Respecification and Application Of a Model For Efficient Estimation In Presence Of Serial Correlation

By

Mohammed I. Chowdhury
and
Nancy Groneman

Abstract

Least Squares estimates are BLUE, provided the error terms satisfy a number of assumptions. In time series data, error terms may be correlated. In presence of serial correlation, OLS estimates and forecasts based on them are unbiased and consistent but become inefficient and as a result, the usual t and F tests become invalid. The model for this study uses per capita revenue air miles as a function of per capita income, average price per mile, accident rate per passenger mile of travel, number of fatalities from airplane accidents, and a period of airline regulation and deregulation. The model is tested for serial correlation and is then respecified to obtain more efficient estimates using Maximum Likelihood Method. Results show that per capita air miles is relatively less sensitive during a deregulatory period compared to a regulatory period; per capita air miles is highly sensitive to per capita income; and deregulation has increased air travel miles.

Introduction

Ordinary Least Square estimators of a linear regression model $Y = Xb + e$ are the best linear unbiased estimators (BLUE) if the regressors (X's) satisfy these conditions: (1) nonstochastic, (2) linearly independent, (3) limiting value of (X'X/T) is finite and nonsingular as the number of observation T increases to infinity, (4) e has a multivariate normal distribution, (5) expected value of the error terms are zero, and (6) the expected value of the variance covariance matrix terms are constant and zero. In particular, the estimates are unbiased, consistent, and most efficient.

Often times, with time series data, problems arise where basic assumptions on the disturbance terms are violated. Error terms being independent implies that they are uncorrelated. Error terms for periods not too far apart may be correlated and this property is known as serial correlation or autocorrelation. The presence of the error terms in the model may be due to (1) omission of variables (2) measurement errors (3) avoidance of non linearities and (4) unpredictable random effects. Serially correlated errors could arise from the first three sources mentioned above. Serial may be caused by misspecification of the functional forms and systematic errors in measurement.

Ordinary Least Square estimates and forecasts based on them are unbiased and consistent even if the error terms are serially correlated but the problem lies with the efficiency of the estimates. In the presence of serial correlation, the error sum of squares becomes smaller than its true value; and R-square value over estimates indicates a better fit than actually it is. Since the estimated variances of the regression coefficients are biased, and hence tests of hypotheses are invalid, the usual t and F tests tend to appear more significant than they really are. To investigate the presence and effect of serial correlation, domestic scheduled air passenger miles linked to price per mile, income per person, accidents, fatalities, regulation/deregulation have been taken. The deregulation of domestic airlines came into effect in 1978. Air passenger mileage has steadily been increasing over the last three decades from 31 billion miles in 1960 to 540 billion miles in 1995. This mileage has more than tripled since 1977. The average nominal price per passenger mile was only 6 cents in the 60's while it was slightly more than 12 cents in the mid 1990's.

The rise in nominal price per passenger mile has been accompanied by a steady rise in per capita income. Per capita income rose from $2,800 in 1960 to $22,536 in 1995. During the last three decades, nominal price of air passenger miles has doubled while nominal per capita income increased eightfold. This has had a positive impact on air passenger miles.

The average number of planes involved in an accident was 54 in the 1960's while it dropped dramatically since then to 29 in the 1970's and to 23 in the 1980's. Also the average number of fatalities dropped from 210 in the 1960's to 111 in the 1980's. This decrease in accident rates coupled with the decrease in fatalities contributed to more scheduled air passenger miles.

In a study, Adrangi et. al (Summer 1997) mentioned that deregulation increased competition, lowered air fares, and increased passenger revenue miles. Morrison et al (Fall 1997) found that under deregulation, travelers benefitted from better service, particularly increased flight frequency. El-Gazzar et al (Jun 1996) found that market favored airline deregulation. Joesch's (Summer 1994) empirical analysis indicated that during the 1980s, the market contestibility may have declined. The demand for air travel, on average, became more own price elastic over the 1980s for the 19 destination cities analyzed. Oum et al (May 1993) found that the price elasticities for the route aggregate demand ranges between 1.24 and 2.34 with an average value of 1.58. This study finds that under regulation, the MLE estimate of price elasticity of per capita air miles is -0.339 and OLS estimate of price elasticity is -0.69. During deregulation, the estimate of price elasticity is -0.43 under ML method and it is -1.59 under OLS method. Both estimation methods, MLE and OLS, indicate higher price elasticity during deregulation compared to regulation. Unlike other studies, this study finds that the income elasticity of air miles is higher under deregulation compared to that of regulation. In a regulatory period, the estimates of income elasticity generated by OLS and MLE correspond to 0.98 and 1.17. Results indicate higher sensitivity of income towards air miles exists during deregulation period than regulation period.

