

PR $\perp$ OP, so PR has a slope of $- a/b$

PR passes through (C,0)

Equation for PR is $y = - a/b\ x + ac/b$

Solving the two equations simultaneously

$$y = \frac{bx}{a} \quad \text{and} \quad y = \frac{ax}{b} + \frac{ac}{b}$$

$$\frac{bx}{a} = - \frac{ax}{b} + \frac{ac}{b}$$

$$bx^2 = -a^2x + a^2c$$

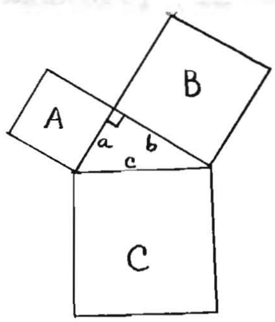
$$= a^2x + b^2x = a^2c$$

$$x(a^2 + b^2) = a^2c$$

$$x = \frac{a^2c}{a^2 + b^2}$$

$$y = b/a \cdot \frac{a^2c}{a^2 + b^2} \cdot y = \frac{abc}{a^2 + b^2}$$

$A = 1/2\ ab$ or $1/2\ kc$



Given: C is the square on the hypotenuse and A and B are the square on the legs of right triangle abc

Prove: $C = A + B$

Proof: $A/C = \frac{a^2}{c^2}$ and $\frac{B}{C} = \frac{b^2}{c^2}$

$$\frac{A+B}{C} = \frac{a^2 + b^2}{c^2}$$

$$a^2 + b^2 = c^2$$

$$\frac{A+B}{C} = \frac{c^2}{c^2}$$

$$A + B = C$$

Modulo systems. This is a new system of arithmetic, so instead of using the = (equal) sign, the congruence sign (≡) is used.

The rules for addition in this system are the same as those for ordinary addition, except that, if the sum is equal to or larger than the particular modulo system, then the sum is divided by the particular modulo used, the quotient discarded, and the remainder is used in place of the ordinary sum. Example: in the mod eight system,

$3 + 4 \equiv 7 \pmod 8$

$5 + 6 \equiv 3 \pmod 8$ (11 divided by eight is equal to one and 3 remainder. Keep only the remainder.)

The rules for multiplication in a particular modulo system are like those of ordinary multiplication, except that, if the product is equal to or larger than the particular modulo used, the product is divided by the particular system and the remainder is used in place of the ordinary product. Example: in the mod four system,

$3 \times 1 \equiv 3 \pmod 4$

$3 \times 2 \equiv 2 \pmod 4$ (6 divided by four is equal to one and 2 remainder. Keep only the remainder.)

There are no negative numbers in the modulo systems because an ordinary number like -5 is the solution of the ordinary equation $x + 5 = 0$. Therefore in the mod 8 system x would be 3.

There are no fractions in the modulo system because the ordinary fraction like 3/4 is the solution of the equation $4x = 3$. Example: $x \equiv 6 \pmod 7$.

The modulo systems are a finite set of numbers.

In some of the modulo systems there exists equations that have no solutions. Example: $X^2 \equiv 6 \pmod 7$.

The following are some addition and multiplication tables in different modulos systems, and also some problems to solve.

Mod 3

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

.	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Mod 4

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

.	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Mod 5

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

.	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Mod 6

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

Mod 7

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

.	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

Mod 8

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

.	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

Mod 9

+	0	1	2	3	4	5	6	7	8
0	0	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8	0
2	2	3	4	5	6	7	8	0	1
3	3	4	5	6	7	8	0	1	2
4	4	5	6	7	8	0	1	2	3
5	5	6	7	8	0	1	2	3	4
6	6	7	8	0	1	2	3	4	5
7	7	8	0	1	2	3	4	5	6
8	8	0	1	2	3	4	5	6	7

.	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8
2	0	2	4	6	8	1	3	5	7
3	0	3	6	0	3	6	0	3	6
4	0	4	8	3	7	2	6	1	5
5	0	5	1	6	2	7	3	8	4
6	0	6	3	0	6	3	0	6	3
7	0	7	5	3	1	8	6	4	2
8	0	8	7	6	5	4	3	2	1

Mod 10

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	0
2	2	3	4	5	6	7	8	9	10	0	1
3	3	4	5	6	7	8	9	10	0	1	2
4	4	5	6	7	8	9	10	0	1	2	3
5	5	6	7	8	9	10	0	1	2	3	4
6	6	7	8	9	10	0	1	2	3	4	5
7	7	8	9	10	0	1	2	3	4	5	6
8	8	9	10	0	1	2	3	4	5	6	7
9	9	10	0	1	2	3	4	5	6	7	8
10	10	0	1	2	3	4	5	6	7	8	9