Model Specification and Estimation

In this research study, the initial model taken to estimate by Ordinary Least Squares is as follows:

(1) ln (x1/x2) = b0 +b1 ln(x3/x1) +b2 ln(x4/x2) + b3ln(x5/x1) +b4 x6 +b5 x7 + e where b's are scalar parameters. In this model, letters represent the following:

x1 represents domestic scheduled air passenger miles in billions
x2 represents U.S. population in millions
x3 represents operating revenue from passengers in millions of dollars
x4 represents gross national product of the United States in billions of dollars
x5 measures number of American planes involved in an accident
x6 is the number of fatalities from airplane accidents
x7 is a dummy variable representing airline regulation or deregulation
e is the random error variable
0 represents airline regulation period from 1960-1978
and 1 represents the deregulation period starting from 1979 and onward.

The ratio of (x1/x2)=x12 is the per capita air passenger miles; (x3/x1)=x31 reflects the average price per air mile; (x4/x2)=x42 measures per capita income; and (x5/x1)=x51 stands for the number of accidents per scheduled air passenger mile of travel.

Since per capita air passenger miles rose faster relative to the average price per mile, and per capita income, these variables are taken in logarithmic form. The model is semi-logarithmic as fatalities and regulation/deregulation are taken without transformation since those variables contain values of magnitude zero. Per capita scheduled air passenger mile is taken as a function of average price per air mile, per capita income, accident rate per air passenger mile, number of fatalities from airplane accidents, and airline regulation/deregulation.

Assuming air travel is a normal good (other things being equal, air travel is inversely related to average price per air mile), the coefficient b1 is expected to be negative as an increase in price in air miles should decrease the per person air travel miles; b2 is expected to be positive as an increase in income would increase travel by airlines. Only disproportionate increases in airplane accidents would probably decrease air travel miles and hence b3 is expected to be negative. b4 is expected to be negative as increases in the number of fatalities would decrease air travel.

Deregulated industries are expected to be more efficient as competition increases. Therefore, with price cut and/or other incentive programs, air travel should rise and so b5 is expected to be positive. However, the sign of b5 cannot be determined for regulated air travel.

The model in equation (1) would be estimated by OLS and errors would be tested for serial correlation. If serial correlation is present, the model would be estimated by the maximum likelihood method using autoregressive order of 1, 2, and 3 successively where the error terms would be expressed as

(2) $e_t = \sum \rho\, e_{t-i} + u_t$; $i = 1, 2, 3$
where u_t is iid with mean zero and variance σ^2 and $Cov(u_t\, u_{t-s})=0$ for t=/ s.

To test for structural change, interaction terms xrp, xrg, and xra are created. The variable xrp is the product term of regulation and logarithm of average price per passenger mile. Similarly, logarithm of per capita income and logarithm of accident rates are multiplied by regulation to create xrg and xra respectively. The model used for the purpose is as follows:

(3) log x12 = c0 + c1 log x31 +c2 xrp + c3 log x42 + c4 xrg + c5 log x51 + c6 xra + c7 x6 +c8 x7 + u
where c's are constant terms, x's are explanatory variables and u has the following structure.

(4) $u_t = \sum \rho\, u_{t-i} + v_t$; $i = 1,2,3$
where v_t is iid with mean zero and variance σ^2 and $Cov(v_t\, v_{t-s})=0$ for t=/ s.

Since Chow test does not give details of structural change, a dummy variable model is used. Introduction of one more interaction term, xrf in equation (3) yields the model as follows:

(5) log x12 = c0 + c1 log x31 +c2 xrp + c3 log x42 + c4 xrg + c5 log x51 + c6 xra + c7 xrf +c8 x6 + u
where xrf is the product of fatalities and a dummy regulation/deregulation variable.