.	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	1	3	5	7	9
3	0	3	6	9	1	4	7	10	2	5	8
4	0	4	8	1	5	9	2	6	10	3	7
5	0	5	10	4	9	3	8	2	7	1	6
6	0	6	1	7	2	8	3	9	4	10	5
7	0	7	3	10	6	2	9	5	1	8	4
8	0	8	5	2	10	7	4	1	9	6	3
9	0	9	7	5	3	1	10	8	6	4	2
10	0	10	9	8	7	6	5	4	3	2	1

Problems:

$3x \equiv 7 \pmod{11}$

$2x \equiv 6 \pmod{8}$

$x^2 \equiv 4 \pmod{8}$

$2x^2 \equiv 5 \pmod{9}$

$x^3 \equiv 6 \pmod{7}$

$2x \equiv 3 \pmod{5}$

$x^2 + x \equiv 1 \pmod{5}$

$2x + 1 \equiv 2 \pmod{3}$

$x^3 \equiv 2 \pmod{3}$

<u>Articles and program materials</u>. A mathematics club can be of invaluable assistance in motivating the pupil's curiosity and imagination in mathematics. Knowing that the club is designed for the purpose of following up the special interests of its members, the pupil feels perfectly free to present his problem to the group where it may be subdivided, if necessary and given to several pupils who have a common interest in that type of problem. The following is a list of topics that could be used for programs in a mathematics club or for newspaper articles.

History of arithmetic, algebra, geometry, trigonometry and other branches of mathematics.

The origin and development of our Arabic Numbers.

Our complete number system: the history of zero, the history of the decimal point.

Development of the abacus: the dust table, wax tablet, line abacus, the slate.

The Abacus of China, Japan, Russia, and Rome.

Cube Root.

Origin and development of the various mathematical signs.

Origin and use of X for the unknown.

Mathematics library.

Mathematics of the Hindus, Arabs, Greeks, Egyptians, and Romans.

Mathematical theory of engineering instruments.

Famous problems of antiquity: trisecting an angle, duplication of a cube, and squaring a circle.

Famous mathematicians and their contributions: Euclid, Plato, Newton, Descartes, Archimedes, Pythagoras, Pascal, Leonardo of Pisa, Stifel, Einstein, Linvitz.

Mathematical Programing.

The measuring instruments of long ago.

General principles of railroad engineering.

Old Greek mathematics and the influence of Christianity.

Women mathematicians.

Mathematical prodigies.

Prime numbers.

Napier and his rods.

Mathematical films.

Archimedes and some of his inventions.

The four fundamental operations with integers.

Origin and development of the idea of "per cent."

Mathematical terms: What is an axiom?

The role of definitions in mathematics.

Graphic records and plotting of curves, types, construction of, uses, and illustrations.

Mathematical concepts and their validity.

Graphical methods.

Nature and uses of formulae.

Theory and examples of indirect measurement.

Approximate nature of a direct measurement.

Mathematics in the training for citizenship.

Study of codes and ciphers.

Mathematics in poetry, literature, art, and music.

"House Project:" buying lot, surveying it, planning the excavation, cooperating with craftsmanship, architecture, blue print, gardening, art and home economics clubs on the plans for the house, estimating the cost of building, finishing, landscaping, insuring, and equipping it with furniture.

The fourth dimension.

A small house may be actually built.

Gambling.

Mathematics and efficiency.

Methods in the calculus.

Mathematics in artillery science.

The binary system.

Marine mathematics: finding position at sea; sue of sextant.

Mathematics of the sun dial.

Mathematics of the calendar.

History of our calendar.

Mathematics of the top.

Mathematics of the carpenter's square.

The theory of equations.

Measurement of the earth.

The cultural values of mathematics.

Proving the curvature of the earth and determining the earth's radius.

Mathematics as a mode of thought.

Mathematical jokes, conundrums.

Mathematics in our everyday life.

Mathematics in nature.

The Peaucellier and allied linkages.

The theory of probability.

Mathematics in war.

Dynamics of geology.

Geometric forms in botany.

Non-Euclidean Geometry.

Mathematics in agriculture, industry, chemestry, art.

Paper folding.

Mathematics and the discovery of Jupiter.

Mathematics in physics.

Mathematics in architecture.

Mathematics and astronomy.

Mathematics in map drawing.

The global concept.

Standardized tests and measurement in mathematics.

Theory of groups.

Determinants.

Concerning the Moebius Band.

The number of significant figures in an indirect measurement.

Highest possible degrees of accuracy.

Degrees of accuracy required in engineering, the sciences, business, manufacturing, and other vocations.

Can we prove that minus 3 (-3) times minus 5 (-5) is equal to plus 15 (15)?

The concept of function and examples.

Discussion of integral and fractional exponents.

Perspective and Projection.

The normal probability or Gaussian curve.

How logarithms are calculated.

Repeating fractions.