Data Description

The data base for this study consists of variables relating to United States population in millions, Gross National Product in billions of dollars, domestic scheduled air carrier passenger miles measured in billions, number of scheduled domestic airline operators, operating revenue from domestic air passengers in millions of dollars, number of American planes involved in an accident, number of fatalities from scheduled passenger air carrier accidents, and a dummy variable to represent airline regulation and deregulation for the period 1960 through 1995. The data has been collected from the Statistical Abstract of The United States.

Results and Conclusion

Six different forms of equation (1) have been estimated and the results are presented in Table 1. Model 1 through Model 3 are estimated by Ordinary Least Squares (OLS) method. Model 4 through Model 6 are autoregressive scheme of order 3 and they are estimated by the maximum likelihood estimation (MLE) method.

The Model 1 of Table 1 is estimated by Ordinary Least Squares using stepwise regression. The parameter estimates of model 1 in equation (1) obtained by OLS are statistically significant at the 5% level for average price per mile and per capita income. The price elasticity of per capita air passenger mileage is relatively inelastic and negative with a magnitude of order 0.64. Per capita income elasticity (1.02) is positive and elastic indicating a percentage change in per capita income will have more than one and one quarter percent change in per capita air passenger mileage. The accident rate elasticity is -0.1768 and it is statistically significant at 5% level of significance.

Increase in airplane accident rates per passenger mile is expected to have a negative impact on air passenger miles and these are shown by the negative signs of the impact multipliers as shown in Table 1. The parameter associated with accident rates is significant only at the 20 percent level. The impact multipliers associated with fatalities and regulation/deregulation remained statistically insignificant. The negative influence of increase in fatalities on scheduled air carriers is shown by the negative sign of the parameter estimates. Since the influence of fatalities on scheduled air carriers is extremely small, stepwise regression did not pick fatality variable. As expected, the explanatory power of the model reflected by adjusted R^2 (0.98) is fairly high and all of the standard errors associated with different parameters except regulation/deregulation are relatively low. The value of F-statistic (437) is high and statistically significant. The presence of first order positive serial correlation is indicated by the Durbin-Watson statistic with a value of 0.965. The presence of systematic error patterns is shown by Cook's D in Chart 1.

The Model 1 has been corrected for serial correlation with autoregressive scheme of order one, two, and three consecutively; and then estimated by the Maximum Likelihood (ML) method. Only two models of order AR(3), Model 5 and Model 6, are reported in Table 1. All price elasticities of air travel under these three autoregressive schemes showed lower absolute value relative to that of the OLS estimate indicating relative insensitivity of price to air travel miles.

Like the OLS estimate, the income elasticities are highly elastic in the ML method under all autocorrelation schemes. The absolute values of the elasticity of accident rate remained close under both ML and OLS but showed different signs. The accident rate elasticities are not significant. The impact of fatalities remained negative on air travel miles under ML with all schemes as well as OLS method. The impact of deregulation is positive on air travel miles under OLS and ML with autoregressive scheme of order AR(2) and AR(3). Unlike accident rate and regulation/ deregulation, the t-statistics associated with price and income are statistically significant.

Since ML method is a non-linear estimation method, R-Square does not have much meaning and so this value cannot be compared with R-Square under OLS. Even though Durbin-Watson statistic under ML method with AR(2) and AR(3) has improved over AR(1) and OLS method, DW statistic is not applicable beyond for AR(1).

Akaike's Information Criterion (AIC) and Schwartz Criterion (SBC) have been taken for model selection criterion. A model with a lower value of criterion statistic is judged to be preferable. The absolute value of SBC associated with model in equation (1) with AR(1) is 77 and it dropped to 69.6 with AR(3) but the absolute value of AIC remained around 88 for both AR(1) and AR(3). The magnitude of price elasticity of air miles, the sign of impact multiplier associated with accident rate, and unchanged absolute value of AIC leaves room for some functional change in the model.

A new functional form of the model is expressed in equation (3) incorporating some interaction terms which would reflect the impact of price, per capita income, and accident rates on air mileage during regulation as well as deregulation. OLS estimation of the Model 2 in Table 1 indicates significant improvement in terms of absence of autocorrelation and explanatory power. DW statistic has a value of 1.07 in model 2 relative to 0.93 in the previous Model 1. The t-statistics associated with average price, per capita income, and accident rates on domestic air miles are statistically significant. Model 2 of Table 1 represents the regulatory period.

During regulation, the impact of nominal average price on air is negative, relatively inelastic, and statistically significant, while during deregulation, the impact of price on air is negative, elastic, and significant. Model 3 in Table 1 represents deregulatory period. This negative impact of price occurred during deregulation because of lower air fare and tougher competition among airlines. Even though nominal price for airfare doubled, air travel tripled, leaving a positive offsetting effect of income on travel miles. The income elasticity is positive, relatively elastic, and significant during a regulatory period while the income elasticity (1.638) is positive and significant during a deregulatory period as travel miles rose proportionately faster than that of income.

Under the deregulatory period, per capita income rose from $11,000 in the late 1970's to less than $22,000 in the late 1980's while the travel miles rose almost two and a half fold during the same period. Contrary to the deregulation period, lower travel miles with higher accident rates during regulation is shown by the negative sign of the parameter estimate as shown by model 2 of Table 1. During 1979 to 1985, the average number of accidents was almost 18 with average travel of 248 billion miles. In the next five years, accidents rose to an average of 26 with an average travel of 416 billion miles. An increase in deaths decreases air travel miles; this is indicated by the negative sign of the impact multipliers of Model 2 through Model 6. Deregulation has a positive impact on the industry in terms of travel miles and these impact multipliers are statistically significant. These multipliers are shown by Model 3 and Model 6.

Also, the functional form expressed in equation (3) with error structure AR(1), AR(2), AR(3) has been estimated, but only models with AR(3) are shown in Table 1. Model 5 represents regulatory period and model 6 is estimated for deregulatory period.

The model with AR(3) is preferable over AR(1) and AR(2) as model selection criterion based on AIC and SBC are lower. The parameter estimates are similar to the model without autoregressive scheme estimates and have meaningful signs. The t-statistics associated with average price per mile, per capita income, and accident rate under both regulation and deregulation, and the impact of deregulation on air travel miles are statistically significant. To explain the structural change in detail, estimates of the model in equation (5) are shown by Model 3 and Model 6 in Table 1. Model 3, estimated by OLS, yields a DW of 1.42 and an R-Square of 0.99. All parameter estimates are statistically significant except accident and fatalities. Income elasticity is positive and relatively inelastic during a regulatory period while it is highly elastic during a deregulatory period. Passengers are less price elastic in a deregulatory period compared to regulatory period. Accident rate elasticities are highly inelastic during the period of study.

Unlike a deregulatory period, an increase in accident rate lowers travel miles under regulation. Travel miles increased proportionately more than accidents and fatalities, causing the signs of the estimates to be positive. Results of Model 4 and Model 5 with AR(3) seems to be similar in terms of estimates and their corresponding significance levels. Model (6) seems to reflect relatively inelastic demand for air travel miles as the average of per capita income during deregulation has tripled compared to the one in the regulatory period. A reduction in both accident rates and fatalities has contributed to the sharp increase in per capita air travel miles.

CHART 1

```
      Student                          Cook's
Obs   Residual      -2-1-0 1 2         D

  1  -1.048    |       **|        |    0.061
  2  -1.175    |       **|        |    0.056
  3  -1.366    |       **|        |    0.047
  4  -0.967    |        *|        |    0.022
  5  -0.462    |         |        |    0.004
  6  -0.078    |         |        |    0.000
  7  -0.192    |         |        |    0.001
  8   0.877    |         |*       |    0.013
  9   1.176    |         |**      |    0.028
 10   1.735    |         |***     |    0.063
 11   1.792    |         |***     |    0.089
 12   1.006    |         |**      |    0.014
 13   0.896    |         |*       |    0.011
 14   0.194    |         |        |    0.001
 15   0.847    |         |*       |    0.075
 16   0.231    |         |        |    0.002
 17  -1.170    |       **|        |    0.045
 18  -1.610    |      ***|        |    0.110
 19  -0.925    |        *|        |    0.052
 20  -0.444    |         |        |    0.022
 21   0.587    |         |*       |    0.021
 22   3.188    |         |******  |    2.316
 23  -1.297    |       **|        |    0.068
 24   0.399    |         |        |    0.004
 25  -0.504    |        *|        |    0.012
 26  -0.238    |         |        |    0.001
 27  -0.569    |        *|        |    0.014
 28   0.274    |         |        |    0.004
 29   0.226    |         |        |    0.002
 30  -0.426    |         |        |    0.007
 31  -0.367    |         |        |    0.008

Sum of Residuals               3.641532E-14
Sum of Squared Residuals           0.3051
Predicted Resid SS (Press)         0.6591
```