The calculation of Pi.

Einstein and his theory.

Mathematical menu.

The laws of algebra.

The nine-point circle.

Guessing contest: of distances, weights, time pitch, etc.

What is a limit?

Solution of problems by intersections of loci.

Zeno's paradoxes.

The Pythagorean Theorem and its various proofs.

Construction of models.

The geometrical representation of complex numbers.

Sexagesimal system.

Modulo systems.

Number systems to different bases.

What does the mathematician mean by infinity?

Why the expression a divided by 0 is absolutely meaningless.

Field trip to study mathematics in nature.

Field trips to aircraft industries, insurance companies, and construction sites.

The practical uses of variation.

Permutations and combinations.

Frequently made errors in Algebra or Geometry.

Members act as teachers' assistants, and also assist pupils needing help.

Vocations in mathematics: engineering, teaching and others.

Preparing for mathematical vocations.

A round table on "How I study mathematics," "What I expect to gain through the study of mathematics."

Mathematical games: Buzz, Simon Says, Old Maid, Blackboard Relay, Math. shark, Math. down, and authors.

Plays and dramatizations given in club meetings.

Short cuts in multiplication. Multiplying by 11, 9, 99, by fractors, by 21, 31, 61, etc., squaring numbers ending in 5, mixed numbers, etc.

Magic Squares, Cubes, and Circles.

Tricks with nines.

Slide Rule.

Suppose our radius were 8 instead of 10.

Debate: Resolved that the metric system should be adopted.

Construction and use of, and practice on and with the mathematical instruments: slide rule, abacus, transit, comptometer, circular slide rule, parallel ruller, compass, protractor, plainimeter, calculating machines, trinometer, harmonic analysis.

Contests in mathematics for speed, accuracy, logic, and other abilities.

Some tricks with the slide rule.

Hilbert's approach to congruence.

Theory of sets.

Fields.

Electrical circuits.

Properties of integers.

Fractions.

The symbols of algebra.

Negative numbers.

The number line.

Irrational numbers.

Mathematical bingo.

Imaginary numbers.

Involution.

Evolution.

Unknown quantities: equations.

Equations of the second degree.

Mathematics information please.

The Binomial Theorem.

Mathematics scavenger hunt.

Parallels.

Triangles.

Quadrilaterials.

Constructions with the use of rules and compasses.

Truth tables.

The circle.

Topology.

The computation of areas.

Bulletin board.

Exhibits.

Trigonometric tables.

The uses of mathematics in other school subjects.

Nomographs.

Ceva's theorem.

Menelaus' theorem.

Euler's line.

The Platonic Solids.

The Golden Section.

Quinxunx.

Optical illusions.

Tower of Hanoi.

Unicursal puzzles.

CHAPTER V

MATHEMATICS CLUBS IN THE STATE OF KANSAS

From the surveys conducted by letter and by personal interviews, it was found that most Kansas High Schools do not have mathematics clubs and haven't any future plans to organize any such club. The few schools that do have a mathematics club are using the club very constructively to influence the students in mathematics.

The following tables give the results of the survey taken in 1959-60 school year in the High Schools of Kansas. One hundred and sixty-one high schools were interviewed. The list of schools is included in the appendix.

13.25% of Class A+ High Schools had a mathematics club.
6.66% of Class A High Schools had a mathematics club.
4.00% of Class B High Schools had a mathematics club.
0.00% of Class C High Schools had a mathematics club.
8.69% of all the schools surveyed had a mathematics club.

Some of the major reasons that were given for not operating a mathematics club were: No time, School too small, Already enough clubs, No need for them, No pupil interest, and No teacher interest.

The "no time" excuse was given by 16.92 per cent of the schools surveyed. Time should be placed aside for a mathematics club, just as time is set aside for sports, music, and other organized classes. Most schools need a mathematics club to help the brighter students from becoming bored, and to keep the students interested in mathematics. In this day and age, mathematics is very important. Many brillant minds are lost to mathematics because the mathematics instructor is not making the subject interesting and exciting to the student.

Many teachers believe that this can be accomplished in the classroom without a mathematics club, but unless the classes are grouped it seems that this cannot be accomplished. In the grouped classroom you can teach to the level of the class and possibly a mathematics club will not be needed, because it can be incorporated into the classroom, but still the students not in this class that still have an interest in mathematics will be left out. This is not fair to all the students.

In the un-grouped classroom we teachers have to contend with all levels of students and therefore the brighter students will become bored unless a mathematics club is organized to give the brighter student a way to explore mathematics on his own and in an advanced degree. This cannot be accomplished in the classroom with all levels of pupils.

At the same time the mathematics club gives the student a sense of pride because he belongs to the organized club. This gives him added initiative to work harder and develop mathematical ideas. This is the greatest advantage over supplement work in the classroom for the superior student.

The "no teacher interest" and "no pupil interest" reasons were given by 38.44 per cent of the schools surveyed. Any teacher who doesn't have an active interest in organizing a mathematics club really doesn't have their heart in the right field. A mathematics teacher must take an active interest in mathematics and a mathematics club is part of mathematics. In some schools the mathematics teacher is also a coach or has other fields to teach, so therefore, they will not be as interested in a club, but they should realize they are denying their students a chance to explore the depths of mathematics. The mathematics club would serve these teachers who teach more than one field, by giving the brighter student the needed push to have them continue in the field of mathematics. Because of the dual duties of teaching in more than one field the instructor doesn't have time to prepare much outside material or supplemental material for the students. With a mathematics club the teacher could allow the students to prepare reports, problems, etc., with his or her supervision, and then present them to the club. In this manner the students would take pride in their accomplishments and proceed much harder to acquire more knowledge of mathematics. Also the busy teacher could have the brighter students in the mathematics club give reports to classes or even teach the class.

The "no pupil interest" reason is a problem brought about by the teacher. With a good mathematics teacher, who is inspiring, willing to work hard for the students, and a good leader, the students should be very interested in a mathematics club. The pupils have to be led at first and when they have had a taste of the problem at hand they will take the lead and go the rest of the way alone. A bright student wants to be challenged, and also wants to accomplish things on his own. This gives the pupil the initiative, interest, and willingness to continue on in mathematics. When the pupils know they are responsible for the clubs continued progress, they are willing to work hard to make a name for their organization. They realize the harder they work the more they gain and the better organization they will construct. This gives the student pride in belonging to such an organization and the students will want to belong. The teacher must develop the interest in the students in mathematics, and if the teacher fails to do this, he has also failed in the classroom teaching of mathematics.

The "already enough clubs" excuse was given by 5.88 per cent of the schools surveyed. This argument is very true from the standpoint of the number of clubs. Like any organization the club is only as good as its leaders. A mathematics club could be organized and planted in the school program with the minimum of problems. There are many clubs that are organized in the high school for the purpose of taking up school time. If one can organize a mathematics club and prove it to be fun and educational it will be a big success. The mathematics club could meet a few times a month in the evenings or during the noon hour to start with, and after a foothold in the school is attained then one can ask for a better or more convenient time to meet. Time can always be found during the school day for the mathematics club meetings if the leaders of the club are forceful enough to really want its success. The mathematics teacher must realize that the mathematics club is just as important, if not more so, as any other club or organization in the school. The teacher must also realize that he is working in a field that is in the spotlight of the world today.

The "school too small" excuse was given by 5.22 per cent of the high schools surveyed. Even if you have one student who is willing to learn above the material in the classroom, a club should be started. After it is started others might decide to join and before long a fully organized mathematics club will be in the school. Teachers must remember that working with only a few is better than not having any to work with. Of course, with only a few the structure will not be as complete as a large mathematics club, but the club will still have the initiative and willingness to learn and work as a large club. These characteristics should be exploited to the utmost degree.

The "have a science club" reason was given by 2.48 per cent of the high schools surveyed. The science club is for the science pupils and therefore it will not take the place of a mathematics club. If it was possible to organize a mathematics and science club, it would be worth-while. The meeting and programs could be alternated between the mathematics department and the science department. This is not the best setup but it could be a good start to expand on in the future.

The "board against it" reason was given by .62 per cent of the high schools surveyed. The schools that gave this reason will have a hard uphill battle to fight. They're going to have to start by educating the board and community and gaining their support first. Giving and showing examples of successful mathematics clubs and what they are accomplishing at Parent Teachers Meetings is an excellent place to start.

CHAPTER VI

CONCLUSION

Mathematics clubs provide an excellent means of stimulating and fostering mathematical study. It can be a rewarding experience to both the students and the sponsor. It provides the time needed to go into the deeper and more exciting phases of mathematics.

The club should be organized to assist its members in learning mathematics. The mathematics club should become the proving ground for the discovery and cultivation of skills and talents based on mathematics, a place to prepare for careers and hobbies in mathematics.

Every member of a mathematics club should have the opportunity to develop the skills, talents, and aptitudes he possesses.

The survey taken shows that an extremely small percentage of schools have a mathematics club, or do any kind of extra-curricular work of a mathematical nature. The possibility or necessity of organizing a club seemingly has received little consideration from many mathematics teachers and school administrators. This may be due mainly to the fact that they have not seen one already set up or have not heard about one. Therefore they have not been convinced of the benefits of a mathematics club to their curriculum.

or they may doubt that a real and lasting interest can be attained in a club of such academic nature.