TABLE 1

Dependent Variable: LX12

	MODELS					
	1	2	3	4	5	6
Methods	OLS	OLS	OLS	ML	ML	ML
Parameter Estimates						
Intercept	-5.73*	-5.61*	-12.70*	-4.56*	-4.94*	-7.21*
LX31	-0.64*	-0.69*	-1.59*	-0.31*	-0.339*	-0.4335*
LX42	1.019*	0.98*	1.638*	1.17*	1.21*	1.47*
LX51	-0.1758*	-0.1859*	-0.0657	0.0384	0.0382	0.0609
X6		$-0.0^{5}644$	$-0.0^{4}8998$	$-0.0^{4}81$	$-0.0^{4}78$	$-0.0^{4}9264$
X7	-0.0704		8.624*	0.0357		5.7*
RXP			1.067*			0.1353
RXG			-0.524			-0.588
RXA			0.1369			-0.01079
DW	0.965	1.073	1.428	2.10	1.98	2.013
R-Sq	0.9804	0.98	0.9905	0.995	0.9955	0.996
Rho1	0.501	0.446	0.270	-0.42*	-0.365	-0.334
Rho2				-0.32*	-0.294	0.204
Rho3				0.22	0.126	-0.093
A(1)				-1.01*	-1.05*	-1.022*
A(2)				-0.1986	-0.1413	-0.1287
A(3)				0.372*	0.354	0.3634
SBC				-75.98	-79.67	-69.60
AIC				-90.233	-92.34	-88.60

*: Statistically significant at 5% level of significance; 0^{4} indicates 0000

References

[1] Adrangi, Braham Chow, Garland Raffiee, Kambiz, Airline deregulation, safety, and profitability in the U.S.", *Transportation Journal*, V36n4, Summer 1997, p. 44-52.

[2] Adrangi, Braham Chow, Garland Raffiee, Kambiz, "Passenger output and labor productivity in the U.S. airline industry after deregulation: A profit function approach", *Logistics and Transportation Review*, V32n4, Dec 1996, p. 389-407.

[3] Antoniou, Andreas, "The Factors Determining the profitability of International Airlines: Some Econometric Results", *Managerial and Decision Economics*, V13n6, Nov/Dec 1992, p. 503-514.

[4] Baltagi, Badi H Griffin, James M. Rich, Daniel P., "Airline deregulation: The cost pieces of the puzzle", *International Economic Review*, V36n1, Feb 1995, p. 245-260.

[5] Box, G.E.P. and Jenkins, G.M., *Time Series Analysis, Forecasting and Control*, San Francisco, Holden Day, 1970.

[6] Beach, C.M. and Mackinnon, J.G., "A Maximum Likelihood Procedure for Regression with Autocorrelated Errors", *Economerica,* 46, p. 51-58.

[7] Bera, A., and C. Jarque, "Efficient Tests for normality, Heteroscedasticity, and Serial Independence of Regression Residuals," *Economic letters*, 6, 1980b, p. 255-259.

[8] Box, G., and D. Pierce, "Distribution of Residual Autocorrelations in Autoregressive Moving Average Time Series Models," *Journal of the American Statistical Association*, 65, 1970, p. 1509-1526.

[9] Brenner, M.A., J.O. Leet, and E. Schott, *Airline Deregulation*, Eno Foundation, Inc., Westport, Connecticut, 1985.

[10] Cochrane, D., and G.H. Orcutt, "Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms," *Journal of the American Statistical Association* 44, p.32-61, 1949.

[11] Durbin, J., and G. Watson, "Testing for Serial Correlation in Least Squares Regression-III," *Biometrica*, 58, 1971, p. 1-42.

[12] El-Gazzar, Samir M Sannella, Alexander J, " The effects of airline deregulation on shareholder wealth: Some additional evidence", *Atlantic Economic Journal*, V24n2, Jun 1996, p. 144-154.