There is no doubt that this interest and enthusiasm developed in the mathematics club motivates interest in the classroom and helps establish in the mind of the pupil a much more receptive attitude toward both its study and its teaching. Attaining results like these, the mathematics club is a valuable teaching aid, and more important still a valuable aid to learning. It is a solution to the problem of the gifted pupil.

In the following paragraph is a quote by Paul Charles Elliott, member of the Garfield High School Mathematics Club, Terre Haute, Indiana. Paul says:

> I sincerely appreciate the opportunity of being a member of the Garfield High School Mathematics Club. Since first joining the organization over two years ago, I have gained much knowledge not only of mathematics but in related fields also. This experience in the mathematics club, I am sure, will be of great assistance to me when I continue my education in college. One of the many values which the mathematics club is to the student is the opportunity which the student is given to express himself when he gives his individual report to the club. Also, the student is given a much greater opportunity for further study in specialized fields of mathematics. The theory of relativity, the fourth dimension, the laws of probability, the slide rule, and topology, to mention only a few, were studied by the students with a great benefit to all. Furthermore, the speakers who come from the various colleges were of great help to the students in the way in which we were enlightened on the subjects for advanced study of mathematics in college. All of these values combined in a spirit of cooperation are sure to provide fun for everyone concerned. But, perhaps, the greatest dividend which we gained from our

work in this organization, has not yet been mentioned. From our association with mathematics teachers in this organization we were able to overcome that age old superstition that teachers of mathematics were supernatural, and we found them to be real live human beings.[1]

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[1] Inez Kelley, "Garfield Mathematics Club," The Mathematics Teacher 45; Page 38, January 1952.

BIBLIOGRAPHY

BIBLIOGRAPHY

BOOKS

Bakst, Aaron. Mathematical Puzzles and Pastimes. New York: D. Van Nostrand Company, 1954.

Ball, W. W. Rouse. Mathematical Recreations and Essays. New York: Macmillan Company, 1905.

Goldberg, Samuel. An Introduction Probability. Englewood Cliffs: Prentice-Hall, 1960.

Jones, Samuel I. Mathematical Wrinkles. Nashville: Samuel I. Jones, Publisher, 1930.

Kemeny, John G., and others. Introduction to Finite Mathematics. Englewood Cliffs: Prentice-Hall, 1957.

PERIODICALS

Ann Agnes, Sister. "Archimedeans," Mathematics Teacher, Vol. 27, May, 1954, pp. 366-7.

Anning, Norman. "High School Mathematics Club," Mathematics Teacher, Vol. 26, February, 1933, pp. 70-76.

Barnes, Augusta. "Making Mathematics Interesting," Mathematics Teacher, Vol. 17, pp. 404-410, November, 1942.

Batz, Muriel. "A Christmas Project," Mathematics Teacher, Vol. 30, December, 1937, pp. 376-7.

Bentley, Byron. "Recreations for the Mathematics Club," Mathematics Teacher, Vol. 23, February, 1930, pp. 95-103.

Boyd, James R. "A Mathematics Club for the Able," Mathematics Teacher, Vol. 45, October, 1952, pp. 457-60.

Giffle, W. J. "Meet the Slop Sticks," Mathematics Teacher, Vol. 51, April, 1958, pp. 306-7.

Gugle, Marei. "Recreational Values Achieved through Mathematics Clubs in Secondary Schools," Mathematics Teacher, Vol. 19, April, 1926, pp. 214-18.

Hatton, Mary Caroline. "A Mathematics Club," Mathematics Teacher, Vol. 20, January, 1927, pp. 39-45.

Hofeditz, P. H. "Sequoria Junior High School Mathematics Team," Mathematics Teacher, Vol. 49, October, 1956, pp. 473-4.

Holroys, Phyllis A. "The Christmas Party," Mathematics Teacher, Vol. 32, December, 1939, pp. 352-3.

Hoog, Ruth. "Sources of Program Material and Some Types of Program Work Which Might Be Undertaken By High School Mathematics Clubs," Mathematics Teacher, Vol. 24, December, 1931, pp. 492-502.

Johnson, Donovon A. "Lets Do Something for the Gifted in Mathematics," Mathematics Teacher, Vol. 48, May, 1953, pp. 322.

Kelley, Inez. "Challenging the Gifted Student," School Life, Vol. 35, November, 1952, pp. 27-8.

Kelley, Inez. "Garfield Mathematics Club," Mathematics Teacher, Vol. 45, January, 1952, pp. 37-8.

Klatz, "Mathematics Clubs for High School," School Activities, Vol. 21, October, 1949, pp. 59-61.

MacKenzie, E. G. "Builder of an Enquiring Mind," Mathematics Teacher, Vol. 48, February, 1955, pp. 109-11.

Manheimer, Wallace. "A Club Project in a Modern Use of Mathematics," Vol. 50, May, 1957, pp. 350-5.

Mary Stephanie, Sister. "Activities for Mathematics Clubs," Catholic School Journal, Vol. 56, September, 1956, pp. 214-5.

Owen, Evelyn L. "A High School Mathematics Club," Mathematics Teacher, Vol. 45, October, 1952, pp. 457-60.

Person, Ruth. "Junior H. S. Mathematics Clubs," Mathematics Teacher, Vol. 34, May, 1941, pp. 228-9.

Porterfield, J. M. "Fun For The Mathematics Club," Mathematics Teacher, Vol. 37, December, 1944, pp. 354-7.

Price, H. Vernon. "Mathematics Clubs," Mathematics Teacher, Vol. 32, November, 1939, pp. 324.

Reed, Zulu. "High School Mathematics Clubs," Mathematics Teacher, Vol. 18, October, 1925, pp. 341-362.

Reflor, Sophia. "Mathematics Clubs in the High School," Mathematics Teacher, Vol. 15, November, 1922, pp. 434-5.

Riter, H. E. "The Enrichment of the Mathematics Course," Mathematics Teacher, Vol. 31, January, 1938, pp. 3-6.

Shelton, Julia B. "A History of Mathematics Chart," Mathematics Teacher, Vol. 52, November, 1959, pp. 49-53.

Snell, C. A. "Mathematics Clubs in the High School," Mathematics Teacher, Vol. 8, December, 1915, pp. 73-8.

Stephens, H. W. "Mathematics Club for Future Mathematicians," School Science and Mathematics, Vol. 54, December, 1954, pp. 715-18.

Sullivan, O. A. "High School Mathematics Club," Mathematics Teacher, Vol. 35, October, 1942, pp. 275-6.

Sweedler, Ester. "The Mathematics Club at Curtis High School," Mathematics Teacher, Vol. 29, December, 1936, pp. 394-5.

Webster, Louisa M. "Mathematics Clubs," Mathematics Teacher, Vol. 10, June, 1917, pp. 203-8.

Weiss, Max F. "The Mathematics Club Streamlined," High Points, Vol. 21, October, 1939, pp. 74-7.

Wheeler, A. Harry. "Mathematics Clubs Program," Mathematics Teacher, Vol. 16, November, 1923, pp. 385-90.

Williams, John Annie. "Organizing a Mathematics Club," Mathematics Teacher, Vol. 49, February, 1956, pp. 149-50.

PRIVATE COMMUNICATION

Brune, Irvin H., of Iowa State College, Cedar Falls, Iowa.

Douglas, Karl R., of Colorado University, Boulder, Colorado.

Fawcett, Harold, of Ohio State University, Columbus, Ohio.

Hunt, George R., of Odessa College, Odessa, Texas.

Kitchens, Nellie M., of Hickman High School, Columbia, Missouri.

Krickenberger, W. R., of The Arsenal Technical High School, Indianapolis, Indiana.

Lewis, Harry, of East Side High School, Newark, New Jersey.

Meder, Jr., Albert E., of Rutgers University, New Brunswick, New Jersey.

Pratt, Virginia Lee, of Central High School, Omaha, Nebraska.

Wren, F. Lynwood, of Geroge Peabody College, Nashville, Tennessee.

APPENDIX

MATHEMATICS CLUB SURVEY OF KANSAS HIGH SCHOOLS

CONDUCTED 1959-60

Name of School	County	Size	Rat-ing	No. of Math. Courses	Math. Club	Reason if "No."
Abilene	Dickinson	460	A+	8	No	No time
Altoona	Wilson	75	C	4	No	No interest
Andale	Sedgwick	170	A	4	No	
Anthony	Harper	240	A+	6	No	No demand
Arkansas City	Cowley	725	A+	6	No	No interest
Ashland	Clark	118	A+	5	No	No reason
Atchison	Atchison	950	A+	10	Yes	Math and Science
Auburn	Shawnee	65	B	4	No	
Augusta	Butler	550	A+	6	No	No need
Barnes	Washington	60	B	4	No	No time
Baxter Springs	Cherokee	312	A+		No	
Belle Plaine	Sumner	170	A+	5	Yes	
Bentley	Sedgwick	40	C	4	No	No interest
Bird City	Cheyenne	100	A+	4	No	No demand
Blue Mound	Linn	110	A	5	No	No interest
Blue Rapids	Marshall	100	A+	5	No	No interest