[13] Ellison, A.P. "The Structural Change of the airline industry Following Deregulation," *Transportation Journal*, Spring, 1982, p. 58-69.

[14] Goldfeld, S.M., and Quandt, *Nonlinear Methods in Econometrics*, Amsterdam, North-Holland, 1972.

[15] Gomez-Ibanez, Jose A., Clinton V. Oster, and Don H. Pickrell, "Airline Deregulation: What's behind the recent losses?" *Journal of Policy Analysis and Management*, Fall, 1983, p. 74-89.

[16] Graham, David R., Daniel P. Kaplan, and David S. Sibley, "Efficiency and Competition in the Airline Industry," *The Bell Journal of Economics*, Spring, 1983, p. 118-138.

[17 James, George W. "Airline Deregulation: Has It Worked?" *Business Economics*, July, 1985, p. 11-14.

[18] Joesch, Jutta M Zick, Cathleen D, "Evidence of changing contestability in commercial airline markets during the 1980s", Journal of Consumer Affairs, V28n1, Summer 1994, p. 1-24.

[19] Jordan, William A., "Results of U.S. Airline Deregulation: Evidence from the Regulated Canadian Airlines", *Logistics & Transportation review*, V22n4, Dec. 1986, p. 297-337.

[20] Hildreth, C. and Lu, J.Y., "Demand Relations with Autocorrelated Disturbances", *Michigan State University Agricultural Experimental Station Technical Bulletin* 276, East Lansing, Michigan, 1960.

[21] Gujarati, D.N., *Basic Econometrics*, Second Edition, McGraw Hill, 1988.

[22] Johnston, J., *Econometric Methods*, 3rd Edition, New York, McGraw Hill, 1984.

[23] Koran, D.W. "The Welfare Effects of Airline Deregulation in the U.S.," *Journal of Transportation Economics and Policy*, May, 1983, p. 177-189.

[24] King, M., "The Durbin-Watson test for Serial Correlation:Bounds for regressions with Trend and/or

Seasonal dummy Variables," *Econometrica*, 49, 1981, p. 1571-1581.

[25] Maddala, G.S., *Introduction to Econometrics*, Second Edition, Macmillian Publishing, 1992.

[26] Morrison, Steven and Clifford Winston, The Economic Effects of Airline Deregulation, *Brookings*, Washington, D.C., 1986.

[27] Morrison, Steven and Clifford Winston, "The Dynamics of Airline Pricing and Competition", *American Economic Review*, V80 n2, May 1990, p. 389-393.

[28] Oum, Tae Hoon Zhang, Anming Zhang, Yimin, "Inter-firm rivalry and firm-specific price elasticities in deregulated airline markets", *Journal of Transport Economics & Policy*, V27n2, May 1993.

[29] Parks, R., and B. Mitchell, "Estimating the Autocorrelated Error Model with Trended Data," *Journal of Econometrics*, 13, 1980, p. 185-201.

[30] Parks, R., "Efficient Estimation of a System of Regression Equations When Disturbances Are Both Serially and Contemporaneously Correlated," *Journal of the American Statistical Association*, 62, 1967, p. 500-509.

[31] Pindyck, R. and Rubinfeld, D., *Econometric Models and Economic Forecasts*, New York, McGraw Hill, 1991, p. 340-341.

[32] Rakowski, James P. And Bejou, David, "Birth, Marriage, Life, and Death: A Life-Cycle Approach for

Examining the Deregulated U.S. Airline Industry", *Transportation Journal*, V32n1, Fall 1992, p. 15-29.

[33] Ramunathan, R., *Introductory Econometrics*, Second Edition, Dryden Press, 1992.

[34] Rose, W. "Three Years After Airline Passenger Deregulation in the U.S. :A Report Card on Trunkline Carriers," *Transportation Journal*, Winter, 1981, p. 51-58.

[35] Schmidt, P., *Econometrics*, Marcel Deckker, 1976.

[36] Theil, H., *Principles of Econometrics*, John Wiley & Sons, 1971.

Mohammad I. Chowdhury is an associate professor and **Nancy Groneman** is a full professor, both in the Department of Business Administration and Education at Emporia State University.