Name of School	County	Size	Rat-ing	No. of Math. Courses	Math. Club	Reason if "No."
Bluff City	Harper	35	C	3	No	
Bonner Springs	Wyandotte	460	A+	6	No	No time
Brewster	Thomas		A		No	
Buhler	Reno	375	A+	5	No	No pupil interest
Burlingame	Osage	114	A	4	yes	
Burlington	Coffee	220	A+	5	No	No interest
Burns	Marion		B	3	No	
Burrton	Harvey	95	A	6	No	No interest
Bushton	Rice	64	A	4	No	No time
Byers	Pratt	28	C	4	No	No time
Caney	Montgomery	256	A+		No	No interest
Cedarvale	Chautaugua	105	A+	4	No	
Chase	Rice	110	A	7	No	
Cheney	Sedgwick	134	A	3	No	No interest
Cherryvale	Montgomery	200	A+		No	No interest
Cimarron	Gray	115	A+	4	No	No time
Clearwater	Sedgwick	215	A+	3	No	
Clyde	Cloud	135	A+	5	No	No interest

Name of School	County	Size	Rat-ing	No. of Math. Courses	Math. Club	Reason if "No."
Codell	Rooks	35	C	3	No	No need
Coffeyville	Montgomery	900	A+	6	No	No need
Derby	Sedgwick	600	A+	11	Yes	
Columbus	Cherokee	550	A+	6	No	No need
Concordia	Cloud	360	A+	5	No	No sponsor
Conway Springs	Summer	145	A	5	No	No time
Coolidge	Hamilton	29	C	3	No	No interest
Courtland	Republic	84	A	4	No	No time
Damar	Rooks	58	B	5	No	No need
Deerfield	Kearny	44	B	5	No	No interest
Delia	Jackson	35	C	5	No	No interest
Dodge City	Ford	550	A+	6	No	No interest
Douglas	Butler	136	A+	6	No	No time
El Dorado	Butler	600	A+	6	No	No interest
Elk City	Montgomery	118	A	5	No	No teacher interest
Elwood	Doniphan	60	C	3	No	No time
Emporia	Lyons	625	A+	5	No	Just don't
Englewood	Clark	35	C	3	No	No interest

Name of School	County	Size	Rat-ing	No. of Math. Courses	Math. Club	Reason if "no."
Enterprise	Dickinson	35	B	4	No	Too many clubs
Ellsworth	Ellsworth	238	A+	5	No	No interest
Eureka	Greenwood	300	A+	5	No	No pupil interest
Everest	Brown	60	B	3	No	No interest
Fredonia	Wilson	240	A+	4	Yes	
Florence	Marion	73	A+	4	No	No time
Ford	Ford	52	A	4	No	
Fowler	Meade	90	A	4	No	No interest
Galena	Cherokee	275	A	5	No	
Galesburg	Neosho	42	B	4	No	No interest
Garden City	Finney	545	A+	6	No	Too many clubs
Garden Plain	Sedgwick	103	C	4	No	No time
Gardner	Johnson	176	A	5	No	No time
Gem	Thomas	30	C	4	No	Small enrollment
Geneseo	Rice	78	B	5	No	never started
Girard	Crawford	252	A+	6	No	
Gorham	Russell	50	B		No	No time

Name of School	County	Size	Rating	No. of Math. Courses	Math. Club	Reason if "No."
Grainfield	Gove	57	C	2	No	No interest
Great Bend	Barton	675	A+	6	No	No time
Greeley	Anderson	69	A	5	No	No time
Grenola	Elk	51	B	4	No	No time
Gypsum	Saline	75	A	4	No	
Hamilton	Greenwood	78	A+	4	No	No time
Hanover	Washington	102	A+	4	No	
Harper	Harper	198	A+	4	No	Too many clubs
Havana	Montgomery	38	B		No	Lack of interest
Hays	Ellis	650	A+	5	Yes	Science and Math.
Healy	Lane	30	C	3	No	Too small
Herndon	Rawlins	49	B	4	No	No time
Hill City	Graham	190	A+	5	Yes	Science and Math.
Hoisington	Barton	378	A+		No	Too many clubs
Holcomb	Finney	87	A	3	No	No interest
Hollenberg	Washington	28	C	3	No	No interest
Horton	Brown	122	A+	4	No	Forming one

Name of School	County	Size	Rat-ing	No. of Math. Courses	Math. Club	Reason if "No."
Hoyt	Jackson	60	C	4	No	No interest
Hudson	Stafford	67	B	7	No	Have projects
Hugoton	Stevens	280	A+	8	No	Too many clubs
Hutchinson	Reno	1250	A+		No	No interest
Iola	Allen	385	A+	5	No	No demand
Ingalls	Gray	54	B	5	No	
Jennings	Decatur	56	B	4	No	None needed
Junction City	Geary	625	A+	5	No	No interest
Kanopolis	Ellsworth	55	B	4	No	never started
Kingman	Kingman	410	A+	9	Yes	
Kinsley	Edwards		A+	6	No	No time
Kiowa	Barber	107	A+	5	No	Too many clubs
Kirwin	Phillips	44	C	2	No	No interest
Kismet	Seward	55	C	3	No	No time
Lacrosse	Rush	157	A+	4	No	No interest
Lacygne	Linn	130	A	5	No	No time
Laharpe	Allen	60	B	3	No	

Name of School	County	Size	Rat-ing	No. of Math. Courses	Math. Club	Reason if "no."
Lamont	Greenwood	32	C	3	No	No interest
Larned	Pawnee	280	A+	4	No	No interest
Lawrence	Douglas	1008	A+	6	No	No demand
Leavenworth	Leavenworth	900	A	6	No	
Lebo	Coffey	105	A	5	No	Organi-zing one
Lenora	Norton	93	A+	6	No	Not nec-essary
Liberal	Seward	512	A+	6	No	No interest
Lyons	Rice	305	A+	5	No	No interest
Manhattan	Riley	700	A+	6	No	No interest
McPherson	McPherson	649	A+	5	No	No interest
Meade		136			No	No interest
Minneola	Clark	50	B	3	No	
Moline	Elk	68	A	5	No	Too many clubs
Moran	Allen	111	A	5	No	No time
Mound City	Linn	100	A	6	yes	
Moundridge	McPherson	185	A	5	No	
Mullinville	Kiowa	72	A	4	No	Lack of interest

Name of School	County	Size	Rat-ing	No. of Math. Courses	Math. Club	Reason if "No."
Mulvane	Sedgwick	310	A+	7	No	Too many clubs
Nashville	Kingman	28	C	4	No	No interest
Natoma	Osborne	78	A+	5	No	
Neodesha	Wilson	511	A+	5	No	Clubs limited by board
Ness City	Ness	178	A+	6	Yes	Science and Math.
Newton	Harvey	600	A+	5	No	No interest
Norwich	Kingman	63	A	4	No	Too many activities
Oberlin	Decatur	245	A+	5	No	
Olathe	Johnson	420	A+	6	Yes	
Ottawa	Franklin	695	A+	6	No	
Paola	Miami	570	A+	7	Yes	
Park	Grove	46	B	4	No	Size of School
Parker	Linn	120	A+	6	No	Why have one
Partridge	Reno	106	A+	4	No	No interest
Pawnee Rock	Barton	100	A	5	No	No time
Peabody	Marion	130	A+		No	No interest

Name of School	County	Size	Rat-ing	No. of Math. Courses	Math. Club	Reason if "No."
Phillipsburg	Phillips	260	A+	6	No	No interest
Pittsburg	Crawford	500	A+	5	No	No interest
Portis	Osborne	36	C	5	No	Too many activities
Powhattan	Brown	84	C	3	No	No time
Pratt	Pratt	484	A+	5	No	
Prescott	Linn	43	B	3	No	No interest
Princeton	Franklin	57	B	3	No	Not feasible
Protection	Comanche	85	A	4	No	No interest
Riverton	Cherokee		A+	6	Yes	
Russell	Russell	450	A+	6	No	No need
Sabetha	Nemaha	190	A+	4	No	No interest
Scranton	Osage	70	B		Yes	
Sedan	Chautauqua	185	A+	4	No	No interest
Syracuse	Hamilton	180	A+		No	Lack of interest
Towanda	Butler	105	B	5	No	No interest
Valley Falls	Jefferson	151	A+	6	No	No interest

Name of School	County	Size	Rating	No. of Math. Courses	Math. Club	Reason if "No."
Waterville	Marshall	88	A+	4	No	
Whiting	Jackson	57	C		No	Too small
Wichita South	Sedgwick	1350	A+	11	No	No interest
Wichita West	Sedgwick	1500	A+	10	No	No time
Wichita Southeast	Sedgwick	1850	A+	11	No	No time
Winchester	Jefferson	90	A	6	No	No time
Winfield	Cowley	1200	A+	8	No	No interest
Yates Center	Woodson	230	A+	4	No	Never considered
Zenda	Kingman	40	B	4	No	No interest

Jimmy G. Merando
Kingman High School
Kingman, Kansas

Dear Sir:

I am writing a thesis on "Mathematics Clubs in the High Schools," and would appreciate your co-operation in answering the following questions.

Name of School____________________________________.

Size of School____________________________________.

Do you have a Mathematics Club?_______________________.

If not why____.

Qualification of members____________________________.

Projects undertaken_________________________________

____.

Do you charge dues?________________________________.

What mathematics courses are taught in your High School?____

____.

Thank you.

LIST OF SUGGESTED MATHEMATICS CLUB NAMES

Alphа

Alphabet

Angle

Archimedian

Blue Triangle

Binary

Circle

Compasses

Cube

Euclidean

Gaussian

Hexagon

Hi Pi

Magic Square

Naperian

Newtonian

Number Sense Club

Octagon

Perigon

Pi

Pyramid

Pythagorean

Sigma

Sphere

Square

Straightedge

The Einsteiners

Triangle

x

x^2